RISK VERSUS AMBIGUITY AND INTERNATIONAL SECURITY DESIGN

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Abstract

We study portfolio allocation and characterize contracts issued by firms in the international financial market when investors exhibit ambiguity aversion and perceive ambiguity in assets issued in foreign locations. Increases in the variance of their risky production process cause firms to issue assets with a higher variable payment (equity). Hikes in investors’ perceived ambiguity have the opposite effect, and lead to less risk-sharing. Entrepreneurs from capital-scarce countries finance themselves relatively more through debt than equity. They are thus exposed to higher volatility per unit of consumption. Such results do not hold when investors exhibit standard risk-averse preferences. New facts uncovered from cross-country firm-level data are consistent with our model.

JEL codes: F21, F34, G11, G15, D81

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1 Introduction

Ambiguity attitude provides a persuasive analysis of the home bias in international asset holdings. As shown in countless experimental studies (Camerer and Weber, 1992; Trautmann et al., 2015) and as captured by a range of recent models (Gilboa, 2009; Gilboa and Marinacci, 2011), many agents perceive and are averse to ambiguity or model uncertainty in certain situations. Incorporation of ambiguity aversion into portfolio models suggests that it can explain the home bias (Uppal and Wang, 2003; Benigno and Nistico, 2012), and is corroborated by recent empirical evidence documenting correlations between ambiguity aversion, foreign stock ownership and underdiversification (Dimmock et al., 2016). However, despite its relevance for portfolio structure, there has been little investigation of more general consequences of ambiguity for international markets. For instance, it is not considered in the literature on capital structure (Frank and Goyal, 2009), much less that on international capital structure. So what does the presence of ambiguity imply about international asset structure? We answer this question by drawing out consequences of perception and attitude to ambiguity in a simple two-country model, and showing moreover that it is consistent with some new facts on firm-level cross-country capital structure that we uncover.

Our model involves risk averse firms, or entrepreneurs, who have access to a risky production technology, and investors with investable wealth that firms can transform into productive capital. Entrepreneurs issue contracts in a monopolistically competitive market that promise a share in the risky outcome and a fixed payout (bond payment). Investors make their portfolio choices after observing the contract terms. The problem is static with no default. The investors may perceive ambiguity or “model uncertainty”, which in this context can be thought of as uncertainty about the proper stochastic distribution from which production shocks are drawn. Beyond being risk averse, they are averse to this ambiguity. Attitudes are symmetric: all investors are assumed to have the same attitudes to both ambiguity and to risk. Moreover, the total uncertainty—risk and ambiguity—perceived by investors is also symmetric: this can be interpreted as them all having access to the same information. The crucial assumption, however, is that ambiguity perception is not symmetric: investors perceive more ambiguity in foreign-issued assets. These assumptions are rendered in a recently proposed parametrisation of the smooth ambiguity model (Klibanoff et al., 2005), which extends
the standard CARA-Normal model to include ambiguity (Gollier, 2011; Maccheroni et al., 2013).

A key finding is that the consequences of risk and ambiguity are different, and on certain parameters even diametrically opposite. In the presence of ambiguity, increases in the riskiness of the production process cause entrepreneurs to offer contracts with a higher risky share and a lower fixed payout—they finance themselves via a capital structure with a higher risky part, hence effectively seeking insurance from investors that are better able to diversify risk by investing in many non-correlated assets. By contrast, increases in the investors’ perceived ambiguity or in their ambiguity aversion cause firms to lower the variable part, therefore insuring (or rather assuring) investors. Whilst the effect on the fixed payout can be both positive and negative, depending on other factors, increases in ambiguity increase the fixed payout/risky share ratio in issued contracts for a range of plausible parameter values. These theoretical findings are consistent with firm-level correlations in the data on capital structure.

A second salient characteristic of the model is the sensitivity of the contract terms set by an entrepreneur to the total wealth of his home investors. By contrast to the standard risk-only model, where contract terms are independent of the country of residence, firms in countries with low levels of investor wealth (relative to others) issue contracts with higher fixed payouts and lower variable parts. Intuitively, they do this to attract some of the significant wealth in the hands of foreign investors, who perceive more ambiguity in their assets.

Combined with other properties of the model, these findings offer a new perspective on several observed regularities in international risk sharing and asset structure, some of which remain puzzling. Ambiguity, it suggests, could provide a unified explanation of the following phenomena.

The first is the relatively low degree of risk-sharing between countries as recently documented by Bengui et al. (2013) or Kose et al. (2007). In particular, despite the perceived volatility of their GDPs, developing countries cannot obtain the insurance from shocks predicted by standard models, which can be attributed to an importance of portfolio debt financing (Kose et al., 2007). This well-documented phenomenon is as predicted by the model: because entrepreneurs in capital-scarce countries issue assets with higher debt / equity ratio, ceteris paribus they obtain less risk-sharing than their counterparts in capital-rich countries. Ambiguity thus provides a complementary me-
mechanism for explaining this phenomenon to existing proposals in the literature, such as optimal contracting in the presence of moral hazard combined with lack of enforcement (Atkeson, 1991), contract enforcement issues (Mendoza et al., 2009) and/or differences in the nature of GDP shocks (transitory or permanent) as in (Aguiar and Gopinath, 2007). Indeed, Bengui et al. (2013) have recently shown, in a standard business cycle model, that portfolio rigidities greatly improve the fit with data on the extent of risk-sharing. Our model suggests that perception and aversion to ambiguity could provide precisely the rigidities required. Moreover, since entrepreneurs in capital-scarce countries promise higher fixed payouts, their consumption is more volatile: the model thus offers an alternative explanation why developing countries (i.e., associated with low investor wealth) may have more volatile consumption streams.

A related phenomenon concerns asset composition, and in particular the issuance of foreign-denominated fixed-rate bonds by developing countries, which has fuelled the long-lasting concern that too much international financing relies on fixed-rate debt (for example, Lessard and Williamson (1985), Rogoff (1999), Borensztein et al. (2004) or Reinhart and Rogoff (2009)). The same mechanism described above—the fact that, in the presence of ambiguity aversion, firms in capital-scarce countries issue assets with a higher fixed payment—is entirely consistent with the prevalence of fixed-rate bonds. Moreover, we can recast the model as that of sovereign countries making a choice about the characteristics of assets issued to borrow funds. Then our conclusion would be that countries would issue more fixed-rate bonds than GDP-linked bonds (of the sort suggested by Schiller (1993)) because investors prefer the former to the latter. Again, this finding complements existing work on asset composition, which focusses on different reasons for fixed/risky asset issuance patterns, though not necessarily in an international context (Gertler and Rogoff, 1990; Mukerji and Tallon, 2004; Caballero and Farhi, 2017).

A third issue is the comparative debt-to-equity ratios across countries. In particular, the share of debt in all public and private external liabilities of capital importers (countries with negative net foreign assets) is markedly higher than that of capital exporters. This holds in cross-country firm-level data that we study in this paper (see Section 4). The pattern is also present for various subsamples of the Lane and Milesi-Ferretti (2007) data for the most recent years characterized by the freest capital flows,
reproduced in Figure 1,\footnote{Debt and equity liabilities of all respective countries were summed up as in Lane and Milesi-Ferretti (2007) to obtain these figures.} and is observed for OECD countries as well as within the Eurozone\footnote{In Eurozone countries, for the 5 year period 2002-2006 (when the foreign exchange risk between these countries was small), capital importers had a debt/equity liabilities ratio on average of 2.95 (a median of 2.80) whereas among the exporters this average was at 1.56 (a median of 1.58). (Capital importers: Austria, Finland, Greece, Italy, Portugal and Spain. Exporters: Belgium, Ireland, France, Germany, Luxembourg and the Netherlands.)}. Since capital-scarce countries are net capital importers in our model, it predicts that firms in capital importing countries issue more debt on average, in line with the data. To our knowledge, the model is among the few to be able to account for this phenomenon, alongside the literature on the effect of moral hazard on capital allocation \citep{GertlerRogoff1990} where debt issuance is a response to a contracting problem and it is implicitly assumed that such problems are more prevalent in capital-scarce countries.

Another regularity documented in the literature \citep{Rey2015} concerns the behavior of capital flows, and in particular the strong negative correlation between global capital flows and indices of market “fear” such as the VIX, a measure of the implied volatility of S & P 500 index options. \cite{ForbesWarnock2012} find that the most important factor associated with capital flow episodes is changes in the VIX, or the related VXO. Another relevant finding is that the home bias increases in times of crises \citep{GiannettiLaeven2016}. Whilst the standard model with risk cannot account for such regularities, our model can, taking the VIX as a proxy of ambiguity. Indeed, according to our model, increases in ambiguity exacerbate the discrepancy in capital attracted by firms in capital-scarce versus capital-abundant countries, hence implying there is less capital flowing from the latter to the former countries when ambiguity is high. Our model predicts the opposite for increases in risk: they reduce the difference in capital attracted. Hence our model suggests that, unlike increases in risk, sudden increases in ambiguity can generate “capital flight”. Taking the VIX as an indication of perceived ambiguity, the model can thus explain the findings of \cite{ForbesWarnock2012}; assuming, as some have suggested \citep{Bernanke2010, CaballeroSimsek2013}, that ambiguity increases during crises, it can also explain the increase in the home bias.

A further phenomenon is the high home bias in developing countries \citep{AhrendSchwellmus2012} and also documented by \cite{SolnikZuo2012}. Our model can account for a higher home bias for capital-scarce countries that other models cannot.
Finally, in our model the marginal effective product of capital can be lower in countries with fewer domestic investors (as a result of the contracts entrepreneurs offer), and hence a lower installed capital stock; what is more, this holds even when the marginal return on physical capital is high. So, in a world with ambiguity and ambiguity aversion, we can have equalized marginal returns of capital (as found for example by Caselli and Feyrer (2007)) even though there would be output gains from reallocated capital. Financial market imperfections may stem from ambiguity aversion and prevent capital flows from “rich” to “poor” countries.

Our simple model thus suggests that ambiguity can provide a unified explanation of a range of phenomena discussed in the literature, which have until now been tackled with different mechanisms, if not left unexplained.

Our model has implications for cross-country firm-level capital structure that have not been explored in existing research. We thus provide data that uncover new facts on this issue, which are consistent with the central messages of the model. Unlike the standard model with risk, say, our model allows opposite effects of risk and ambiguity on the debt / equity ratio in firm asset issuance. Taking, as above, the VIX as a proxy for ambiguity and the standard deviation of GDP as a proxy for risk, our data exhibits the expected patterns: risk correlates negatively, and ambiguity correlates positively with standard measures of firm-level leverage such as debt to total asset value. Firms in capital-importing countries are on average nearly twice as sensitive to the effects of ambiguity in their leverage decisions. These correlations, which are robust to several checks, indicate that ambiguity may play an important role in determining asset structure across different countries.

Organization of the paper

In Section 2 we lay down the assumptions and develop the investors’ and firms’ problems. In Section 3.1, we provide the general solution to our model. Next, in Section 3.2, we analyze the “standard model with risk” – a benchmark case where investors are not ambiguity averse. We discuss the cases when countries are symmetric in wealth in Section 3.3 and when they are not in Section 3.4. Section 4 presents and analyses data on the effects of risk and ambiguity on firm asset issuance. Section 5 provides further discussion, specifically of the robustness of our conclusions, Section 6
considers related literature while Section 7 concludes. Proofs are in the Appendix with derivations available online.³

2 The model

There are two countries, ⁴ dubbed country 1 and country 2, M firms, of which Mₖ reside in country c. There are N investors, each of which resides in a single country (with Nₖ investors residing in country c), and each with the same wealth w > 0 that can be turned into productive capital by firms.

Each firm (entrepreneur) residing in a particular country has access to a technology that is governed by an i.i.d. stochastic process; a firm n has a risky project with a stochastic return xₙ.

The productivity draws across firms in a particular country and across countries are independent, and this is common knowledge. A typical firm issues a contract containing two elements. The first, vₙ, describes the share of the proceeds (or participation in losses) from the risky project. The second is a riskless return (or payment demanded) of Rₙ. One interpretation of this capital structure (given the stochastic processes considered) is as a reflection of the standard distinction between debt (the Rₙ factor) and equity (the vₙ part).⁵ Like several other static international portfolio choice models, we abstract from exchange rate risk for the sake of simplicity.⁶ Hence in our model all investors, irrespective of their country of residence, perceive the contract terms in the same way.

There is one period within which the timing of events is as follows. First, each firm communicates to investors the contract terms of hiring capital that they will irrevocably honor. They do this noncooperatively, and they compete monopolistically for funds.

³https://studies2.hec.fr/jahia/webdav/site/hec/shared/sites/hill/acces_anonyme/Articles/Hill_Michalski_040216_supplementary_material.pdf

⁴The model developed below can be extended to a multicountry case without gaining significant insight on the qualitative results of interest here.

⁵Modeling default or limited liability does not allow us to obtain tractable closed form solutions.

⁶Exchange rate movements could theoretically be a source of home bias, especially in dynamic models (see the important contribution of Benigno and Nistico (2012) in the presence of ambiguity aversion). However, the evidence so far is mixed. Hedging short-run real exchange rate fluctuations per se has been shown not to be a plausible channel for the home bias (exchange rate covariances with the foreign-domestic equity excess returns are small as in van Wincoop and Warnock (2010)) but Fidora et al. (2007) claim that real exchange rate volatility can explain up to 20% of the cross-country variation in home biases.
Investors observe these contract terms and make their portfolio decisions. Then wealth is invested, transformed into capital by firms, productivity draws are realized, output produced, and payouts and consumption take place.

2.1 Investor’s problem

We begin by describing the investor’s problem. An investor $l$ will allocate a fraction of wealth $0 \leq \alpha_{ln} \leq 1$ into assets issued by firm $n$, with $\sum_n \alpha_{ln} = 1$. The value of an investor’s portfolio is $w \sum \alpha_{ln}(v_n x_n + R_n)$.

Investors often feel surer in their judgements about assets from their own country than foreign assets. This intuition can be translated by the fact that investors perceive more ambiguity with respect to events concerning foreign assets (such as the realisation of the stochastic return $x_n$) than events concerning domestic firms. If investors have a non-neutral attitude to ambiguity, then the difference in ambiguity may have effects on investment behavior (see the review of the empirical literature in Section 6).

To capture this we adopt the smooth ambiguity model proposed by Klibanoff et al. (2005). Rather than assuming that agents have a single (“known”) probability distribution $P$ for the returns of an uncertain asset $x$, the model allows uncertainty about the “true” distribution governing returns, which is represented by a (second-order) probability distribution over the possible distributions. Letting $\pi$ denote this second-order distribution, decision makers choose assets $x$ to maximise:

$$V(x) = \int_\Delta \varphi(\mathbb{E}_P(u(x))) \, d\pi = \mathbb{E}_\pi \varphi(\mathbb{E}_P(u(x)))$$

(1)

where $u$ is a standard (von Neumann-Morgenstern) utility function, $\varphi$ is a strictly increasing real-valued function, and $\Delta$ is the space of probability distributions over values of $x$. As standard, $u$ represents the decision maker’s risk attitude; by contrast, $\varphi$ represents the decision maker’s ambiguity attitude (in the sense of Klibanoff et al. 2005, §3). Concave $\varphi$ corresponds to ambiguity aversion.

This decision model has a natural interpretation in terms of model uncertainty (see eg. Klibanoff et al. (2005), Hansen (2007) and Marinacci (2015)). The set $\Delta$ can represent the set of possible parameter estimates for a particular model of the stochastic

\[ \text{In Section 5, we show that our results extend to the case where there is a risk-free asset in which investors from both countries invest.} \]
process determining asset returns. The decision maker may be unsure as to which of the parameter values is correct: there may be a set all of which are plausible given the data. This model uncertainty is represented by the second-order distribution \( \pi; \) \( \varphi \) represents attitudes to model uncertainty (Marinacci, 2015). The functional form, with a concave \( \varphi \), can be thought of as one way of incorporating considerations of robustness of one’s choice across the possible parameter values. Note that the model neatly separates ambiguity or model uncertainty—\( \pi \)—from ambiguity attitude—\( \varphi \).

We thus allow an investor \( l \) to consider that there are several possible distributions for the stochastic return \( x_n \) ran by a firm \( n \). For tractability, we use the specification of Gollier (2011), and assume that the investor is sure that the returns follow a normal distribution. He is sure about the variance (volatility) of the return, but not necessarily about the mean. So he considers plausible only distributions \( \tilde{x}_n \sim N(m_n, \sigma^2_n) \) for some fixed \( \sigma^2_n \), and some set of possible expected returns \( m_n \). His second-order prior over this set of distributions is itself normally distributed, with mean \( \mu_{ln} \) and variance \( \tau^2_{ln} \): \( \tilde{m}_n \sim N(\mu_{ln}, \tau^2_{ln}) \). Following standard terminology, we shall call \( \sigma^2_{ln} \) (the variance of the underlying stochastic process) his (perceived) risk. In this specification, the “extent” of the (model) uncertainty about the parameters of the stochastic process is summarized by \( \tau^2_{ln} \), which we call his (perceived) ambiguity. When \( \tau^2_{ln} > 0 \), we shall say that the investor perceives ambiguity; otherwise, he perceives no ambiguity.

The two-stage distribution just described induces a reduced distribution over returns (obtained by taking expectations according to the second-order distribution); in this case, this is a normal distribution for \( x_n \) with mean \( \mu_{ln} \) and variance \( \sigma^2_{ln} + \tau^2_{ln} \). This distribution can be thought of as characterising the total uncertainty faced by the agent, comprising both his perceived risk and ambiguity (Maccheroni et al. (2013)). Accordingly, we call \( \sigma^2_{ln} + \tau^2_{ln} \) the uncertainty. A Bayesian decision maker would rely solely on this reduced distribution to evaluate options. By contrast, under the smooth ambiguity model, decision makers are sensitive to how much of the uncertainty is ambiguity (i.e. the decomposition of \( \sigma^2_{ln} + \tau^2_{ln} \) into \( \sigma^2_{ln} \) and \( \tau^2_{ln} \); see Maccheroni et al. (2013) for an in-depth study).

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8Maccheroni et al. (2013) shows that this specification yields a ‘robust’ version of mean-variance preferences, incorporating ambiguity.

9As Boyle et al. (2012, footnote 8) observe, this is justified by the fact that estimation of expected returns is much more difficult than estimation of second moments; see Merton (1980).

10Such an agent acts as if there is no model uncertainty or ambiguity; all uncertainty has been “bundled together” and is treated as standard stochastic uncertainty, or risk.
Following Gollier’s specification, each investor has a constant absolute risk aversion utility function of the form $u(z) = -(1/\theta)e^{-\theta z}$ where $\theta > 0$ represents the degree of (absolute) risk aversion. Each investor is ambiguity averse and has constant relative ambiguity aversion; the transformation function is thus of the form $\varphi(U) = -\frac{(-U)^{1+\gamma}}{1+\gamma}$, where $\gamma \geq 0$ represents the (degree of) ambiguity aversion. When $\gamma > 0$, we shall say that there is ambiguity aversion.

Hence, for an investor $l$, his optimisation problem is to find:

$$\arg \max_{1 \geq \alpha_{ln} \geq 0, \sum_n \alpha_{ln} = 1} V_l(\alpha_l)$$

where $V_l$ is defined in equation (1) with $u$, $\varphi$ and $\pi$ as specified above. (Recall that $\alpha_{ln}$ denotes the portfolio allocation of investor $l$ in firm $n$.) Using a standard technique (see e.g. Gollier (2001)), one obtains the following expression for the expected utility of portfolio $\alpha_l$ for investor $l$ under the distribution $P$ with $x_n \sim N(m_n, \sigma_{ln}^2)$ for all $n$.

$$E_P(u(\alpha_l)) = -(1/\theta)e^{-\theta w \sum_n[\alpha_{ln}(v_n m_n + R_n) - (\theta w \sigma_{ln}^2/2)v_n^2 \alpha_{ln}]}$$

This is the inside term in equation (1); plugging it into that equation under the specification set out above yields:

$$V_l(\alpha_l) = -\frac{1}{\theta^{1+\gamma}(1+\gamma)}e^{-\theta w (1+\gamma) \left( \sum_{i=1}^{M} \left( \alpha_{li}(\mu_{li} v_i + R_i) - (\theta w \sigma_{li}^2/2) \alpha_{lj}^2 v_{lj}^2 \right) \right)}$$

as the evaluation of $\alpha_l$ by an investor $l$. Hence the investor’s problem is to maximise this function, under the constraints that $1 \geq \alpha_{ln} \geq 0$ for all $n$ and $\sum_n \alpha_{ln} = 1$.

**Domestic and foreign asset characteristics perceived by investors.** We assume that all investors use the same mean and have the same uncertainty about each firm, irrespective of their country of residence; i.e., for each firm $n$, and investors $l$ and $l'$, $\mu_{ln} = \mu_{l'n}$ and $\sigma_{ln}^2 + \tau_{ln}^2 = \sigma_{l'n}^2 + \tau_{l'n}^2$. This assumption is consistent with investors having the same information, and hence distinguishes this model from analyses in terms of differences in information between home and foreign investors (as in Gehrig (1993)). However, identical uncertainty does not imply that investors perceive the same amounts of ambiguity: it may be that $\tau_{ln} \neq \tau_{l'n}$. Our base assumption is that
this is precisely the difference between home and foreign investors: whilst an investor $l'$ residing in a different country from firm $n$ perceives ambiguity with respect to its stochastic return $x_n (\tau_{ln} > 0)$, an investor $l$ residing in the same country perceives none ($\tau_{ln} = 0$). For the latter investor, all the uncertainty concerning firm $n$ is perceived as risk: $\sigma^2_{ln} = \sigma^2_{ln'} + \tau^2_{ln}$, and $\tau_{ln} = 0$. Whilst, for home firms, this investor resembles a Bayesian decision maker (who relies entirely on the reduced distribution), he differs from the Bayesian when it comes to firms that do not reside in his country: for a foreign firm $n'$, he may perceive ambiguity in its return—$\tau_{ln'} > 0$.\footnote{Refined versions of the setup in which investors also perceive ambiguity towards domestic assets are discussed in Section 5: as long as the ambiguity perceived in domestically-issued assets is lower than towards the foreign-issued ones, there is no significant change in the qualitative results.} Finally, we assume no asymmetries in the risk and ambiguity attitudes among investors: risk aversion ($\theta$) and ambiguity aversion ($\gamma$) are the same for all investors.

Our leading interpretation of these assumptions is as stated: there are no asymmetries in uncertainty (and, relatedly, information), risk attitudes or ambiguity attitudes among investors, but only an asymmetry in ambiguity perception—investors perceive no ambiguity for home firms though they may perceive some concerning foreign ones. This is entirely coherent with the interpretation proposed by Klibanoff et al. (2005) and others of the smooth model: $u$ and $\varphi$ reflect risk and ambiguity attitude respectively; the reduced distribution (with variance $\sigma^2 + \tau^2$, in our case) sums up the total uncertainty, and the second-order distribution (with variance $\tau^2$) captures “model uncertainty” or ambiguity. Note that, consistently with its distinction between ambiguity and ambiguity attitude ($\pi$ and $\varphi$ in (1)), there are two distinct special cases in which the smooth ambiguity model collapses to the Bayesian model: one in which there is no ambiguity though there may be non-neutral ambiguity attitude—ie. $\pi$ is a degenerate second-order distribution though $\varphi$ may be non-linear—and one in which ambiguity attitude is neutral, though the decision maker may perceive ambiguity—ie. $\pi$ is non-trivial, but $\varphi$ is linear. As intimated above, an investor treats home firms in a way reminiscent of a Bayesian: under the suggested interpretation, this is analogous to the first special case, with no ambiguity. This interpretation has the advantage of allowing the investor to have the same ambiguity attitude—as captured by $\varphi$—concerning all firms, be they at home or abroad. Indeed, the smooth ambiguity model (in its standard form, at least) assumes that the ambiguity attitude adopted by the decision maker is
independent of the assets under consideration, just as the Bayesian model uses a single utility function (reflecting risk attitudes) to evaluate all prospects. An alternative interpretation of our assumptions would postulate that investors have different ambiguity attitudes to assets issued in different countries, interpreting the differences in $\tau_n$ among investors as reflecting a difference in ambiguity attitudes. Since it breaks with the standard account of the smooth ambiguity model, we shall not discuss this interpretation further here.\footnote{A model that is both faithful to the standard interpretation of (1) and involves differing ambiguity attitudes depending on the asset’s country of issuance would differ from (1), insofar as it would require two functions $\varphi$: one for attitude towards home assets and one for attitude to foreign assets. We are not aware of an existing study of such a model.}

Furthermore, we assume symmetry in the perception of risk and ambiguity among investors residing in the same country: in particular, if firm $n$ is in a foreign country for investors $l'$ and $l''$, then $\sigma^2_{l'n} = \sigma^2_{l''n}$ and $\tau^2_{l'n} = \tau^2_{l''n}$. Hence, for a firm $n$, we can drop subscripts and use $\sigma^2_n$ to denote the risk perceived by an investor in the foreign country, and $\tau^2_n$ for the ambiguity perceived by foreign investors. In the interests of brevity, we henceforth call $\sigma^2_n$ the risk and $\tau^2_n$ the ambiguity of the return for firm $n$’s stochastic process. Note that, despite this terminology, $\tau^2_n$ only captures the ambiguity perceived by investors in a different country from the firm; home investors perceive no ambiguity (their $\tau_n$ is 0), perceiving all the uncertainty as risk ($\sigma^2_{ln} = \sigma^2_n + \tau^2_n$). Hence an increase in ambiguity $\tau^2_n$ corresponds to an increase in the ambiguity perceived by foreign investors. For home investors, an increase in $\tau^2_n$ does not lead to an increase in the ambiguity they perceive—for they perceive no ambiguity with respect to that firm. However, since an increase in $\tau^2_n$ implies ceteris paribus an increase in uncertainty, an increase in $\tau^2_n$ implies an increase in uncertainty for home investors, and hence an increase in the risk they perceive ($\sigma^2_n + \tau^2_n$). This is a consequence of the assumption that uncertainty is identical across investors. Whilst we retain this assumption to focus on the specific contribution of our approach, in particular with respect to an account based on information asymmetries across investors, the results are robust to altering or weakening it in the scenarios we investigated. In particular, as we show in Section 5, the results generally remain valid under the diametrically opposite assumption that

\footnote{Note that the separation of ambiguity attitude and ambiguity perception, permitted by the smooth ambiguity model, is relatively rare in the decision-theoretic literature. The Maximin-EU model of Gilboa and Schmeidler (1989) does not support such a separation, and so the question of whether ambiguity (perception) or ambiguity attitude differs cannot be posed in the model. See Section 5 on the extent to which our conclusions go through for such models.}
every difference in ambiguity perception translates to a difference in uncertainty—ie.
where all investors perceive the same risk $\sigma_n^2$, but home investors perceive no ambiguity
and hence less uncertainty ($\sigma_n^2$ as opposed to $\sigma_n^2 + \tau^2_n$ for foreign investors).

Under the given specifications, (4) reduces, for an investor $l$ in country 1, to

$$V_{ll}(\alpha_l) = -\frac{1}{\theta^{1+\gamma}(1+\gamma)} e^{-\theta w(1+\gamma)} \left( \sum_{i=1}^{M_1} \left( \alpha_{li} (\mu_i v_i + R_i) - \frac{(\sigma_i^2 + \tau_i^2) \theta w}{2} \left[ \alpha_{li} v_i \right]^2 \right) \right)$$

$$+ \sum_{j=M_1+1}^{M} \left( \alpha_{lj} (\mu_j v_j + R_j) - \frac{(\sigma_j^2 + \tau_j^2 (1+\gamma)) \theta w}{2} \left[ \alpha_{lj} v_j \right]^2 \right)$$

(5)

There is a similar expression for residents of country 2.

2.2 The firm’s problem

Each firm must choose the two contract terms $v_n$ and $R_n$. Once a firm $n$ issues contracts
and investors make their portfolio choices, it obtains capital $k_n$, which it invests in a
risky production process. The proceeds can be described by $y_n = x_n f(k_n)$ where
$f(k_n) = k_n$ and $x_n$ is stochastic productivity that is unknown prior to investment. We
assume that the entrepreneur has the same information as investors and, like home
investors, perceives no ambiguity with respect to its stochastic process; hence it treats
the stochastic return as being distributed according to the normal distribution with
$x_n \sim N(\mu_n, \sigma_n^2 + \tau_n^2)$.

The entrepreneur acts as a standard Bayesian decision maker who has a CARA
utility function with a degree of absolute risk aversion $A$: that is, his utility function
is $u(z) = -(1/A) e^{-Az}$. The entrepreneur understands that the capital that can be
raised in the international capital market will be a function of the contract terms
offered by all firms (denote it by a matrix $\mathbf{C} = [\mathbf{v}, \mathbf{R}]$ where $\mathbf{v}$ and $\mathbf{R}$ are vectors
of contract terms) and the world distribution of investor wealth $wN$. If the entrepreneur’s
productivity draw is $x$, he will thus obtain $(1 - v_n) x f(k_n(\mathbf{C}, wN)) - R_n k_n(\mathbf{C}, wN)$
after payment of dues to investors. The entrepreneur chooses contract terms so as to

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14For ease of presentation, we assume that the first $M_1$ firms are in country 1 and the remaining
$M_2$ are in country 2.

15Entrepreneur risk aversion is a convenient simplification to allow modeling capital structure choices
and gives a meaningful reason for their issuance of outside assets (see a wide finance literature as Jensen
and Meckling (1976), Fama (1980) or Fama and Jensen (1983)).
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maximise his expected utility from consumption. Hence the problem of an entrepreneur \( n \) is as follows:

\[
\arg \max_{\{v_n, R_n\}} -(1/A) \int e^{-A[(1-v_n)x_n f(k_n(C,wN))-R_nk_n(C,wN)]} \frac{1}{\sqrt{2\pi(\sigma_n^2 + \tau_n^2)}} e^{-(x_n-\mu_n)^2/(2(\sigma_n^2+\tau_n^2))} dx
\]

Plugging in for \( f(\cdot) \), we rewrite this problem as

\[
\arg \max_{\{v_n, R_n\}} -\frac{1}{A} e^{-A\left(\[(1-v_n)\mu_n-R_nk_n(C,wN)\left./\sigma_n^2 + \tau_n^2\right)\left./(1-v_n)\right]^{2}}
\]

Note that a natural requirement is that \( v_n \in [0, 1] \); this does not need to be explicitly imposed as a constraint, because it will turn out to be already satisfied in equilibrium in the cases we consider. Similarly, simple contracts such as \( v_n = 0 \) and \( R_n > 0 \) are allowed, but they turn out not to be an equilibrium outcome. The firm can offer \((0, 0)\) or \((1, 0)\) and obtain a utility of at least \(-1/A\).\(^{16}\)

**Symmetry assumptions** Throughout we assume symmetry among firms in a given country. That is, we assume that for any firms \( n, n' \) in country 1, \( \sigma_n^2 = \sigma_{n'}^2 = \sigma_1^2 \), \( \tau_n^2 = \tau_{n'}^2 = \tau_1^2 \), and similarly for firms in country 2. Moreover, we also set the means of the stochastic process to be equal across firms and countries \( \mu_n = \mu_{n'} = \mu \) for any firms \( n, n' \) (independently of the country). For this reason, all subscripts \( n \) are dropped in the sequel. Note finally that the clearing conditions for this market are trivial given that the firms issue only as many contracts as are demanded.

\(^{16}\)To be able to solve the model, we need to assume that both the entrepreneurs and the investors can have negative consumption and that a firm can make payouts to investors even if negative productivity is realized. Given investor and entrepreneur preferences that admit negative consumption and the widespread use of the CARA-Normal framework in the finance literature we find the assumptions on the productivity generating process to be awkward but not inadmissible. In Section 5 (and Appendix A.7), we report simulations suggesting that our main results are robust to dropping this assumption.
3 Analytic results

3.1 The general case

Solving the maximization problem for an investor in country 1 gives the following portfolio allocations to a typical firm $i$ in country 1 and $j$ in country 2:

$$\alpha_{1i} = S_{1i} + \frac{1 - M_1 S_{1i} - M_2 S_{1j}}{M_1 + M_2 \left( \frac{\sigma^2_1 + \tau^2_1}{\sigma^2_1 + \tau^2_1 (1 + \gamma)} \right) \frac{v^2_i}{v^2_j}}$$

$$\alpha_{1j} = S_{1j} + \frac{1 - M_1 S_{1i} - M_2 S_{1j}}{M_1 + M_2 \left( \frac{\sigma^2_2 + \tau^2_2 (1 + \gamma)}{\sigma^2_1 + \tau^2_1} \right) \frac{v^2_i}{v^2_j}}$$

where $S_{1i} = \frac{R_i + \mu v_i}{\left( \frac{\sigma^2_1 + \tau^2_1}{\sigma^2_1 + \tau^2_1 (1 + \gamma)} \right) \theta w v_i}$ and $S_{1j} = \frac{R_j + \mu v_j}{\left( \frac{\sigma^2_2 + \tau^2_2 (1 + \gamma)}{\sigma^2_1 + \tau^2_1} \right) \theta w v_j}$, and $v_i$ and $R_i$ ($v_j$ and $R_j$ respectively) are the $v$ and $R$ set by a typical firm $i$ in country 1 (firm $j$ in country 2).

The first term in each equation is reminiscent of the standard solution of the unconstrained optimization problem in the CARA-Normal model: it is the expected return normalized by a variance. In the case of home assets, the relevant variance is $\sigma^2_1 + \tau^2_1$, the variance of the “reduced” distribution, whereas in the case of the foreign assets, it is a “distorted” variance $\sigma^2_2 + \tau^2_2 (1 + \gamma)$. Given that the investor perceives ambiguity towards foreign assets, one could think of these as the “effective” variances (incorporating the ambiguity) he perceives towards home and foreign assets. The second term is a correction term due to the fact that there is a competition for funds among firms. The capital obtained by a firm issuing a contract $\{v_i, R_i\}$ is $(N_1 \alpha_{1i} + N_2 \alpha_{2i}) w$.

A representative entrepreneur $i$ located in country 1 maximizes utility given in (7) anticipating the portfolio choices of investors and offers the following contract terms:

$$v_i = \frac{A}{\theta} \left( N_1 + \frac{\sigma^2_1 + \tau^2_1}{\sigma^2_1 + \tau^2_1 (1 + \gamma)} N_2 \right) \left( \frac{2 + A}{\theta} \left( N_1 + \frac{\sigma^2_1 + \tau^2_1}{\sigma^2_1 + \tau^2_1 (1 + \gamma)} N_2 \right) \right) \left( N_1 + \frac{\sigma^2_1 + \tau^2_1}{\sigma^2_1 + \tau^2_1 (1 + \gamma)} N_2 \right)$$

$$R_i = \mu (1 - v_i) - w \theta \Upsilon_i$$

where $\Upsilon_i$ is an (involved) function of the parameters of the model (see Appendix A.1). $R_i$ has two components: $(1 - v_i) \mu$, which moves in the opposite direction to $v_i$; and $w \theta \Upsilon_i$, where $\Upsilon_i$ is positive, which changes with the competition in the market for
investor funds. The expected return to investors on a unit of invested capital is then
\[ R_i + v_i \mu = \mu - w\theta \lambda_i: \] investors get the mean of the stochastic production process minus an adjustment.

Similar expressions are obtained for investors and firms located in country 2. Note that the contract terms are such that \( v_i \in [0, 1) \), as one would expect. Moreover, given the symmetry in the risk and ambiguity of firms in the same country, they issue contracts with the same terms. Henceforth we replace the firm subscript with a country one, using \( v_1 = v_i \) and \( R_1 = R_i \) for any firm \( i \) in country 1 and \( v_2 = v_j \) and \( R_2 = R_j \) for any \( j \) in country 2. Similarly, we write \( \alpha_{11} = \alpha_{1i} \) for every firm \( i \) in country 1, and similarly for \( \alpha_{12}, \alpha_{21} \) and \( \alpha_{22} \).

We can thus obtain closed form solutions for the general case. Already on the basis of these, we can conduct basic comparative statics for the equity part of the contracts offered (the \( v \)). To the extent that the equity participation captures the share in the risky process that the firm offloads to investors, this gives some first results on the effects of the various parameters on the risk sharing entrepreneurs obtain. First of all, note that the level of a firm’s end exposure to risk in equilibrium depends negatively on the ratio of firm to investor risk aversion \( A/\theta \) and the total number of investors (whether home or foreign), as one would expect. Risk and ambiguity have opposite effects on risk sharing. On the one hand, as the risk increases, the \( v \) increases as well \( (\frac{\partial v_1}{\partial \sigma^2_1} > 0) \): as the production process becomes more risky, firms decide to reduce their equity exposure and issue contracts with higher investor participation. On the other hand, increases in ambiguity \( (\tau^2_1) \) lead to a reduction in \( v \) \( (\frac{\partial v_1}{\partial \tau^2_1} < 0) \): to attract (foreign) investment, firms offload less risk on investors. The same is true for ambiguity aversion \( (\gamma) \): as investors become more ambiguity averse, the amount of risk sharing in the proposed contracts falls \( (\frac{\partial v_1}{\partial \gamma} < 0) \).

**Proposition 1.** If \( N_2 > 0, \gamma > 0 \) and \( \tau^2_1 > 0 \), then risk sharing for firms in country 1 \( (v_1) \) falls as either ambiguity \( (\tau^2_1) \) or ambiguity aversion \( (\gamma) \) increase. By contrast, risk sharing increases with an increase in risk \( (\sigma^2_1) \).

It is the presence of foreign investors that makes firms change the contracts they offer: in the absence of foreign investors, changes in ambiguity \( (\tau^2_1) \) have the same effect as changes in risk. In fact, there is a strong dependence on the number of home investors and foreign investors, as can be seen in the limit risk sharing as risk, ambiguity
or ambiguity aversion increase.

**Proposition 2.** If \( \gamma > 0, \tau_1^2 > 0 \), \( \lim_{\gamma \to \infty} v_1 = \frac{4N_1}{(2+\frac{\gamma}{\theta}N_1)} \), \( \lim_{\tau_1^2 \to \infty} v_1 = \frac{4(N_1+1/N_2)}{(2+\frac{\gamma}{\theta}(N_1+1/N_2))} \), and \( \lim_{\sigma_1^2 \to \infty} v_1 = \frac{4(N_1+N_2)}{(2+\frac{\gamma}{\theta}(N_1+N_2))} \).

As ambiguity aversion becomes very large, firms rely almost exclusively on home investors, who do not perceive any ambiguity with respect to their production processes. Risk has the opposite effect: in cases of high risk, the risk sharing is determined by the total number of investors in the world, and is insensitive to the distribution of wealth between the countries. The limit effect of ambiguity lies between these two cases: in general, firms rely on foreign investors less than in cases of large risk, but *ceteris paribus* more than in cases of extreme ambiguity aversion. In fact, as ambiguity becomes large, it constitutes the bulk of the uncertainty, and the extent to which firms rely on foreign investors depends entirely on the ambiguity aversion: higher ambiguity aversion means a stronger dependence on home investors. So, in cases of high ambiguity and ambiguity aversion, if there are many home investors, firms may easily and cheaply obtain insurance from them, and their risk sharing remains high: ambiguity has a mitigated effect. By contrast, if there are few home investors, then it becomes very difficult for entrepreneurs to get risk sharing in such contexts. This could provide an explanation for some of the regularities mentioned in the Introduction: the model predicts that risk sharing is lower for countries with a small amount of home wealth when ambiguity and ambiguity aversion are significant.

Given the complexity of the formulas, it is difficult to do more comparative statistics at this general level; this is specifically the case for the interest rate \( R \) and the debt / equity ratio. For instance, whilst the first component of \( R \) from Equation (11) will move in the opposite direction to the expected return from the risky part of the contract \( v\mu \), reflecting rebalancing of the firms’ risk mix, the general effect on \( R \) will depend on the trade-off with the second term, \( \Upsilon \), which is sensitive to competition for funds. As suggested in the Introduction, the case of countries with asymmetric wealth is of particular interest; in Section 3.4, we consider in detail a simplified, tractable but relevant special case. Before, to provide appropriate comparisons but also to illustrate the power of the model, we shall analyze two other families of special cases: the standard model with only risk—a benchmark case where investors perceive no ambiguity, which corresponds to a standard CARA-Normal model under our assumptions on the
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economy—and the symmetric case when firms are identical in their stochastic processes and countries are symmetric in the number of investors and number of firms residing in each country.

### 3.2 The standard model with risk and no ambiguity

We begin our analysis with the benchmark case with no ambiguity; i.e. \( \tau^2 = 0 \). In this case, as noted previously, investors act like expected utility maximizers and the portfolio choice problem collapses to a standard CARA-normal problem. We call this case “the standard model with risk”.

We can solve the model for the \( v \)’s and \( R \)’s to yield

\[
v_1 = v_2 = v = \frac{\frac{4}{\theta} (N_1 + N_2)}{(2 + \frac{4}{\theta} (N_1 + N_2))}
\]

and

\[
R_1 = R_2 = R = \mu - \frac{2}{(2 + \frac{4}{\theta} (N_1 + N_2))} - wA \left( \frac{4}{\theta} (N_1 + N_2) \right) \left( \frac{1}{\sigma_1^2} M_1 + \frac{1}{\sigma_2^2} M_2 \right) \left( 2 + \frac{4}{\theta} (N_1 + N_2) \right)^2
\]

The expected return is then

\[
R + \mu v = \mu - wA \left( \frac{4}{\theta} (N_1 + N_2) \right) \left( \frac{1}{\sigma_1^2} M_1 + \frac{1}{\sigma_2^2} M_2 \right) \left( 2 + \frac{4}{\theta} (N_1 + N_2) \right)^2
\]

for firms in both countries.

Portfolio shares for investors from country 1 are then:

\[
\alpha_{11} = S_{11} + \frac{1}{M_1 + M_2} \left( \frac{\sigma_1^2}{\sigma_2^2} \right) S_{12}
\]

and

\[
\alpha_{12} = S_{12} + \frac{1}{M_1 + M_2} \left( \frac{\sigma_1^2}{\sigma_2^2} \right) S_{11}
\]

where

\[
S_{11} = \frac{R + \mu v}{\theta w \sigma_1^2 \sigma_2^2} \quad \text{and} \quad S_{12} = \frac{R + \mu v}{\theta w \sigma_2^2 \sigma_1^2}.
\]

After substitutions we find that

\[
\alpha_{11} = \alpha_{21} = \frac{1}{M_1 + \frac{\sigma_2^2}{\sigma_1^2} M_2} \quad \text{while} \quad \alpha_{12} = \alpha_{22} = \frac{1}{M_1 + \frac{\sigma_1^2}{\sigma_2^2} M_2}.
\]

In particular, if \( \sigma_1^2 = \sigma_2^2 \), we have

\[
\alpha_{11} = \alpha_{12} = \alpha_{21} = \alpha_{22} = \frac{1}{M_1 + M_2}.
\]

Then, a firm in country 1 issuing an asset with characteristics \((v, R)\) will obtain

\[
k_1 = \frac{N_1 + N_2}{M_1 + \frac{\sigma_1^2}{\sigma_2^2} M_2} w
\]

while a firm in country 2 issuing an asset with characteristics \((v, R)\) will obtain

\[
k_2 = \frac{N_1 + N_2}{\frac{\sigma_1^2}{\sigma_2^2} M_1 + M_2} w. \quad \text{We have} \quad \frac{k_2}{k_1} = \frac{\sigma_2^2}{\sigma_1^2}.
\]

The result is that all firms issue assets with the same contract terms irrespective of their residence, and investors hold identical portfolios. End capital allocation does not
depend at all on where investors or firms reside, but only on the risk of the stochastic production process: there are no frictions in the capital market. If firms in one country have more risky production processes, then they will choose to obtain less capital but offer the same level of equity. In that sense, in equilibrium the firms offload the same share of their idiosyncratic risk. If the stochastic processes are the same in terms of mean and variance, the firms obtain the same level of consumption insurance and the same Sharpe ratio of consumption no matter the country in which they reside. Moreover, in this case, expected returns on all assets are the same in both countries.

As the environment becomes more risky ($\sigma_1^2$ or $\sigma_2^2$ increase), the interest rates offered by firms in equilibrium on their risk-free bonds ($R$) fall (as investors value the sure return more). The overall effect of more risk on expected real asset returns is negative while the ratio of debt / equity-financed capital ($\frac{dR}{k\theta}$) declines.

### 3.3 Symmetric countries

Now we solve and analyze the case where all countries are identical in terms of the number of investors ($N_1 = N_2 = \frac{N}{2}$) and the number of firms ($M_1 = M_2 = \frac{M}{2}$), and all firms (irrespective of the country of origin) are perceived as having the same stochastic properties of their production process. Therefore we assume that $\sigma_1^2 = \sigma_2^2 = \sigma^2$ and $\tau_1^2 = \tau_2^2 = \tau^2$.

Drawing on (10) and (11), imposing symmetry and using the notation above we obtain

$$v = \frac{\frac{4}{\theta} N}{4 + \frac{4}{\theta} N} \left(1 + \frac{\sigma^2 + \tau^2}{\sigma^2 + \tau^2(1+\gamma)}\right)$$

$$R = \mu (1 - v) - w\theta \frac{(2 - \nu)v}{\frac{M}{2} \left(\frac{1}{\sigma^2 + \tau^2} + \frac{1}{\sigma^2 + \tau^2(1+\gamma)}\right)}$$

The expected return is then $R + \nu \mu = \mu - \theta \frac{(2 - \nu)v}{\frac{M}{2} \left(\frac{1}{\sigma^2 + \tau^2} + \frac{1}{\sigma^2 + \tau^2(1+\gamma)}\right)}$.

We can now analyze the comparative statics of the interest rate $R$. An increase in the mean of the stochastic process ($\mu$) increases the interest rate as project returns become more attractive; as the number of firms $M$ increases, competition for funds gets tougher, and the equilibrium $R$ grows (as $M \to \infty$, $R \to \mu (1 - v)$). As the number of investors $N$ increases, their wealth $w$ increases (so more funds are available), or
risk aversion $\theta$ grows (so that investors seek the risk-free payment more), interest rates decrease. Such tendencies are also present in the standard model with risk.

The interesting comparisons are with respect to risk, ambiguity and ambiguity aversion. The interest rate decreases as the production processes become more risky ($\frac{\partial R}{\partial \sigma^2} < 0$), just as in the standard model with only risk. Investors in a more risky environment are willing to accept a lower risk-free return. By contrast, no such general trend exists for the effect of increases in ambiguity ($\tau^2$) or ambiguity aversion ($\gamma$) on the interest rate. There, two opposing forces are in play. First, an increase in ambiguity causes investors to value the risk-free return more, and so leads to a drop in interest rates. However, an increase in ambiguity also makes home assets less attractive to foreign investors; this will lead firms to increase the rate offered. (Similar points hold for increases in ambiguity aversion.) Only the first effect is present in the standard model with risk, the latter being specific to the case where investors perceive ambiguity towards foreign assets. So, as one would expect, as one tends to the benchmark case—when ambiguity aversion is low or ambiguity is insignificant—the first force will prevail: $\left[\frac{\partial R}{\partial \tau^2}\right]_{\gamma=0} < 0$ and $\left[\frac{\partial R}{\partial \gamma}\right]_{\tau^2=0} < 0$. The first effect also dominates when one is far from the benchmark case: if the ambiguity or ambiguity aversion is high (so that the share of foreign investors’ participation in financing is low) further increases in ambiguity decrease the interest rate: $\left[\frac{\partial R}{\partial \tau^2}\right]_{\tau^2 \to \infty} < 0$ and $\left[\frac{\partial R}{\partial \gamma}\right]_{\gamma \to \infty} < 0$. In intermediate cases, no tendency can be identified in general: for medium values of ambiguity aversion $\gamma$, for instance, the effect of increases of ambiguity, even from small levels, is strongly dependent on the other parameters: the derivative $\left[\frac{\partial R}{\partial \tau^2}\right]_{\tau^2=0}$ cannot be unequivocally signed. The mean of the firm’s production process, for example, may be important: if it is high ($\mu$ is high enough) and ambiguity ($\tau^2$) is small enough, then increases in ambiguity lead to increases in the interest rate ($\left[\frac{\partial R}{\partial \tau^2}\right]_{\tau^2=0} > 0$). This is because increases in ambiguity lead firms to assure investors, transferring risky remuneration into sure compensation, and the higher the expected value of the risky return ($\mu v$), the more leeway they have to increase $R$ (this is clear in the first term in equation (11)). Or, to take another example, when competition is high, due to there being many firms, little wealth or low risk aversion ($M$ high, or $w$ or $\theta$ low), and ambiguity is small enough ($\tau^2$ small), firms are more reluctant to lose foreign investors, and so offer higher interest rates as ambiguity rises: $\left[\frac{\partial R}{\partial \tau^2}\right]_{\tau^2=0} > 0$.

In general, the reaction of the interest rate to an increase in ambiguity may have
a “hump” shape. As the ambiguity increases from low levels, entrepreneurs try to keep foreign investors allocating their wealth in their firms and so increase the interest rate; but with high “model uncertainty”, as the markets become very segmented, firms increasingly turn towards their home investors only, and so their reaction resembles that of the standard model with risk. An example of such a reaction of the interest rate is shown in the upper left panel of Figure 2. This figure also suggests that for typical risk and ambiguity aversion parameter values, and intermediate values of ambiguity (up to 3 times higher than risk), ambiguity has a positive effect on interest rates (for the parameter values used in Figure 2, interest rates are increasing in ambiguity until it is around three times as large as risk, and afterwards, they decrease very slowly).

We summarize these patterns in the following proposition.

**Proposition 3.** Assume $N_1 = N_2$ and $M_1 = M_2$. If $\gamma > 0$, $\tau > 0$, then interest rates ($R$) fall with an increase in risk ($\sigma^2$). No general trend exists for the effect of increases in ambiguity ($\tau^2$) or ambiguity aversion ($\gamma$) on the interest rates. However, for $\mu$ or $M$ high enough or $w$ or $\theta$ low enough, and $\tau^2$ small enough, interest rates ($R$) increase with an increase in ambiguity ($\tau^2$) or ambiguity aversion ($\gamma$).

It follows that risk and ambiguity may have opposite effects on the firms’ capital mix. As risk increases, the variable part ($v$) of the contract payout increases while the fixed payout ($R$) falls. However, in the presence of ambiguity and ambiguity attitudes, the opposite may be true: firms may seek to assure investors and offer them a lower variable part and a higher fixed payout, taking more risk on themselves. Thus, whereas the debt / equity ratio ($kR_k\mu$) declines with risk, it may increase with ambiguity or ambiguity aversion, especially in the presence of attractive (high $\mu$) returns from projects, or intensive firm competition for funds. The bottom left panel of Figure 2 illustrates the common pattern for ambiguity up to 5 times higher than risk: note that, as for interest rates, debt / equity ratio increases with ambiguity, at small to intermediate ambiguity values.

Like increases in risk in the standard model with risk, increases in investor’s ambiguity aversion or in ambiguity unequivocally depress expected real asset returns, despite any positive effect they may have on the fixed payments offered in the contracts.

With symmetric countries and entrepreneurs, the level of wealth invested will be
the same across firms, as in the benchmark risk-only case. However, the shares of domestic and foreign assets in investors’ portfolios will differ depending on the levels of risk ($\sigma^2$), ambiguity ($\tau^2$) and ambiguity aversion ($\gamma$). For the symmetric case, we can obtain simple results for the portfolio shares of representative firms from country 1 and 2 in a country 1 investor’s portfolio:

$$\alpha_{11} = \frac{2}{M \left(1 + \frac{\sigma^2 + \tau^2}{\sigma^2 + \tau^2(1+\gamma)}\right)}$$  \hspace{1cm} (16)$$

$$\alpha_{12} = \frac{2}{M \left(\frac{\sigma^2 + \tau^2(1+\gamma)}{\sigma^2 + \tau^2} + 1\right)}$$  \hspace{1cm} (17)$$

As an indication of the difference in investment at home and abroad, we adopt a standard measure of the degree of home bias (see for example Coeurdacier and Rey (2012); Solnik and Zuo (2012)) as one minus the ratio between actual share of foreign holdings and share of foreign equities in the world market portfolio. In this simple symmetric case, where half the world capital is invested in each country, this can be calculated using (16) and (17) as

$$HB = 1 - \frac{2 (\sigma^2 + \tau^2)}{2 (\sigma^2 + \tau^2) + \tau^2 \gamma}$$  \hspace{1cm} (18)$$

This quantity takes the value 0 when there is no home bias (in this case, when the investor invests precisely half of his wealth abroad). When it is higher than 0, then there is home bias; when it is 1, the investor holds no foreign assets at all. The observation that this quantity is higher than 0 whenever $\gamma > 0$ and $\tau^2 > 0$ yields the following proposition.

**Proposition 4.** Ambiguity and ambiguity aversion towards foreign assets cause home bias in asset holdings.

The fact that in the symmetric case there may be significant home bias without any effect on the total wealth invested in each country (with respect to the benchmark risk-only case) is reminiscent of the conclusions of Solnik and Zuo (2012), who find the possibility of significant home bias in a symmetric case with no effect on asset prices.

Notice that the portfolio shares depend neither on the risk aversion of investors nor on that of firms; what matters is the ratio of “effective” variances of the stochastic
production processes of home and foreign firms, as perceived by the investor. This
in turn depends on the levels of risk, ambiguity and ambiguity aversion. We have
that \( \frac{\partial}{\partial \sigma} (HB) < 0, \frac{\partial}{\partial \tau} (HB) > 0 \) and \( \frac{\partial}{\partial \gamma} (HB) > 0 \). As the ambiguity or ambiguity
aversion increase, the markets become more segmented, exacerbating the home bias;
on the other hand, as the risk increases, the differences between home and foreign
assets get drowned out by the risk factor (which is perceived in the same way by home
and foreign investors), thus mitigating the home bias. One can easily obtain high
levels of the home bias for reasonable parametrizations. For example, if \( \tau^2 = 4\sigma^2 \) and
\( \gamma = 2.5 \) then the holdings of foreign assets will only make up 1/4 of the portfolio,
and \( HB = 0.5 \). The same home bias of 0.5 will be obtained if \( \tau^2 = \sigma^2 \) and \( \gamma = 4 \) or
\( \tau^2 = 0.5\sigma^2 \) and \( \gamma = 6 \).\(^{17}\)

The effect of ambiguity and ambiguity aversion on portfolio holdings affects the
correlation of consumption of investors. In our setup it only makes sense to compare
the consumption of investors, as entrepreneurs always have an idiosyncratic compo-
nent stemming from their own production process that they do not diversify away (as
we prevent them from investing in any securities). The correlation of consumption
among the investors from the two countries in the symmetric case is \( \text{Corr}(c_1, c_2) = \frac{2(\sigma^2 + \tau^2)(\sigma^2 + \tau^2(1+\gamma))}{(\sigma^2 + \tau^2(1+\gamma))^2 + (\sigma^2 + \tau^2)^2} \). This yields the following Corollary to Proposition 4.

**Corollary 1.** Ambiguity and ambiguity aversion lower the correlation of consumption
of investors from the two countries relative to the benchmark case.

For example, for the sets of parameters displayed above (\( \tau^2/\sigma^2 = 4, \gamma = 2.5; \)
\( \tau^2/\sigma^2 = 1, \gamma = 4; \) or \( \tau^2/\sigma^2 = 0.5, \gamma = 6 \) ) the correlation of consumption for investors
falls to 0.6.

### 3.4 Asymmetric countries

As was made clear in the Introduction, a major motivation for integrating ambiguity
into security structure is to analyze the differential effects on firms in countries with
differing amounts of wealth. In this section, we carry out this analysis. To focus on
this question, we assume that the countries are identical as concerns risk, ambiguity
and the number of firms (\( \sigma_1^2 = \sigma_2^2 = \sigma^2, \tau_1^2 = \tau_2^2 = \tau^2, \) and \( M_1 = M_2 = \frac{M}{2} \)), though

\(^{17}\)Solnik and Zuo (2012) report the average HB in developed countries in 2008 at 55%.
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may differ in the number of investors. We assume that $N_1 > N_2$, and refer to country 2 as the capital-scarce country.

First of all, it can be shown that the overall wealth invested in the country with a larger number of investors is higher ($\frac{M}{2} k_1 > \frac{M}{2} k_2$). Moreover, the country with fewer investors is always a net capital importer ($\frac{M}{2} \alpha_{12} N_1 w > \frac{M}{2} \alpha_{21} N_2 w$). This latter fact allows us to identify the capital-importing country as the one with less domestic wealth.

We have the following result concerning asset structure and expected returns.

**Proposition 5.** Suppose that $\tau^2 > 0$, $\gamma > 0$ and $N_1 > N_2$. Then a) $v_1 > v_2$ and b) $R_1 < R_2$.

The situation in the presence of ambiguity is markedly different from the benchmark risk-only case considered in Section 3.2. First of all, whereas in a world with no ambiguity aversion, all firms would offer the same contract irrespective of the investor distribution, in the presence of ambiguity, both contract terms are different between the two countries. On the one hand, entrepreneurs from the capital-scarce country issue contracts that “insure” investors more, bearing more risk themselves. On the other hand, they pay a higher interest rate per unit of capital obtained. Both of these factors can be explained by the attempt to have access to the richer capital market in the foreign country.

Proposition 5 has two immediate consequences.

**Corollary 2.** Firms in the capital-scarce country issue relatively more bonds than equity relative to those in the capital-rich country.

The value of the bond part of a contract issued by a typical firm is $kR$ while the equity part is $kv\mu$. Hence our model implies that countries that are capital importers will have a higher bond/equity ratio in outstanding assets; this is consistent with the patterns exhibited in Figure 1\textsuperscript{18} and confirmed by the data examined in Section 4.2.

**Corollary 3.** The Sharpe ratio of consumption of entrepreneurs in the capital-rich country is higher than in the capital-scarce one.

\textsuperscript{18}Obviously the group of capital-importing countries is very heterogenous. For example, global investors probably have a much better understanding of the stochastic processes governing production in a country like the US than say Greece or Peru.
The nature of the contracts issued by entrepreneurs in the capital-scarce country also renders entrepreneurial consumption more variable when measured by the Sharpe ratio. This is a consequence of the ambiguity perceived by foreign investors, which renders it more difficult for firms from capital-importing countries to obtain insurance from shocks. Accordingly, to attract the desired capital, they must propose contracts that hinder risk sharing (with relatively low equity part \( v \)) and exacerbate the variance of consumption (since more capital is obtained through bond issuance).

**Proposition 6.** Suppose that \( \gamma > 0 \) and \( N_1 > N_2 > 0 \). Then a) As \( \tau \to 0 \) or \( \gamma \to 0 \) the expected return on assets is higher in country 1, and b) As \( \gamma \to \infty \) the expected return on assets is higher in country 2.

The intuition behind Proposition 6 is that as ambiguity aversion grows, firms turn primarily to domestic investors for funds. In autarky the expected returns are higher in the capital-scarce country as the competition for funds is fiercer (given the same number of firms in both countries). On the other hand, when ambiguity aversion is low, and foreign investors are ready to invest, the firms in the capital-scarce country compete with those from the capital-abundant country. Indeed, when \( N_2 = 0 \), and country 2 firms compete with country 1 firms for capital, the expected return on assets is always higher in country 1 though the risk mix offered by firms from the two countries differs. These patterns are in stark contrast to the case without ambiguity studied in Section 3.2, where the expected returns are independent of the distribution of wealth.

Combining the insight that firms in capital-scarce countries offer lower expected returns when investors are not too ambiguity averse\(^{19}\) with the aforementioned fact that the capital invested in a representative firm is higher for the capital-rich country, it follows that, even with a lower installed capital stock, the marginal effective product of capital can be lower in countries with fewer domestic investors, without the marginal return on *physical* capital necessarily being low (as for example in Matsuyama (2004) because of low contract enforcement). In a world with ambiguity and ambiguity aversion, we can thus have equalized marginal returns of capital as measured by Caselli and Feyrer (2007), even in the presence of output gains from reallocating capital. Moreover, the financial market imperfections stemming from ambiguity aversion may play a major role in preventing capital flows from “rich” to “poor” countries.

\(^{19}\)In all simulations that we performed this was true for plausible values of \( \gamma \) (for example, < 50); given the complexity of the expressions involved, a bound of \( \gamma \) could not be found.
We can extend the analysis in Section 3.3 to examine the effects of asymmetry in country wealth on the home bias. Taking as the world market portfolio the total capital invested in both countries that would be observed under our model, the home bias for country 1 and country 2 respectively are
\[ HB_1 = 1 - \frac{M_{2}^{\alpha_{12}}}{M_{1}^{\alpha_{11}}M_{2}^{\alpha_{12}}} \] and
\[ HB_2 = 1 - \frac{M_{1}^{\alpha_{21}}}{M_{1}^{\alpha_{21}}M_{2}^{\alpha_{22}}} \]. Comparing these two expressions and substituting for the portfolio shares and invested capital, one can show that \( HB_2 > HB_1 \) if and only if \( \alpha_{21}N_2 < \alpha_{12}N_1 \). Since, as noted above, the capital-scarce country is a net capital importer, then \( HB_2 > HB_1 \). Country 2 has a higher home bias because its firms rely relatively more on foreign capital for investment, and so the relative share of country 2 firms in the portfolios of country 1 investors is going to be closer to the relative global capital share invested in country 2. To the extent that capital-importing countries in the model can be associated with less developed countries (which for financially open countries is substantiated by Reinhardt et al. (2013)), our model is thus consistent with the empirical analysis by Ahrend and Schwellnus (2012), which concludes that (financially) less developed countries experience a higher home bias. This conclusion is at odds with the view that investors from such countries should have a higher demand for safe assets – and in particular for foreign-issued assets – than those from financially developed countries (for example, Caballero et al. (2008)).

Further analysis of the model is intractable at this level of generality. Therefore we consider the special yet suggestive case where one country has all the wealth.

### 3.4.1 One country with no wealth

Beyond the assumptions mentioned at the beginning of Section 3.4, we assume that \( N_2 = 0 \). We have the following solutions for the contact terms offered by firms in the two countries.

\[ v_1 = \frac{\frac{\alpha}{\beta}N_1}{(2 + \frac{\alpha}{\beta}N_1)} \quad (19) \]

\[ R_1 = \mu (1 - v_1) - w\theta \left( \frac{2 - v_1}{\sigma^2 + \gamma^2} \frac{M_1}{v_1} + \frac{1}{\sigma^2 + \gamma^2(1+\gamma)} \frac{M_2}{v_2} \right) \quad (20) \]
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\[ v_2 = \frac{\frac{A}{\theta} \sigma^2 + \tau^2}{\left(2 + \frac{A}{\theta} \sigma^2 + \tau^2 \frac{1}{1+\gamma}\right) N_1} \]  \hspace{1cm} (21)

\[ R_2 = \mu (1 - v_2) - w \theta \left( \frac{1}{\sigma^2 + \tau^2} \frac{M_1}{v_1} + \frac{1}{\sigma^2 + \tau^2 \frac{1}{1+\gamma}} \frac{M_2}{v_2} \right) \]  \hspace{1cm} (22)

The portfolio shares are then \( \alpha_{11} = \frac{1}{M_1 + \frac{A}{\theta} \sigma^2 + \tau^2 \frac{1}{1+\gamma} \frac{v_1 M_1}{v_2} M_2} \) and \( \alpha_{12} = \frac{\frac{A}{\theta} \sigma^2 + \tau^2 \frac{1}{1+\gamma}}{\frac{A}{\theta} \sigma^2 + \tau^2 \frac{1}{1+\gamma}} M_1 + M_2 \)

while the ratio of capital obtained by firms is \( \frac{k_2}{k_1} = \frac{2 + \frac{A}{\theta} \sigma^2 + \tau^2 \frac{1}{1+\gamma} N_1}{2 + \frac{A}{\theta} \sigma^2 + \tau^2 \frac{1}{1+\gamma}} \).

The observations made above carry over to this case. For example, risk sharing is lower for firms in country 2: indeed, it follows from Proposition 2 that, with high levels of ambiguity aversion, there will be no risk sharing possible for firms in country 2 \( (\lim_{\gamma \to \infty} v_2 = 0) \). As in the general case discussed above, the Sharpe ratio of consumption of firms is higher in country 1, as is the capital invested.

Now let us consider in detail the comparative statics of the elements of capital structure as the risk, ambiguity and ambiguity aversion change. Consider first a change in the risk \( (\sigma^2) \) of the production processes. In terms of the issued contracts in the two countries, \( \frac{\partial v_1}{\partial \sigma^2} = 0, \frac{\partial R_1}{\partial \sigma^2} < 0 \) while \( \frac{\partial v_2}{\partial \sigma^2} > 0, \frac{\partial R_2}{\partial \sigma^2} < 0 \). As observed in Proposition 1, increases in risk increase equity participation as long as there are foreign investors, because it mitigates the (relative) importance of ambiguity. Moreover, as in the standard model with risk, an increase in the global riskiness of the underlying production processes increases investors’ willingness to hold risk-free assets whether at home or abroad, thus leading to a drop in interest rate. On the other hand, if the investors’ ambiguity aversion \( (\gamma) \) changes it is as if they perceive foreign assets as carrying a higher effective variance. Then \( \frac{\partial v_1}{\partial \gamma} = 0 \) and \( \frac{\partial R_1}{\partial \gamma} < 0 \), but \( \frac{\partial v_2}{\partial \gamma} < 0 \), and \( \frac{\partial R_2}{\partial \gamma} > 0 \) whenever \( R_2 > 0 \). As ambiguity aversion increases, entrepreneurs from the capital-importing country offer lower equity stakes (Proposition 1) and “insure” foreign investors by offering higher interest rates on the bond part of the contract. Moreover, the “weakening” of foreign competition allows the firms in the capital-rich countries to offer lower interest rates.

The picture is more complex, however, as concerns changes of ambiguity \( (\tau^2) \). Qualitatively, an increase in ambiguity is an intermediate scenario between an increase in risk and an increase in ambiguity aversion: the “effective” variance of production processes perceived by investors increases for both home and foreign assets, though more
strongly for the latter. Then $\frac{\partial v_1}{\partial \tau_2} = 0$, $\frac{\partial R_1}{\partial \tau_2} < 0$. As in the case of increased ambiguity aversion, firms from the capital-rich country obtain lower interest rates as investors seek more “safety”. Also, as shown in Proposition 1, $\frac{\partial v_2}{\partial \tau_2} < 0$. However, the effect on the interest rate cannot be signed in general ($\frac{\partial R_2}{\partial \tau_2} > 0$). This is due to the interplay between two forces. On the one hand, an increase in ambiguity leads to an increase in the variance perceived by all investors for all assets, so they are willing to accept a lower remuneration for holding bonds and firms do not compete as strongly for funds; this is the mechanism driving the interest rate down when the risk increases. On the other hand, the “effective” variance perceived by country-1 investors towards country-2 firms increases faster than the variance perceived towards country-1 firms, due to ambiguity aversion; this is the same as the mechanism that drives the interest rates in country 2 up when the ambiguity aversion increases. Which of these effects prevails depends on other parameter values. For instance, just as in the symmetric-country case (Section 3.3), for $\mu, M$ high enough or $w$ or $\theta$ low enough, for increases of $\tau^2$ from small levels will lead to increases in $R_2$. An example of the behavior of the interest rates as $\tau^2$ increases is given by the upper right panel of Figure 2. This figure also suggests that, just like the symmetric case (Section 3.3), for typical risk aversion and ambiguity aversion parameter values, and intermediate values of ambiguity (up to 5 times risk), ambiguity has a positive effect on interest rates. Indeed, the positive effect of ambiguity on interest rates for intermediate parameter values appears to extend beyond the ‘extreme’ cases of perfect symmetry and complete asymmetry (one country with no wealth): the upper middle panel of Figure 2 shows that the effect remains in an intermediate case of moderate asymmetry in country wealth.

**Proposition 7.** If $\gamma > 0$, $\tau > 0$ and $R_2 > 0$, then interest rates in country 1 ($R_1$) fall as any of risk ($\sigma^2$), ambiguity ($\tau^2$) or ambiguity aversion ($\gamma$) increase. Although interest rates in country 2 ($R_2$) fall as risk increases ($\sigma^2$), it increases with increases of ambiguity aversion ($\gamma$). No general trend exists for the effect of increases in ambiguity ($\tau^2$) on interest rates in country 2 ($R_2$). However, for $\mu$ or $M$ high enough or $w$ or $\theta$ low enough, and $\tau^2$ small enough, interest rates ($R$) increase with an increase in ambiguity ($\tau^2$).

Combining these effects, it is clear that changes in risk, ambiguity and ambiguity aversion may have different consequences for the debt / equity ratio ($\frac{kR}{k\nu\mu}$). For both
countries, increases in risk lead to a fall in this ratio, as interest rates fall while the equity share rises in country 2. For the capital-scarce country, increases in ambiguity aversion have the opposite effect: they lead to an increase in the ratio. Increases in ambiguity do not have a blanket effect: depending on the other parameter values, they may lead to an increase or a decrease in the debt / equity ratio. In particular, for low levels of $\tau^2$, if the projects are attractive (high $\mu$) or there exists intensive competition for funds (high $M$), $\frac{kR}{k\mu}$ will increase with increases in ambiguity, just like in the symmetric country case. The lower right panel of Figure 2 shows the behaviour of the debt / equity ratio as ambiguity changes. Just as for the symmetric case, and unsurprisingly given the previous observations concerning interest rates, ambiguity typically has a positive effect on the debt / equity ratio, when it is not too high. Again, the middle right panel of Figure 2 that this pattern generalises to (non-extreme) asymmetric countries.

Note that Figure 2 also suggests that the blanket negative effect of ambiguity on interest rates offered in the capital-rich country is particular to the case of one country having all the wealth. For the intermediate asymmetric country case displayed in the middle panel of the figure, there is a ‘hump shape’ in the reaction of interest rates to ambiguity in both countries; and similarly for the debt-equity ratio. However, both the interest rates and the debt / equity ratio are less sensitive to ambiguity in the capital-rich country than in the capital-scarce one, especially when ambiguity is not too high (the graph for the capital-rich country is less steep on the left). This and the other patterns are borne out by explorations of further sets of parameter values.

Such opposite effects are also found in the end capital distribution and interest rate differential across countries. The end capital distribution between firms in the two countries becomes more even with the increase in risk ($\frac{\partial}{\partial \sigma^2} \left( \frac{k_2}{k_1} \right) > 0$), and the interest differential between their firms’ bonds decreases ($\frac{\partial}{\partial \sigma^2} (R_2 - R_1) < 0$). The opposite is true for increases in ambiguity aversion: the interest differential offered by firms in the two countries grows ($\frac{\partial}{\partial \gamma} (R_2 - R_1) > 0$). Despite this, the capital invested in the capital-scarce country decreases, and so the capital distribution becomes more uneven. The Sharpe ratio of entrepreneurial consumption in the two countries also diverges: entrepreneurs in country 2 need to accept more risk per unit of expected consumption. As for increases in ambiguity, although they do not necessarily have a monotonic effect on the interest differential $R_2 - R_1$ (for example, it may have a hump shape), they do
have an unequivocal effect on the discrepancy between the wealth invested in the firms of the capital-rich and capital-scarce country: it increases. More generally, as long as the proportional increase in ambiguity is greater than that for risk, the firm-level capital distribution becomes more uneven.

**Proposition 8.** Suppose that $\sigma^2$ increases by a factor $a$ and $\tau^2$ increases by a factor $b$. If $b > a$, then $\frac{k_2}{k_1}$ decreases.

This observation suggests an interesting interpretation of global crisis events. If investors become suddenly less sure about the stochastic properties of productivity for all assets – if their ambiguity or “model uncertainty” increases – the model would predict relatively less investment in capital-scarce countries which would empirically manifest itself as “capital flight” (“sudden stops”) from countries that are capital importers. This could be an explanation for what happens during global crises, such as the one in 2008, and is consistent with the evidence presented in Forbes and Warnock (2012). Giannetti and Laeven (2016), who document increases in home bias during financial crises, favor a behavioral explanation like this. It does not assume asymmetric shocks to peripheral capital importers, increases in risk, ambiguity aversion, transaction or information costs: a general unanticipated shock in ambiguity (or “model uncertainty”) about the stochastic properties of economic fundamentals would produce the observed effects. Notice that increases in risk alone cannot explain the data; according to the model, they would lead to more investment flowing to countries with relatively fewer investors. It is simultaneous increases in risk and ambiguity, as long as the proportional increase in ambiguity is larger than that of risk, that lead to such effects. Indeed, global crises arguably correspond more precisely to situations of this sort: the environments have not got significantly more risky in the technical sense—of having higher variances for the underlying stochastic processes for production – but there have been large increases in ambiguity—people are less sure about the parameters governing those stochastic processes. Moreover, recent evidence documented by Rey (2015) may support the model’s prediction of a relationship between capital flows and ambiguity. She shows that global capital flows (which are strongly correlated across asset classes like equity and debt) can be explained well by one global factor that is negatively correlated to an index of “market fear” called VIX (a measure of the implied volatility of S & P 500 index options). Taking this index as a proxy for ambiguity, this finding is
precisely as the model would suggest.

4 Ambiguity, risk and asset structure patterns

Our model has specific consequences for the effects of risk and ambiguity on elements of the capital structure: Proposition 1, for instance, indicates that they have opposite effects on the equity part, whereas Propositions 3 and 7 suggest that they may have differing effects on the bond part, depending, for instance, on the relative abundance of investors and hence investable wealth. In this section we uncover patterns in cross-country firm-level capital structure data that are consistent with our model.

The capital structure literature eschews direct measures of the equity or debt issuance, preferring leverage measures instead. As noted in Sections 3.3 and 3.4.1, our model has specific consequences for the debt / equity ratio—it falls with rises in risk, but, unlike the standard model with risk alone, may rise with increases in ambiguity—and the same patterns hold for leverage. However, whilst there have been studies of the effect of risk on capital structure (see Section 6), we are aware of no firm-level study comparing the effects of ambiguity and risk. We present such a study in this section. It also allows us to check other predictions of the model, concerning leverage differences between firms in capital exporting and importing countries, and the comparative response of firms in capital-scarce countries to changes in ambiguity.

4.1 Empirical setup

Ambiguity and Risk. We first specify proxies for risk and ambiguity. Recall that decision makers are said to perceive ambiguity—or “model uncertainty”—if they cannot assign a single probability measure to the relevant events on which to base their decisions. By contrast, when they do act on the basis of a single measure, they perceive no ambiguity: in the terminology adopted here, there is only risk. One instance of this latter case is where there is a constant stochastic process whose distribution individuals have had the chance to learn; more generally, what individuals know about the distribution of the underlying stochastic process can be assimilated with what we have called their (perceived) risk. To the extent that facts about the distribution can be learnt from past data, a natural variable to take as a proxy for risk is variance of a rele-
vant parameter on the basis of past data. This is relatively standard in the literature: Frank and Goyal (2009), for instance, use variance of stock returns. Since we will not have data on stock returns, we will take moving measures of world and country-level standard deviation of GDP—denoted $WVAR$ and $CVAR$ respectively—as our proxy for risk.

On the other hand, a paradigmatic instance of ambiguity is the residual uncertainty concerning aspects of a stochastic process that the decision maker has not had the data or the time to learn or that are unlearnable (see Epstein and Schneider (2007) for an example). One possible proxy for such uncertainty could be indices of expected market volatility such as the VIX. Roughly speaking, this a measure of the variance of the market’s expected variance, and hence indicates the degree of agreement among investors about the stochastic properties of future market behaviour. If, as seems reasonable, investors are less sure about the stochastic fundamentals when there is more disagreement on them in the market, this measure should co-vary with the ambiguity perceived by the average investor. In the following analysis, we thus take the VIX as a proxy for ambiguity; in so doing, we join other recent studies, such as Giannetti and Laeven (2016)).

**Data.** We take firm level data on capital structure from ORBIS, provided by Bureau van Dijk that gathers various accounting information for firms worldwide and standardizes it to one format. We combined it with standardly available data for 2006-2015 and perform usual cleaning procedures (detailed in Appendix A.5). After these, our base Full sample contains data for 142,902 firms from 86 countries for 10 years of consecutive data 2006-2015. For robustness checks, we also use subsamples where we retain countries with better developed financial markets. We consider a sample with OECD countries in 2005 only (30 countries) and one with countries that are characterized by an above-median measure “Domestic credit to private sector (% of GDP)” in 2005 provided in the WDI data set by the World Bank (this sample contains 43 countries, with the median value for the indicator 64%). Although we impose such restrictions, these subsamples still contain 90% and 84% of the Full sample observations respectively. Specifications of how all our variables are obtained are given in Appendix A.5.

The summary statistics of the variables used are shown in Table IV.
Leverage ratio. We follow the capital structure literature as close as possible, whilst retaining parsimony as far as possible. As a proxy for the debt / equity ratio, we take two out of the four most popular measures of leverage in this literature: debt to book value of total assets ($DTA$) and long-term debt to book value of total assets ($LTDTA$).\textsuperscript{20} Frank and Goyal (2009) claim that these measures typically give similar results to the measures based on the market values of assets, which give a direct measure of debt / equity choice. They have, however, the advantage that they do not conmove with stock market fluctuations for example along the business cycles (that could inflate them in “crisis” times) but rather directly reflect firm-level capital structure decisions (and constraints).

Control Variables. Frank and Goyal (2009), an oft-cited reference that studies exhaustively the determinants of capital structure of U.S. stock-market firms, find that six variables are strongly and robustly correlated with various measures of leverage and important in terms of explaining the existing variation in the data. Two of them, industry median leverage and firm size will be absorbed by fixed effects in our setup. We cannot measure and account for one of them—market-to-book assets ratio—as we do not have stock market prices for most companies in our sample (see Appendix A.5). We include all three others: asset tangibility ($TANGTOT$, tangible assets to total assets ratio), profitability ($PROFITS$, profits to total assets ratio) and expected inflation at the country level ($INFLEXP$).\textsuperscript{21} We lag the two measures $TANGTOT$ and $PROFITS$ (to obtain $L.TANGTOT$ and $L.PROFITS$ respectively) so that they are not jointly determined with our dependent variables and take country-level inflation expectations that would be known to market participants during the entire calendar year. As we are dealing with international data, where all items are expressed in national currency, we also include the logarithm of local currency to SDR exchange rate ($\lnSDR$). This controls for the possibility that the debt / asset ratios fluctuate with the local exchange rate, which could have an impact on our results if firms would borrow

\textsuperscript{20}For most of the firms in our sample we do not have their market value so we cannot calculate the other two measures – debt to market value of assets and long term debt to market value of assets.

\textsuperscript{21}Frank and Goyal (2009) are able to generate a firm-level measure of risk by calculating a moving volatility of firm stock prices. In our case, we have too few year observations (10) to do so in any meaningful way, whether at the firm or industry level. Given our setup with fixed effects, however, inherent variability at the sectoral or firm-level will be absorbed in the fixed effects.
extensively abroad in foreign currencies\(^{22}\).

**Empirical Specification.** Our basic regression specification is thus

\[
y_{i,t} = \alpha_0 + \beta_1 VIX_t + \beta_2 WVAR_t + \beta_3 CVAR_{c,t} + \beta_4 L.TANGTOT_{i,t} + \beta_5 L.PROFITS_{i,t} + \beta_6 INFLEXP_{c,t} + \beta_7 \ln SDR_{c,t} + \delta_i + \varepsilon_{i,t}
\]

(23)

where \(y_{i,t} \in (DTA, LTDTA)\) are the yearly firm-level debt / asset ratio and long-term debt / asset ratio, \(\delta_i\) is the firm-level fixed effect and subscript “\(c\)” denotes country-level variables. We estimate equation (23) with firm-level fixed effects which helps us to account for unobservables that do not change with time.\(^{23}\) Following Frank and Goyal (2009), we double cluster the standard errors at the firm and year level.

### 4.2 Correlations in the data

**Means comparisons.** First of all, we compare the means and medians for the variables of interest between capital exporters (net foreign assets > 0 in 2005) and importers in Table I. Recall that our model implies that firms in capital importing countries should have a higher debt / equity in the presence of ambiguity (Section 3.4, Corollary 2). For the Full sample and the subsamples considered, this is indeed the case: the medians and means for \(DTA\) and \(LTDTA\) are always higher for capital importers. For example, for the Full sample, the mean \(DTA\) ratio is 0.157 and 0.223 for firms in countries with positive and negative NFA in 2005 respectively. The differences in means are always statistically significant at the 1\% level. This firm-level data is in line with country-level findings from the Lane and Milesi-Ferretti (2007) data exhibited in Figure 1.

**Regression analysis.** We turn now to regression analysis of the effects of ambiguity and risk on capital structure, presented in Tables II (for \(DTA\)) and III (for \(LTDTA\)).

\(^{22}\)We also ran the regressions using the local currency / U.S. dollar rate with similar results. The nominal effective exchange rates were available for a smaller subset of countries, so we did not use them and opted for SDR rates instead.

\(^{23}\)We opted not to include the numerous variables used in the capital structure literature except for the main ones used by Frank and Goyal (2009). The reason is that the sample shrinks dramatically with additional restrictions as the accounting data in ORBIS data set is incomplete for many companies.
Our base specification is shown in column 1. First, we observe that the control variables drawn from Frank and Goyal (2009) enter with the same signs as found by these authors: this is a consistency check on our data and analysis. On average, firms with higher tangible assets to total assets ratio ($L.TANGTOT$) issue more debt while the more profitable ones ($L.PROFITS$) have lower leverage. Both measures are statistically significant at the 1% level. Our measure of inflation expectations ($INFLEXP$), though correctly signed (higher expected inflation leads firms to issue more debt), is not statistically significant. According to our estimates, a depreciation of the domestic exchange rate is correlated with a fall of the debt to asset ratios, though it is statistically significant from zero only for $DTA$.

As concerns the variables of interest, the coefficients on our proxy for ambiguity—the $VIX$—are equal to 0.001 in Tables II and III and both statistically significant at the 1% level. As ambiguity increases, firms tend to increase both their debt- and long-term debt- to asset ratios. On average, a one standard deviation growth in $VIX$ (a change of 6.75 in this indicator) implies that both $DTA$ and $LTDTA$ increase by 0.67 percentage points. The opposite seems to be true for the world and country-level variances—our proxy for risk. Although on average firms decrease $DTA$ and $LTDTA$ in response to increases in world GDP variance ($WVAR$), this finding is not statistically significant. Country level variance ($CVAR$), however, does correlate negatively with firms’ capital structure in a statistically significant way at the 1% level. A one standard deviation increase of $CVAR$ (1.82) can be associated with a 0.73 and 0.6 percentage point falls in the $DTA$ and $LTDTA$ respectively. The changes implied by the movements of $VIX$ and $CVAR$ are substantial given that the average sample values for $DTA$ and $LTDTA$ are .187 and .124 respectively.

In column 2 of each table we add the interactions of our variables with the negative of the countries’ net foreign assets / GDP ratio in 2005 ($VIX \times NFAneg$, a continuous variable). The positive coefficient on the interaction of $VIX \times NFAneg$ means that as $VIX$ increases firms in capital-importing countries increase their leverage more than firms in countries with positive NFA. In column 3 we rework the formulation in column 2 and interact our variables with an indicator variable $VIX \times D.NFA < 0$ showing whether the firm’s domicile country’s net foreign asset position in 2005 was positive (coded as “0”) or negative (coded as “1”).
Robustness The remaining columns in Tables II and III are robustness checks. In columns 4 and 5 we keep either the measure of variance of GDP at the world or the country level only. This does not change our findings qualitatively in either table. In columns 6 and 7 respectively we confirm our results restricting our sample to include either only OECD countries or those where domestic credit / GDP ratio in 2005 is above the median (where presumably financial markets are more developed). In column 8 we reestimate the specification from column 1 clustering both at the country and year level to address potential concerns about error structure.

Discussion Data patterns, and more generally the fact that leverage correlates in opposite ways with ambiguity and risk variables, is consistent with our model. One striking property of our model (in comparison with the standard model with risk, for instance) is to permit such contrasting effects: as is clear from Propositions 3 and 7, whilst risk has a negative effect on the debt / equity ratio, and hence on leverage, ambiguity may have a positive effect. Indeed, as noted in Sections 3.3 and 3.4.1, calculations suggest that, no matter the distribution of wealth between countries, the model predicts positive effects of ambiguity for intermediate parameter values. For instance, for a reasonable set of parameter values, Figure 2 shows that the effect will be positive (for at least one of the countries) whenever ambiguity is less than around three times as high as risk.

Another pattern in the data is that firms in capital-importing countries are on average nearly twice as sensitive to the effects of ambiguity in their leverage decisions (columns 2 and 3 of each table). Again, this is in line with what the model would suggest. As noted in Section 3.4.1, the impact of ambiguity on the debt / equity ratio may be strongly country-dependent when countries are asymmetric. In particular, as noted there and suggested by the middle panel of Figure 2, for intermediate cases of country asymmetry, the debt / equity ratio in the country-rich country will be less sensitive to ambiguity.

So our model is particularly consistent with the data. Note that this is not the case for the standard model with risk alone, which cannot explain the correlation between ambiguity and leverage. Indeed, although some existing theories can explain some of the aforementioned aspects, we are not aware of any than explain them all. For instance, the negative effects of risk on $DTA$ and $LTDTA$ can be explained by the
trade-off theory of capital structure, but then this theory cannot explain why ambiguity has the opposite effect.

5 Discussion and robustness checks

Inclusion of ambiguity and ambiguity aversion changes the properties of the standard model with risk and makes it more realistic. Increases in risk, ambiguity and ambiguity aversion have radically different, and in many cases opposite effects on the capital invested internationally, the extent of risk sharing given by $v$ and the interest rates $R$. This allows us to rationalize stylized facts about international capital flows with a simple model; in fact, the model can explain the different patterns of debt/equity issuance at the macroeconomic level. Moreover, as argued in Section 1, this allows the model to explain a range of puzzling phenomena.

Note that the model supports several alternative interpretations beyond the proposed one. The firms in our model can be interpreted as different idiosyncratic sources of risk in the relevant country. The case where $M_1 = M_2 = 1$ can be interpreted as there being one source of risk in each country or alternatively in terms of an assumption that all stochastic production processes within a country are perfectly correlated: one could have several separate entities issuing assets with the same characteristics (this would be an equilibrium outcome) that would be treated by investors as if they were the same asset. An alternative interpretation of the case $M_1 = M_2 = 1$ is in terms of holding a market portfolio of equity and bonds from that country. Generalising this interpretation, a version of the model can be developed with more than two countries that non-cooperatively compete for funds, yielding qualitatively similar results to those reported above.

Whilst we have deliberately adopted a simple model, in order to bring out more clearly the essential points about the possible consequences of ambiguity and ambiguity aversion for international capital structure, our main conclusions are robust to many refinements of or variations in the model, such as the following\textsuperscript{24}.

Differing risk and ambiguity  Whilst it was assumed in Section 3.4 that the countries are identical as concerns risk and ambiguity ($\sigma_1^2 = \sigma_2^2 = \sigma^2$ and $\tau_1^2 = \tau_2^2 = \tau^2$),

\textsuperscript{24}Details of each extension are available upon request.
the literature indicates that many capital-scarce countries experience higher volatility of their production processes. It is thus worth asking what happens when risk or ambiguity are higher in the capital-scarce country – so that either $0 < \sigma_1^2 < \sigma_2^2$ or $0 < \tau_1^2 < \tau_2^2$. In the case when $N_2 = 0$ (as in Section 3.4.1), these cases are relatively easy to analyze, yielding expressions similar to those in equations (19–22). Generally speaking, the phenomena identified above carry over to this case. Just as in the case studied in Section 3.4.1, entrepreneurs in the capital-scarce country obtain less risk sharing ($v_1 > v_2$) and they offer high interest rates ($R_2 > R_1$); Moreover, as the production process becomes more risky the entrepreneurs in the capital-scarce country obtain more risk sharing, whereas the opposite is true when ambiguity increases ($\frac{\partial v_2}{\partial \sigma_2^2} > 0$ while $\frac{\partial v_2}{\partial \tau_2^2} < 0$). Also, just as in Section 3.4.1, the interest rate decreases with increases in risk ($\frac{\partial R_2}{\partial \sigma_2^2} < 0$) while the effect of ambiguity ($\frac{\partial R_2}{\partial \tau_2^2}$) cannot be signed in general.

**Asymmetric uncertainty; Symmetric risk.** As explained in Section 2.1, to focus on the role of ambiguity, we have assumed throughout that uncertainty (that is, risk plus ambiguity) is symmetric across countries, though ambiguity and risk may vary. However, the main thrust of our results do not depend on the symmetry assumption; in particular, they continue to hold in the distinct case where risk is symmetric, and every difference in ambiguity between investors corresponds to a difference in uncertainty. Suppose that both home and foreign investors perceive a risk $\xi_n^2$ in firm $n$’s process, while foreign investors perceive in addition ambiguity $\zeta_n^2$. It can be shown that in this case, and under the other symmetry assumptions set out in Section 2, the investments by a country-1 investor in typical firms in countries 1 and 2 respectively are:

$$\alpha_{11} = S_{11} + \frac{1 - M_1 S_{11} - M_2 S_{12}}{M_1 + M_2 \left( \frac{s_1}{s_2 + t_2} \right) \frac{v_2^2}{v_1^2}}$$

$$\alpha_{12} = S_{12} + \frac{1 - M_1 S_{11} - M_2 S_{12}}{\left( \frac{v_2^2}{v_1^2} \right) \left( \frac{s_1 + t_2}{s_1} \right) M_1 + M_2}$$

where $S_{11} = \frac{R_1 + \mu v_1}{s_1 \theta \omega v_1^2}$ and $S_{12} = \frac{R_2 + \mu v_2}{(s_2 + t_2) \theta \omega v_2^2}$ and

$$s_1 = \xi_1^2, \quad t_1 = \xi_1^2 (1 + \gamma)$$
with $s_2$ and $t_2$ defined similarly (see Appendix A.3).

Noting that (8) and (9) can be put in the form of equations (24) and (25) with:

\[
s_1 = \sigma_1^2 + \tau_1^2 \\
t_1 = \tau_1^2 \gamma
\]

(27)

it is clear that most of the findings in the symmetric-uncertainty case will carry over immediately to the symmetric-risk case. In particular, since changes in $\sigma^2$ and $\gamma$ affect the $s$ and $t$ terms respectively in both cases, all the comparative static results on risk and ambiguity aversion go through immediately. Moreover, in the symmetric-risk case changes in $\zeta^2$ only affect the $t$ term, just like $\gamma$ in the symmetric-uncertainty case, so the comparative statics of $\zeta^2$ in the former case will behave exactly as those for $\gamma$ in the latter case studied above. Indeed, all Propositions and findings go through directly in the symmetric-risk case, modulo the obvious corrections following from what has just been said.

We conclude that our central findings go through even when uncertainty is not symmetric.

**Ambiguity with respect to home assets.** One might consider the assumption that investors perceive no ambiguity with respect to home assets to be too strong, and wonder to what extent the results are sensitive to it. In Appendix A.3 we show how the current framework can incorporate investor perception of ambiguity in all assets, with more ambiguity perceived for foreign assets than for domestic ones. (Recall that all investors have the same ambiguity aversion, irrespective of the source of the ambiguity, i.e. the asset.) More specifically, we assume that all investors perceive the same risk $\sigma^2$, but whereas home investors perceive an ambiguity $\tau_h^2$, foreign investors perceive ambiguity $\tau_f^2 > \tau_h^2$. The firm perceives only risk $\sigma^2$ towards its production process.

Then portfolio allocations are as in (24) and (25) with:

\[
s_1 = \sigma^2 + \tau_h^2(1 + \gamma) \\
t_1 = (\tau_f^2 - \tau_h^2)(1 + \gamma)
\]

(28)

\[25\text{More precisely: in Proposition 2, } \lim_{\zeta^2 \to \infty} v_1 = \lim_{\gamma \to \infty} v_1 = \frac{N_i}{2 + N_i}; \text{ in Proposition 6, part a) no longer holds for } \gamma \to 0 \text{ (since this no longer corresponds to } t \to 0, \text{ under (26), though part b) holds for } \zeta^2 \to \infty; \text{ finally, the comparative statics for } \zeta^2 \text{ in the one country with no wealth case (Section 3.4.1) are as for } \gamma. \text{ In particular, } \frac{\partial R_2}{\partial \zeta^2} > 0 \text{ whenever } R_2 > 0.\]
The comparisons in this case do not follow automatically from the Propositions in Section 3 (as in the symmetric-risk case just discussed) because of the difference in the way home investors and firms treat the firm’s production process: the former perceive some ambiguity in it, to which they are averse, whereas the latter perceive no ambiguity. Nevertheless, it is easy to deduce for example that Propositions 1 and 5 hold, and that Propositions 4, 6a and 8 hold, when reformulated in terms of ambiguity differences \((\tau^2_{f} - \tau^2_{h})\) instead of the foreign investors’ ambiguity while Proposition 2 does not apply. The comparative statics of Propositions 3 and 7 are different: in particular, a change in \(\gamma\) here has the same effect as a general change in \(\tau^2\) in the original setup.

**A different decision model.** Are the results specific to the decision model used, namely the smooth ambiguity model of Klibanoff et al. (2005), or do they continue to hold for other ambiguity models? In Appendix A.4 we study the same questions using the maxmin expected utility model of Gilboa and Schmeidler (1989), of which the Hansen and Sargent (2001) constraint model is a special case. The general qualitative properties identified above continue to hold, with the sole exception that there is no parameter in the maxmin EU model that, in the context of this problem, yields the same behavior as the ambiguity parameter \(\tau^2\). This is related to a specificity of the smooth ambiguity model that was noted at the outset: it permits a distinction between ambiguity and ambiguity aversion that is absent in most other ambiguity models, and in particular in the maxmin EU one. The ambiguity-ambiguity aversion parameter in the maxmin EU model behaves more similarly to the ambiguity aversion parameter in the smooth model \(\gamma\) than to the ambiguity one \(\tau^2\).

**A comparison with a standard model with borrower moral hazard.** Another potential factor in the explanation of our target phenomena is moral hazard. Rather than integrating it into the general form of our model, we considered, for the sake of comparison, the consequences of adding moral hazard to the standard model with risk, in a simple yet comprehensive case. Add to the standard model with risk (Section 3.2) the assumption that production requires unobservable and uncontractable effort \(\xi\) that is costly for the entrepreneur (described by some function \(Z(\xi, k)\)) and affects the obtained return for production so that the production function is now \(f(k, \xi) = \xi k\). Could this drive changes in the issued contracts? In particular, could it explain
an asymmetric propensity to invest in some forms of contract versus others across countries? It turns out that firms’ contracts in all countries are going to be affected equally if $\sigma_1^2 + \tau_1^2 = \sigma_2^2 + \tau_2^2$. Interestingly $v$, the equity part offered in equilibrium to investors, does not change relative to the benchmark, though $R$ does. The intuition for this is that, since there are no frictions in the capital market, a firm in a capital-scarce country is not penalized in any way and has exactly the same incentives as a firm in the capital-rich country: the investor composition does not matter. Given the CARA-Normal formulation, any changes in the properties of the production returns (that are affected by effort) impact the interest rate. This is true no matter what cost-of-effort function one takes. If, contrary to the main models examined in this paper, there is a difference in uncertainty between countries (so $\sigma_1^2 + \tau_1^2 < \sigma_2^2 + \tau_2^2$, say, though we are not assuming that country 2 is the one with fewer investors), the result for the $R$ term depends in general on the effort function chosen. If the cost of effort increases linearly with the capital that is obtained from investors, the result remains the same: there is no difference in the contract composition between firms from different countries. In general, the presence of moral hazard does not cause (in the absence of other frictions) the contracts between countries to differ: moral hazard in itself does not seem capable of explaining the existence of the home bias. Whilst we by no means wish to suggest that such effects are absent, this suggests that the range of phenomena discussed here cannot all be explained by moral hazard alone.

**Different assumptions on the stochastic production process.** To derive our results we used a specification based on the standard workhorse setup with a CARA utility and a Normal distribution. This setup allows for closed-form solutions, but by construction permits negative output draws. Unfortunately, to our knowledge, there exists no other set of assumptions on the utility and/or the distribution of production shocks that allows for tractable solutions for both the firm problem and/or the portfolio choice problem under ambiguity. Therefore, we conducted a series of numerical simulations to check whether the main properties of the model carry through in the absence of negative productivity draws. To stay close to the original framework, we assumed truncated normal distributions with a left bound at zero for the stochastic production processes. An investor considers distributions $\tilde{x}_n \sim TrunNorm(m_n, \sigma_{ln}^2)$ for some fixed $\sigma_{ln}^2$, and some set of possible expected returns $m_n$, rather than standard
normal distributions as in the bulk of the paper. As in Section 2.1, he has a second-order prior over this set of distributions that is itself normally distributed, with mean $\mu_{\ln}$ and variance $\tau_{\ln}^2$: $\tilde{m}_n \sim N(\mu_{\ln}, \tau_{\ln}^2)$. The rest of the assumptions are as specified in Section 2; in particular, the firm uses the same distribution as a home investor, and hence gives zero weight to negative productivity draws. (See Appendix A.7 for technical details.) With such assumptions, we cannot obtain closed form solutions either for the firm or the investor problem.

Examples of obtained equilibrium contract terms, portfolio shares and derived items are given in Appendix A.7. We numerically exhibit Proposition 1 and Proposition 4 with an assumption of symmetric countries (Figures 3-5), and Proposition 5, Corollary 2 and Proposition 6a with an assumption that there are no investors in country $j$ (Figures 6-8). Given the different assumptions on the distribution of productivity, not all propositions are amenable to confrontation with numerical simulations (such as Proposition 2). We conclude that the central messages of the model are not dependent on the use of the normal distribution.

Adding a risk-free asset to the model. Whilst it has been assumed that all investor wealth is invested in the firms, one could weaken this assumption by allowing a risk-free asset, for instance. The analysis above can be interpreted as concerning the case where there is a risk-free asset, but a scarcity of investors relative to firms: indeed, if investors invested significantly in the risk-free asset, then, as long as entry costs were not too high, one would expect more firms to enter the market until the sums to be invested were “absorbed” by firms, with none going to the risk-free asset (assuming that shorting the risk-free asset is not possible). The issue turns out to be immaterial to the basic points made here, because the central findings go through when there is a risk-free asset that is used by investors, as long as its rate of return is less than the mean $\mu$ of the firms’ stochastic process. The solution for $v$ does not change (see Appendix A.8). In this case, we have the following implication of the model, which strengthens Propositions 1, 3 and 7:

**Observation.** Suppose $N_1 > 0, N_2 > 0, \gamma > 0$ and $\tau_1^2 > 0$. Then $\frac{\partial v_1}{\partial \tau_1^2} < 0$, $\frac{\partial v_1}{\partial \gamma} < 0$, $\frac{\partial v_1}{\partial \sigma_1^2} > 0$, $\frac{\partial R_1}{\partial \tau_1^2} > 0$, $\frac{\partial R_1}{\partial \gamma} > 0$, $\frac{\partial R_1}{\partial \sigma_1^2} < 0$, $\frac{\partial}{\partial \gamma} \left( \frac{R_1}{\nu_1 \mu} \right) > 0$, $\frac{\partial}{\partial \sigma_1^2} \left( \frac{R_1}{\nu_1 \mu} \right) < 0$.

Proposition 2, 4, 5, 8 and all Corollaries follow while, as concerns Proposition 6,
the expected return is always higher in the capital-rich country. Therefore, the major findings concerning the differing impacts of ambiguity and risk on asset composition, as well as the importance of home wealth, are unaffected by the addition of a risk-free asset to the model.

Allowing for firm entry. One cannot obtain closed-form solutions on the equilibrium number of firms assuming some sunk cost of entry or some threshold utility for entrepreneurs. We conducted simulations to verify the robustness of the model in this regard. Firstly, the solution for $v$ does not change as it does not depend on the number of firms. Next, we numerically confirm that all our central findings continue to hold with firm entry, except for the Propositions 2 and 6, that are not amenable to such treatment. Furthermore, we observe some interesting patterns on firm entry. In the simulations, the number of entering firms increases as ambiguity or ambiguity aversion increases. Such increases segment the capital markets in the two countries and depress returns available to local investors; as a result they make it more profitable for firms to enter. It is also more difficult for firms to enter in capital-scarce countries. The simulations thus suggest that capital market imperfections may have important implications on the equilibrium number of firms, a theme which is underresearched in the literature, and one that we leave as topic for future research.

Multiple contracts. Another robustness check concerns the extent to which the findings depend on the assumption that entrepreneurs offer a single contract. After all, they could try to discriminate between domestic and foreign investors by offering a menu of contracts, with investors able to self-select and obtain a desired contract by differentially investing in different contracts. In general, the solution to a model where firms offer multiple contracts is much more involved than the one treated above. It is, however, possible to deduce some properties of the equilibrium contracts. Suppose that a firm offered a succession of contracts with different $(v, R)$ (though all based on its, single, stochastic process). Since the contracts offered by a firm involve the same source of risk, by mixing contracts from a given firm, investors have more choice over the equity / bond mix they will face from a given stochastic process. It turns out that, in the general case, when $\gamma > 0$, domestic investors would choose a bundle of contracts (mix) that is riskier, in the sense of involving a higher effective $v$, but that yields more
in expectation. Moreover, the mix chosen by domestic investors will have a lower debt to equity ratio. Thus firms in the country with the lower number of domestic investors (relatively capital scarce) will have a higher debt-equity ratio. We can thus ascertain that at least some of our basic results (Propositions 1 and 5) carry through to the multi-contract case.

Differing wealth levels. Whilst it is assumed that all investors have the same wealth, one might expect wealth accumulation effects and ask what consequences different wealth levels of investors between countries would have for the results (in particular those in Section 3.4).\textsuperscript{26} It turns out that the results concerning the equity and bond elements of proposed contracts hold even when investors have different wealth levels in the two countries, as long as the investor-wealth differences are compatible with any differences in investor population (e.g. the country with poorer investors does not have more investors, so this country can still be unambiguously referred to as the capital-scarce one). Moreover, the other conclusions concerning returns and capital attracted all go through when capital-scarcity and capital-abundancy of a country is characterised in terms of the (lesser, respectively greater) wealths of the resident investors rather than how many there are.

6 Related literature

This paper is motivated by the abundant evidence that ambiguity and ambiguity aversion may play a significant role in financial market decisions. First of all, a significant body of experimental evidence documents non-neutral attitudes to ambiguity (e.g. Camerer and Weber 1992; Trautmann et al. 2015). General findings include the preponderance of ambiguity aversion, especially with respect to events that are not very unlikely. Bossaerts et al. (2010) identify specific consequences of varying ambiguity attitude among investors for market variables—in particular concerning state price/probability ratios—and find, in an experimental asset market, such patterns of market variables. This attests both to the existence on non-trivial ambiguity attitude in such markets and to their importance for market factors.

\textsuperscript{26}Note that the in-depth consideration of cases where there are differing numbers of investors covers a certain type of wealth accumulation: namely accumulation resulting in changes in the investor population.
A crucial study for us is Dimmock et al. (2016), that uses survey data on a large sample of households to investigate the consequences of ambiguity “in the field”. By inserting a question eliciting ambiguity attitude into a standard household survey, they obtain information on household ambiguity attitudes, which they can compare to their investment behaviour. They find significant heterogeneity of ambiguity attitudes, with a predominance of ambiguity aversion, confirming laboratory findings. Moreover ambiguity aversion is negatively correlated to stock market participation (for the less-informed) and equity holdings. By contrast, they find no relationship between participation and their (similarly elicited) risk aversion measure. Importantly, the authors find direct evidence for a relationship between ambiguity attitude and the home bias: ambiguity aversion is negatively related to foreign stock ownership (and positively to home company ownership), and to underdiversification. As they note, this undermines any interpretation of their ambiguity aversion as a indication of risk aversion, for the relationship would have the opposite sign in that case. Again, these findings not only suggest the relevance of ambiguity attitude for understanding investment behaviour, but are entirely coherent with our model of investor portfolio choice. This general message is confirmed by another study (Dimmock et al., 2015), employing a finer analysis of ambiguity attitudes. Finally, they examined equity holders’ reaction to the 2008-9 crisis—which is associated with a sharp increase in ambiguity of future asset returns—showing that investors with higher ambiguity aversion were significantly more likely to actively sell equities during the crisis. Again, this is in sync with the interpretation of crisis episodes that we suggest in Section 3.4.1, and its consequences under our model.

The data and analysis provided in Section 4 is, to our knowledge, the first empirical study of the effect of ambiguity on (international) capital structure. The most related study is doubtless Korajczyk and Levy (2003) where one of their explanatory variables is the commercial paper spread (the annualized rate on three-month commercial paper over the rate on the three-month Treasury bill). This could arguably be another reasonable proxy for “model uncertainty” or ambiguity in the literature—it is positively correlated with high yield fixed income investments and correlated with the VIX (see below)\textsuperscript{27}. Their findings are coherent with ours: as commercial paper spread increases,

\textsuperscript{27}The correlation of this measure with the VIX over the period 1997-2016 for which data in Datastream is available weekly is 0.36 while it is 0.51 for 2008 alone.
so does leverage for unconstrained firms.

By contrast, the existing capital structure literature does discuss several channels through which risk could have an impact on debt / equity ratios, which are generally different from ours. The trade-off theory (see Frank and Goyal (2009)) implies that firms with more volatile cash flows have higher expected costs of default and consequently would issue less debt. Moreover, volatile cash flows lower the possibilities to use tax shields and are less desirable for risk-averse shareholders. However, an alternative view states that firms that have more volatile cash flows could be touched more by adverse selection. Then according to the pecking order theory, riskier stock-market firms could have higher leverage. In a comprehensive empirical study of Frank and Goyal (2009) find that higher stock variance is negatively related (in a statistically significant manner) with leverage most of the time for all measures of leverage. Graham et al. (2015) find a negative but non-robust relation between GDP growth volatility or earnings volatility and aggregate leverage for U.S. data. Our observations concerning our proxy for risk (Section 4) echo these findings.

As noted in the Introduction, the question of the composition of the contracts offered across countries has long been a subject of concern in international economics and finance. Empirical investigations include for example Claessens et al. (2003), Burger and Warnock (2006) or Kose et al. (2007). Alfaro and Kanczuk (2009, 2010) study the macroeconomic tradeoffs of issuing various type of (fixed versus floating, short vs. long term) debt or GDP-linked securities in calibration exercises after imposing a financial structure. Gertler and Rogoff (1990) provide a moral hazard explanation of some of the debt/equity composition facts considered here based on differences in entrepreneur wealth across countries under agency problems. Entrepreneurs in a ‘poor’ country would need to borrow more than their rich-country counterparts to obtain the efficient level of investment, and the moral hazard problem manifests itself more severely there. Their theory does not explain, however, why empirically observed increases in ambiguity could drive debt/equity composition or change the observed

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28 The pecking order theory, first formalized by Myers (1984), stresses the role of adverse selection. From the point of view of an outside investor, because of agency problem considerations equity is strictly riskier than debt. Still, the insiders (owners) prefer to finance themselves with retained earnings. Therefore, the influence of risk on debt / equity ratio is, according to this theory, unclear at best.

29 These authors also find no evidence of a relation between proxies for managerial incentives (that would direct attention to moral moral hazard explanations of capital structure) and aggregate leverage.
home bias. Other papers study different aspects of international contracting such as the preference for foreign currency-rather than local currency-denominated debt financing (Caballero and Krishnamurthy, 2003) or the maturity composition of debt issues (Broner et al., 2013; Jeanne, 2009). More generally, in a closed economy, if there is ambiguity aversion, then fixed-rate bonds would seem to be the best solution to insure uncertain investments. Mukerji and Tallon (2004) argue that ambiguity-averse agents may nevertheless prefer non-indexed bonds, claiming that with inflation-indexed bonds, investors are still not covered from the relative price risk in the bundles that they consume. An emerging literature studies the implications of the importance of demand for safe assets, based on special models that incorporate ambiguity averse agents (“locally Knightian agents” in Caballero and Farhi (2017)). However, as noted in Section 3.4 such demand for safe assets is at odds with the empirical evidence on high home bias of developing countries.

As suggested in the introduction, our model is related to the literature on the home bias. There, ambiguity has been suggested as an explanation for the home bias, and its consequences for asset pricing have been investigated. Of course, our focus is different, in that we are interested in the asset structure (which is typically assumed as exogenous), capital allocation among countries with unequal wealth and the risk-sharing and consumption volatility implied by the contracts. A related strand of literature on the home bias concentrates on informational frictions, concentrated primarily on the equity home bias. Gehrig (1993) first incorporates an assumption

30See Dow and da Costa Werlang (1992), Garlappi et al. (2007) and Boyle et al. (2012) that use the multiple priors approach of Gilboa and Schmeidler (1989) or Uppal and Wang (2003) with intertemporal portfolio choice when investors take into account model misspecification, in the style of Hansen and Sargent (2001, 2007). Benigno and Nistico (2012) use the latter approach to study the home bias puzzle under the assumption that investors have different beliefs about the characteristics of foreign and domestic assets. A general conclusion of this research is that “high” ambiguity leads to portfolio underdiversification. In addition, there is a sizable literature on asset pricing involving ambiguity models. Examples include Epstein and Wang (1994), Cao et al. (2011); see Epstein and Schneider (2010) for a survey. Solnik and Zuo (2012) study asset pricing using regret preferences.

31There is abundant empirical evidence in favor of informational frictions. Gelos and Wei (2005) show that, in times of crisis, funds invest less in and exit faster from countries that are less transparent. In Andrade and Chhaochharia (2010) past U.S. FDI positions in a country are found to be positively correlated with larger stock portfolio engagements at a later period. Bae et al. (2008) find that local analysts know more about local stocks than foreigners. Mondria et al. (2010) find using web browsing data that investors prefer assets from familiar countries. With an exogenous increase in information about a country, investors increase the asset holdings there. These results can be interpreted as resulting from differing ambiguity perception with respect to foreign or unfamiliar countries, rather than simple differences of information.
that investors from different countries have different information sets. He assumes that foreign securities are considered as more risky by expected utility maximizers and derives a home bias in equities. This line of modeling is pursued also by Brennan and Cao (1997) and Kang and Stulz (1997). Nieuwerburgh and Veldkamp (2009) discuss a model where potential investors can learn additional statistical properties of various assets at home and abroad by incurring some costs, showing that investors that have better information about locally-issued assets prefer to learn more about their home assets because they profit more from information that the others don’t know. Learning amplifies the initial information asymmetry. The asset structure in their model is exogenous. By contrast, as discussed in Sections 2.1 and 5, whilst our framework can encompass informational differences, they are not necessary to obtain our results. Under the leading interpretation of our model all investors have the same information about all assets; however, foreign investors may perceive more ambiguity than home investors towards a given asset. The distinction is important when one tries to understand periods of financial crises. One does not need increases in ambiguity aversion, transaction or information costs to generate “capital flight to safety”—the tendency of capital to move to low-risk capital markets during such events—a general increase in ambiguity about the stochastic properties of economic fundamentals suffices (see Section 3.4.1).

7 Conclusions

We have studied the implications of ambiguity and ambiguity aversion on the international allocation of capital, risk sharing and security design. Our conclusion is that with ambiguity aversion, risk sharing is impaired. Moreover, firms from capital-scarce countries offer more fixed-income securities relative to those from capital-rich ones. There are marked differences in the contract terms and the composition of capital allocation across the world, which depend on the initial distribution of capital. These differences have consequences for expected returns and capital allocation, and suggest a unified explanation for seemingly puzzling phenomena in international asset structure.

There are a number of extensions that we would wish to pursue in future research. An interesting avenue concerns the idea that investors may be more ambiguity averse towards events further in the future; hence there could be a natural tendency for
countries with relatively little domestic capital (owned by “savvy” investors) to issue short-term securities. Another direction would be to look in more detail at firm entry across countries: ambiguity and ambiguity aversion introduce a market segmentation that may have interesting implications on this front. A natural continuation would be to consider a calibration exercise where one would analyse a dynamic general equilibrium system that could match real-world data, which was clearly beyond the scope of this paper. Another possible extension of the model would be to incorporate exchange rate risk by, for example, allowing the bond part of the contract to be treated as risky by foreign investors. This might be able to explain why economies with few domestic investors issue many foreign-denominated bonds.

The problems created by ambiguity in the model could be addressed by producing more high quality and reliable data so that investors feel surer about the environment they invest in (which is the conclusion of empirical studies like Gelos and Wei (2005)). In our context, this calls for more transparency on the side of statistical agencies or central banks, especially in the capital-importing countries. Another solution to the problem of lack of risk sharing and underinvestment caused by ambiguity would be to encourage foreign direct investment and multinational companies: entrepreneurs from different countries could jointly issue contracts and self-insure within such an entity.

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References


Figure 1: Ratios of debt to equity of external liabilities among net capital importers (solid line) and capital exporters (dashed line). Data from the Lane and Milesi-Ferretti (2007) data set. Upper left panel: a sample of all 177 countries available for years 1999-2007. Upper right panel: a sample of 29 OECD members (as of end 1996). Lower left panel: 15 European Union members as of end of 1995. Lower right panel: 11 Euro zone members as of beginning of 1999.
Figure 2: Upper panels: The reaction of the interest rate to increases in ambiguity. Lower panels: The reaction of the debt/equity ratio to increases in ambiguity. Left panels: symmetric case, \(N_1 = N_2 = 100\). Right panels: one country with no wealth, \(N_1 = 200, N_2 = 0\). Middle panels: intermediate asymmetric case, \(N_1 = 120, N_2 = 80\). Solid line: country 1, dashed line: country 2. Base parameters: \(\mu = 10, \theta = 2, A = 1, \gamma = 1.5, \sigma = 1, \sigma = 1, M_1 = 500, M_2 = 500, w = 1\).
### Table I: Means of leverage ratios

This Table presents the medians, means and standard deviations of the firm-level *D*TA and *LT*D*TA* leverage measures for the Full sample and two subsamples with either only OECD countries or those with countries with Domestic credit / GDP in 2005 higher than the median in the Full sample. The difference in means is shown. *** indicates that the t-test of difference in means is statistically significant at a 1% level.

<table>
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<th>debt / total assets</th>
<th>long-term debt / total assets</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>median</td>
<td>mean</td>
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<tr>
<td><strong>Full sample (86 countries)</strong></td>
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|                            | **VIX × NFA<0**     |                                             |                               |                          |                          |             |                               |                                 |
|                            | 0.0007***           |                                             |                               |                          |                          |             |                               |                                 |
|                           | (2.90)              |                                             |                               |                          |                          |             |                               |                                 |
| **WVAR × NFA<0**          | -0.0206             |                                             |                               |                          |                          |             |                               |                                 |
|                           | (1.26)              |                                             |                               |                          |                          |             |                               |                                 |
| **CVAR × NFA<0**          | 0.0004              |                                             |                               |                          |                          |             |                               |                                 |
|                           | (0.33)              |                                             |                               |                          |                          |             |                               |                                 |
| **LTANGTOT × NFA<0**      | -0.1483***          |                                             |                               |                          |                          |             |                               |                                 |
|                           | (12.36)             |                                             |                               |                          |                          |             |                               |                                 |
| **LPROFITS × NFA<0**      | -0.0082*            |                                             |                               |                          |                          |             |                               |                                 |
|                           | (1.73)              |                                             |                               |                          |                          |             |                               |                                 |
| **INFLEXP × NFA<0**       | -0.0035             |                                             |                               |                          |                          |             |                               |                                 |
|                           | (1.28)              |                                             |                               |                          |                          |             |                               |                                 |

|                            | **VIX × D_NFA<0**   |                                             |                               |                          |                          |             |                               |                                 |
|                            | 0.0007***           |                                             |                               |                          |                          |             |                               |                                 |
|                           | (2.90)              |                                             |                               |                          |                          |             |                               |                                 |
| **WVAR × D_NFA<0**        | -0.0133             |                                             |                               |                          |                          |             |                               |                                 |
|                           | (0.85)              |                                             |                               |                          |                          |             |                               |                                 |
| **CVAR × D_NFA<0**        | -0.0029             |                                             |                               |                          |                          |             |                               |                                 |
|                           | (1.04)              |                                             |                               |                          |                          |             |                               |                                 |
| **LTANGTOT × D_NFA<0**    | -0.1083***          |                                             |                               |                          |                          |             |                               |                                 |
|                           | (17.11)             |                                             |                               |                          |                          |             |                               |                                 |
| **LPROFITS × D_NFA<0**    | -0.0071*            |                                             |                               |                          |                          |             |                               |                                 |
|                           | (1.81)              |                                             |                               |                          |                          |             |                               |                                 |
| **INFLEXP × D_NFA<0**     | -0.0047*            |                                             |                               |                          |                          |             |                               |                                 |
|                           | (1.76)              |                                             |                               |                          |                          |             |                               |                                 |

| Number of observations   | 1286118             | 1286118                                    | 1286118                       | 1286118                  | 1286118                   | 1169316     | 1085742                        | 1286118                         |
| Fixed effects            | firm                 | firm                                       | firm                          | firm                     | firm                       | firm        | firm                           | firm                            |
| Clustered std. errors    | firm and year        | firm and year                              | firm and year                 | firm and year            | firm and year              | firm and year| firm and year                  | country and year                |

**Table II: Behavior of Debt to Total Assets Ratio**
This table presents the estimates of regression equation:

\[ DT_A_{i,t} = \alpha_0 + \beta_1 VIX_t + \beta_2 WVAR_{c,t} + \beta_3 CVAR_{c,t} + \beta_4 L.TANGTOT_{i,t} + \beta_5 L.PROFITS_{i,t} + \beta_6 INFLEXP_{c,t} + \beta_7 \ln SDR_{c,t} + \delta_i + \epsilon_{i,t} \]

where subscripts \( i, c, t \) pertain to firm, country and year respectively; \( DT_A \) is the firm-level (loans + long-term debt) / total assets, \( VIX \) the average yearly VIX indicator, \( WVAR \) and \( CVAR \) world and country-level variances of GDP, \( L.TANGTOT \) is the one-year lagged tangible assets to total assets ratio, \( L.PROFITS \) the one-year lagged EBIT / total assets, \( INFLEXP \) are country-level inflation forecasts in percentage points, \( NFA \) is country-level net foreign asset / GDP ratio in 2005, \( \ln SDR \) is the logarithm of the country exchange rate to SDR and \( \delta \) is the firm-level fixed effect. Interactions of the above variables are with \( NFA \) and an indicator variable whether a country’s \( NFA < 0 \); \( \epsilon \) is the error term. T-statistics are provided in the parentheses. The regressions standard errors are double clustered at the firm and year level, with the exception of column 8 where they are double clustered at the country and year level. ∗∗∗, ∗∗, and ∗ denote statistical significance at the 1%, 5%, and 10% levels, respectively.
## Table III: Behavior of Long-Term Debt to Total Assets Ratio

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<th>OECD sample</th>
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Number of observations 1286118 1286118 1286118 1286118 1286118 1169316 1085742 1286118

Fixed effects firm firm firm firm firm firm firm firm

Clustered std. errors firm and year firm and year firm and year firm and year firm and year firm and year firm and year country and year

ctd.
This table presents the estimates of regression equation:

\[ LTDTA_{i,t} = \alpha_0 + \beta_1 VIX_t + \beta_2 WVAR_t + \beta_3 CVAR_t + \beta_4 L.TANGTOT_{i,t} + \beta_5 L.PROFITS_{i,t} + \beta_6 INFLEXP_{c,t} + \beta_7 \ln SDR_{c,t} + \delta_i + \epsilon_{i,t} \]

where subscripts \( i, c, t \) pertain to firm, country and year respectively; \( LTDTA \) is the firm-level long-term debt / total assets, \( VIX \) the average yearly VIX indicator, \( WVAR \) and \( CVAR \) world and country-level variances of GDP, \( L.TANGTOT \) is the one-year lagged tangible assets to total assets ratio, \( L.PROFITS \) is the one-year lagged EBIT / total assets, \( INFLEXP \) are country-level inflation forecasts in percentage points, \( NFA \) is country-level net foreign asset / GDP ratio in 2005, \( \ln SDR \) is the logarithm of the country exchange rate to SDR and \( \delta \) is the firm-level fixed effect. Interactions of the above variables are with \( NFA(\times NFAneg) \) and an indicator variable whether a countrys \( NFA \) was negative in 2005 \( (\times D.NFA < 0) \); \( \epsilon \) is the error term. T-statistics are provided in the parentheses. The regressions standard errors are double clustered at the firm and year level, with the exception of column 8 where they are double clustered at the country and year level. \(**, ***, \) and \(*\) denote statistical significance at the 1%, 5%, and 10% levels, respectively.
A For Online Publication: Technical Appendix

Detailed derivations of all results are available in an online appendix, downloadable at https://studies2.hec.fr/jahia/webdav/site/hec/shared/sites/hill/acces_anonyme/Articles/Hill_Michalski_040216_supplementary_material.pdf.

A.1 General expression of R

The general expression for the interest rate in (11) is

$$ R_i = \mu (1 - v_i) - w \theta \gamma_i $$

where

$$ \gamma_i \left( \sigma_1^2, \sigma_1^2, \sigma_2^2, \sigma_2^2, \gamma_i, v_i, v_j, N_1, N_2, M_1, M_2 \right) $$

$$ = \left( 2v_i^{-1} - 1 \right) \left( \frac{N_1}{\sigma_1^2 + \tau_1^2(1+\gamma)} + \frac{N_2}{\sigma_2^2 + \tau_2^2(1+\gamma)} \right) \left( \frac{\hat{X}_1 N_1 + \hat{Y}_1 N_2}{\sigma_2^2 + \tau_2^2(1+\gamma)} + \frac{\hat{X}_1 N_1}{\sigma_2^2 + \tau_2^2(1+\gamma)} \right) $$

and $\hat{X}_1 = \left( \frac{M_1}{v_i} + \frac{\sigma_1^2 + \tau_1^2}{\sigma_2^2 + \tau_2^2(1+\gamma)} \frac{M_2}{v_j} \right)^{-1}$, $\hat{Y}_1 = \left( \frac{M_1}{v_i} + \frac{M_2}{v_j} \frac{\sigma_1^2 + \tau_1^2(1+\gamma)}{\sigma_2^2 + \tau_2^2} \right)^{-1}$.

A.2 Proofs of Propositions

Calculations for Proposition 3 and Proposition 7 are in the online appendix.

**Proof of Proposition 1.** Taking respective derivatives from (10) one obtains $\frac{\partial \gamma_i}{\partial \gamma} > 0$, $\frac{\partial \gamma_i}{\partial \tau_i} < 0$ and $\frac{\partial \gamma_i}{\partial \gamma} < 0$.

**Proof of Proposition 2.** Immediate from taking the appropriate limits in (10).

**Proof of Proposition 4.** Immediate from (18) where $HB = 0$ if $\gamma = 0$ or $\tau^2 = 0$.

**Proof of Proposition 5.**
Proof of statement a). Rewrite the expressions for $v_1$ and $v_2$ as

$$v_1 = \frac{\eta^2 A \left[ \frac{1}{\eta^2} N_1 + \frac{1}{h^2} N_2 \right]}{2 + \eta^2 A \left[ \frac{1}{\eta^2} N_1 + \frac{1}{h^2} N_2 \right]}$$

$$v_2 = \frac{\zeta^2 A \left[ \frac{1}{\zeta^2} N_1 + \frac{1}{z^2} N_2 \right]}{2 + \zeta^2 A \left[ \frac{1}{\zeta^2} N_1 + \frac{1}{z^2} N_2 \right]}$$

where $\eta^2 = \sigma_1^2 + \tau_1^2, h^2 = \sigma_1^2 + \tau_1^2 (1 + \gamma), \zeta^2 = \sigma_2^2 + \tau_2^2, z^2 = \sigma_2^2 + \tau_2^2 (1 + \gamma)$.

Proof by contradiction. Suppose that $N_1 > N_2$ but $v_1 < v_2$. Then by a direct comparison of $v_1$ and $v_2$ we obtain

$$N_1 \left( 1 - \frac{\zeta^2}{z^2} \right) < N_2 \left( 1 - \frac{\eta^2}{h^2} \right)$$

and we assumed that $\eta^2 = \zeta^2, h^2 = z^2, \gamma > 0$ so a contradiction.

Proof of part b). We can rewrite

$$R_2 - R_1 = \mu \left( 1 - v_2 \right) - w\theta \Upsilon_2 - \mu \left( 1 - v_1 \right) + w\theta \Upsilon_1$$

$$= \mu \left( v_1 - v_2 \right) - w\theta \left( \Upsilon_2 - \Upsilon_1 \right) \quad (29)$$

Clearly $(v_1 - v_2) > 0$ by part a) of the Proposition but it turns out that it might be the case that $(\Upsilon_2 - \Upsilon_1) > 0$ so we need to impose conditions on $\mu$.

We rely on the assumption (see Section 2.1) that the constraint concerning how much capital an investor can invest is tight: that is, she would not want to invest some of her assets in a zero-yielding risk-free asset if given the chance. This means that, given the initial parameters, she would like to invest all wealth in risky (firm issued) assets. The assumption that we are in this “constrained” case implies that $\mu$ is high enough (or the number of firms $M_1$ and $M_2$ high enough); we now calculate the precise condition on $\mu$ implied.

The portfolio shares are given by (8) and (9), where $S_{11} = \frac{R_1 + \mu v_1}{\eta^2 \theta wv_1^2}$ and $S_{12} = \frac{R_2 + \mu v_2}{h^2 \theta wv_2^2}$. Let $W_1 = M_1 S_{11} + M_2 S_{12}$ and $W_2 = M_1 S_{21} + M_2 S_{22}$. It is clear from the derivation of (8) and (9) that investors in country 1 are in the constrained case when $W_1 \geq 1$, and similarly for investors in country 2 and $W_2 \geq 1$. This implies, for $W_1$:

$$M_1 \frac{R_1 + \mu v_1}{\eta^2 \theta wv_1^2} + M_2 \frac{R_2 + \mu v_2}{h^2 \theta wv_2^2} \geq 1$$

or

$$M_1 \frac{\mu - w\theta \Upsilon_1}{\eta^2 \theta wv_1^2} + M_2 \frac{\mu - w\theta \Upsilon_2}{h^2 \theta wv_2^2} \geq 1$$
which gives a condition on the minimum $\mu_{1,\text{min}}$ for investors from country 1 to be constrained:

$$\mu_{1,\text{min}} \geq \theta w \left(1 + \frac{M_1}{\eta^2 v_1^2} Y_1 + \frac{M_2}{h^2 v_2^2} Y_2 \right) \left(\frac{M_1}{\eta^2 v_1^2} + \frac{M_2}{h^2 v_2^2}\right).$$

For $W_2$, the minimum $\mu_{2,\text{min}}$ for investors of country 2 to be constrained is then $\mu_{2,\text{min}} \geq \theta w \left(1 + \frac{M_1}{\eta^2 v_1^2} Y_1 + \frac{M_2}{h^2 v_2^2} Y_2 \right) \left(\frac{M_1}{\eta^2 v_1^2} + \frac{M_2}{h^2 v_2^2}\right)$. Because both class of investors are assumed to be constrained, it suffices to take any of the minimum $\mu$'s to evaluate.

Returning to the evaluation of (29), the equation can be rewritten, assuming that $M_1 = \frac{M}{2}$ and $\eta^2 = \zeta^2$, $h^2 = \varepsilon^2$ (note that, under these assumptions, $\tilde{X}_1 = \left(\frac{M}{2}\right)^{-1} \left(\frac{1}{v_1^2} + \frac{\eta^2}{h^2 v_2^2}\right)^{-1}$, $\tilde{Y}_1 = \left(\frac{M}{2}\right)^{-1} \left(\frac{1}{v_1^2} + \frac{1}{v_2^2}\right)^{-1}$, $\tilde{X}_2 = \left(\frac{M}{2}\right)^{-1} \left[\frac{h^2}{\eta^2 v_1^2} + \frac{1}{v_2^2}\right]^2$, $\tilde{Y}_2 = \left(\frac{M}{2}\right)^{-1} \left(\frac{1}{v_1^2} + \frac{1}{v_2^2}\right)^{-1}$):

$$R_2 - R_1 = \mu (v_1 - v_2) - w \theta \left[ \left(\tilde{X}_2 N_1 + \tilde{Y}_2 N_2\right) \left[\frac{1}{\eta^2} N_1 + \frac{1}{\zeta^2} N_2\right] - \left(\tilde{X}_1 N_1 + \tilde{Y}_1 N_2\right) \left[\frac{1}{\zeta^2} N_1 + \frac{1}{\eta^2} N_2\right] \right]$$

$$+ \left[\frac{1}{\eta^2} N_1 + \frac{1}{\zeta^2} N_2\right] \left[\frac{1}{\zeta^2} \tilde{X}_1 N_1 + \frac{1}{\eta^2} \tilde{Y}_1 N_2\right] \frac{M_2}{v_2^2}$$

$$+ \left[\frac{1}{\eta^2} \tilde{X}_2 N_1 + \frac{1}{\zeta^2} \tilde{Y}_2 N_2\right] \left[\frac{1}{\eta^2} N_1 + \frac{1}{\zeta^2} N_2\right] \frac{M_2}{v_1^2}$$

We evaluate this expression with the minimum $\mu$ for investors from country 1 and, after substitutions we obtain that

$$\left(1 + \frac{\left(\frac{M}{2}\right)}{\eta^2 v_1^2} Y_1 + \frac{\left(\frac{M}{2}\right)}{h^2 v_2^2} Y_2 \right) (v_1 - v_2) - (\tilde{Y}_2 - \tilde{Y}_1) > 0$$

**Proof of Proposition 6.**

**For part a.** The difference in expected return on assets between countries 1 and 2 is given by $(\mu v_1 + R_1) - (\mu v_{1,2} + R_{1,2}) = w \theta (Y_2 - Y_1)$. 

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Then, from (11)

\[
(\Upsilon_2 - \Upsilon_1) = \left( (2v_2^{-1} - 1) \left( \hat{X}_2 N_1 + \hat{Y}_2 N_2 \right) \left[ \frac{1}{\sigma_1^2 + \tau_1^2(1+\gamma)} N_1 + \frac{1}{\sigma_1^2 + \tau_1^2(1+\gamma)} N_2 \right] \right. \\
- \left. (2v_1^{-1} - 1) \left( \hat{X}_1 N_1 + \hat{Y}_1 N_2 \right) \left[ \frac{1}{\sigma_2^2 + \tau_2^2(1+\gamma)} N_1 + \frac{1}{\sigma_2^2 + \tau_2^2(1+\gamma)} N_2 \right] \right)
\]

(30)

where \( \hat{X}_2 = \frac{\sigma_2^2 + \tau_2^2}{\sigma_2^2 + \tau_2^2(1+\gamma)} \hat{X}_1 \) and \( \hat{Y}_2 = \frac{\sigma_2^2 + \tau_2^2}{\sigma_2^2 + \tau_2^2(1+\gamma)} \hat{Y}_1 \).

The sign of \((\Upsilon_2 - \Upsilon_1)\) depends on the numerator as the denominator is always positive. We can transform the numerator and, assuming \(M_1 = M_2\), \(\sigma_1^2 = \sigma_2^2 = \sigma^2\), \(\tau_1^2 = \tau_2^2 = \tau^2\) and simplifying, obtain the following expression:

\[
\left( \frac{(2v_2 - v_2^2)}{v_2} \left( N_1 v_1^2 + N_2 v_2^2 + N_1 \frac{\sigma_2^2 + \tau_2^2}{\sigma_2^2 + \tau_2^2(1+\gamma)} v_2^2 + N_2 \frac{\sigma_2^2 + \tau_2^2(1+\gamma)}{\sigma_2^2 + \tau_2^2(1+\gamma)} v_1^2 \right) \left[ \frac{1}{\sigma_2^2 + \tau_2^2(1+\gamma)} N_1 + \frac{1}{\sigma_2^2 + \tau_2^2(1+\gamma)} N_2 \right] \right)
\]

(31)

Note that, when \(\gamma = 0\) or \(\tau^2 = 0\), \(\Upsilon_2 - \Upsilon_1 = 0\), so the sign of \((\Upsilon_2 - \Upsilon_1)\) around \(\gamma = 0\) (respectively, \(\tau^2 = 0\)) is indicated by the derivative of \(\Upsilon_2 - \Upsilon_1\) \(\frac{\partial (\Upsilon_2 - \Upsilon_1)}{\partial \gamma}\) and \(\frac{\partial (\Upsilon_2 - \Upsilon_1)}{\partial \tau^2}\) respectively. By the form of (30) and (31), these are derivatives of the form \(\frac{\partial}{\partial \gamma} \left( f(x) - g(x) \right) = \left[ \frac{f'(x) - g'(x)}{h'(x)} \right]\), where, when \(\gamma = 0\) or \(\tau^2 = 0\), \(f(x) = g(x)\). Hence, to sign these derivatives, it suffices to evaluate the derivative of (31).

Evaluating the derivative of the numerator (31) with respect to \(\gamma\), with \(\gamma = 0\), substituting for \(\frac{\partial v_1}{\partial \gamma} = \frac{2\hat{X}_2 N_2}{(2 + \hat{X}_2)(N_1 + N_2)} \frac{\sigma_2^2 + \tau_2^2}{\sigma_2^2 + \tau_2^2(1+\gamma)} \frac{\partial (v_2)}{\partial \gamma} = \frac{2\hat{X}_2 N_2}{(2 + \hat{X}_2)(N_1 + N_2)} \frac{\sigma_2^2 + \tau_2^2}{\sigma_2^2 + \tau_2^2(1+\gamma)}\), and noting that when \(\gamma = 0\) we have \(v_1 = v_2 = v\), we arrive at the condition

\[
\left( \frac{N_1 - N_2}{2(2 + \frac{4}{N_1 + N_2})^2} \right)^2 \left[ \frac{1}{\sigma_2^2 + \tau_2^2(1+\gamma)} \right]^2 > 0
\]

Since \(N_1 > N_2\), this is positive, so \(\Upsilon_2 - \Upsilon_1\) is negative close to \(\gamma = 0\), as required.

For \(\tau^2 \to 0\) we do the same; we have \(\frac{\partial v_1}{\partial \tau^2} = \frac{2\hat{X}_2 N_2}{(2 + \hat{X}_2)(N_1 + N_2)} \frac{\sigma_2^2 + \tau_2^2}{\sigma_2^2 + \tau_2^2(1+\gamma)} \frac{\partial v_2}{\partial \tau^2} = \frac{2\hat{X}_2 N_2}{(2 + \hat{X}_2)(N_1 + N_2)} \frac{\sigma_2^2 + \tau_2^2}{\sigma_2^2 + \tau_2^2(1+\gamma)}\). Taking the derivative of (31), setting \(\tau^2 = 0\), substituting for \(\frac{\partial v}{\partial \tau^2}\) and noting that when \(\tau^2 = 0\)
we have \( v_1 = v_2 = v \), we arrive at the condition

\[
\left( [N_1 - N_2] 4 \frac{2A}{(2 + \frac{A}{b} (N_2 + N_1))^2} \gamma (N_1 + N_2) [N_1 + N_2] \right) \frac{1}{(\sigma^2)^2} > 0
\]

Since \( N_1 > N_2 \), this is positive, so \( \Upsilon_2 - \Upsilon_1 \) is negative close to \( \tau^2 = 0 \), as required.

**Proof of part b).** At the limit as \( \gamma \to \infty \), financial autarky is reached and investors do not invest abroad. Then

\[
v_{1,\text{AUT}} = \frac{A}{b} \frac{N_1}{2 + \frac{A}{b} N_1}
\]

\[
R_{1,\text{AUT}} = \mu \frac{2}{2 + \frac{A}{b} N_1} - wA \frac{(\sigma^2 + \tau^2) (4 + \frac{A}{b} N_1) N_1}{M_1 (2 + \frac{A}{b} N_1)^2}
\]

\[
v_{2,\text{AUT}} = \frac{A}{b} \frac{N_2}{2 + \frac{A}{b} N_2}
\]

\[
R_{2,\text{AUT}} = \mu \frac{2}{2 + \frac{A}{b} N_2} - wA \frac{(\sigma^2 + \tau^2) (4 + \frac{A}{b} N_2) N_2}{M_2 (2 + \frac{A}{b} N_2)^2}
\]

If \( M_1 = M_2 = \frac{M}{2} \) then

\[
(\mu v_1 + R_1) - (\mu v_i,2 + R_i,2) = \frac{wA (\sigma^2 + \tau^2)}{M} \left( \frac{4 + \frac{A}{b} N_2}{(2 + \frac{A}{b} N_2)^2} \right) - \left( \frac{4 + \frac{A}{b} N_1}{(2 + \frac{A}{b} N_1)^2} \right)
\]

Since \( \left[ \frac{(4 + \frac{A}{b} N_2)}{(2 + \frac{A}{b} N_2)^2} - \frac{(4 + \frac{A}{b} N_1)}{(2 + \frac{A}{b} N_1)^2} \right] < 0 \) the expression is negative.

**Proof of Proposition 8.** Express \( k_2 \)

\[
k_2 = \left( 2 + \frac{A}{b} \frac{1 + \chi}{1 + \chi(1 + \gamma)} N_1 \right) (2 + \frac{A}{b} N_1)^{-1} \text{ where } \chi = \frac{\tau^2}{\sigma^2}.
\]

Then \( \frac{\partial}{\partial \chi} \left( \frac{k_2}{k_1} \right) < 0. \)

### A.3 Allowing for different assumptions on risk and ambiguity perception

Let us write the investor and firm problem in a more general way to show how one can accommodate different assumptions on the investor and firm risk and ambiguity perceptions.
A.3.1 Investor’s problem.

If the evaluation of $V_1 (\alpha_1)$ for an investor based in country 1 gives:

$$V_1 (\alpha_1) = -\frac{1}{\theta (1 + \gamma) (1 + \gamma)} e^{-\theta w (1 + \gamma)} \left( \sum_{i=1}^{M_1} \left( \alpha_{li} (\mu_i v_i + R_i) - \frac{s_1 \theta w}{2} [\alpha_{li} v_i^2] \right) + \sum_{j=M_1+1}^{M} \left( \alpha_{lj} (\mu_j v_j + R_j) - \frac{(s_2 + t_2) \theta w}{2} [\alpha_{lj} v_j^2] \right) \right)$$

where the parameters $s_1$ and $s_2 + t_2$ represent the risk and ambiguity that the investor perceives respectively towards home and foreign assets after the evaluation of the expected utility, the first order conditions are then of the sort

$$-\omega - \gamma e^\kappa (\mu_i v_i + R_i) - s_1 \theta w \alpha_{li} v_i^2 + \lambda = 0$$

for a representative domestic asset $i$ and

$$-\omega - \gamma e^\kappa ((\mu_j v_j + R_j) - (s_2 + t_2) \theta w \alpha_{lj} v_j^2) + \lambda = 0$$

for a representative foreign asset $j$ where

$$\kappa = -\theta w (1 + \gamma)$$

and

$$\sum_{k=1}^{M} \alpha_{lk} = 1$$

From these conditions, rearranging the first order conditions we can arrive at the formulation for the allocation into a home asset $s$ and a foreign asset $d$

$$\alpha_{ls} = \frac{R_s + \mu v_s}{s_1 \theta w v_i^2} + \left[ 1 - M_1 \frac{R_i + \mu v_i}{s_1 \theta w v_i^2} - M_2 \frac{R_j + \mu v_j}{(s_2 + t_2) \theta w v_j^2} \right] \left[ M_1 + M_2 \frac{s_1}{(s_2 + t_2)} \frac{v_i^2}{v_j^2} \right]^{-1}$$

$$\alpha_{ld} = \frac{R_d + \mu v_d}{(s_2 + t_2) \theta w v_j^2} + \left[ 1 - M_1 \frac{R_i + \mu v_i}{s_1 \theta w v_i^2} - M_2 \frac{R_j + \mu v_j}{(s_2 + t_2) \theta w v_j^2} \right] \left[ \frac{(s_2 + t_2)}{s_1} M_1 \frac{v_i^2}{v_j^2} + M_2 \right]^{-1}$$
A.3.2 Firm’s problem.

Suppose a firm \( s \) facing a choice of \( v_s \) and \( R_s \) wants to maximize its utility

\[
-\frac{1}{A} e^{-A \left[ \mu(1-v_s) - R_s \right] k_s - \frac{z_s A(1-v_s)^2 k_s^2}{2}} \tag{35}
\]

where \( z_s \) is its perceived risk of its own production process.

It is then straightforward to show by working out the first order conditions that for a representative firm in country 1, applying the notation from Section 3.1:

\[
v_1 = \frac{A}{\theta} z_1 \left( \frac{N_1}{s_1} + \frac{N_2}{s_1 + t_1} \right) \tag{36}
\]

while the interest rate can be written as before as a function

\[
R_1 = \mu (1 - v_i) - \theta w \sigma_{i,1} (\sigma_1^2, \tau_1^2, \sigma_2^2, \tau_2^2, \gamma, v_j, N_1, N_2, M_1, M_2) \tag{37}
\]

(not directly with \( z_1 \) or \( z_2 \)).

A.3.3 Different cases.

**Base formulation.** If \( s_1 = z_1 = \sigma_1^2 + \tau_1^2 \), \( t_1 = \tau_1^2 \gamma \), \( s_2 = z_2 = \sigma_2^2 + \tau_2^2 \) and \( t_2 = \tau_2^2 \gamma \) we are back to the base case analyzed in our paper and (32) is (5).

**Symmetric risk and asymmetric uncertainty.** Suppose that both home and foreign investors perceive a risk \( \xi_2^2 \) in firm \( n \)'s process while foreign investors perceive in addition ambiguity \( \zeta_2^2 \). Then one can derive (32) and (35) following exactly the same steps as described in Sections 2.1-2.2 and obtain a formulation with \( s_1 = z_1 = \xi_1^2 \), \( t_1 = \xi_1^2 (1 + \gamma) \), \( s_2 = z_2 = \xi_2^2 \) and \( t_2 = \xi_2^2 (1 + \gamma) \).

**Ambiguity with respect to home assets.** Suppose that both home and foreign investors perceive ambiguity towards all assets although the ambiguity perceived for foreign assets is higher than domestic ones. Suppose all investors (and the firm itself) perceive risk \( \sigma_n^2 \) in firm \( n \)'s production process, the home investors perceive ambiguity \( \tau_h^2 \) while foreign investors perceive ambiguity \( \tau_f^2 \). Then we can derive (32) and (35) following exactly the same steps as described in Sections 2.1-2.2 with \( z_1 = \sigma_1^2, s_1 = \sigma_1^2 + \tau_h^2 (1 + \gamma) \) and \( t_1 = \left( \tau_f^2 - \tau_h^2 \right) (1 + \gamma) \).
A.4 An Analysis using the Gilboa and Schmeidler (1989) model

A similar analysis could be conducted using the Gilboa and Schmeidler (1989) Maxmin Expected Utility ambiguity model, yielding similar results. In this model, decision makers do not have a single probability distribution $P$ for the relevant issues (in this case, the return of the uncertain asset), but a set of such distributions, $C$. Decision makers choose an asset $x$ to maximize:

$$V^{GS}(x) = \min_{P \in C} \mathbb{E}_P(u(x)) \quad (38)$$

Similarly to the interpretation of the smooth model given in Section 2.1, the set $C$ can be thought of as representing the decision maker’s uncertainty about the correct parameter values governing the stochastic process determining asset returns, or his model uncertainty. Unlike the smooth model, however, there is no (clear) separation of ambiguity and ambiguity attitude in this model, there being only one relevant “parameter” in the model, $C$.

To conduct the analysis, we adopt the following specification. As in the case studied in the main text, we assume that an investor considers that there are several possible distributions for the stochastic return $x_n$ ran by a firm $n$ in a foreign country, all of which follow a normal distribution. However, we assume that he is sure about the expected return $\mu_n$, but not about the variance $\sigma^2_n$. We assume that the set of variances he envisages are those which are within a distance $\delta_n$ from a particular “reference value” $\sigma_n$. So, the set $C$ is the set of distributions with $x_n \sim N(\mu_n, \sigma^2_n + \epsilon)$ with $\epsilon \in [-\delta_n, \delta_n]$. $\sigma^2_n$ can be thought of as the investor’s “best guess” for the variance; $\delta_n$, which parametrizes the “size” of the set $C$, can be thought of as representing the decision maker’s ambiguity aversion (or ambiguity; as noted above, there is no distinction in this model). We assume, as in the case studied above, that investors perceive no ambiguity (or, equivalently here, are not ambiguity averse towards) home assets: for a home asset $m$, $\delta_m = 0$ and the investor uses a single probability distribution. Moreover, we assume that home and foreign investors have the same $\sigma^2_n$ (as well as the same $\mu_n$): this is the equivalent of the assumption in Section 2.1 that they have the same reduced distribution. As was noted, this is consistent with all investors having the same information, but there being a difference in ambiguity (or ambiguity attitude) with respect to foreign assets. As above, we assume that each investor has constant absolute risk aversion, and hence a utility function of the form $u(z) = -(1/\theta)e^{-\theta z}$ where $\theta > 0$ represents the degree of (absolute) risk aversion. All investors are assumed to have the same risk aversion.

Using equation (3), the investor’s problem is to maximize:
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\[ V_{II}^{GS}(\alpha_1) = \min_{\epsilon_j \in [-\delta_j, \delta_j], M_1 + 1 \leq j \leq M} -(1/\theta)e^{-\theta w \left( \sum_{i=1}^{M_1} (\alpha_{i}(v_i \mu_i + R_i) + (\theta w \sigma_i^2/2)v_i^2 \alpha_{i}^2) + \sum_{j=M_1+1}^{M} (\alpha_{ij}(v_j \mu_j + R_j) + (\theta w (\sigma_j^2 + \epsilon_j)/2)v_j^2 \alpha_{ij}^2) \right)} \]  

(39)

Solving for the portfolio allocation for an investor in country 1, under the other assumptions made and notation used in Section 3.1, gives:

\[ \alpha_{II}^{GS} = S_{II}^{GS} + \frac{1 - M_1 S_{II}^{GS} - M_2 S_{I2}^{GS}}{M_1 + M_2} \left( \frac{\sigma_i^2}{\sigma_j^2 + \delta_{12}} \right) \]  

(40)

\[ \alpha_{I2}^{GS} = S_{I2}^{GS} + \frac{1 - M_1 S_{I1}^{GS} - M_2 S_{I2}^{GS}}{M_1 + M_2} \left( \frac{\sigma_j^2}{\sigma_i^2 + \delta_{12}} \right) \]  

(41)

where \( S_{II}^{GS} = \frac{R_i + \mu v}{\sigma_i^2 \theta w v_i^2} \) and \( S_{I2}^{GS} = \frac{R_i + \mu v}{\sigma_j^2 \theta w v_j^2} \).

Noting that \( S_{II}^{GS} \) and \( S_{I2}^{GS} \) are equal to \( S_{11} \) and \( S_{12} \) (Section 3.1) once one replaces \( \sigma_i^2 + \tau_i^2 \) by \( \sigma_i^2 \) and \( \tau_i^2 \gamma \) by \( \delta_1 \), it is clear that the solutions and results in the previous sections translate immediately into results for the maxmin expected utility model. In particular, the behavior of parameters with changes in risk (\( \sigma^2 \)) is the same in the smooth and the maxmin expected utility models; and changes in \( \delta \) have precisely the same effects as changes in \( \gamma \). There is no natural equivalent of the behavior obtained from changes in \( \tau^2 \) in the maxmin expected utility model; this is to be expected, given that the distinction between ambiguity and ambiguity aversion is absent in this model. That there are interesting effects of changes in \( \tau^2 \) (see for example Sections 3.3 and 3.4.1) may indicate the economic utility of the analysis using the smooth ambiguity model, which does incorporate such a distinction.

A.5 Data Appendix

For data on capital structure at the firm level we use the ORBIS data set, provided by Bureau van Dijk that gathers various accounting information for firms worldwide and streamlines and standardizes it to one format. Data is available for the period 2006-2015. We retain firms for which detailed capital structure (total assets, tangible assets, current and non-current liabilities, loans, long-term debt) and profits data is available for the entire 10 years. To avoid outliers or errors, we drop firms with even one observation with negative total
assets\textsuperscript{32}, tangible assets, current liabilities, loans or long-term debt. We keep only firms that have tangible assets / total assets < 1, maximal debt/total assets < 10, maximal long-term debt/total assets < 10 and maximal current liabilities / total assets < 10 to avoid extreme cases. We remove the (few) countries for which no information about net foreign assets are available and the NACE Rev. 2 sectors related to finance, real estate and predominantly dominated by public services (like hospitals and education), households and extraterritorial organizations. We exclude thus NACE Rev. 2 sectors such as 64-68, 84-88, 91 and 97-99.

Given the findings of Covas and Haan (2011) that the largest firms 1% are predominantly responsible for the movements in the aggregate leverage we trim the data, dropping the top 1% of observations in terms of maximal total assets within the data span ($\text{TOT}_{\text{ASST}}\text{MAX}$). We also drop the lowest 1% in terms of minimum total assets to minimize the effect of noise that small firms might have on our results ($\text{TOT}_{\text{ASST}}\text{MIN}$). For the Full sample, those are firms below 1,000USD in terms of total assets. The characteristics of the variables in the resulting sample are shown in Table IV.

Our variables are obtained and constructed in the following way, as is standardly done in the capital structure literature. $\text{LTDTA}$ is defined as long-term debt / total assets while $\text{DTA} = \text{loans} / \text{total assets} + \text{LTDTA}$. “Long-term debt” in the ORBIS data set refers to liabilities with a maturity longer than a year while loans are classified as a part of current liabilities (maturity below 1 year). $\text{TANGTOT}$ is the tangible assets to total assets ratio while $\text{PROFITS}$ are EBIT / total assets. From the available World Economic Outlook historical databases for 1990-2016 we construct world and country-level variance of GDP (variables $\text{WVAR}$ and $\text{CVAR}$ respectively, calculated from the WEO variable “ngdp rpch”) and obtain country-level inflation forecasts in percentage points ($\text{INFLEXP}$, from the variable “pcpi pch”). These variables are calculated from what would be known to a decision maker at the beginning of each calendar year: that is, the variance with a lag of the previous year and inflation expectation from the previous year for the year to come. Given that the wide WEO historical data set starts with data from 1988, we have for the first sample year of 2006 at most 17 years of data to calculate the variance. To control for changes in variance due to the size of the data set, we carry a constant window for each country. This has also a benefit that the most recent and hence relevant information about the variance of the GDP process for each country is kept\textsuperscript{33}. We obtain daily VIX from Datastream and average it over each year. The net foreign asset / GDP ratios (variable $\text{NFA}$) required to determine the capital

\textsuperscript{32}For total assets, we drop also firms that indicate a zero value of total assets.

\textsuperscript{33}We re-estimated our base specification taking the variances calculated over all available data (i.e., without the moving window) or taking inflation expectations for the current year; neither of these produce qualitative changes in our estimates.
exporting and importing countries before our sample starts in 2006 are calculated from the Lane and Milesi-Ferretti (2007) data. SDR exchange rates are obtained from the IMF and the Taiwanese central bank.

A.6 Additional tables
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<td>1.335346</td>
<td>0.133429</td>
<td>1.08472</td>
<td>1.463971</td>
</tr>
<tr>
<td>CVAR</td>
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<td>2.32501</td>
<td>1.826597</td>
<td>0.550712</td>
<td>32.47319</td>
</tr>
<tr>
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<td>0.257173</td>
<td>0.262109</td>
<td>0</td>
<td>0.999963</td>
</tr>
<tr>
<td>L.PROFITS</td>
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<td>0.688624</td>
<td>-297</td>
<td>234.75</td>
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<td>1.071093</td>
<td>-3.52736</td>
<td>43.5</td>
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<tr>
<td>NFA</td>
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<td>0.318276</td>
<td>-1.83156</td>
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<tr>
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<td>0.463458</td>
<td>0.498663</td>
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<tr>
<td>VIX × NFAneg</td>
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<td>6.84401</td>
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<tr>
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<td>1.224118</td>
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<td>0.866608</td>
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<td>11.17029</td>
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<td>0.672055</td>
<td>0</td>
<td>1.463971</td>
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<td>2.262058</td>
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<tr>
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<td>0.252373</td>
<td>0</td>
<td>0.999963</td>
</tr>
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<td>0.023939</td>
<td>0.44517</td>
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<td>96.21429</td>
</tr>
<tr>
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<td>1.473783</td>
<td>-3.52736</td>
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<td>420566.4</td>
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<tr>
<td>TOT_ASST_MIN</td>
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<td>30225.78</td>
<td>188211.3</td>
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<td>4623562</td>
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Table IV: Summary statistics.

This Table presents the summary statistics of the Full sample used. DTA is the firm-level (loans + long-term debt) / total assets, LTDTA the firm-level long-term debt / total assets, VIX the average yearly VIX indicator, WVAR and CVAR world and country-level variances of GDP, L.TANGTOT is the one-year lagged tangible assets to total assets ratio, L.PROFITS is the one-year lagged EBIT / total assets, INFLEXP are country-level inflation forecasts in percentage points, NFA is country-level net foreign asset / GDP ratio in 2005, lnSDR is the logarithm of the country exchange rate to SDR, TOT_ASST_MAX is the firm-level maximum value of total assets between 2006-2016 and TOT_ASST_MIN is the firm-level minimum value of total assets between 2006-2016. Interactions of the above variables with NFA (× NFAneg) and an indicator variable whether a countrys NFA was negative in 2005 (× D_NFA<0) are also shown.
A.7 Simulations with the Truncated Normal distribution

A.7.1 General setup

We adopt the setup set out in Section 2.1, with the sole difference that, where in Section 2.1 it is assumed that investors are sure that returns follow a normal distribution with given variance, we now assume that they are sure that the returns follow a truncated normal, truncated at 0. That is, the distributions considered possible are of the form

\[ P(x) = \begin{cases} 
\frac{1}{\sqrt{2\pi}\sigma^2} 1 - \Phi\left(-\frac{m}{\sigma}\right) e^{-\frac{(x-m)^2}{2\sigma^2}} & \text{if } x \geq 0 \\
0 & \text{if } x < 0 
\end{cases} \]  

(42)

where \( \Phi() \) is the cdf of the normal distribution. We write \( x \sim \text{TrunNorm}(m, \sigma, 0, \text{inf}) \) to express that \( x \) is distributed according to the truncated normal with \( m \) and \( \sigma \). Like in the paper, the \( \sigma^2 \) is the same across all such distributions, whereas \( m \) may vary. The second-order prior is a normal distribution with mean \( \mu \) and variance \( \tau^2 \).

As shown below, there is no closed-form solution under these assumptions. Therefore we conducted numerical simulations in MATLAB with the derivations below serving as a basis. The precision for the convergence of contract terms \( v \) and \( R \) was set at \( 1 \times e^{-6} \).
A.7.2 Expected utility with a truncated normal

Evaluating expected utility with one asset with the productivity process \( x \sim \text{TrunNorm}(m, \sigma, 0, \inf) \)

\[
\begin{align*}
\int_0^\infty & e^{-\theta w_{\alpha_i}(v_i x_i + R_i)} \frac{1}{\left(1 - \frac{1}{2}(1 + erf\left(-\frac{m_i}{\sigma_i \sqrt{2}}\right))\right) \sqrt{2\pi \sigma_i^2}} e^{\frac{(x_i - m_i)^2}{2\sigma_i^2}} dx_i \\
= & \left(\frac{1}{2} \left(1 - erf\left(-\frac{m_i}{\sigma_i \sqrt{2}}\right)\right)\right) \sqrt{2\pi \sigma_i^2} e^{-\theta w_{\alpha_i} R_i} \int_0^\infty e^{-\theta w_{\alpha_i} v_i x_i} e^{-\frac{x_i^2 + m_i^2 - 2x_i m_i}{2\sigma_i^2}} dx_i \\
= & \left(\frac{2}{erfc\left(-\frac{m_i}{\sigma_i \sqrt{2}}\right)}\right) \sqrt{2\pi \sigma_i^2} e^{-\theta w_{\alpha_i} R_i} e^{\frac{m_i^2 - 2m_i x_i}{2\sigma_i^2}} \int_0^\infty \frac{2}{erfc\left(-\frac{m_i - \sigma_i^2 \theta w_{\alpha_i} v_i}{\sigma_i \sqrt{2}}\right)} \sqrt{2\pi \sigma_i^2} e^{-\frac{x_i^2}{2\sigma_i^2}} dx_i \\
= & \frac{erfc\left(-\frac{m_i - \sigma_i^2 \theta w_{\alpha_i} v_i}{\sigma_i \sqrt{2}}\right)}{erfc\left(-\frac{m_i}{\sigma_i \sqrt{2}}\right)} e^{-\theta w_{\alpha_i} [R_i + m_i v_i] - \frac{\sigma_i^2}{2} \theta w_{\alpha_i} (v_i)^2}
\end{align*}
\]

A.7.3 Investing in home assets

As discussed in Section 2.1, we assume that total uncertainty—comprising risk and ambiguity—is symmetric across investors, but that investors do not perceive ambiguity with respect to home assets. It follows that they perceive all uncertainty concerning home assets as risk. As discussed in Section 2.1, the uncertainty concerning a firm (and hence the risk perceived by home investors) is characterised by the reduced distribution over returns; since, in the case considered here, the first order distributions are truncated normal, this reduced distribution needs to be explicitly re-calculated.
Given the definition of truncated normals, its density function is 0 for \( x < 0 \); otherwise, it is given by:

\[
\frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{\sqrt{2\pi\tau^2}} \int_{-\infty}^{\infty} \frac{1}{1 - \Phi(-\frac{m}{\sigma})} e^{-\left[\frac{(x-m)^2}{2\sigma^2} + \frac{(m-\mu)^2}{2\tau^2}\right]} \, dm
\]

\[
= \frac{1}{\sqrt{2\pi(\sigma^2 + \tau^2)}} \int_{-\infty}^{\infty} \frac{1}{1 - \Phi(-\frac{m}{\sigma})} e^{-\left[\frac{1}{2\sigma^2} + \frac{1}{2\tau^2}\right] \left(\frac{m-\mu}{\sigma^2 + \frac{1}{\tau^2}}\right)^2} \, dm
\]

\[
= \frac{1}{\sqrt{2\pi(\sigma^2 + \tau^2)}} e^{-\left[\frac{(x-\mu)^2}{2\sigma^2 + 2\tau^2}\right]} \int_{-\infty}^{\infty} \frac{1}{1 - \Phi(-\frac{m}{\sigma})} e^{-\left[\frac{1}{2\sigma^2} + \frac{1}{2\tau^2}\right] \left(\frac{m-\mu}{\sigma^2 + \frac{1}{\tau^2}}\right)^2} \, dm
\]

\[
= \frac{1}{\sqrt{2\pi(\sigma^2 + \tau^2)}} \Upsilon(x) e^{-\frac{(x-\mu)^2}{2\sigma^2 + 2\tau^2}}
\]

where \( \Upsilon(x) = \int_{-\infty}^{x} \frac{1}{1 - \Phi(-\frac{m}{\sigma})} e^{-\left[\frac{1}{2\sigma^2} + \frac{1}{2\tau^2}\right] \left(\frac{m-\mu}{\sigma^2 + \frac{1}{\tau^2}}\right)^2} \, dm \). Let \( \Psi(y) = \frac{1}{\sqrt{2\pi(\sigma^2 + \tau^2)}} \int_{0}^{\infty} e^{-\frac{(y-\mu)^2}{2\sigma^2 + 2\tau^2}} \Psi(x) \, dx \). Note that, by the fact that the truncated normals are normalised, \( \Psi(\mu) = 1 \), but this is not necessarily the case for \( \Psi(y) \), when \( y \neq \mu \).

To summarise, the distribution will be 0 for \( x < 0 \) and otherwise:

\[
\frac{1}{\sqrt{2\pi(\sigma^2 + \tau^2)}} \Upsilon(x) e^{-\frac{(x-\mu)^2}{2\sigma^2 + 2\tau^2}}
\]
Then, expected utility for the investment in one home asset is:

$$
-\frac{1}{\theta} \int_{0}^{\infty} e^{-\theta w \alpha_{i} (v_{i} x_{i} + R_{i})} \frac{1}{\sqrt{2\pi (\sigma_{i}^{2} + \tau_{i}^{2})}} e^{-\frac{(x_{i} - \mu_{i})^{2}}{2(\sigma_{i}^{2} + \tau_{i}^{2})}} Y(x) dx_{i}
$$

$$
= -\frac{1}{\theta} \int_{0}^{\infty} e^{-\theta w \alpha_{i} v_{i} x_{i}} e^{-\frac{x_{i}^{2} + \mu_{i}^{2} - 2x_{i} \mu_{i}}{2(\sigma_{i}^{2} + \tau_{i}^{2})}} Y(x) dx_{i}
$$

$$
= -\frac{1}{\theta} \int_{0}^{\infty} e^{-\theta w \alpha_{i} R_{i}} e^{-\frac{(\mu_{i} - (\sigma_{i}^{2} + \tau_{i}^{2}) \theta w \alpha_{i} v_{i})^{2} - \mu_{i}^{2}}{2(\sigma_{i}^{2} + \tau_{i}^{2})}} Y(x) dx_{i}
$$

$$
= -\frac{1}{\theta} \psi(\mu_{i} - (\sigma_{i}^{2} + \tau_{i}^{2}) \theta w \alpha_{i} v_{i}) e^{-\theta w \left(\alpha_{i} [R_{i} + \mu_{i} v_{i}] - \frac{\sigma_{i}^{2} + \tau_{i}^{2}}{2} \theta w (\alpha_{i} v_{i})^{2}\right)}
$$

### A.7.4 Expected utility for the agent

Putting this together with the calculation of expected utility from investing in assets with a production process that follows a truncated normal, we get (when there are $M_1$ home firms and $M_2$ foreign ones) as expected utility:

$$
-\frac{1}{\theta} e^{-\theta w \sum_{i=1}^{M_1} \left(\alpha_{i} [R_{i} + \mu_{i} v_{i}] - \frac{\sigma_{i}^{2} + \tau_{i}^{2}}{2} \theta w (\alpha_{i} v_{i})^{2}\right)} \prod_{i=1}^{M_1} \psi(\mu_{i} - (\sigma_{i}^{2} + \tau_{i}^{2}) \theta w \alpha_{i} v_{i}) e^{-\theta w \sum_{j=1}^{M_2} \left(\alpha_{j} [R_{j} + m_{j} v_{j}] - \frac{\sigma_{j}^{2} + \tau_{j}^{2}}{2} \theta w (\alpha_{j} v_{j})^{2}\right)} \prod_{j=1}^{M_2} erfc\left(-\frac{m_{j} - \sigma_{j}^{2} \theta w \alpha_{j} v_{j}}{\sigma_{j} \sqrt{2}}\right)
$$

$$
\prod_{j=1}^{M_2} erfc\left(-\frac{m_{j}}{\sigma_{j} \sqrt{2}}\right)
$$
Risk versus ambiguity and international security design

Going to the second level, we have

\[ V(\alpha) = \int_{-\infty}^{\infty} e^{-\alpha(\gamma + 1)} \prod_{i=1}^{M_1} \left( \frac{\Psi(\mu_i - (\sigma_i^2 + \tau_i^2) \sigma_i) e^{-\frac{\sigma_i^2 \theta_i}{2}}}{\sqrt{\pi} \beta_i} \right) \prod_{j=1}^{M_2} \left( \frac{\Psi(\mu_j - (\sigma_j^2 + \tau_j^2) \sigma_j) e^{-\frac{\sigma_j^2 \theta_j}{2}}}{\sqrt{\pi} \beta_j} \right) \]

We evaluate numerically the following expression to find the maximum of the investor’s problem:

\[ V(\alpha) = \int_{-\infty}^{\infty} e^{-\alpha(\gamma + 1)} \prod_{i=1}^{M_1} \left( \frac{\Psi(\mu_i - (\sigma_i^2 + \tau_i^2) \sigma_i) e^{-\frac{\sigma_i^2 \theta_i}{2}}}{\sqrt{\pi} \beta_i} \right) \prod_{j=1}^{M_2} \left( \frac{\Psi(\mu_j - (\sigma_j^2 + \tau_j^2) \sigma_j) e^{-\frac{\sigma_j^2 \theta_j}{2}}}{\sqrt{\pi} \beta_j} \right) \]

\[ = \int_{-\infty}^{\infty} e^{-\alpha(\gamma + 1)} \prod_{i=1}^{M_1} \left( \frac{\Psi(\mu_i - (\sigma_i^2 + \tau_i^2) \sigma_i) e^{-\frac{\sigma_i^2 \theta_i}{2}}}{\sqrt{\pi} \beta_i} \right) \prod_{j=1}^{M_2} \left( \frac{\Psi(\mu_j - (\sigma_j^2 + \tau_j^2) \sigma_j) e^{-\frac{\sigma_j^2 \theta_j}{2}}}{\sqrt{\pi} \beta_j} \right) \]

\[ = \int_{-\infty}^{\infty} e^{-\alpha(\gamma + 1)} \prod_{i=1}^{M_1} \left( \frac{\Psi(\mu_i - (\sigma_i^2 + \tau_i^2) \sigma_i) e^{-\frac{\sigma_i^2 \theta_i}{2}}}{\sqrt{\pi} \beta_i} \right) \prod_{j=1}^{M_2} \left( \frac{\Psi(\mu_j - (\sigma_j^2 + \tau_j^2) \sigma_j) e^{-\frac{\sigma_j^2 \theta_j}{2}}}{\sqrt{\pi} \beta_j} \right) \]

\[ = \int_{-\infty}^{\infty} e^{-\alpha(\gamma + 1)} \prod_{i=1}^{M_1} \left( \frac{\Psi(\mu_i - (\sigma_i^2 + \tau_i^2) \sigma_i) e^{-\frac{\sigma_i^2 \theta_i}{2}}}{\sqrt{\pi} \beta_i} \right) \prod_{j=1}^{M_2} \left( \frac{\Psi(\mu_j - (\sigma_j^2 + \tau_j^2) \sigma_j) e^{-\frac{\sigma_j^2 \theta_j}{2}}}{\sqrt{\pi} \beta_j} \right) \]
A.7.5 The firm's problem

As in Section 2.2, firms perceive the same risk as home investors. So, for a typical firm, it will use the truncated normal distribution with a pdf

\[ \frac{1}{\sqrt{2\pi(\sigma_i^2 + \tau_i^2)}} \mathcal{Y}(x)e^{-\frac{(x-x_i)^2}{2(\sigma_i^2 + \tau_i^2)}} \] as a distribution too. This would give for the firm problem:

\[
- \frac{1}{A} \int_0^\infty e^{-A(x_i(1-v_i)-R_i)k_i} \frac{1}{\sqrt{2\pi(\sigma_i^2 + \tau_i^2)}} e^{-\frac{(x-x_i)^2}{2(\sigma_i^2 + \tau_i^2)}} \mathcal{Y}(x)dx_i
\]

\[
= - \frac{1}{A\sqrt{2\pi(\sigma_i^2 + \tau_i^2)}} e^{AR_i k_i} \int_0^\infty e^{-A(1-v_i)k_ix_i} e^{-\frac{x_i^2+\mu_i^2-2x_i\mu_i}{2(\sigma_i^2 + \tau_i^2)}} \mathcal{Y}(x)dx_i
\]

\[
= - \frac{1}{A\sqrt{2\pi(\sigma_i^2 + \tau_i^2)}} e^{AR_i k_i} \int_0^\infty e^{-\frac{(\mu_i-(\sigma_i^2 + \tau_i^2)(1-v_i)A_k_i)^2-\mu_i^2}{2(\sigma_i^2 + \tau_i^2)}} e^{-\frac{x_i^2+\mu_i^2-2x_i\mu_i}{2(\sigma_i^2 + \tau_i^2)}} \mathcal{Y}(x)dx_i
\]

\[
= - \frac{1}{A\sqrt{2\pi(\sigma_i^2 + \tau_i^2)}} e^{AR_i k_i} e^{-(1-v_i)A_k_i \mu_i} \frac{(\sigma_i^2 + \tau_i^2)(1-v_i)^2 A^2 k_i^2}{2} \int_0^\infty \mathcal{Y}(x)e^{-\frac{x_i^2+\mu_i^2-2x_i\mu_i-(\sigma_i^2 + \tau_i^2)(1-v_i)A_k_i\mu_i}{2(\sigma_i^2 + \tau_i^2)}} dx_i
\]

\[
= - \frac{1}{A} \Psi(\mu_i-(\sigma_i^2 + \tau_i^2)(1-v_i)A_k_i)e^{-(1-v_i)R_iA_k_i} \frac{(\sigma_i^2 + \tau_i^2)(1-v_i)^2 A^2 k_i^2}{2}
\]
A.7.6 Simulation examples

Figure 3: Symmetric countries, effects of changes in $\sigma^2$. Parameters for the simulation: $N_1 = N_2 = 10$, $M_1 = M_2 = 50$, $\gamma = 1.5$, $\tau^2 = 1$, $A = 1$, $\theta = 2$, $\mu = 10$, $w = 1$. 
Figure 4: Symmetric countries, effects of changes in $\gamma$. Parameters for the simulation: $N_1 = N_2 = 10$, $M_1 = M_2 = 50$, $\sigma^2 = 1$, $\tau^2 = 1$, $A = 1$, $\theta = 2$, $\mu = 10$, $w = 1$. 
Figure 5: Symmetric countries, effects of changes in $\tau^2$. Parameters for the simulation: $N_1 = N_2 = 10$, $M_1 = M_2 = 50$, $\gamma = 1.5$, $\sigma^2 = 1$, $A = 1$, $\theta = 2$, $\mu = 10$, $w = 1$. 
Figure 6: Asymmetric countries, effects of changes in $\sigma^2$. Parameters for the simulation: $N_1 = 20$, $N_2 = 0$, $M_1 = M_2 = 50$, $\gamma = 1.5$, $\tau_1^2 = \tau_2^2 = 1$, $A = 1$, $\theta = 2$, $\mu = 10$, $w = 1$. 
Figure 7: Asymmetric countries, effects of changes in $\gamma$. Parameters for the simulation: $N_1 = 20$, $N_2 = 0$, $M_1 = M_2 = 50$, $\sigma^2 = 1$, $\tau_1^2 = \tau_2^2 = 1$, $A = 1$, $\theta = 2$, $\mu = 10$, $w = 1$. 
Figure 8: Asymmetric countries, effects of changes in $\tau^2 = \tau_1^2 = \tau_2^2$. Parameters for the simulation: $N_1 = 20$, $N_2 = 0$, $M_1 = M_2 = 50$, $\gamma = 1.5$, $\sigma^2 = 1$, $\hat{A} = 1$, $\theta = 2$, $\mu = 10$, $w = 1$. 
A.8 Allowing for a risk-free asset

Let there be a risk free asset yielding $z$ risk-free and let the portfolio share of wealth invested there be $\alpha_{l0}$. The investor problem (5) becomes

$$
\max V = -\frac{1}{\theta(1+\gamma)(1+\gamma)} e^{-(1+\gamma)\theta w} \times \\
\left[ -e^{-\theta w \sum_i \left( \alpha_i [v_i \mu_i + R_i] - \frac{\theta w v_i^2 \sigma_i^2}{2} \right)} \right] \\
- (1+\gamma) e^{-\theta w \sum_j \left[ \alpha_j [v_j \mu_j + R_j] - \frac{\theta w v_j^2 \sigma_j^2 + \tau_j^2}{2} \right]} \\
+ \lambda \left( \sum \alpha - 1 \right)
$$

The first order conditions yield after reworking the portfolio shares

$$
\alpha_i = \frac{(v_i \mu_i + R_i) - z}{\theta w v_i^2 \left( \sigma_i^2 + \tau_i^2 \right)}
$$

$$
\alpha_{lj} = \frac{(v_j \mu_j + R_j) - z}{\theta w v_j^2 \left( \sigma_j^2 + \tau_j^2 \left(1+\gamma\right) \right)}
$$

The returns offered by firms now have to be high enough so that investors allocate positive shares $\alpha$ of their wealth (the risk free return $z$ is the threshold) while

$$
\alpha_{l0} = 1 - \left( \sum \frac{(v_i \mu_i + R_i) - z}{\theta w v_i^2 \left( \sigma_i^2 + \tau_i^2 \right)} + \sum \frac{(v_j \mu_j + R_j) - z}{\theta w v_j^2 \left( \sigma_j^2 + \tau_j^2 \left(1+\gamma\right) \right)} \right)
$$

There are multiple cases that arise. If $z$ is too high relatively to what is offered investors would like to short the risky assets to go long into the risk-free assets. If $\alpha_{l0} < 0$ then investors wish to borrow the risk-free asset to invest into risky firm assets. If $\alpha_{l0}, \alpha_{li}$ and $\alpha_{lj}$ are positive, then it is as if the investors are unconstrained (i.e. have too much wealth) and do not wish to invest all their wealth in risky assets. We shall confine our analysis to the latter.
Capital attracted by a representative firm $s$ in country 1 issuing a contract $(v_s, R_s)$ is now

$$k_s = \left( N_1 \alpha_1 w + N_2 \alpha_2 w \right)$$
$$= \left( N_1 w \frac{(v_s \mu + R_s) - z}{\theta w v_s^2 (\sigma_i^2 + \tau_i^2)} + N_2 w \frac{(v_s \mu + R_s) - z}{\theta w v_s^2 (\sigma_i^2 + \tau_i^2 (1 + \gamma))} \right)$$
$$= \left( \frac{(v_s \mu + R_s) - z}{\theta v_s^2} \right) \left( \frac{N_1}{\sigma_i^2 + \tau_i^2} + \frac{N_2}{\sigma_i^2 + \tau_i^2 (1 + \gamma)} \right)$$
$$= \left( \frac{(v_s \mu + R_s) - z}{\theta v_s^2} \right) \Gamma$$

where $\Gamma = \left( \frac{N_1}{(\sigma_i^2 + \tau_i^2)} + \frac{N_2}{\sigma_i^2 + \tau_i^2 (1 + \gamma)} \right)$. Then

$$\frac{\partial k_s}{\partial R_s} = \frac{1}{v_s^2} \frac{\Gamma}{\theta}$$
$$\frac{\partial k_s}{\partial v_s} = \left[ \frac{v_s \mu + 2R_s - 2z}{v_s} \right] \frac{\partial k_s}{\partial R_s}$$

$$k_s = \left( \frac{(v_s \mu + R_s) - z}{\theta v_s^2} \right) \Gamma = \frac{\partial k_s}{\partial R_s}$$

Substituting into the firm’s problem FOCs

$$\frac{\partial k_s}{\partial R_s} \mu - \frac{\partial k_s}{\partial v_s} - (\sigma_i^2 + \tau_i^2) A\frac{\partial k_s}{\partial R_s} [1 - v_s] k_s = 0 \quad (44)$$
$$\frac{\partial k_s}{\partial v_s} [1 - v_s] - \frac{\partial k_s}{\partial R_s} R_s - k_s = 0, \quad (45)$$

reworking, assuming $v_s \mu + R_s - z \neq 0 \forall v_s \neq 0$ and applying notation $v_1 = v_i$ and $R_1 = R_i$ for any firm $i$ in country 1 given the symmetry in the risk and ambiguity of firms in the same country, we obtain

$$v_1 = \frac{\frac{\mu}{\sigma} \left( N_1 + \frac{(\sigma^2 + \tau_i^2)}{(\sigma_i^2 + \tau_i^2 (1 + \gamma))} N_2 \right)}{2 + \frac{\mu}{\sigma} \left( N_1 + \frac{(\sigma^2 + \tau_i^2)}{(\sigma_i^2 + \tau_i^2 (1 + \gamma))} N_2 \right)}$$

the same expression as equation (10).

For $R_1$,

$$R_1 = z - \frac{v_1 (z + \mu)}{2} > \frac{z - \mu}{2}$$

while expected return

$$v_1 \mu + R_1 = \frac{v_1 \mu + (2 - v_1) z}{2}$$
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and debt/equity ratio

\[ \frac{R_1}{v_1 \mu} = \frac{z}{v_1 \mu} - \frac{(z + \mu)}{2\mu}. \]

For investment in the risky assets to be positive it has to be the case that \( v_1 \mu + R_1 - z > 0 \) or \( \mu > z \).

**Implications for the results derived for the base model.** These results mean that all comparative statics of \( v_1 \) with respect to \( \sigma^2_1, \tau^2_1 \) and \( \gamma \) are as in the base model. The comparative statics on \( R_1 \) and the debt/equity ratio \( \frac{R_1}{v_1 \mu} \) are now clear and move exactly in the opposite direction than those of \( v_1 \). Hence, we can state a stronger and extended version of Proposition 1 which overrides Propositions 3 and 7:

Suppose \( N_1 > 0, N_2 > 0, \gamma > 0 \) and \( \tau^2_1 > 0 \). Then \( \frac{\partial v_1}{\partial \tau^2_1} < 0, \frac{\partial v_1}{\partial \gamma} < 0, \frac{\partial v_1}{\partial \sigma^2_1} > 0, \frac{\partial R_1}{\partial \tau^2_1} > 0, \frac{\partial R_1}{\partial \gamma} < 0, \frac{\partial R_1}{\partial \sigma^2_1} > 0, \frac{\partial R_1}{\partial \gamma} < 0, \frac{\partial R_1}{\partial \sigma^2_1} < 0, \frac{\partial}{\partial \tau^2_1} \left( \frac{R_1}{v_1 \mu} \right) > 0, \frac{\partial}{\partial \gamma} \left( \frac{R_1}{v_1 \mu} \right) > 0, \frac{\partial}{\partial \sigma^2_1} \left( \frac{R_1}{v_1 \mu} \right) < 0. \)

Proposition 2, 4, 5, 8 and all Corollaries follow while Proposition 6 is changed as the expected return is always higher in the capital-abundant country.

**Discussion.** The undesirable feature of this version of the model is that investors with such utility are willing to invest a positive fixed amount in any additional risky asset that is available as risky assets are viewed as valuable. There is therefore no competition between firms issuing assets for attracting capital which leads to the simplicity of the results (in particular, the term \( w \theta \Upsilon \) is absent in the evaluation of \( R \)). Another observation regarding this formulation is the following. If free entry of firms were allowed, and it was profitable for at least one firm to enter, firms would have an incentive to enter and raise capital so as to exhaust the wealth of investors, so that there would be no more positive investment into the risk-free security (and investors could actually short the risk-free security...). Since incorporating and analyzing firm entry and the supply and the return of the risk-free security etc. is unfortunately not tractable, we prefer to analyze our base model in the paper with the assumption that \( \alpha_{l0} = 0 \) and \( \sum \alpha_{li} + \sum \alpha_{lj} = 1. \)