Confidence, consensus and aggregation

Brian Hill

hill@hec.fr

www.hec.fr/hill

CNRS & HEC Paris

TUS XI
2025

Dedicated to the memory of Philippe Mongin

Belief aggregation . . . for subsequent decision

i (honest, well-intentioned) 'experts'; 0 group.

Belief aggregation . . . for subsequent decision

i (honest, well-intentioned) ‘experts’; 0 group.

Probability aggregation

- **Input**: p^i (probabilities)
- **Output belief:** p^0 (group probability)
- **Output decision(s)** SEU with p^0

Belief aggregation . . . for subsequent decision

i (honest, well-intentioned) ‘experts’; 0 group.

Probability aggregation

- **Input**: p^i (probabilities)
- **Output belief:** p^0 (group probability)
- **Output decision(s)** SEU with p^0

E.g. Linear opinion pooling:

$$p^0(E) = \sum_{i=1}^{n} w^i p^i(E)$$

Pareto (Mongin, 1995)

Belief aggregation . . . for subsequent decision

i (honest, well-intentioned) 'experts'; 0 group.

Probability aggregation

- **Input**: p^i (probabilities)
- **Output belief:** p^0 (group probability)
- **Output decision(s)** SEU with p^0

Model misspecification

- **Input**: p^i (probabilities)
- **Output belief:** ?
- **Output decision(s)**: E.g.

$$\min_{p \in \Delta} \left\{ \mathbb{E}_p u(f) + \lambda \min_{p^i} R(p \| p^i) \right\}$$

E.g. Linear opinion pooling:

$$p^0(E) = \sum_{i=1}^{n} w^i p^i(E)$$

(Hansen and Sargent, 2022; Cerreia–Vioglio et al., 2025)

Pareto (Mongin, 1995)

Belief aggregation . . . for subsequent decision

i (honest, well-intentioned) 'experts'; 0 group.

Probability aggregation $\overset{?}{\leftrightarrow}$ Model misspecification

Probability aggregation

- **Input**: p^i (probabilities)
- **Output belief:** p^0 (group probability)
- **Output decision(s)** SEU with p^0

E.g. Linear opinion pooling:

$$p^0(E) = \sum_{i=1}^{n} w^i p^i(E)$$

Pareto (Mongin, 1995)

Model misspecification

- **Input**: p^i (probabilities)
- **Output belief:** ?
- **Output decision(s)**: E.g.

$$\min_{p \in \Delta} \left\{ \mathbb{E}_p u(f) + \lambda \min_{p^i} R(p \| p^i) \right\}$$

(Hansen and Sargent, 2022; Cerreia–Vioglio et al., 2025)

Challenges

Example

Challenges
Example

Probability certain interest rate rise has limited effect on:

	Labour	Real estate	Both
Laura	0.9	0.1	0.09
Ray	0.1	0.9	0.09
Lin. pool.	$0.1 + 0.8w^L$	$0.9 - 0.8w^L$	0.09

$$p^0(E) = w^L p^L(E) + (1 - w^L) p^R(E)$$

Challenges

Example

Probability certain interest rate rise has limited effect on:

	Labour	Real estate	Both
Laura	0.9	0.1	0.09
Ray	0.1	0.9	0.09
Lin. pool.	$0.1 + 0.8w^L$	$0.9 - 0.8w^L$	0.09

$$p^0(E) = w^L p^L(E) + (1 - w^L) p^R(E)$$

Challenges
Example

Probability certain interest rate rise has limited effect on:

	Labour	Real estate	Both
Laura	0.9	0.1	0.09
Ray	0.1	0.9	0.09
Lin. pool.	$0.1 + 0.8w^L$	$0.9 - 0.8w^L$	0.09

$$p^0(E) = w^L p^L(E) + (1 - w^L)p^R(E)$$

Spurious Unanimity

- Why respect spurious consensus?

(Mongin, 2016; Bradley, 2017b; Mongin and Pivato, 2020; Dietrich, 2021; Bommier et al., 2021)

Challenges

Example

Probability certain interest rate rise has limited effect on:

	Labour	Real estate	Both
Laura	0.9	0.1	0.09
Ray	0.1	0.9	0.09
Lin. pool.	$0.1 + 0.8w^L$	$0.9 - 0.8w^L$	0.09

$$p^0(E) = w^L p^L(E) + (1 - w^L)p^R(E)$$

Spurious Unanimity

- Why respect spurious consensus?

Diverse (intra-agent) expertise

- Why is Laura's judgement on Labour treated the same as her judgement on Real-estate?

(Genest and Zidek, 1986; French, 1985)

Challenges
Example

Probability certain interest rate rise has limited effect on:

	Labour	Real estate	Both
Laura	0.9	0.1	0.09
Ray	0.1	0.9	0.09
Lin. pool.	$0.1 + 0.8w^L$	$0.9 - 0.8w^L$	0.09

$$\min_{p \in \Delta} \left\{ \mathbb{E}_p u(f) + \lambda \min_{p^i} R(p \| p^i) \right\}$$

Spurious Unanimity

- Why respect spurious consensus?

Diverse (intra-agent) expertise

- Why is Laura's judgement on Labour treated the same as her judgement on Real-estate?

Challenges

Summary

Probability aggregation

- **Input**: p^i (probabilities)
- **Output belief:** p^0 (group probability)
- **Output decision(s)** SEU with p^0

Model misspecification

- **Input**: p^i (probabilities)
- **Output belief:** ?
- **Output decision(s)**: E.g.

$$\min_{p \in \Delta} \left\{ \mathbb{E}_p u(f) + \lambda \min_{p^i} R(p|p^i) \right\}$$

Desiderata A belief **aggregation for decision** procedure that:

0. generalises and situates these
1. respects the right consensuses
   - avoiding spurious unanimities
2. can do justice to varying expertise

Challenges

Summary

Probability aggregation

- **Input**: p^i (probabilities)
- **Output belief:** p^0 (group probability)
- **Output decision(s)** SEU with p^0

Model misspecification

- **Input**: p^i (probabilities)
- **Output belief:** ?
- **Output decision(s)**: E.g.

$$\min_{p\in\Delta}\left\{\mathbb{E}_p u(f) + \lambda \min_{p^i} R(p|p^i)\right\}$$

Insight

- New aggregation rule for **confidence in beliefs**
- (+ known approach to confidence in decision)

skip insight

Plan

Proposal

Insights

Issue-level consensus

Spuriousness

Desideratum A belief aggregation procedure that:

1. respects the right consensus(es)
   - avoiding spurious unanimities

Proposal

Insights

Issue-level consensus

Spuriousness

↓

Evidence, reasons, information

$p^i(E)$ does not exhaust the elements of belief states pertaining to event E relevant for aggregation ...

Proposal

Insights

Issue-level consensus

Spuriousness

↓

Evidence, reasons, information

↓

Confidence

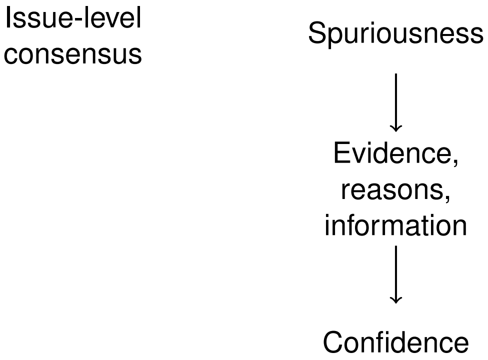
$p^i(E)$ does not exhaust the elements of belief states pertaining to event E relevant for aggregation ...

Confidence in beliefs

Hill, 2013, 2019b,a; Bradley, 2017a ... Klibanoff et al., 2005; Maccheroni et al., 2006; Hansen and Sargent, 2008; Chateauneuf and Faro, 2009

Proposal

Insights



Desideratum A belief aggregation procedure that:

1. respects the right consensus(es)
   - avoiding spurious unanimities

Proposal

Insights

Issue-level consensus

↓

Corpus-level consensus

Spuriousness

↓

Evidence, reasons, information

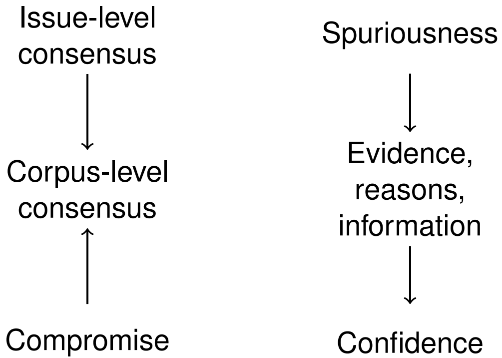
↓

Confidence

Corpus: (coherent) set of probability judgements $\equiv$ set of priors.

Proposal

Insights



Corpus-level consensus: Everyone is willing to ‘leave off the table’ or compromise any potential disagreement.

Proposal

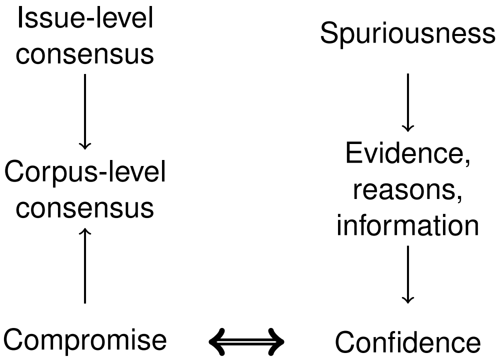
Insights

Issue-level consensus
↓
Corpus-level consensus
↑
Compromise

Spuriousness
↓
Evidence, reasons, information
↓
Confidence

What compromises are agents willing to make?

Proposal
Insights



Confidence and Compromise

The more confident an individual is in a belief, the less willing she is to compromise on it.

Proposal
Insights

In aggregation:

Respect corpus-level consensus

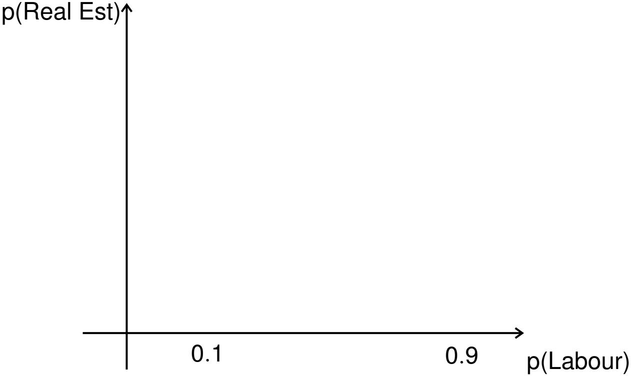
where

The more confident an individual is in a belief,
the less willing she is to compromise on it.

Plan

Plan

Confidence in beliefs

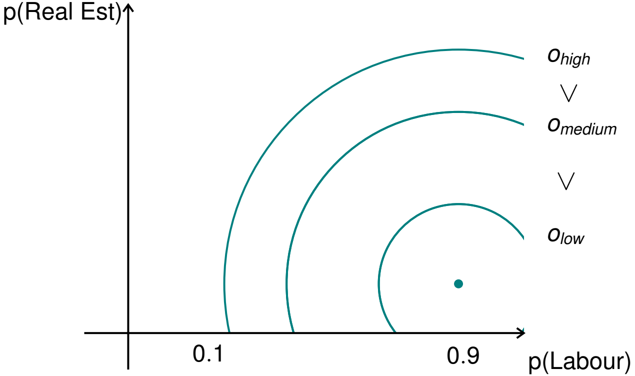


Preliminaries

Δ probability measures (over states Ω)

$(O, >)$ confidence levels

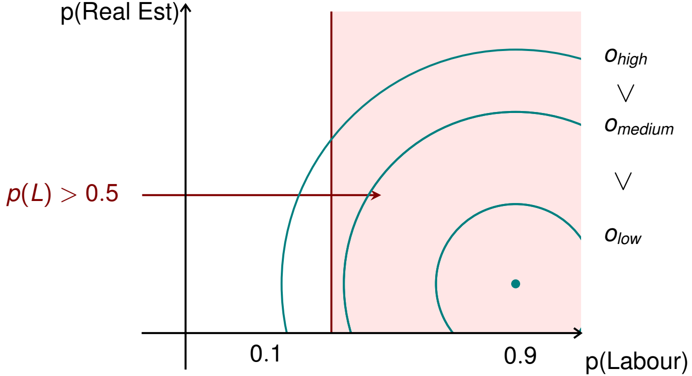
Confidence in beliefs



Confidence ranking: Increasing $c^i : O \to 2^\Delta \setminus \emptyset$.

(Hill, 2013, 2019b; Manski, 2013; Bradley, 2017a)

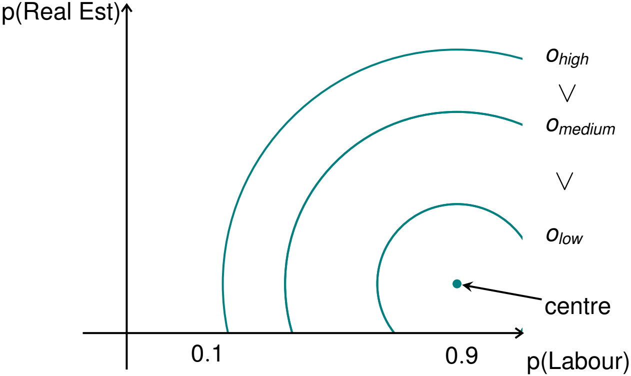
Confidence in beliefs



Confidence ranking: Increasing $c^i : O \to 2^\Delta \setminus \emptyset$.

(Hill, 2013, 2019b; Manski, 2013; Bradley, 2017a)

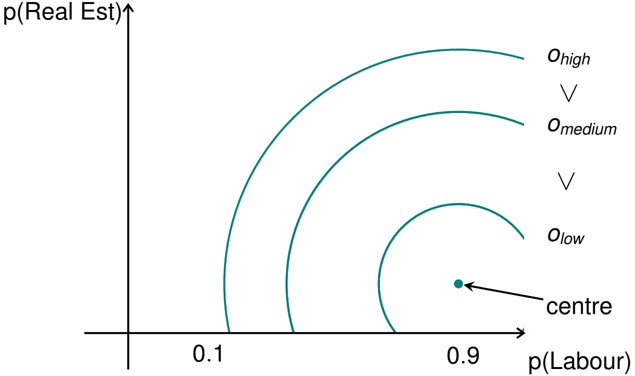
Confidence in beliefs



Confidence ranking: Increasing $c^i : O \to 2^{\Delta} \setminus \emptyset$.

(Hill, 2013, 2019b; Manski, 2013; Bradley, 2017a)

Confidence in beliefs



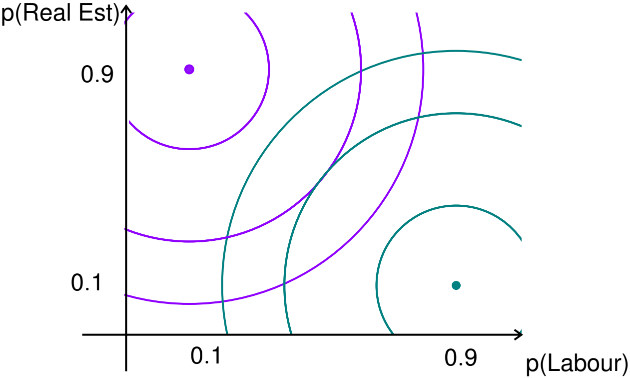
Confidence ranking: Increasing $c^i : O \to 2^\Delta \setminus \emptyset$.

Implausibility fn $\iota_{c^i} : \Delta \to O \cup \emptyset$

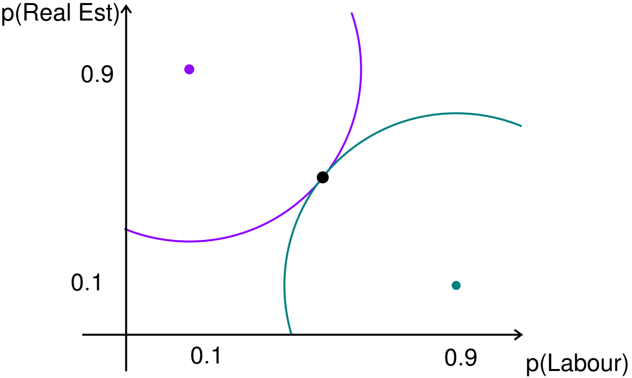
(Hill, 2013, 2019b; Manski, 2013; Bradley, 2017a)[a]

[a] $\iota_{c^i}(p) = \min\{o : p \in c^i(o)\}$; Reduced form for Klibanoff et al., 2005; Maccheroni et al., 2006; Hansen and Sargent, 2008; Chateauneuf and Faro, 2009 . . .

Consensus



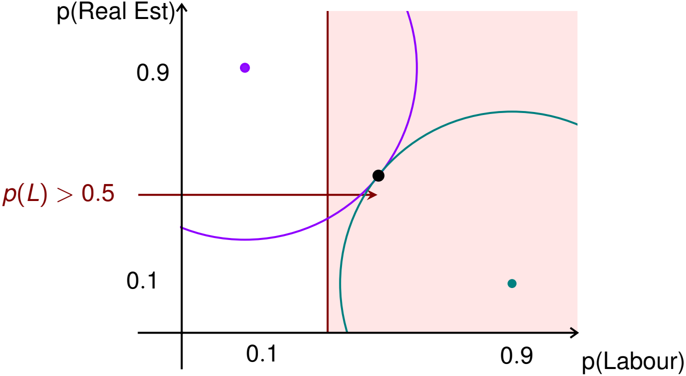
Consensus



Consensus: a coherent set of probability judgements rejected by no-one (at the relevant confidence levels).

$$\bigcap_i c^i(o)$$

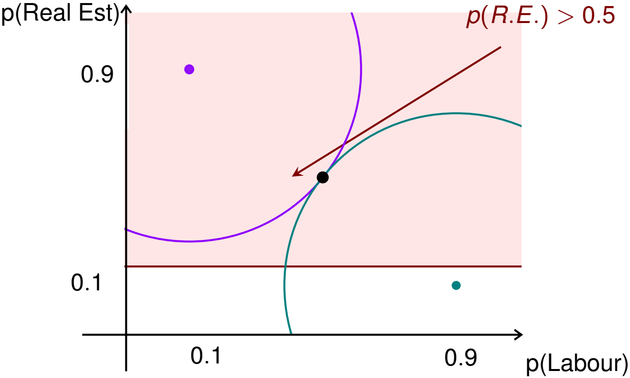
Consensus



Consensus: a coherent set of probability judgements rejected by no-one (at the relevant confidence levels).

$$\bigcap_i c^i(o)$$

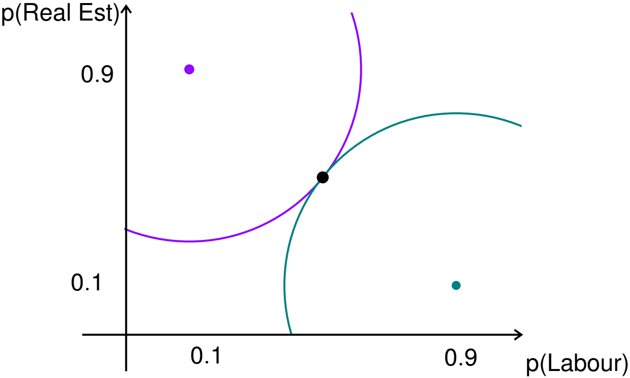
Consensus



Consensus: a coherent set of probability judgements rejected by no-one (at the relevant confidence levels).

$$\bigcap_i c^i(o)$$

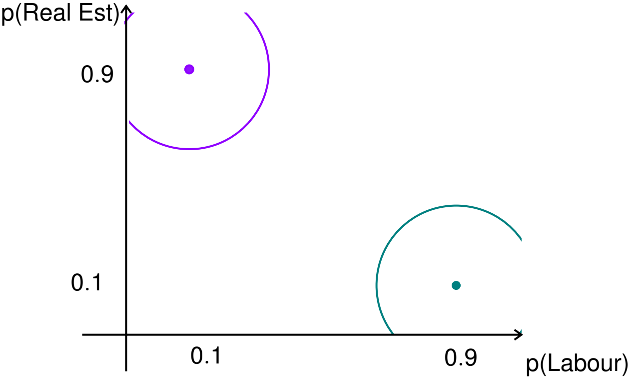
Consensus



Consensus: a coherent set of probability judgements rejected by no-one (at the relevant confidence levels).

Maxim If more confident in a belief, less willing to compromise it.

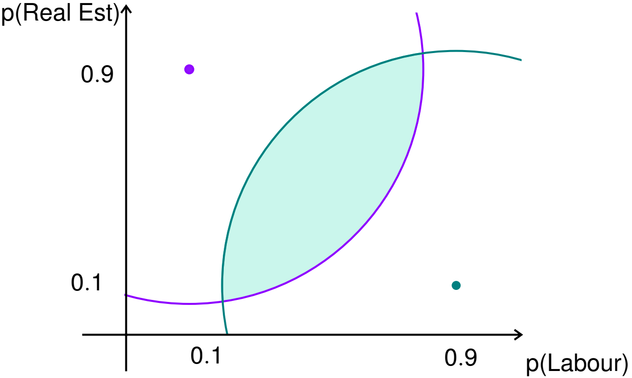
Consensus



Consensus: a coherent set of probability judgements rejected by no-one (at the relevant confidence levels).

Maxim If more confident in a belief, less willing to compromise it.

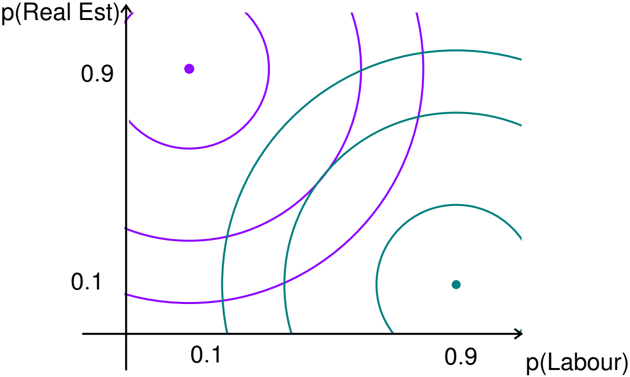
Consensus



Consensus: a coherent set of probability judgements rejected by no-one (at the relevant confidence levels).

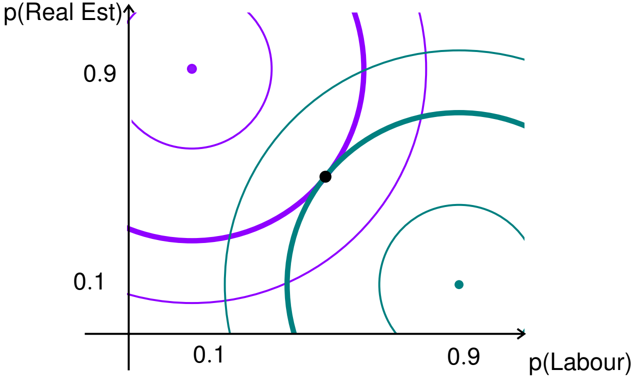
Maxim If more confident in a belief, less willing to compromise it.

Confidence aggregation



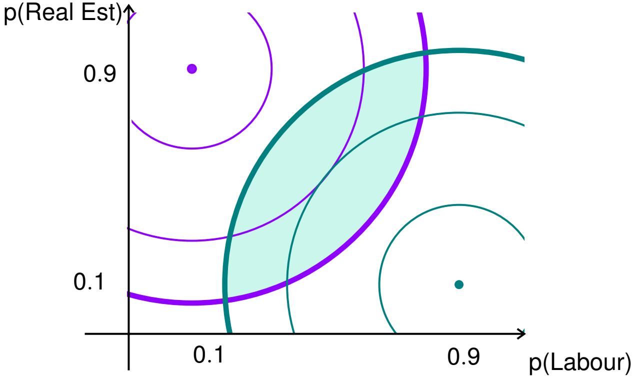
Confidence aggregation

$$\underbrace{\bigcap_i c^i(o_i)}_{\textit{Consensus}}$$



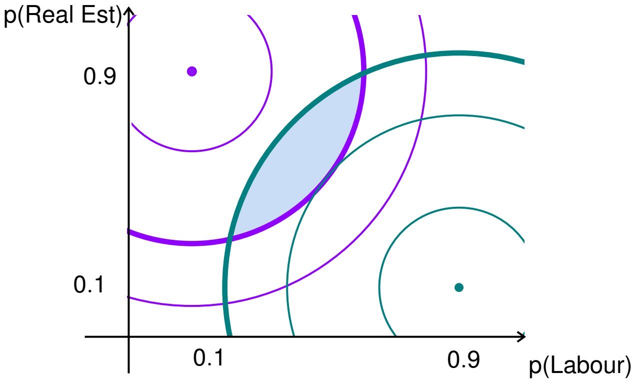
Confidence aggregation

$$\underbrace{\bigcap_i c^i(o'_i)}_{\textit{Consensus}}$$



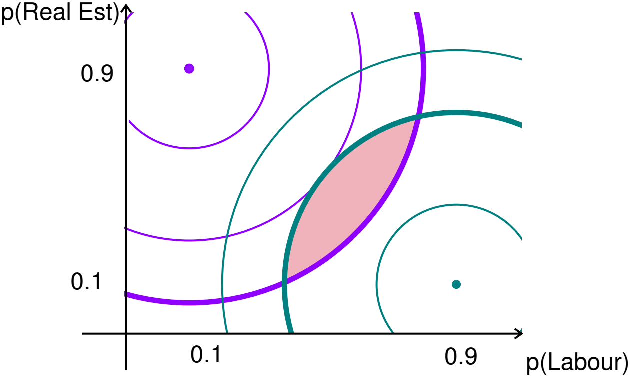
Confidence aggregation

$$\underbrace{\bigcap_i c^i(o_i'')}_{\textit{Consensus}}$$



Confidence aggregation

$$\underbrace{\bigcap_i c^i(o_i''')}_{Consensus}$$



Confidence aggregation

$$\underbrace{\bigcap_i c^i(o_i)}_{Consensus}$$

$\otimes : O^n \to O$: confidence level aggregator.

- $\otimes\mathbf{o}$: group confidence in consensus judgements in $\mathbf{o}$
- monotonic

Confidence aggregation

$$\underbrace{\bigcap_i c^i(o_i)}_{Consensus}$$

$\otimes : O^n \to O$: confidence level aggregator.

- $\otimes\mathbf{o}$: group confidence in consensus judgements in $\mathbf{o}$
- monotonic

E.g.

Maximum agg. $\otimes\mathbf{o} = \max\{o_i\}$

Minimum agg. $\otimes\mathbf{o} = \min\{o_i\}$

Average agg. $\otimes\mathbf{o} = \sum \frac{1}{n} o_i + \chi$

Confidence aggregation

$$F_{\otimes}(c^1, \ldots, c^n)(o) = \bigcup_{\mathbf{o}:\otimes\mathbf{o}\leq o} \underbrace{\bigcap_i c^i(o_i)}_{\textit{Consensus}}$$

for **o** with $\bigcap_i c^i(o_i) \neq \emptyset$

Group judgement: held in all consensuses with that level of confidence.

Confidence aggregation

Consensus-preserving confidence aggregation:

$$F_{\otimes}(c^1, \ldots, c^n)(o) = \bigcup_{\mathbf{o}:\otimes\mathbf{o} \leq o} \bigcap_i c^i(o_i)$$

The more individual confidence there is in a consensus judgement, the more confidence the group has in it.

Confidence aggregation

Consensus-preserving confidence aggregation:

$$F_{\otimes}(c^1, \ldots, c^n)(o) = \bigcup_{\mathbf{o}: \otimes \mathbf{o} \leq o} \bigcap_i c^i(o_i)$$

Equivalently:

$$F_{\otimes}(\iota^1, \ldots, \iota^n)(p) = \otimes(\iota^1(p), \ldots, \iota^n(p))$$

So

$$\mathrm{Centre}_{F_{\otimes}(c^1,\ldots,c^n)} = \arg\min_{p \in \Delta} \otimes(\iota^1(p), \ldots, \iota^n(p))$$
$$\overset{avge\ \otimes}{=} \arg\min_{p \in \Delta} \sum_{i=1}^{n} \iota^i(p)$$

Plan

Pooling & Confidence

Probabilities p^i

Pooling & Confidence

Probabilities p^i: *stipulate* confidence rankings centred on p^i

$$c^i(o) = \left\{ q \in \Delta : w^i d(q, p^i) \leq o \right\}$$
$$\iota_{c^i}(q) = w^i d(q, p^i)$$

d: (classical) statistical distance[a]

E.g.

Euclidean $d(q,p) = \sum_{\omega \in \Omega} (q(\omega) - p(\omega))^2$

Relative Entr. $d(q,p) = R(q \| p)$

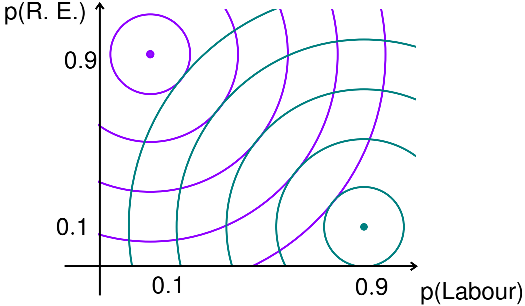
Reverse Rel. Entr. $d(q,p) = R(p \| q)$

[a] lower semicts; $\rho(q,p) = 0 \Leftrightarrow p = q$.

Pooling & Confidence

Probabilities p^i: *stipulate* confidence rankings centred on p^i

$$c^i(o) = \left\{ q \in \Delta : w^i d(q, p^i) \leq o \right\}$$

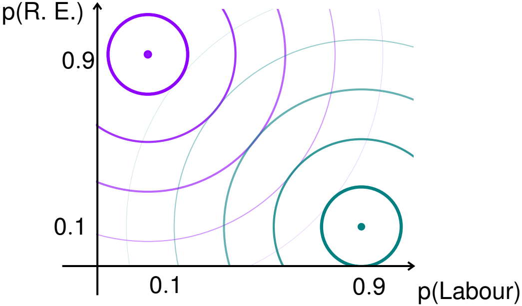


d: Euclidean

Pooling & Confidence

Probabilities p^i: *stipulate* confidence rankings centred on p^i

$$c^i(o) = \left\{ q \in \Delta : w^i d(q, p^i) \leq o \right\}$$



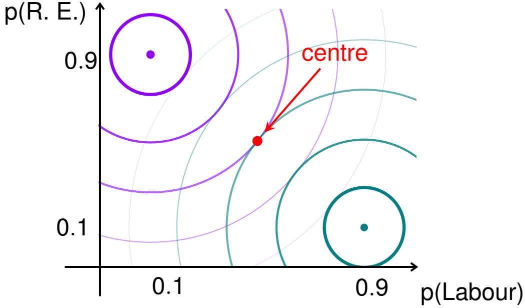
$w^L = w^R$

d: Euclidean

Pooling & Confidence

Probabilities p^i: *stipulate* confidence rankings centred on p^i

$$c^i(o) = \left\{ q \in \Delta : w^i d(q, p^i) \leq o \right\}$$



Confidence aggregation
$w^L = w^R$

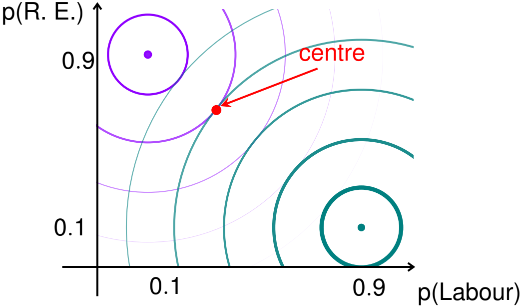
d: Euclidean

Average $\otimes$

Pooling & Confidence

Probabilities p^i: *stipulate* confidence rankings centred on p^i

$$c^i(o) = \left\{ q \in \Delta : w^i d(q, p^i) \leq o \right\}$$



Confidence aggregation
$w^L < w^R$

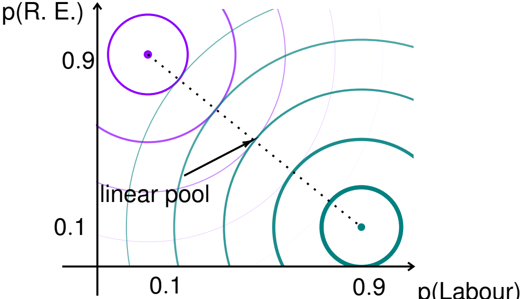
d: Euclidean

Average $\otimes$

Pooling & Confidence

Probabilities p^i: *stipulate* confidence rankings centred on p^i

$$c^i(o) = \left\{ q \in \Delta : w^i d(q, p^i) \leq o \right\}$$



Confidence aggregation
$w^L < w^R$

d: Euclidean

Average $\otimes$

Pooling & Confidence

Probabilities p^i: *stipulate* confidence rankings centred on p^i

$$c^i(o) = \left\{ q \in \Delta : w^i d(q, p^i) \leq o \right\}$$

Theorem

Centre of confidence aggregation = result of pooling rule

Generating distance	*Pooling rule*
Euclidean	*Linear*
Relative Entropy	*Geometric*
Reverse Rel. Entr.	*Linear*

with weights $\frac{w^i}{\sum_{i=1}^{n} w^i}$.

Pooling & Confidence

Probabilities p^i: *stipulate* confidence rankings centred on p^i

$$c^i(o) = \left\{ q \in \Delta : w^i d(q, p^i) \leq o \right\}$$

Theorem

Centre of confidence aggregation = result of pooling rule

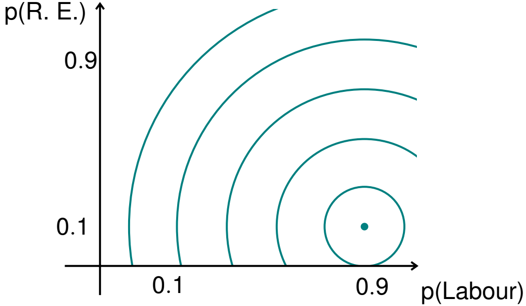
Moral Linear pooling =

- special case of confidence aggregation
- corresponding to assumptions on individuals' confidence.

Expertise

Euclidean generated confidence ranking:

$$c^L(o) = \left\{ q \in \Delta : w^L \sum_{s \in S'} (q(s) - p^L(s))^2 \leq o \right\}$$



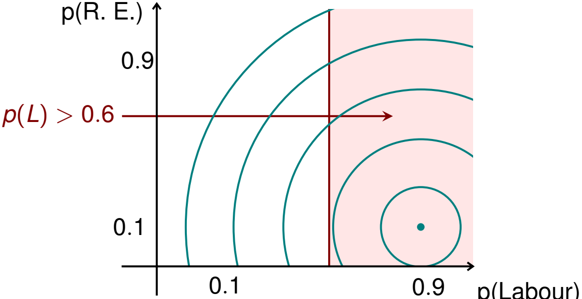
Expertise

Euclidean generated confidence ranking:

$$c^L(o) = \left\{ q \in \Delta : w^L \sum_{s \in S'} (q(s) - p^L(s))^2 \leq o \right\}$$

Fact Euclidean: assumes

- divergence from p^i on Labour $\equiv$ divergence on R. E.



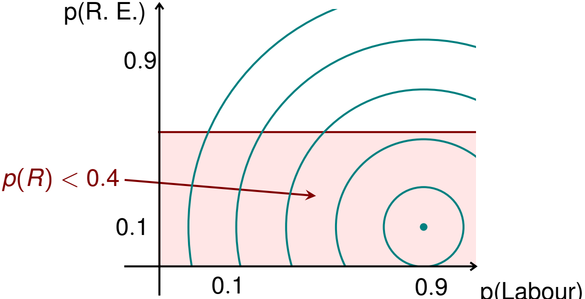
Expertise

Euclidean generated confidence ranking:

$$c^L(o) = \left\{ q \in \Delta : w^L \sum_{s \in S'} (q(s) - p^L(s))^2 \leq o \right\}$$

Fact Euclidean: assumes

- divergence from p^i on Labour $\equiv$ divergence on R. E.



Expertise

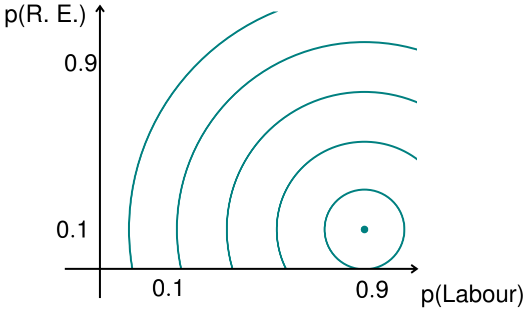
Euclidean generated confidence ranking:

$$c^L(o) = \left\{ q \in \Delta : w^L \sum_{s \in S'} (q(s) - p^L(s))^2 \leq o \right\}$$

Fact Euclidean: assumes

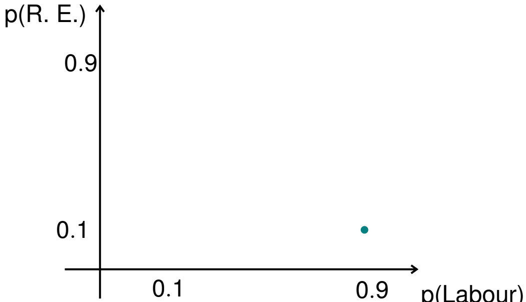
- divergence from p^i on Labour $\equiv$ divergence on R. E.

$\rightarrow$ same confidence on all issues!



Expertise

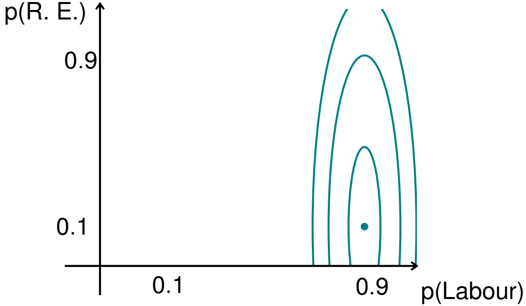
$$c^L(o) = \left\{ q \in \Delta : \begin{array}{l} w^L_L(q(L) - p^L(L))^2 \\ +w^L_R(q(R) - p^L(R))^2 \end{array} \leq o \right\}$$



Expertise

$$c^L(o) = \left\{ q \in \Delta : \begin{array}{l} w^L_L(q(L) - p^L(L))^2 \\ + w^L_R(q(R) - p^L(R))^2 \end{array} \leq o \right\}$$

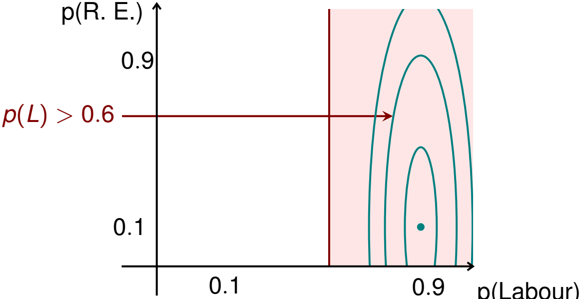
$w^L_L > w^L_R$



Expertise

$$c^L(o) = \left\{ q \in \Delta : \begin{array}{l} w^L_L(q(L) - p^L(L))^2 \\ + w^L_R(q(R) - p^L(R))^2 \end{array} \leq o \right\}$$

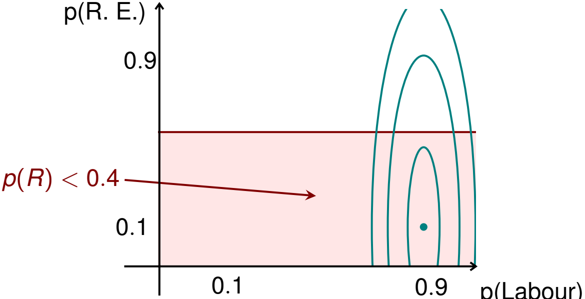
Fact $w^L_L > w^L_R$: more confident in Labour judgements



Expertise

$$c^L(o) = \left\{ q \in \Delta : \begin{array}{l} w_L^L (q(L) - p^L(L))^2 \\ + w_R^L (q(R) - p^L(R))^2 \end{array} \leq o \right\}$$

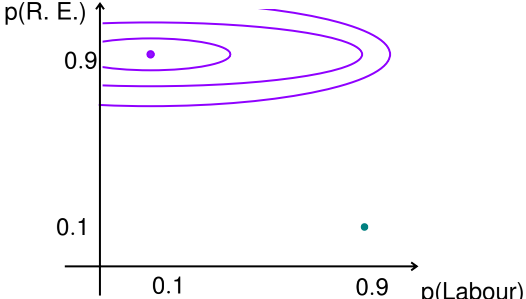
Fact $w_L^L > w_R^L$: more confident in Labour judgements



Expertise

$$c^R(o) = \left\{ q \in \Delta : \begin{array}{l} w^R_L(q(L) - p^L(L))^2 \\ +w^R_R(q(R) - p^L(R))^2 \end{array} \leq o \right\}$$

$w^R_L < w^R_R$: more confident in Real-Estate judgements



Expertise

Confidence: rich enough to capture diverse expertise.

A family: $(\mathbf{w}^i, d, p)$-generated confidence ranking:

$$c^i(o) = \left\{ q \in \Delta : \sum_{j=1}^{m} w_j^i d(q|_{\mathcal{P}_j}, p|_{\mathcal{P}_j}) \leq o \right\}$$

where:

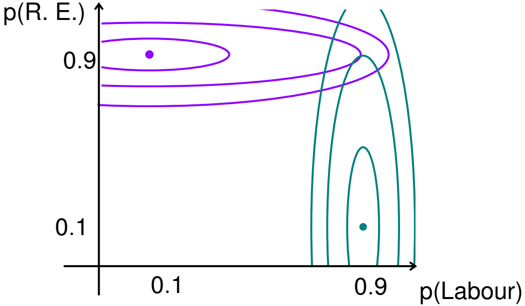
- $\mathcal{P}_j$ Issues: partitions of Ω
- $\mathbf{w}^i$ vector of weights
- d distance, for each partition

Often, can rewrite e.g.

$$c^i(o) = \left\{ q \in \Delta : (\mathbf{q} - \mathbf{p^i})^T \mathbf{D}^i (\mathbf{q} - \mathbf{p^i}) \leq o \right\}$$

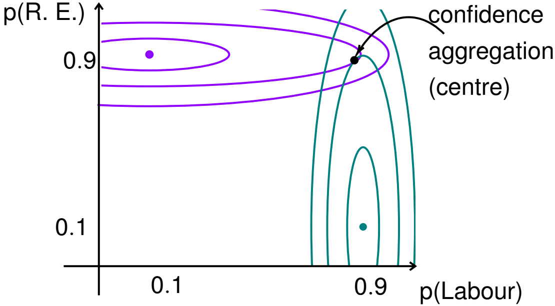
Expertise in Aggregation

Confidence: rich enough to capture diverse expertise.



Expertise in Aggregation

Confidence: rich enough to capture diverse expertise.

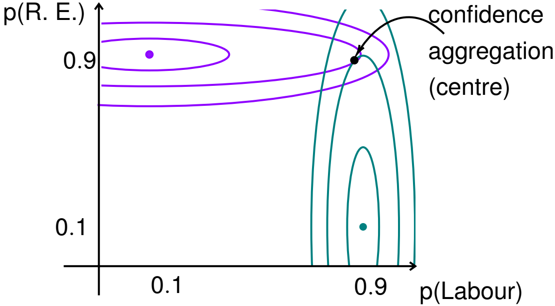


Expertise in Aggregation

Confidence: rich enough to capture diverse expertise.

Confidence aggregation:

- Does justice to varying expertise (Desideratum 2)

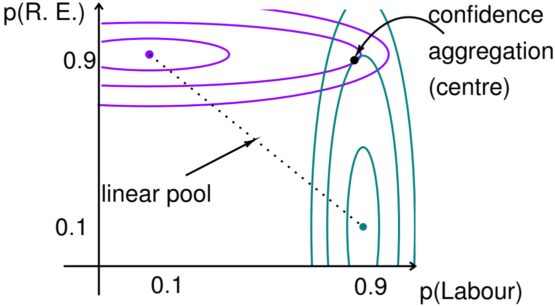


Expertise in Aggregation

Confidence: rich enough to capture diverse expertise.

Confidence aggregation:

- Does justice to varying expertise (Desideratum 2)
- Does not necessarily respect spurious unanimities (Desideratum 1)



Aggregation and Expertise

More generally:

Theorem

Confidence aggregation of $(\mathbf{w}^i, d, p^i)$*-generated confidence rankings:*

$$\text{Centre} = \arg\min_{p \in \Delta} \sum_{i=1}^{n} \sum_{j=1}^{m} w_j^i d(p|_{\mathcal{P}_j}, p^i|_{\mathcal{P}_j})$$

- Within-person expertise diversity & spurious unanimity
- Some cases: $\arg\min_{\mathbf{Aq} \leq \mathbf{r}} \sum_{i=L,R} (\mathbf{q} - \mathbf{p^i})^T \mathbf{D^i} (\mathbf{q} - \mathbf{p^i})$
- Always non-empty (even when issue-dependency)

Aggregation and Expertise

More generally:

Theorem

Confidence aggregation of $(\mathbf{w}^i, d, p^i)$-generated confidence rankings:

$$\text{Centre} = \arg\min_{p \in \Delta} \sum_{i=1}^{n} \sum_{j=1}^{m} w_j^i d(p|_{\mathcal{P}_j}, p^i|_{\mathcal{P}_j})$$

When d convex and the issues sufficiently rich: single probability.

- Within-person expertise diversity & spurious unanimity
- Some cases: $\arg\min_{\mathbf{Aq} \leq \mathbf{r}} \sum_{i=L,R} (\mathbf{q} - \mathbf{p^i})^T \mathbf{D^i} (\mathbf{q} - \mathbf{p^i})$
- Always non-empty (even when issue-dependency)

Aggregation and Expertise

Corollary Generates a new probability aggregation rule:

Expertise-sensitive pooling

$$F^{d}_{\mathcal{P}_1,\ldots,\mathcal{P}_m}(p^1,\ldots,p^n) = \arg\min_{p\in\Delta} \sum_{i=1}^{n}\sum_{j=1}^{m} w^i_j d(p|_{\mathcal{P}_j}, p^i|_{\mathcal{P}_j})$$

for convex d and rich $\{\mathcal{P}_j\}$.

- Within-person expertise diversity & spurious unanimity
- Tractable cases
- Well-defined

Aggregation and Expertise

Corollary Generates a new probability aggregation rule:

Expertise-sensitive pooling

$$F^{d}_{\mathcal{P}_1,\ldots,\mathcal{P}_m}(p^1,\ldots,p^n) = \arg\min_{p\in\Delta} \sum_{i=1}^{n}\sum_{j=1}^{m} w^i_j d(p|_{\mathcal{P}_j}, p^i|_{\mathcal{P}_j})$$

for convex d and rich $\{\mathcal{P}_j\}$.

- Within-person expertise diversity & spurious unanimity
- Tractable cases
- Well-defined
- Resolves a long-standing challenge (Genest and Zidek, 1986; French, 1985).

Plan

Introduction

Confidence Aggregation

Probablity aggregation & Expertise

Deciding (with models)

Characterisation

Dynamic Rationality

Conclusion

Deciding with Experts and Models

Experts p^i

Aggregation Confidence aggregation with ρ_i and $\otimes$:

$$c(o) = \left\{ q : \otimes \rho_i(q, p^i) \leq o \right\}$$

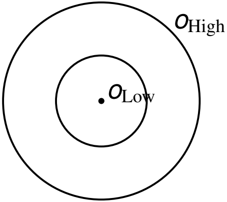
E.g. $\rho_i = w^i d$

Deciding with Experts and Models

Experts p^i

Aggregation Confidence aggregation with ρ_i and $\otimes$:

$$c(o) = \left\{ q : \otimes \rho_i(q, p^i) \leq o \right\}$$



Deciding with Experts and Models

Experts p^i

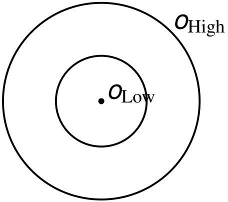
Aggregation Confidence aggregation with ρ_i and $\otimes$:

$$c(o) = \left\{ q : \otimes \rho_i(q, p^i) \leq o \right\}$$

Decision E.g.*

$$\min_{q \in c(D(f))} \mathbb{E}_q u(f)$$

Confidence ranking



*Hill (2013, 2019b)

Deciding with Experts and Models

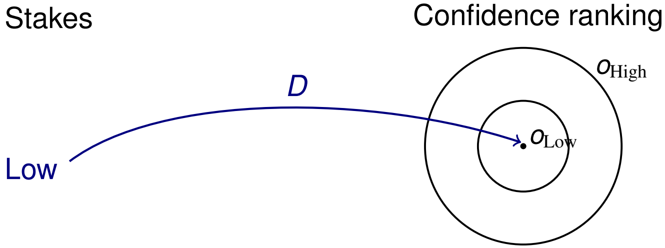
Experts p^i

Aggregation Confidence aggregation with ρ_i and $\otimes$:

$$c(o) = \left\{ q : \otimes \rho_i(q, p^i) \leq o \right\}$$

Decision E.g.*

$$\min_{q \in c(D(f))} \mathbb{E}_q u(f)$$



*Hill (2013, 2019b)

Deciding with Experts and Models

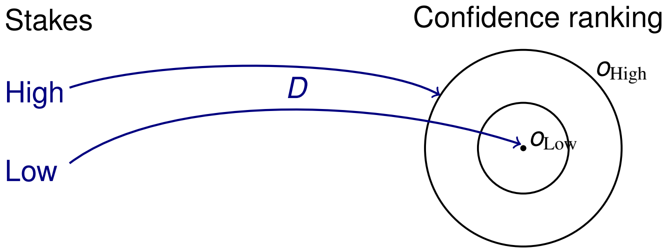
Experts p^i

Aggregation Confidence aggregation with ρ_i and $\otimes$:

$$c(o) = \left\{ q : \otimes \rho_i(q, p^i) \leq o \right\}$$

Decision E.g.*

$$\min_{q \in c(D(f))} \mathbb{E}_q u(f)$$



*Hill (2013, 2019b)

Deciding with Experts and Models

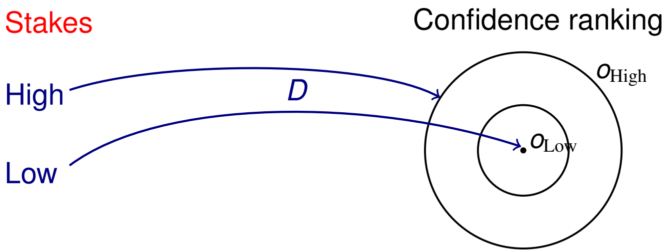
Experts p^i

Aggregation Confidence aggregation with ρ_i and $\otimes$:

$$c(o) = \left\{ q : \otimes \rho_i(q, p^i) \leq o \right\}$$

Decision E.g.

$$\min_{q \in c(D(f))} \mathbb{E}_q u(f)$$



'Overall' Decision rule: ρ, $\otimes$, Stakes

Deciding with Experts and Models

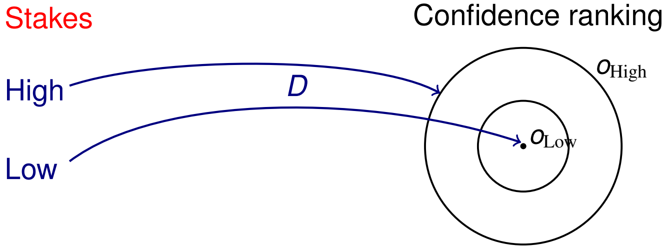
Models $\mathcal{M}$

Aggregation Confidence aggregation with ρ_m and $\otimes$:

$$c(o) = \{q : \otimes_{m \in \mathcal{M}} \rho_m(q, m) \leq o\}$$

E.g. $\rho_m = w(m)d$

$$\min_{q \in c(D(f))} \mathbb{E}_q u(f)$$



'Overall' Decision rule: ρ, $\otimes$, Stakes

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average		

Deciding with Models

ρ	$\bigotimes$	Stakes	'Overall'
Rev. RE	Average		

- Centre = linear pool

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	$\mathbb{E}_{\sum_{\mathcal{M}} \frac{w(m)}{\sum_{m \in \mathcal{M}} w(m)} m} u(f)$

- SEU

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging & SEU[a] $\mathbb{E}_{\sum_{\mathcal{M}} \frac{w(m)}{\sum_{m\in\mathcal{M}} w(m)} m} u(f)$

[a](Steel, 2020)

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging
RE	Average	Low	

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging
RE	Average	Low	Geometric pooling & SEU[a] $\mathbb{E}_{\chi \prod_{\mathcal{M}} m^{\frac{w(m)}{\sum_{\mathcal{M}} w(m)}}} u(f)$

[a](Dietrich, 2021)

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging
RE	Average	Low	Geometric pooling & SEU
RE	Average	High	$\min_{\substack{q\in\Delta: \\ \sum_{\mathcal{M}} w(m)R(q\|m)\leq\eta}} \mathbb{E}_q u(f)$

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging
RE	Average	Low	Geometric pooling & SEU
RE	Average	High	'Average' robust control $\min_{\substack{q\in\Delta: \\ \sum_{\mathcal{M}} w(m)R(q\|m)\leq\eta}} \mathbb{E}_q u(f)$

Variational form

$$\min_{q\in\Delta}\left(\mathbb{E}_q u(f) + \lambda \sum_{\mathcal{M}} w(m) R(q\|m)\right)$$

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging
RE	Average	Low	Geometric pooling & SEU
RE	Average	High	'Average' robust control $\min_{\substack{q\in\Delta: \\ \sum_{\mathcal{M}} w(m)R(q\|m)\leq\eta}} \mathbb{E}_q u(f)$

Variational form

$$\min_{q\in\Delta}\left(\mathbb{E}_q u(f) + \lambda \sum_{\mathcal{M}} w(m)R(q\|m)\right)$$

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging
RE	Average	Low	Geometric pooling & SEU
RE	Average	High	'Average' robust control
RE	Min	High	$\min_{\substack{q\in\Delta: \\ \min_{m\in\mathcal{M}} R(q\|\|m)\leq\eta}} \mathbb{E}_q u(f)$

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging
RE	Average	Low	Geometric pooling & SEU
RE	Average	High	'Average' robust control
RE	Min	High	'Minimum' robust control $\min_{\substack{q\in\Delta: \\ \min_{m\in\mathcal{M}} R(q\|m)\leq\eta}} \mathbb{E}_q u(f)$

Variational form*

$$\min_{q\in\Delta}\left(\mathbb{E}_q u(f) + \lambda \min_{\mathcal{M}} R(q\|m)\right)$$

*(Hansen and Sargent, 2022; Cerreia–Vioglio et al., 2025)

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging
RE	Average	Low	Geometric pooling & SEU
RE	Average Min	High	Model misspecification

- Separate aggregation & decision

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging
RE	Average	Low	Geometric pooling & SEU
RE	Average Min	High	Model misspecification

- Models: not equally good on all issues

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging
RE	Average	Low	Geometric pooling & SEU
RE	Average Min	High	Model misspecification
Exp-S R. RE	Average	Low	Expertise-sensitive pooling & SEU $\mathbb{E}_{\arg\min_{q\in\Delta}\sum_{\mathcal{M}}\sum_{j=1}^{l} w(m,l)R(q\|_{\mathcal{P}_j}\|m\|_{\mathcal{P}_j})}u(f)$

- Models: not equally good on all issues

Deciding with Models

ρ	$\bigotimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging
RE	Average	Low	Geometric pooling & SEU
RE	Average Min	High	Model misspecification
Exp-S R. RE	Average	Low	Expertise-sensitive pooling & SEU
Exp-S RE	Min	High	Expertise-sensitive 'min.' robust control $\min_{\substack{q\in\Delta: \\ \min_{\mathcal{M}} \sum_{j=1}^{l} w(m,l) R(q\|_{\mathcal{P}_j} \|\| m\|_{\mathcal{P}_j}) \leq \eta}} \mathbb{E}_q u(f)$

- Models: not equally good on all issues

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging
RE	Average	Low	Geometric pooling & SEU
RE	Average Min	High	Model misspecification
Exp-S R. RE	Average	Low	Expertise-sensitive pooling & SEU
Exp-S RE	Min	High	Expertise-sensitive 'min.' robust control

Variational form

$$\min_{q \in \Delta} \left(\mathbb{E}_q u(f) + \lambda \min_{\mathcal{M}} \sum_{j=1}^{l} w(m,l) R(q|_{\mathcal{P}_j} \| m|_{\mathcal{P}_j}) \right)$$

Deciding with Models

ρ	$\otimes$	Stakes	'Overall'
Rev. RE	Average	Low	Bayesian Model Averaging
RE	Average	Low	Geometric pooling & SEU
RE	Average Min	High	Model misspecification
Exp-S R. RE	Average	Low	Expertise-sensitive pooling & SEU
E-S RE	Average Min	High	Expertise-sensitive model misspecification

Deciding with models

Confidence aggregation $+$ decision:

- separates aggregation from decision
- recoups existing approaches to deciding with (multiple) models
- reveals hitherto unnoticed relationships
- resolves the expertise-diversity challenge for model misspecification

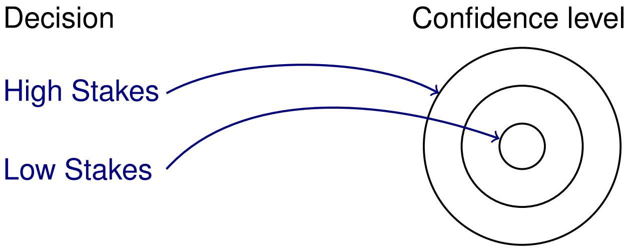
Plan

Decision Setup

AA-like framework $\mathcal{A}$ acts: $\Omega \to \mathcal{X}$ $[f, g, \ldots]$

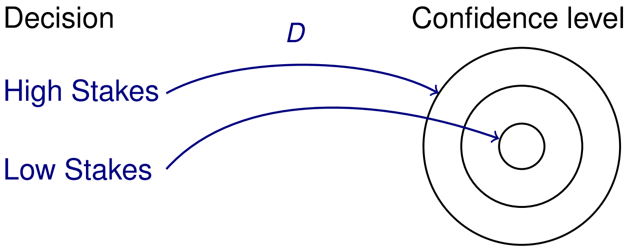
Decision Setup

AA-like framework $\mathcal{A}$ acts: $\Omega \to \mathcal{X}$ $[f, g, \ldots]$



Decision Setup

AA-like framework $\mathcal{A}$ acts: $\Omega \rightarrow \mathcal{X}$ $[f, g, \ldots]$



Cautiousness coefficient *D*:

- Reflects uncertainty attitude

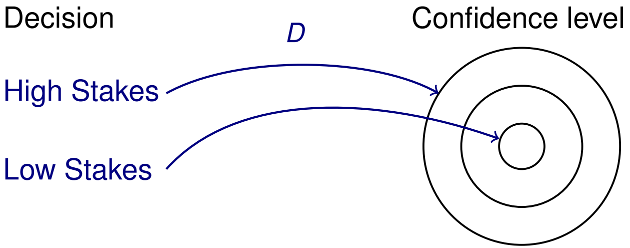
Decision Setup

AA-like framework $\mathcal{A}$ acts: $\Omega \to \mathcal{X}$ $[f, g, \ldots]$

Assumption 1 (Confidence Preferences) For all agents: $f \succ g$ iff

$$\mathbb{E}_p u^i(f) > \mathbb{E}_p u^i(g) \qquad \text{for all } p \in c(D^i(f,g))$$

$D^i : \mathcal{A}^2 \to O$



Cautiousness coefficient D:

- Reflects uncertainty attitude

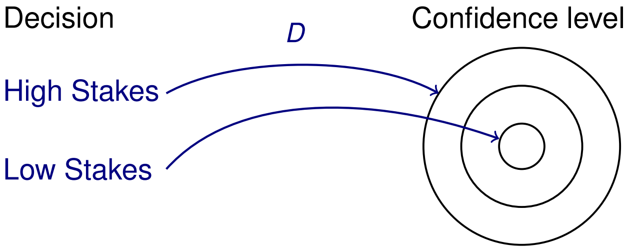
Decision Setup

AA-like framework $\mathcal{A}$ acts: $\Omega \to \mathcal{X}$ $[f, g, \dots]$

Assumption 1 (Confidence Preferences) For all agents: $f \succ g$ iff

$$\mathbb{E}_p u^i(f) > \mathbb{E}_p u^i(g) \qquad \text{for all } p \in c(D^i(f,g))$$

$D^i : \mathcal{A}^2 \to O$



Cautiousness coefficient D:

- Reflects uncertainty attitude

Decision Setup

AA-like framework $\mathcal{A}$ acts: $\Omega \to \mathcal{X}$ $[f, g, \dots]$

Assumption 1 (Confidence Preferences) For all agents: $f \succ g$ iff

$$\mathbb{E}_p u^i(f) > \mathbb{E}_p u^i(g) \qquad \text{for all } p \in c(D^i(f, g))$$

$D^i : \mathcal{A}^2 \to O$

Assumption 2 Identical tastes: same u^i, D^i.

Decision Setup

AA-like framework $\mathcal{A}$ acts: $\Omega \to \mathcal{X}$ $[f, g, \ldots]$

Assumption 1 (Confidence Preferences) For all agents: $f \succ g$ iff

$$\mathbb{E}_p u^i(f) > \mathbb{E}_p u^i(g) \qquad \text{for all } p \in c(D^i(f,g))$$

$D^i : \mathcal{A}^2 \to O$

Assumption 2 Identical tastes: same u^i, D^i.

Facts Can define

- stakes levels $\mathcal{S}$ $[s_1, s_2 \ldots]$
- preferences $\succ_s$ at stakes level s
    - $f \succ_s g$: f preferred when evaluated 'as if' stakes s

Characterisation

Main axiom

Definition For stakes levels $\mathbf{s} = (s_1, \dots s_n)$:

- $\mathbf{s}$ *exhibits consensus* if $\succ_{\mathbf{s}} = \bigcup_{i=1}^{n} \succ_{s_i}^{i}$ non-contradictory.
- $\succ^0$ *respects the consensus* $\succ_{\mathbf{s}}$ *at* s if $\mathbf{s}$ exhibits consensus and $\succ_s^0 \subseteq \succ_{\mathbf{s}}$.

Characterisation

Main axiom

Definition For stakes levels $\mathbf{s} = (s_1, \ldots s_n)$:

- $\mathbf{s}$ *exhibits consensus* if $\succ_{\mathbf{s}} = \bigcup_{i=1}^{n} \succ_{s_i}^{i}$ non-contradictory.
- $\succ^0$ *respects the consensus* $\succ_{\mathbf{s}}$ *at s* if $\mathbf{s}$ exhibits consensus and $\succ_s^0 \subseteq \succ_{\mathbf{s}}$.

Axiom (Issue-wise Pareto)

$$f \succ^i g \text{ for all } i \qquad \Rightarrow \qquad f \succ^0 g$$

Characterisation

Main axiom

Definition For stakes levels $\mathbf{s} = (s_1, \dots s_n)$:

- $\mathbf{s}$ *exhibits consensus* if $\succ_{\mathbf{s}} = \bigcup_{i=1}^{n} \succ_{s_i}^{i}$ non-contradictory.
- $\succ^0$ *respects the consensus* $\succ_{\mathbf{s}}$ *at* s if $\mathbf{s}$ exhibits consensus and $\succ_s^0 \subseteq \succ_{\mathbf{s}}$.

Axiom (Issue-wise Pareto)

$f \succ^i g$ for all i $\Rightarrow$ $f \succ^0 g$

Axiom (Corpus-wise Pareto)

$f \succ_{\mathbf{s}} g$ for all $\mathbf{s}$ such that $\succ^0$ respects the consensus at s $\Rightarrow$ $f \succ_s^0 g$

Preference held under all respected consensuses $\Rightarrow$ held by the group.

Characterisation

Result

Theorem

Corpus-wise Pareto *Technical ax.*	$\Leftrightarrow$	*There exists $\otimes$ such that* *Confidence aggregation* *up to convex closure*

Furthermore, unique minimal $\otimes$.

$\rightarrow$ Confidence aggregation respects the right consensuses (Desideratum 1)

Plan

Dynamic Rationality

Another desideratum for aggregation:

Dynamic Rationality Aggregation commutes with update (Genest and Zidek, 1986; Dietrich, 2021)

*Generalises Rényi, 1955; Blume et al., 1991; Ortoleva, 2012.

Dynamic Rationality

Another desideratum for aggregation:

Dynamic Rationality Aggregation commutes with update (Genest and Zidek, 1986; Dietrich, 2021)

Here: confidence update (Hill, 2022)*

- $c|_{\rho_E}$: update of c by E.

Theorem

$$F_{\otimes}(c_1|\rho_E, \ldots, c_1|\rho_E) = F_{\otimes}(c_1, \ldots, c_n)|\rho_E$$

$\rightarrow$ Dynamic rationality satisfied by confidence aggregation.

*Generalises Rényi, 1955; Blume et al., 1991; Ortoleva, 2012.

Summing up

Confidence aggregation:

- respects the right consensuses (Corpus-wise Pareto)
- avoids respecting spurious unanimities
- can integrate within-person expertise diversity
- generates a new pooling rule accounting for expertise diversity
- recoups model misspecification approaches
- suggests more refined ones
- satisfies dynamic rationality
- outperforms linear pooling in a Cog Psy-inspired situations

Thank you.

hill@hec.fr

www.hec.fr/hill

Further details on confidence:

- Confidence in Beliefs and Rational Decision Making, *Economics and Philosophy*, 32, 2019.
- Confidence and Decision, *Games and Economic Behavior*, 82, 2013.
- Incomplete Preferences and Confidence, *Journal of Mathematical Economics*, 65, 2016.
- Climate Change Assessments: Confidence, Probability and Decision, *Philosophy of Science*, 84, 2017 (with R. Bradley, C. Helgeson).
- Combining probability with qualitative degree-of-certainty metrics in assessment, *Climatic Change* 149, 2018 (with R. Bradley, C. Helgeson)
- Confidence in belief, weight of evidence and uncertainty reporting. *Proceedings of Machine Learning Research*, 103, 2019.
- Updating Confidence in Beliefs, *Journal of Economic Theory*, 2022.

Confidence Elicitation Web Tool http://confidence.hec.fr/app/

Confidence, consensus and aggregation

Brian Hill

hill@hec.fr

www.hec.fr/hill

CNRS & HEC Paris

TUS XI
2025

Dedicated to the memory of Philippe Mongin

References I

Blume, L., Brandenburger, A., and Dekel, E. (1991). Lexicographic probabilities and equilibrium refinements. *Econometrica: Journal of the Econometric Society*, pages 81–98.

Bommier, A., Fabre, A., Goussebaïle, A., and Heyen, D. (2021). Disagreement aversion. *Available at SSRN 3964182*.

Bradley, R. (2017a). *Decision theory with a human face*. Cambridge University Press.

Bradley, R. (2017b). Learning from others: conditioning versus averaging. *Theory and Decision*, pages 1–16.

Cerreia–Vioglio, S., Hansen, L. P., Maccheroni, F., and Marinacci, M. (2025). Making decisions under model misspecification. *Review of Economic Studies*, page rdaf046. Publisher: Oxford University Press.

Chateauneuf, A. and Faro, J. H. (2009). Ambiguity through confidence functions. *J. Math. Econ.*, 45:535–558.

References II

Dietrich, F. (2021). Fully Bayesian Aggregation. *Journal of Economic Theory*, 194:105255. Publisher: Elsevier.

French, S. (1985). Group consensus probability distributions: A critical survey in Bayesian statistics. *Bayesian statistics*, 2.

Genest, C. and Zidek, J. V. (1986). Combining Probability Distributions: A Critique and an Annotated Bibliography. *Statistical Science*, 1(1):114–135.

Hansen, L. P. and Sargent, T. J. (2008). *Robustness*. Princeton university press.

Hansen, L. P. and Sargent, T. J. (2022). Structured ambiguity and model misspecification. *Journal of Economic Theory*, 199:105165.

Hill, B. (2013). Confidence and decision. *Games and Economic Behavior*, 82:675–692.

References III

Hill, B. (2019a). Confidence in Belief, Weight of Evidence and Uncertainty Reporting. In *Proceedings of the Eleventh International Symposium on Imprecise Probabilities: Theories and Applications*, pages 235–245. PMLR. ISSN: 2640-3498.

Hill, B. (2019b). Confidence in Beliefs and Rational Decision Making. *Economics & Philosophy*, 35(2):223–258.

Hill, B. (2022). Updating confidence in beliefs. *Journal of Economic Theory*, 199:105209.

Klibanoff, P., Marinacci, M., and Mukerji, S. (2005). A Smooth Model of Decision Making under Ambiguity. *Econometrica*, 73(6):1849–1892.

Maccheroni, F., Marinacci, M., and Rustichini, A. (2006). Ambiguity Aversion, Robustness, and the Variational Representation of Preferences. *Econometrica*, 74(6):1447–1498.

References IV

Manski, C. F. (2013). *Public Policy in an Uncertain World : Analysis and Decisions*. Harvard University Press, Cambridge, Mass.

Mongin, P. (1995). Consistent Bayesian Aggregation. *Journal of Economic Theory*, 66(2):313–351.

Mongin, P. (2016). Spurious unanimity and the Pareto principle. *Economics & Philosophy*, 32(3):511–532. Publisher: Cambridge University Press.

Mongin, P. and Pivato, M. (2020). Social preference under twofold uncertainty. *Economic Theory*, 70(3):633–663. Publisher: Springer.

Ortoleva, P. (2012). Modeling the Change of Paradigm: Non-Bayesian Reactions to Unexpected News. *American Economic Review*, 102(6):2410–2436.

Rényi, A. (1955). On a new axiomatic theory of probability. *Acta Mathematica Hungarica*, 6(3-4):285–335.

References V

Steel, M. F. (2020). Model averaging and its use in economics. *Journal of Economic Literature*, 58(3):644–719.

Technical axioms

Axiom (Consensus-based beliefs)

For every stakes level $s \in \mathcal{S}$ and acts $f, g \in \mathcal{A}$, if $f \not\succ^0_{s'} g$ for every stakes level s' such that some consensus $\succ_{\mathbf{s}}$ is respected at s', then $f \not\succ^0_s g$.

Axiom (Non-degeneracy)

There exists a tuple of stakes levels $\mathbf{s}$ exhibiting consensus.

back