Confidence in Beliefs and Rational Decision Making

Brian Hill

hill@hec.fr

www.hec.fr/hill

GREGHEC, CNRS & HEC Paris

2016
(Presentation prepared for an interdisciplinary audience)

Flooding or drought defence (2016)

	Floods	Droughts
Flood Defence	100	0
Drought Defence	0	100

Flooding or drought defence (2016)

	Floods	Droughts
Flood Defence	100	0
Drought Defence	0	100

Flooding or drought defence (1946)

	Floods	Droughts
Flood Defence	100	0
Drought Defence	0	100

Flooding or drought defence (2016)

	Floods	Droughts
Flood Defence	100	0
Drought Defence	0	100

Flooding or drought defence (1946)

	Floods	Droughts
Flood Defence	100	0
Drought Defence	0	100

- Which decision would you prefer to take?

Ellsberg (1961)

Red or Black

- Unknown urn: 100 balls, each red or black.
- Known urn: 100 balls, 50 red, 50 black.

	Unknown urn		Known urn	
	Red	Black	Red	Black
I	\$ 100	\$ 0		
II	\$ 0	\$ 100		
III			\$ 100	\$ 0
IV			\$ 0	\$ 100

Ambiguity

- Ellsberg behaviour is inconsistent with **Bayesianism.**

What is it? Three tenets:

Belief Beliefs can be represented by **probabilities.**

Decision Decision Makers maximise **expected utility.**
Subjective Expected Utility (SEU)

Belief Formation Update by conditionalisation.

Expected utility

$$\mathbb{E}_p(u(f)) = \sum_{s \in S} u(f(s)).p(s)$$

- u: utility function, representing desires or tastes
- p: probability measure, representing beliefs

What does it do? Purportedly:

Descriptive it accurately captures peoples' choice behaviour.

Economic Modelling it is a tractable approximate model of peoples' choice behaviour.

Normative it is the canon of rationality for belief and decision making.

Prescriptive it provides a tractable framework for taking complex decisions (properly).

Why? Purportedly:

1. it corresponds to a reasonable (pre-theoretical) intuition
2. it has acceptable choice-theoretical consequences
3. it is conceptually clear about the roles of different mental attitudes
4. it has a certain tractability for eg. decision analysis

Why? Purportedly:

1. it corresponds to a reasonable (pre-theoretical) intuition
2. it has acceptable choice-theoretical consequences
3. it is conceptually clear about the roles of different mental attitudes
4. it has a certain tractability for eg. decision analysis

- Dutch Book Arguments, Representation Theorems

i.e. results of the following form:

Properties of preferences / choice	$\Leftrightarrow$	$\exists\ p, u$ s.t. choice maximises $\mathbb{E}_p(u(f))$

And the p, u are appropriately unique.

Why? Purportedly:

1. it corresponds to a reasonable (pre-theoretical) intuition
2. it has acceptable choice-theoretical consequences
3. it is conceptually clear about the roles of different mental attitudes
4. it has a certain tractability for eg. decision analysis

- u: utility function, representing desires or tastes
- p: probability measure, representing beliefs

Ellsberg (1961)
Red or Black

- Unknown urn: 100 balls, each red or black.
- Known urn: 100 balls, 50 red, 50 black.

	Unknown urn		Known urn	
	Red	Black	Red	Black
I	\$ 100	\$ 0		
II	\$ 0	\$ 100		
III			\$ 100	\$ 0
IV			\$ 0	\$ 100

Which would you prefer (or would you be indifferent) between:

- *I* and *III*?
- *I* and *II*?
- *III* and *IV*?

Ellsberg (1961)

Red or Black

- Unknown urn: 100 balls, each red or black.
- Known urn: 100 balls, 50 red, 50 black.

	Unknown urn		Known urn	
	Red	Black	Red	Black
I	\$ 100	\$ 0		
II	\$ 0	\$ 100		
III			\$ 100	\$ 0
IV			\$ 0	\$ 100

Which would you prefer (or would you be indifferent) between:

- *I* and *III*? $\rightsquigarrow$ $p(Red_K) > p(Red_U)$
- *I* and *II*?
- *III* and *IV*?

Ellsberg (1961)
Red or Black

- Unknown urn: 100 balls, each red or black.
- Known urn: 100 balls, 50 red, 50 black.

	Unknown urn		Known urn	
	Red	Black	Red	Black
I	\$ 100	\$ 0		
II	\$ 0	\$ 100		
III			\$ 100	\$ 0
IV			\$ 0	\$ 100

Which would you prefer (or would you be indifferent) between:

- *I* and *III*? $\rightsquigarrow$ $p(Red_K) > p(Red_U)$
- *I* and *II*? $\rightsquigarrow$ $p(Red_K) = p(Black_K) = 0.5$
- *III* and *IV*? $\rightsquigarrow$ $p(Red_U) = p(Black_U) = 0.5$

Severe Uncertainty

- Ellsberg behaviour is inconsistent with Bayesianism

Severe Uncertainty

- Ellsberg behaviour is inconsistent with Bayesianism
- Much work on finding models that:
  - are descriptively accurate and / or
  - are sufficiently tractable for use in economic models

Severe Uncertainty

- Ellsberg behaviour is inconsistent with Bayesianism
- Much work on finding models that:
  - are descriptively accurate and / or
  - are sufficiently tractable for use in economic models

How about models that help us decide in these difficult decisions?

Aim

To formulate and defend:

- Normatively valid models to give guidance for rational decision making under uncertainty.

Proposal in a nutshell

The concept . . .

> *to express the proper state of belief, not* one *number but* two *are requisite, the first depending on the inferred probability, the second on the amount of knowledge on which that probability is based. (Peirce, 1878, p179)*

> *there may be another respect in which some kind of quantitative comparison between arguments is possible. This comparison turns upon a balance, not between the favourable and the unfavourable evidence, but between the* absolute *amounts of relevant knowledge and of relevant ignorance respectively. (Keynes, 1921, p71)*

Proposal in a nutshell

The concept . . .

> *to express the proper state of belief, not* one *number but* two *are requisite, the first depending on the inferred probability, the second on the amount of knowledge on which that probability is based. (Peirce, 1878, p179)*

Beyond the degree to which one endorses a particular proposition

. . . there is the degree to which one is confident in this endorsement.

Proposal in a nutshell

The concept . . .

> *to express the proper state of belief, not* one *number but* two *are requisite, the first depending on the inferred probability, the second on the amount of knowledge on which that probability is based. (Peirce, 1878, p179)*

Beyond the degree to which one endorses a particular proposition

. . . there is the degree to which one is confident in this endorsement.

If the former is one's beliefs, the latter is one's

confidence in one's beliefs

Confidence and Decision

There appear to be many significant decisions where confidence in beliefs do, and should, play a role.

The action which follows upon an opinion depends as much upon the amount of confidence in that opinion as it does upon the favorableness of the opinion itself. (Knight, 1921, p226-227)

Confidence and Decision

There appear to be many significant decisions where confidence in beliefs do, and should, play a role.

The action which follows upon an opinion depends as much upon the amount of confidence in that opinion as it does upon the favorableness of the opinion itself. (Knight, 1921, p226-227)

that course of action is preferable which involves least risk, and about the results of which we have the most complete knowledge. . . . the coefficients of weight and risk as well as that of probability are relevant to our conclusion [as to the preferable course of action]. (Keynes, 1921, p315)

Confidence and Decision

There appear to be many significant decisions where confidence in beliefs do, and should, play a role.

The action which follows upon an opinion depends as much upon the amount of confidence in that opinion as it does upon the favorableness of the opinion itself. (Knight, 1921, p226-227)

that course of action is preferable which involves least risk, and about the results of which we have the most complete knowledge. . . . the coefficients of weight and risk as well as that of probability are relevant to our conclusion [as to the preferable course of action]. . . . the only difficulty in this being the lack of any principle for the calculation of the degree of their influence. (Keynes, 1921, p315)

Confidence and Decision

There appear to be many significant decisions where confidence in beliefs do, and should, play a role.

The action which follows upon an opinion depends as much upon the amount of confidence in that opinion as it does upon the favorableness of the opinion itself. (Knight, 1921, p226-227)

But what role?

Proposal in a nutshell

. . . and the intuition

- would we like decisions about climate change policy to be taken on the basis of “best hunch” estimates?

Proposal in a nutshell

. . . and the intuition

- would we like decisions about climate change policy to be taken on the basis of “best hunch” estimates?
- and what about wagers between us?

Proposal in a nutshell

. . . and the intuition

Maxim

The higher the stakes involved in a decision, the more confidence is needed in a belief for it to play a role in the decision.

Proposal in a nutshell

. . . and the intuition

Maxim

The higher the stakes involved in a decision, the more confidence is needed in a belief for it to play a role in the decision.

Aim

To formulate and defend:

- Normatively valid models to give guidance for rational decision making under uncertainty.

Desiderata

The criteria evoked in favour of Bayesianism:

1. it corresponds to a reasonable (pre-theoretical) intuition
2. it has acceptable choice-theoretical consequences
3. it is conceptually clear about the roles of different mental attitudes
4. it has a certain tractability for eg. decision analysis

Desiderata

The criteria evoked in favour of Bayesianism:

1. it corresponds to a reasonable (pre-theoretical) intuition
2. it has acceptable choice-theoretical consequences
3. it is conceptually clear about the roles of different mental attitudes
4. it has a certain tractability for eg. decision analysis

And of course:

5. it can fruitfully deal with “severe uncertainty” situations such as those above.

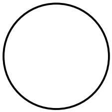
Plan

1. develop a theory based on these claims
   - model of confidence in beliefs
   - family of decision rules incorporating confidence
2. defence and consequences of the theory:
   - conceptual and choice-theoretic properties
   - consequences and applications

Modelling confidence

Idea (First attempt)

- Represent beliefs by a set of probability measures (à la Levi (1986); Gilboa and Schmeidler (1989); Walley (1991); Bewley (2002) and . . .).

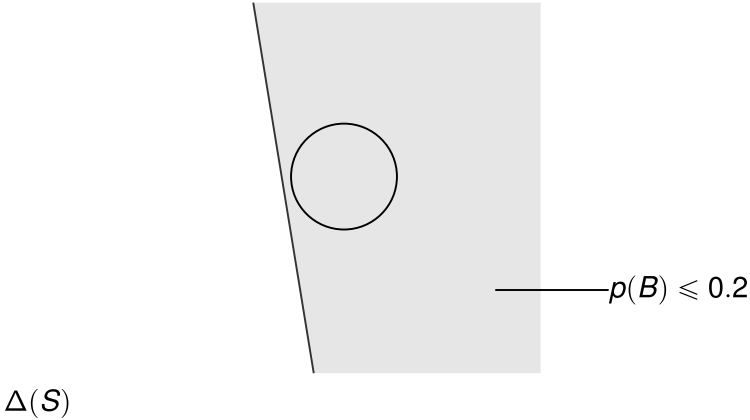


$\Delta(S)$

Modelling confidence

Interpretation

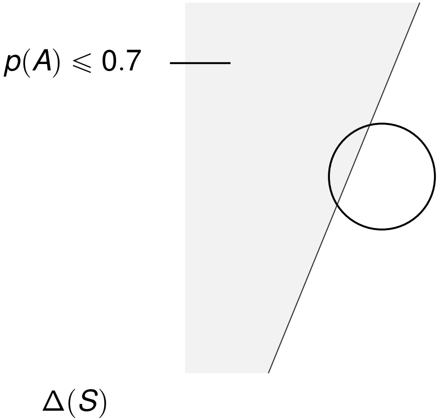
- confident that the probability of B is less than 0.2



Modelling confidence

Interpretation

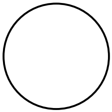
- unsure whether the probability of A is less than 0.7



Modelling confidence

Problem

- confidence is represented as “binary”.
- in reality, it comes in degrees.

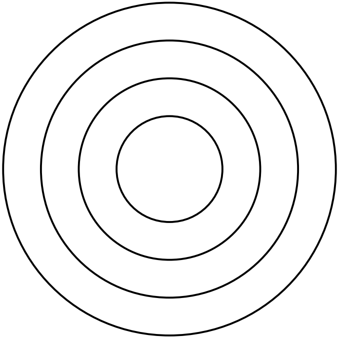


$\Delta(S)$

Modelling confidence

Idea

- Represent beliefs by a **nested family of sets** of measures

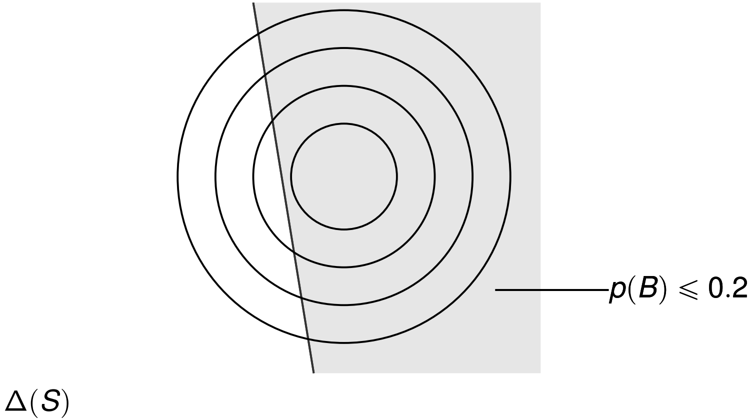


$\Delta(S)$

Modelling confidence

Idea

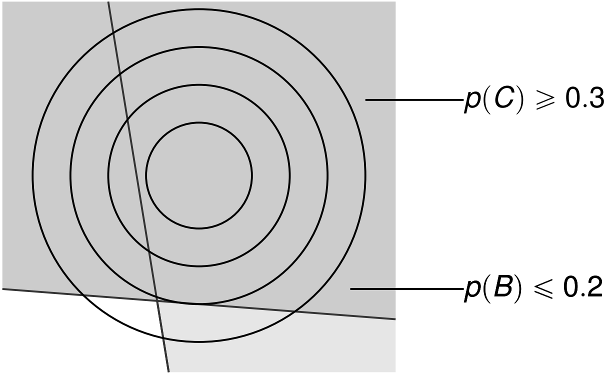
- Represent beliefs by a **nested family of sets** of measures



Modelling confidence

Interpretation

- **more confident** in $p(C) \geqslant 0.3$ than $p(B) \leqslant 0.2$



$\Delta(S)$

Modelling confidence

Definition

A **confidence ranking** Ξ is a nested family of closed subsets of $\Delta(S)$.

It is **convex** if every $C \in \Xi$ is convex.
It is **continuous** if, for every $C \in \Xi$, $C = \overline{\bigcup_{C' \subsetneq C} C'} = \bigcap_{C' \supsetneq C} C'$.
It is **centered** if it contains a singleton set.

Modelling confidence

Definition

A **confidence ranking** Ξ is a nested family of closed subsets of $\Delta(S)$.

It is **convex** if every $C \in \Xi$ is convex.
It is **continuous** if, for every $C \in \Xi$, $C = \overline{\bigcup_{C' \subsetneq C} C'} = \bigcap_{C' \supsetneq C} C'$.
It is **centered** if it contains a singleton set.

C.f.

- Belief revision, conditionals, non-monotonic logics [though over $\Delta(S)$] ...
- Gärdenfors and Sahlin (1982) ...

The role of confidence in choice

Maxim

The higher the stakes involved in a decision, the more confidence is needed in a belief for it to play a role in the decision.

The role of confidence in choice

Maxim

The higher the stakes involved in a decision, the more confidence is needed in a belief for it to play a role in the decision.

Decision

f

The role of confidence in choice

Maxim

The higher the stakes involved in a decision, the more confidence is needed in a belief for it to play a role in the decision.

Decision | Confidence level

$$f \longrightarrow \mathcal{C}_f \in \Xi$$

The role of confidence in choice

Maxim

The higher the stakes involved in a decision, the more confidence is needed in a belief for it to play a role in the decision.

Decision	Confidence level

$$f \xrightarrow{\quad D \quad} D(f) \in \Xi$$

A **cautiousness coefficient** for a confidence ranking Ξ is a surjective function $D : \mathcal{A} \to \Xi$

The role of confidence in choice

Maxim

The higher the stakes involved in a decision, the more confidence is needed in a belief for it to play a role in the decision.

Decision | Confidence level

$$f \xrightarrow{D} D(f) \in \Xi$$

A **cautiousness coefficient** for a confidence ranking Ξ is a surjective function $D : \mathcal{A} \to \Xi$

- the higher the stakes, the larger $D(f)$

A family of decision models

For each model in the family:

Ingredients:

- utility function u
- confidence ranking Ξ
- cautiousness coefficient D

General form:

preferences concerning f are a function of

$u(f(s))$ and $D(f)$

according to decision rule I

Confidence and choice

Examples

Decision rules using a (closed) set of probabilities $\mathcal{C}$:

- unanimity rule

  $f \preceq g$ iff $\mathbb{E}_p(u(f)) \leqslant \mathbb{E}_p(u(g))$ for all $p \in \mathcal{C}$

- maxmin expected utility

  $f \preceq g$ iff $\min_{p \in \mathcal{C}} \mathbb{E}_p(u(f)) \leqslant \min_{p \in \mathcal{C}} \mathbb{E}_p(u(g))$

  $\rightarrow$ Belief functions / convex capacities

- Hurwicz or α-maxmin rule

  $\alpha \min_{p \in \mathcal{C}} \mathbb{E}_p(u(f)) + (1 - \alpha) \max_{p \in \mathcal{C}} \mathbb{E}_p(u(f))$

- E-admissibility
- etc.

On stakes

Plan

1. develop a theory based on these claims
   - model of confidence in beliefs
   - family of decision rules incorporating confidence
2. defence and consequences of the theory:
   - conceptual and choice-theoretic properties
   - consequences and applications

Desiderata

1. it corresponds to a reasonable (pre-theoretical) intuition
2. it has acceptable choice-theoretical consequences
3. it is conceptually clear about the roles of different mental attitudes
4. it has a certain tractability for eg. decision analysis

And of course:

5. it can fruitfully deal with “severe uncertainty” situations such as those above.

Desiderata

1. it corresponds to a reasonable (pre-theoretical) intuition
2. it has acceptable choice-theoretical consequences
3. it is conceptually clear about the roles of different mental attitudes
4. it has a certain tractability for eg. decision analysis

And of course:

5. it can fruitfully deal with “severe uncertainty” situations such as those above.

Careful Preferences Incomplete Preferences

Conceptual properties

Careful preferences

- “Maxmin EU” decision rule

Conceptual properties

Careful preferences

Choose to maximise:

$$\min_{p \in D(f)} \mathbb{E}_p(u(f))$$

Conceptual properties

Careful preferences

Choose to maximise:

$$\min_{p \in D(f)} \mathbb{E}_p(u(f))$$

Under such a rule:

- for higher stakes, one is effectively only relying on beliefs in which one has sufficient confidence.
- behaviour is as “pessimistic” as one’s confidence: the more confident in appropriate beliefs or the lower the stakes, the less pessimistic.

Conceptual properties

Careful preferences

Choose to maximise:

$$\min_{p \in D(f)} \mathbb{E}_p(u(f))$$

Conclusion This yields the following advice:

Choose boldly if one has sufficient confidence; choose cautiously if not.

Conceptual properties

Careful preferences

Choose to maximise:

$$\min_{p \in D(f)} \mathbb{E}_p(u(f))$$

Conclusion This yields the following advice:

Choose boldly if one has sufficient confidence; choose cautiously if not.

Comparison Few “non-EU” rules correspond so closely to plausible maxims of this sort.

Comparison This rule is not as extreme as maxmin EU.

Ellsberg Can accommodate Ellsberg behaviour in the same way as the “standard maxmin EU rule”.

Conceptual properties

Incomplete preferences

- "Unanimity" decision rule

Conceptual properties

Incomplete preferences

$f \preceq g$ if and only if:

$$\mathbb{E}_p(u(f)) \leqslant \mathbb{E}_p(u(g)) \qquad \text{for all } p \in D((f,g))$$

Conceptual properties

Incomplete preferences

$f \preceq g$ if and only if:

$$\mathbb{E}_p(u(f)) \leqslant \mathbb{E}_p(u(g)) \qquad \text{for all } p \in D((f, g))$$

Under such a rule:

- choices made at low stakes may be suspended (but not reversed) at higher stakes.
- for higher stakes, one is decisive only if one is confident enough in appropriate beliefs.

Conceptual properties

Incomplete preferences

$f \preceq g$ if and only if:

$$\mathbb{E}_p(u(f)) \leqslant \mathbb{E}_p(u(g)) \qquad \text{for all } p \in D((f,g))$$

Under such a rule:

- choices made at low stakes may be suspended (but not reversed) at higher stakes.
- for higher stakes, one is decisive only if one is confident enough in appropriate beliefs.

One Interpretation of no preference between f and g:

- Deferral

Conceptual properties

Incomplete preferences

$f \preceq g$ if and only if:

$$\mathbb{E}_p(u(f)) \leqslant \mathbb{E}_p(u(g)) \qquad \text{for all } p \in D((f,g))$$

Conclusion This yields the following advice:

Defer when one's confidence in relevant beliefs is insufficient to match the importance of the decision.

Conceptual properties

Incomplete preferences

$f \preceq g$ if and only if:

$$\mathbb{E}_p(u(f)) \leqslant \mathbb{E}_p(u(g)) \qquad \text{for all } p \in D((f, g))$$

Conclusion This yields the following advice:

Defer when one's confidence in relevant beliefs is insufficient to match the importance of the decision.

Comparison Few "incomplete preference" rules defended by invoking plausible maxims of this sort.

Comparison It is not as extreme as the unanimity rule.

Ellsberg Can accommodate Ellsberg behaviour in the same way as the "standard unanimity rule".

Desiderata

✓ it corresponds to a reasonable (pre-theoretical) intuition
2. it has acceptable choice-theoretical consequences
3. it is conceptually clear about the roles of different mental attitudes
4. it has a certain tractability for eg. decision analysis

And of course:

✓ it can fruitfully deal with “severe uncertainty” situations such as those above.

Desiderata

✓ it corresponds to a reasonable (pre-theoretical) intuition

2. it has acceptable choice-theoretical consequences
3. it is conceptually clear about the roles of different mental attitudes
4. it has a certain tractability for eg. decision analysis

And of course:

✓ it can fruitfully deal with “severe uncertainty” situations such as those above.

▸ Dutch Books ▸ Representation Theorems

▸ Neither in detail

A typical framework

S	non-empty finite set of states
$\Delta(S)$	set of probability measures on S
C	set of consequences
$\mathcal{A}$	set of acts (functions $S \to C$)
$\preceq$	preference relation on $\mathcal{A}$

A typical framework

S	non-empty finite set of states
$\Delta(S)$	set of probability measures on S
C	set of consequences [Convex subset of a vector space]
$\mathcal{A}$	set of acts (functions $S \to C$)
$\preceq$	preference relation on $\mathcal{A}$

Notation:

- $f_\alpha g$: shorthand for $\alpha f + (1-\alpha)g$.

A typical framework

S	non-empty finite set of states
$\Delta(S)$	set of probability measures on S
C	set of consequences [Convex subset of a vector space]
$\mathcal{A}$	set of acts (functions $S \to C$)
$\preceq$	preference relation on $\mathcal{A}$

Notation:

- $f_\alpha g$: shorthand for $\alpha f + (1 - \alpha)g$.

Special case (Anscombe-Aumann framework)

$C =$ set of lotteries (probability distributions with finite support) over a set X of prizes.

Careful preferences

Axioms

Expected utility (Anscombe and Aumann):

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0,1)$:

Non triviality and weak order $\preceq$ is non-trivial, reflexive, transitive and complete.

Independence $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$.

Continuity $\{\alpha \in [0,1] \mid f_\alpha h \preceq g\}$ and $\{\alpha \in [0,1] \mid f_\alpha h \succeq g\}$ are closed.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Careful preferences

Axioms

Expected utility (Anscombe and Aumann):

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0, 1)$:

Non triviality and weak order $\preceq$ is non-trivial, reflexive, transitive and complete.

Independence $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$.

Continuity $\{\alpha \in [0,1] | \ f_\alpha h \preceq g\}$ and $\{\alpha \in [0,1] | \ f_\alpha h \succeq g\}$ are closed.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Careful preferences

Axioms

Standard maxmin EU model (Gilboa-Schmeidler):

For all $f, g, h \in \mathcal{A}$, $c \in C$, $\alpha \in (0,1)$:

Non triviality and weak order $\preceq$ is non-trivial, reflexive, transitive and complete.

C-Independence $f \preceq g$ iff $f_\alpha c \preceq g_\alpha c$.

Continuity $\{\alpha \in [0,1] | f_\alpha h \preceq g\}$ and $\{\alpha \in [0,1] | f_\alpha h \succeq g\}$ are closed.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Uncertainty Aversion if $f \sim g$ then $f_\alpha g \succeq f$.

Careful preferences

Axioms

Standard maxmin EU model (Gilboa-Schmeidler):

For all $f, g, h \in \mathcal{A}$, $c \in C$, $\alpha \in (0,1)$:

Non triviality and weak order $\preceq$ is non-trivial, reflexive, transitive and complete.

C-Independence $f \preceq g$ iff $f_\alpha c \preceq g_\alpha c$.

Continuity $\{\alpha \in [0,1] | \; f_\alpha h \preceq g\}$ and $\{\alpha \in [0,1] | \; f_\alpha h \succeq g\}$ are closed.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Uncertainty Aversion if $f \sim g$ then $f_\alpha g \succeq f$.

Careful preferences

Axioms

Confidence-based careful preference model:

For all $f, g, h \in \mathcal{A}$, $c, d \in C$, $\alpha \in (0,1)$:

Non triviality and weak order $\preceq$ is non-trivial, reflexive, transitive and complete.

S-Independence (i) if $f \succeq c$, then $f_\alpha d \succeq c_\alpha d$ whenever the stakes are lower.
(ii) if $f \preceq c$, then $f_\alpha d \preceq c_\alpha d$ whenever the stakes are higher.

Continuity $\{\alpha \in [0,1] | \ f_\alpha h \preceq g\}$ and $\{\alpha \in [0,1] | \ f_\alpha h \succeq g\}$ are closed.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Uncertainty Aversion For all $f, g \in \mathcal{A}$, $\alpha \in (0,1)$, if $f \sim g$ then $f_\alpha g \succeq f$.

Careful preferences

Axioms

Confidence-based careful preference model:

For all $f, g, h \in \mathcal{A}$, $c, d \in C$, $\alpha \in (0,1)$:

Non triviality and weak order $\preceq$ is non-trivial, reflexive, transitive and complete.

S-Independence (i) if $f \succeq c$, then $f_\alpha d \succeq c_\alpha d$ whenever the stakes are lower.
(ii) if $f \preceq c$, then $f_\alpha d \preceq c_\alpha d$ whenever the stakes are higher.

Continuity $\{\alpha \in [0,1] | \ f_\alpha h \preceq g\}$ and $\{\alpha \in [0,1] | \ f_\alpha h \succeq g\}$ are closed.

Monotonicity applies to acts of the same stakes

Uncertainty Aversion applies to acts of the same stakes

Careful preferences

Representation theorem

Theorem

$\preceq$ satisfies axioms above

$\Leftrightarrow$ *there exist $u : X \to \Re$, Ξ and $D : \mathcal{A} \to \Xi$ such that, for all $f, g \in \mathcal{A}$, $f \preceq g$ iff*

$$\min_{p \in D(f)} \mathbb{E}_p u(f(s)) \leqslant \min_{p \in D(g)} \mathbb{E}_p u(f(s))$$

Furthermore u is unique up to positive affine transformation, and Ξ and D are unique.

Enough

Incomplete preferences

Axioms

Expected utility (Anscombe and Aumann):

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0,1)$:

Non triviality and reflexivity $\preceq$ is non-trivial and reflexive.

Completeness $f \preceq g$ or $f \succeq g$.

Transitivity if $f \preceq g$ and $g \preceq h$, then $f \preceq h$.

Independence $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$.

Continuity $\{(\alpha, \beta) \in [0,1]^2 |\ f_\alpha h \preceq g_\beta h\}$ is closed.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Incomplete preferences

Axioms

Expected utility (Anscombe and Aumann):

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0, 1)$:

Non triviality and reflexivity $\preceq$ is non-trivial and reflexive.

Completeness $f \preceq g$ or $f \succeq g$.

Transitivity if $f \preceq g$ and $g \preceq h$, then $f \preceq h$.

Independence $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$.

Continuity $\{(\alpha, \beta) \in [0, 1]^2 | \; f_\alpha h \preceq g_\beta h\}$ is closed.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Incomplete preferences

Axioms

Standard unanimity model (Bewley):

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0,1)$:

Non triviality and reflexivity $\preceq$ is non-trivial and reflexive.

Completeness $f \preceq g$ or $f \succeq g$ whenever f, g are constant acts.

Transitivity if $f \preceq g$ and $g \preceq h$, then $f \preceq h$.

Independence $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$.

Continuity $\{(\alpha, \beta) \in [0,1]^2 | \ f_\alpha h \preceq g_\beta h\}$ is closed.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Incomplete preferences

Axioms

Confidence-based incomplete preference model:

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0,1)$:

Non triviality and reflexivity $\preceq$ is non-trivial and reflexive.

Completeness $f \preceq g$ or $f \succeq g$ whenever f, g are constant acts.

Transitivity if $f \preceq g$ and $g \preceq h$, then $f \preceq h$.

Independence $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$.

Continuity $\{(\alpha, \beta) \in [0,1]^2 \mid f_\alpha h \preceq g_\beta h\}$ is closed.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Incomplete preferences

Axioms

Confidence-based incomplete preference model:

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0, 1)$:

Non triviality and reflexivity $\preceq$ is non-trivial and reflexive.

Completeness $f \preceq g$ or $f \succeq g$ whenever f, g are constant acts.

S-Transitivity if $f \preceq g$ and $g \preceq h$ when the stakes are higher than for (f, h), then $f \preceq h$.

Independence $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$, whenever both preferences are determinate.

Continuity $\{(\alpha, \beta) \in [0, 1]^2 | \ f_\alpha h \preceq g_\beta h\}$ is closed.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Incomplete preferences

Axioms

Confidence-based incomplete preference model:

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0,1)$:

Non triviality and reflexivity $\preceq$ is non-trivial and reflexive.

Completeness $f \preceq g$ or $f \succeq g$ whenever f, g are constant acts.

S-Transitivity if $f \preceq g$ and $g \preceq h$ when the stakes are higher than for (f, h), then $f \preceq h$.

Independence $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$, whenever both preferences are determinate.

Continuity $\{(\alpha, \beta) \in [0,1]^2 | \ f_\alpha h \preceq g_\beta h\}$ is closed.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Consistency when the stakes decrease, one cannot suspend (determinate) preferences.

Incomplete preferences

Representation theorem

Theorem

$\preceq$ satisfies axioms above

$\Leftrightarrow$ *there exists affine $u : C \to \Re$, Ξ and $D : \mathcal{A}^2 \to \Xi$ such that, for all $f, g \in \mathcal{A}$, $f \preceq g$ iff*

$$\sum_{s \in S} \mathbb{E}_p(u(f)) \leqslant \sum_{s \in S} \mathbb{E}_p(u(g)) \quad \forall p \in D((f, g))$$

Furthermore u is unique up to positive affine transformation, and Ξ and D are unique.

General moral

	Expected utility
Remove "precision"	↓
	"Set of priors" rule
Remove stakes-independence	↓
	Confidence rule

Conclusion

The basic issue between the confidence family and fixed set of priors (aka imprecise probability) models:

- is stakes-independence a rationality constraint?

▸ Enough

Dutch Books

The standard argument (approximately):

Conditions on bets $\Leftrightarrow$ betting quotients are probabilities

Where:

Bet on A with stakes S yields €S if A and €0 if not A.

Betting quotient $q(A)$ value such that you are indifferent between buying and selling the bet at stakes S for €$q(A)S$.

Dutch Books

The standard argument (approximately):

Conditions on bets $\Leftrightarrow$ betting quotients are probabilities

Assumptions:

For any bets on events A with stakes S:

- €$q(A)S$ is the price at which you are indifferent between buying and selling the bet
- €$q(A)S$ is the buying / selling price for all stakes S

Dutch Books

The standard argument (approximately):

Conditions on bets $\Leftrightarrow$ betting quotients are probabilities

Assumptions:

For any bets on events A with stakes S:

- €$q(A)S$ is the price at which you are indifferent between buying and selling the bet
- €$q(A)S$ is the buying / selling price for all stakes S

Dutch Books

Removing it gives:

Conditions on bets $\Leftrightarrow$ buying / selling prices are minimal / maximal probabilities of fixed set

Assumptions:

For any bets on events A with stakes S:

- you have a buying price €$\underline{q_S}(A)S$ and a selling price €$\overline{q_S}(A)S$
- €$q(A)S$ is the buying / selling price for all stakes S

Dutch Books

Removing it gives:

Conditions on bets ⇔ buying / selling prices are minimal / maximal probabilities of fixed set

Assumptions:

For any bets on events A with stakes S:

- you have a buying price €$\underline{q_S}(A)S$ and a selling price €$\overline{q_S}(A)S$
- €$q(A)S$ is the buying / selling price for all stakes S

Dutch Books

Which gives:

Conditions on bets $\Leftrightarrow$ buying / selling prices are minimal / maximal probabilities of a confidence ranking

Assumptions:

For any bets on events A with stakes S:

- you have a buying price €$\underline{q_S}(A)S$ and a selling price €$\overline{q_S}(A)S$
- quotients $\underline{q_S}(A)$, $\overline{q_S}(A)$ may depend on stakes

Desiderata

- ✓ it corresponds to a reasonable (pre-theoretical) intuition
- ✓ it has acceptable choice-theoretical consequences

3. it is conceptually clear about the roles of different mental attitudes
4. it has a certain tractability for eg. decision analysis

And of course:

- ✓ it can fruitfully deal with “severe uncertainty” situations such as those above.

Separation of beliefs and desires

The model contains three elements:

- Utility function
- Confidence ranking
- Cautiousness coefficient

Separation of beliefs and desires

The model contains three elements:

- Utility function = Desires over outcomes
- Confidence ranking
- Cautiousness coefficient

Separation of beliefs and desires

The model contains three elements:

- Utility function = Desires over outcomes
- Confidence ranking = Beliefs and confidence in beliefs
- Cautiousness coefficient

Separation of beliefs and desires

The model contains three elements:

- Utility function = Desires over outcomes
- Confidence ranking = Beliefs and confidence in beliefs
- Cautiousness coefficient = Attitude to choosing on the basis of limited confidence

Separation of beliefs and desires

The model contains three elements:

- Utility function = Desires over outcomes
- Confidence ranking = Beliefs and confidence in beliefs
- Cautiousness coefficient = Attitude to choosing on the basis of limited confidence

And there is a natural comparison of attitude to uncertainty

- DM 1 is more averse to uncertainty than DM 2 if $\forall f$, constant c, $f \succeq_1 c \Rightarrow f \succeq_2 c$.

Separation of beliefs and desires

The model contains three elements:

- Utility function = Desires over outcomes
- Confidence ranking = Beliefs and confidence in beliefs
- Cautiousness coefficient = Attitude to choosing on the basis of limited confidence

And there is a natural comparison of attitude to uncertainty

that corresponds precisely to differences in the cautiousness coefficient.

For DMs with the same u and Ξ

- 1 is more averse to uncertainty

$\Leftrightarrow$ $D_1(f) \supseteq D_2(f)$ for all acts f.

Separation of beliefs and desires

The model contains three elements:

- Utility function = Desires over outcomes
- Confidence ranking = Beliefs and confidence in beliefs
- Cautiousness coefficient = Attitude to choosing on the basis of limited confidence

There is a natural comparison of decisiveness

- DM 1 is more indecisive than DM 2 if $\forall f, g \; f \succeq_1 g \;\Rightarrow f \succeq_2 g$.

Separation of beliefs and desires

The model contains three elements:

- Utility function = Desires over outcomes
- Confidence ranking = Beliefs and confidence in beliefs
- Cautiousness coefficient = Attitude to choosing on the basis of limited confidence

There is a natural comparison of decisiveness

that corresponds precisely to differences in the cautiousness coefficient

For DMs with the same u and Ξ

1 is more indecisive

$\Leftrightarrow$ $D_1((f,g)) \supseteq D_2((f,g))$ for all pairs f and g.

Separation of beliefs and desires

The model contains three elements:

- Utility function = Desires over outcomes
- Confidence ranking = Beliefs and confidence in beliefs
- Cautiousness coefficient = Attitude to choosing on the basis of limited confidence

Conclusion There is a clean separation between beliefs and desires (attitudes to outcomes and to choosing in the absence of confidence).

Comparison All other "incomplete preference" rules we are aware of do not exhibit such a separation.

Comparison Maxmin EU, as well as many other "non-EU" models of decision making, do not exhibit such a separation.

An example: Maxmin EU

$$\min_{p \in \mathcal{C}} \mathbb{E}_p u(f(s))$$

For DMs with the same u, DM 1 is more averse to uncertainty iff $\mathcal{C}_1 \supseteq \mathcal{C}_2$. (Ghirardato and Marinacci, 2002)

An example: Maxmin EU

$$\min_{p \in \mathcal{C}} \mathbb{E}_p u(f(s))$$

For DMs with the same u, DM 1 is more averse to uncertainty iff $\mathcal{C}_1 \supseteq \mathcal{C}_2$. (Ghirardato and Marinacci, 2002)

Flood / drought example

Governor 1's set of priors, $\mathcal{C}_1 \supseteq \mathcal{C}_2$, Governor 2's set.

- Does Gov 2 have further information / beliefs?
- Or is he just less cautious?

An example: Maxmin EU

$$\min_{p \in \mathcal{C}} \mathbb{E}_p u(f(s))$$

For DMs with the same u, DM 1 is more averse to uncertainty iff $\mathcal{C}_1 \supseteq \mathcal{C}_2$. (Ghirardato and Marinacci, 2002)

Flood / drought example

Governor 1's set of priors, $\mathcal{C}_1 \supseteq \mathcal{C}_2$, Governor 2's set.

- Does Gov 2 have further information / beliefs?
- Or is he just less cautious?

The point

Maximin EU can't decide the question ...or even properly represent the possibilities.

Desiderata

✓ it corresponds to a reasonable (pre-theoretical) intuition
✓ it has acceptable choice-theoretical consequences
✓ it is conceptually clear about the roles of different mental attitudes
4. it has a certain tractability for eg. decision analysis

And of course:

✓ it can fruitfully deal with "severe uncertainty" situations such as those above.

What does the decision maker / expert need to provide?

The confidence ranking

What does the decision maker / expert need to provide?

The confidence ranking is an ordinal second-order structure.

Needed ordinal confidence comparisons over probabilities

Eg. I am more confident that $p(A) \geqslant 0.5$ than $p(B) \leqslant 0.3$

What does the decision maker / expert need to provide?

The confidence ranking is an ordinal second-order structure.

Needed ordinal confidence comparisons over probabilities

Comparison Many recent “belief-like parameters” used in the literature are cardinal.

Needed numerical confidence comparisons over probabilities

Eg. I am confident to degree x that $p(A) \geqslant 0.5$ but only confident to degree y that $p(B) \leqslant 0.3$

What does the decision maker / expert need to provide?

The confidence ranking

Needed ordinal confidence comparisons over probabilities

Comparison Many recent “belief-like parameters” used in the literature

Needed numerical confidence comparisons over probabilities

Conclusion The confidence framework is much more parsimonious than other recent models . . .

and hence easier to apply to analysis and guidance of actual decisions!

In summary

1. it embodies plausible maxims of choice
2. it has acceptable choice-theoretical consequences
3. it involve a neat separation of beliefs and tastes
4. it is involves an ordinal notion of confidence

And of course:

5. it can fruitfully deal with “severe uncertainty” situations such as those above.

Plan

1. develop a theory based on these claims
   - model of confidence in beliefs
   - family of decision rules incorporating confidence
2. defence and consequences of the theory:
   - conceptual and choice-theoretic properties
   - consequences and applications

Plan

1. develop a theory based on these claims
    - model of confidence in beliefs
    - family of decision rules incorporating confidence
2. defence and consequences of the theory:
    - conceptual and choice-theoretic properties
    - consequences and applications

Climate uncertainty

Current state of scientific knowledge about the climate: reported in IPCC's uncertainty language (Mastrandrea et al., 2010).

How can it be used to inform decision making on climate policy?

IPCC uncertainty language

Two metrics for communicating degree of uncertainty:

Confidence in the validity of a finding, based on the type, amount, quality, and consistency of evidence (e.g., mechanistic understanding, theory, data, models, expert judgment) and the degree of agreement.

It is expressed qualitatively (five qualifiers, ranging from *very low* to *very high*).

Quantified measures of uncertainty in a finding expressed probabilistically (based on statistical analysis of observations or model results, or expert judgment).

IPCC uncertainty language

Table 1. Likelihood Scale	
Term*	**Likelihood of the Outcome**
Virtually certain	99-100% probability
Very likely	90-100% probability
Likely	66-100% probability
About as likely as not	33 to 66% probability
Unlikely	0-33% probability
Very unlikely	0-10% probability
Exceptionally unlikely	0-1% probability

* Additional terms that were used in limited circumstances in the AR4 (*extremely likely* – 95-100% probability, *more likely than not* – >50-100% probability, and *extremely unlikely* – 0-5% probability) may also be used in the AR5 when appropriate.

IPCC uncertainty language

Often, both metrics appear together:

‘In the Northern Hemisphere, 1983–2012 was *likely* the warmest 30-year period of the last 1400 years (medium confidence).’

‘Multiple lines of evidence provide high confidence that an [Equilibrium Climate Sensitivity] value less than 1°C is extremely unlikely.’

IPCC uncertainty language

Confidence in the validity of a finding, based on the type, amount, quality, and consistency of evidence (e.g., mechanistic understanding, theory, data, models, expert judgment) and the degree of agreement.

It is expressed qualitatively (five qualifiers, ranging from *very low* to *very high*).

Quantified measures of uncertainty in a finding expressed probabilistically (based on statistical analysis of observations or model results, or expert judgment).

IPCC uncertainty language

Confidence in the validity of a finding, based on the type, amount, quality, and consistency of evidence (e.g., mechanistic understanding, theory, data, models, expert judgment) and the degree of agreement.

It is expressed qualitatively (five qualifiers, ranging from *very low* to *very high*).

Quantified measures of uncertainty in a finding expressed probabilistically (based on statistical analysis of observations or model results, or expert judgment).

so it is perhaps not surprising that . . .

IPCC uncertainty language

- IPCC assessments can easily be translated into confidence rankings
- They are difficult to connect to other recent representations of uncertainty used in decision theory

IPCC uncertainty language

- IPCC assessments can easily be translated into confidence rankings
- They are difficult to connect to other recent representations of uncertainty used in decision theory

In fact, the confidence framework:

- is a pragmatic vindication of IPCC practices
- shows how they can be used to guide decision making
- provides practical recommendations for the future use of the language

Conclusion

The higher the stakes involved in a decision, the more confidence is needed in a belief for it to play a role in the decision.

We have:

- a maxim concerning the role of confidence in choice
- a formal model of confidence in beliefs and a family of decision rules embodying the maxim
- these rules are unique in having attractive conceptual and choice-theoretic properties
- and potentially interesting consequences for high-stakes decision making.

Thank you.

Material drawn from:

- Confidence and Decision, *Games and Economic Behavior*, 82: 675–692, 2013.
- Incomplete Preferences and Confidence, *Journal of Mathematical Economics*, 65: 83-103, 2016.
- Confidence as a Source of Deferral, HEC Paris Research Paper No. ECO/SCD-2014-1060.
- Climate Change Assessments: Confidence, Probability and Decision, *Philosophy of Science*, forthcoming (with R. Bradley, C. Helgeson).
- Confidence in Beliefs and Rational Decision Making, mimeo HEC Paris.

Confidence in Beliefs and Rational Decision Making

Brian Hill

hill@hec.fr

www.hec.fr/hill

GREGHEC, CNRS & HEC Paris

2016
(Presentation prepared for an interdisciplinary audience)

References I

Bewley, T. F. (2002). Knightian decision theory. part i. *Decisions in Economics and Finance*, 25:79–110.

Chateauneuf, A. and Faro, J. H. (2009). Ambiguity through confidence functions. *J. Math. Econ.*, 45:535–558.

Ellsberg, D. (1961). Risk, ambiguity, and the Savage axioms. *Quart. J. Econ.*, 75:643–669.

Gärdenfors, P. and Sahlin, N.-E. (1982). Unreliable probabilities, risk taking, and decision making. *Synthese*, 53(3):361–386. Reproduced in *Decision, Probability and Utility: Selected Readings, Gärdenfors, P. and Sahlin, N.-E. (eds), CUP, 1988*.

Ghirardato, P. and Marinacci, M. (2002). Ambiguity made precise: A comparative foundation. *J. Econ. Theory*, 102(2):251 – 289.

References II

Gilboa, I. and Schmeidler, D. (1989). Maxmin expected utility with non-unique prior. *J. Math. Econ.*, 18(2):141–153.

Keynes, J. M. (1921). *A Treatise on Probability*. Dover Publications.

Klibanoff, P., Marinacci, M., and Mukerji, S. (2005). A smooth model of decision making under ambiguity. *Econometrica*, 73(6):1849–1892.

Knight, F. H. (1921). *Risk, Uncertainty, and Profit*. Houghton Mifflin, Boston and New York.

Levi, I. (1986). *Hard Choices. Decision making under unresolved conflict*. Cambridge University Press, Cambridge.

Maccheroni, F., Marinacci, M., and Rustichini, A. (2006). Ambiguity aversion, robustness, and the variational representation of preferences. *Econometrica*, 74:1447–1498.

References III

Mastrandrea, M. D., Field, C. B., Stocker, T. F., Edenhofer, O., Ebi, K. L., Frame, D. J., Held, H., Kriegler, E., Mach, K. J., Matschoss, P. R., Plattner, G.-K., Yohe, G. W., and Zwiers, F. W. . . (2010). Guidance note for lead authors of the IPCC fifth assessment report on consistent treatment of uncertainties. Technical report, Intergovernmental Panel on Climate Change (IPCC). Available at <http://www.ipcc.ch>.

Peirce, C. S. (1878). *Philosophical Writings of Peirce*. Dover, New York.

Savage, L. J. (1954). *The Foundations of Statistics*. Dover, New York. 2nd edn 1971.

Schmeidler, D. (1989). Subjective probability and expected utility without additivity. *Econometrica*, 57(3):571–587.

Walley, P. (1991). *Statistical reasoning with imprecise probabilities*. Chapman and Hall, London.

Belief functions / capacities

Capacity An increasing function $\nu : 2^S \to [0,1]$ s.t. $\nu(\emptyset) = 0$, $\nu(S) = 1$.

- A (Dempster-Shafer) belief function is a (convex) capacity.

Convex: $\nu(E) + \nu(F) \leqslant \nu(E \cup F) + \nu(E \cap F)$

Belief functions / capacities

Capacity An increasing function $\nu : 2^S \to [0, 1]$ s.t. $\nu(\emptyset) = 0$, $\nu(S) = 1$.

- A (Dempster-Shafer) belief function is a (convex) capacity.

Decision

The appropriate decision rule for capacities is maximisation of the **Choquet integral**, $\int u(f(s))d\nu$ (Schmeidler, 1989).

$$\int u(f(s))d\nu = \int_0^\infty \nu\left(\{s \in S : u(f(s)) \geqslant t\}\right) dt$$

Belief functions / capacities

Capacity An increasing function $\nu : 2^S \to [0,1]$ s.t. $\nu(\varnothing) = 0$, $\nu(S) = 1$.

- A (Dempster-Shafer) belief function is a (convex) capacity.

Decision

The appropriate decision rule for capacities is maximisation of the **Choquet integral**, $\int u(f(s))d\nu$ (Schmeidler, 1989).

For convex capacities,

$$\int u(f(s))d\nu = \min_{p \in \mathcal{C}_\nu} \mathbb{E}_p u((f))$$

$$\mathcal{C}_\nu = \left\{ p \in \Delta(S) : p(E) \geqslant \nu(E) \quad \forall E \in 2^S \right\}$$

Belief functions / capacities

Capacity An increasing function $\nu : 2^S \to [0, 1]$ s.t. $\nu(\emptyset) = 0$, $\nu(S) = 1$.

- A (Dempster-Shafer) belief function is a (convex) capacity.

Decision (in brief)

The decision rule for convex capacities is (equivalent to) maxmin expected utility.

back

Confidence, stakes and choice

Examples

Stakes involved in the choice:

- of an act;

Eg. utility of worst consequence of the act

Back

Confidence, stakes and choice

Examples

Stakes involved in the choice:

- of an act;
- from a menu;

Eg. utility of worst consequence of any act on the menu

Back

Confidence, stakes and choice

Examples

Stakes involved in the choice:

- of an act;
- from a menu;
- in a context . . .

Back

Violations and abstentions

	Black	White
f	15	-10
0	0	0
g	1.5 M	-1 M
f^n	$15 \times n$	$-10 \times n$

Bets on colour of next ball drawn from urn where:

- only black and white balls
- observed 15 draws (with replacement): 9 black, 6 white.

Back

Violations and abstentions

	Black	White
f	15	-10
0	0	0
g	1.5 M	-1 M
f^n	$15 \times n$	$-10 \times n$

Bets on colour of next ball drawn from urn where:

- only black and white balls
- observed 15 draws (with replacement): 9 black, 6 white.

$$f \text{ vs. } 0 \; / \; f^{n+1} \text{ vs. } f^n \; / \; g \text{ vs. } 0?$$

Back

Violations and abstentions

	Black	White
f	15	-10
0	0	0
g	1.5 M	-1 M
f^n	$15 \times n$	$-10 \times n$

Bets on colour of next ball drawn from urn where:

- only black and white balls
- observed 15 draws (with replacement): 9 black, 6 white.

$$f \text{ vs. } 0 \ / \ f^{n+1} \text{ vs. } f^n \ / \ g \text{ vs. } 0?$$

Standard Independence $f \succeq 0 \;\Leftrightarrow\; g \succeq 0$

Standard Transitivity $f^{n+1} \succeq f^n \;\forall n \;\Rightarrow\; g \succeq 0$

Back

Violations and abstentions

	Black	White
f	15	-10
0	0	0
g	1.5 M	-1 M
f^n	$15 \times n$	$-10 \times n$

Bets on colour of next ball drawn from urn where:

- only black and white balls
- observed 15 draws (with replacement): 9 black, 6 white.

$$f \text{ vs. } 0 \;/\; f^{n+1} \text{ vs. } f^n \;/\; g \text{ vs. } 0?$$

Standard Independence $f \succeq 0 \;\Leftrightarrow\; g \succeq 0$

Pure Independence allows $f \succeq 0$ and $g \not\succeq 0$

Back

Violations and abstentions

	Black	White
f	15	-10
0	0	0
g	1.5 M	-1 M
f^n	$15 \times n$	$-10 \times n$

Bets on colour of next ball drawn from urn where:

- only black and white balls
- observed 15 draws (with replacement): 9 black, 6 white.

$$f \text{ vs. } 0 \;/\; f^{n+1} \text{ vs. } f^n \;/\; g \text{ vs. } 0?$$

Standard Transitivity $f^{n+1} \succeq f^n \; \forall n \;\Rightarrow\; g \succeq 0$

Stakes Transitivity allows $f^{n+1} \succeq f^n \; \forall n$ and $g \not\succeq 0$

Back

Violations and abstentions

	Black	White
f	15	-10
0	0	0
g	1.5 M	-1 M
f^n	$15 \times n$	$-10 \times n$

Bets on colour of next ball drawn from urn where:

- only black and white balls
- observed 15 draws (with replacement): 9 black, 6 white.

However

- Pure Independence $\Leftrightarrow$ Standard Independence
- Stakes Transitivity $\Leftrightarrow$ Standard Transitivity

whenever preferences are determinate.

Back

Choice theoretic properties

S-independence

	Blue	Red
f	10 000	-1 M
g	10 M	-1 000 M
0	0	0

Bets on colour of next ball drawn from urn where:

- 1 M balls, each of which is either red or blue
- Sure that at least 990000 blue, and at least 1 red.
- Experts say: at most 10 red.

Choice theoretic properties

S-independence

	Blue	Red
f	10 000	-1 M
g	10 M	-1 000 M
0	0	0

Bets on colour of next ball drawn from urn where:

- 1 M balls, each of which is either red or blue
- Sure that at least 990000 blue, and at least 1 red.
- Experts say: at most 10 red.

$$f \text{ vs. } 0 \,/\; g \text{ vs. } 0?$$

Story back Main

Choice theoretic properties

S-independence

	Blue	Red
f	10 000	-1 M
g	10 M	-1 000 M
0	0	0

Bets on colour of next ball drawn from urn where:

- 1 M balls, each of which is either red or blue
- Sure that at least 990000 blue, and at least 1 red.
- Experts say: at most 10 red.

$$f \text{ vs. } 0 \,/\, g \text{ vs. } 0?$$

C-independence $f \succeq 0 \Leftrightarrow g \succeq 0$.

Choice theoretic properties

S-independence

	Blue	Red
f	10 000	-1 M
g	10 M	-1 000 M
0	0	0

Bets on colour of next ball drawn from urn where:

- 1 M balls, each of which is either red or blue
- Sure that at least 990000 blue, and at least 1 red.
- Experts say: at most 10 red.

$$f \text{ vs. } 0 \,/\, \; g \text{ vs. } 0?$$

C-independence $f \succeq 0 \Leftrightarrow g \succeq 0$.

S-independence $g \succeq 0 \Rightarrow f \succeq 0$, but not vice versa.

Why confidence?

Public decision making

The governor must decide whether to allow a factory project

- fumes from the factory could affect district farming area.
- probabilities controversial, but he retains estimate of 10^{-5}.
- with this probability, project retained.

Why confidence?

Public decision making

The governor must decide whether to allow a factory project

- fumes from the factory could affect district farming area.
- probabilities controversial, but he retains estimate of 10^{-5}.
- with this probability, project retained.

The governor must decide whether to allow a GM crops project

- probability of infecting non-GM area same as probability of fumes arriving there.
- consequences are larger by a factor of a thousand, in governor's opinion.

Why confidence?

Public decision making

The governor must decide whether to allow a factory project

- fumes from the factory could affect district farming area.
- probabilities controversial, but he retains estimate of 10^{-5}.
- with this probability, project retained.

The governor must decide whether to allow a GM crops project

- probability of infecting non-GM area same as probability of fumes arriving there.
- consequences are larger by a factor of a thousand, in governor's opinion.
- Yet it is not *prima facie* unreasonable to turn down the project!

back

Why confidence?

Public decision making

Stereotyped version

Urn with 10^6 balls; at least 990000 blue and at least 1 red.
Advisers' estimate: at most 10 are red.

	Colour of ball drawn from urn	
	Blue	Red
f	10 000	-1 M
g	10 M	-1 000 M
p_0	0	0

f: factory; g: GM crops.

Preferences: $f \succ p_0$ and $g \prec p_0$.

back

Second-order models

	"Belief" representation	Rule
Smooth Ambiguity (Klibanoff et al., 2005)	second-order probability $\mu \in \Delta(\Delta(S))$	$\int_{\Delta(S)} \phi\left(\mathbb{E}_p u(f)\right) \mu(p)$
Variational preferences (Maccheroni et al., 2006)	$c : \Delta(S) \to [0, \infty]$	$\min_{p \in \Delta(S)} \left(\mathbb{E}_p u(f) + c(p)\right)$
Chateauneuf and Faro (2009)	$\varphi : \Delta(S) \to [0, 1]$	$\min_{p \in \Delta(S)} \frac{1}{\varphi(f)} \mathbb{E}_p u(f)$

Back