Profs. Lovo & Ors **Financial Economics** **HEC Paris**

Final Exam – November 2, 2009

2 hours / Open-book & open-notes / Calculators are allowed / No laptops or cell phones

LAST NAME: _____ FIRST NAME: _____

Question 1 [30 points]: circle the correct answer AND DO NOT FORGET TO RETURN THIS PAGE WITH YOUR EXAM BOOK!

You invest your savings in a bank account paying an annual rate of 3 %. The compounding frequency is every quarter (three months). This means that in one year your bank account increases by

- Exactly 3%
- Slightly more than 3%
- Slightly less than 3%
- About 9%
- About 12%

If the discount rate is positive, would you rather prefer to pay € 10 000 today or to pay an ordinary monthly annuity of € 1 000 lasting ten months?

- The options are equivalent.
- € 10 000 today.
- The annuity.
- Impossible to tell.

When computing the Net Present Value of a project, one should include all past non-recoverable cash-flows only if these costs are actually related to the project.

- True
- False
- Impossible to tell

The fact that the internal rate of return of a project is lower than the opportunity cost of capital, necessarily implies that the NPV of the project is negative.

- True
- False
- Impossible to tell

A strategy consisting in investing in many different risky assets allows to

- Obtain a risk-less portfolio.
- Eliminate the systemic risk of the portfolio.
- Minimize the idiosyncratic risk of the portfolio.
- Maximize the expected return of the portfolio

Portfolio D is positively correlated with the market portfolio, whereas portfolio E is negatively correlated with market portfolio. Then, according to CAPM,

- Portfolio D is efficient
- The expected return of portfolio E is negative.
- The risk premium on portfolio D is larger than the risk premium on portfolio E.
- Portfolio E is below the SML.

Final Exam questions continue on page 3

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Final Exam questions continue on the next page.

Question 2 [40 points]:

You are asked to evaluate an investment project that will last exactly 10 years. Undertaking this project requires an investment of 30 million in equipment at date t=0. The sales projections are based on a market-demand assessment study that was conducted a year ago and for which 500 thousand was paid previously. The annual operating expenses, which the company will start to incur with date t=1, are 6 million per year, and will remain constant through the years. The company uses straight-line depreciation (i.e., the company depreciates the equipment that was bought at a rate of 3 million per year). The company operates in a part of the country where there are no taxes for 10 years for any new investment made. The opportunity cost of capital for this project is 12% per year.

a) If the sales revenue is 10 million per year, what are the annual cash flows for this project?

b) Should you undertake this project? Why? Clearly show all of your calculations.

c) Suppose that the sales at the end of the first-year (at t=1) will be 10 million, and that the sales in subsequent years will grow at a constant rate of 5% per year. Would your answer to part b) change? Clearly show all of your calculations.

d) Suppose now that the sales at the end of the first-year (at t=1) will be 10 million, that the sales in subsequent years will grow at a constant rate of 5% per year for the first 4 years (i.e., from t=1 to t=5), but that the sales thereafter will decrease at a constant rate of 5% until t=10. Would your answer change: would you still undertake this project? Clearly show all of your calculations.

Final Exam questions continue on the next page

Question 3 [45 points]:

Consider an economy with two risky assets s_1 and s_2. Suppose that the expected rate of return of asset 1 is 10% and that σ_1=10%. The expected rate of return of asset 2 is 5% and σ_2=20%. The correlation coefficient between s_1 and s_2 is $\rho_{1,2} = 0.875$.

a) What is the composition, the expected return and the standard deviation of the minimum variance portfolio?

b) Represent in a graph all the combination risk/return that can be obtained with portfolios containing s_1 and s_2. Clearly identify s_1, s_2 and the minimum variance portfolio.

c) Harry is a mean-variance investor with risk aversion A=3. Harry is not allowed to short-sell asset s_2 while he can short-sell asset s_1.

c.1) In a new graph, represent risk/return combinations that are available to Harry.

c.2) What is the composition of Harry's optimal portfolio?

d) Let us introduce a risk free asset s_f with return r_f =10%. Now all short-sales are allowed:

d.1) In a third graph represent, s_1, s_2 and s_f , the set of risk/return combination that can be obtained with portfolios composed only of s_1 and s_2, the tangency portfolio T, and the capital allocation line.

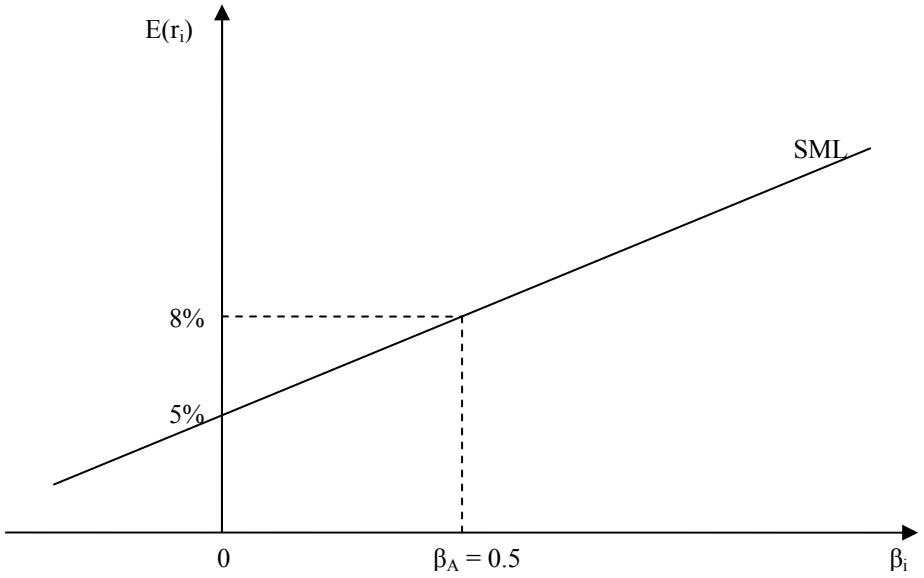
d.2) Let portfolio D has the composition X_D ={x_1=1/3, x_2=1/3, x_f=1/3,}. Is portfolio D efficient? Explain carefully but concisely why it is or why it is not.

d.3) What is the composition of Harry's optimal portfolio in this case (recall that Harry cannot short-sell asset s_2)

Final Exam questions continue on the next page

Question 4 [45 points]:

Consider an economy with many risky assets and one risk-free asset. Suppose that CAPM assumptions hold. You are given the following graph:



a) What is the risk-free asset's return in this economy?

b) What is the expected return of the market portfolio in this economy?

c) What must be the composition of the efficient portfolio B that has a beta of 1.5?

d) Knowing that $\rho_{A,M} = 0.25$ and $\sigma_A = 40\%$, clearly indicate the portfolio B's position in the total risk – expected return space (i.e., in the (σ , E(r)) space). Mark your graph's horizontal and vertical axes; clearly label all the important curves, lines, line-segments; indicate the numerical values that correspond to portfolio B's position along the horizontal and vertical axes.

e) Can you also show portfolio A in the graph you drew in part (d)? If yes, clearly indicate point A on the same graph the same way you did portfolio B. If no, briefly explain why.

End of the Exam

Profs. Lovo & Ors **Financial Economics** **HEC Paris**

Final Exam – November 2, 2009 – ANSWER KEY

Question 1 [30 points]: circle the correct answer AND DO NOT FORGET TO RETURN THIS PAGE WITH YOUR EXAM BOOK!

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- Exactly 3%
- Slightly more than 3% ←
- About 9%
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If the discount rate is positive, would you prefer to pay € 10 000 today or to pay an ordinary monthly annuity of €1 000 lasting ten months?

- It is the same.
- € 10 000 today.
- The annuity. ←
- Impossible to tell.

When computing the Net Present Value of a project, one should include all past non-recoverable cash-flows only if these costs are actually related to the project.

- True
- False ←
- Impossible to tell

The fact that the internal rate of return of a project is lower than the opportunity cost of money, necessarily implies that the NPV of the project is negative.

- True
- False ←
- Impossible to tell

A strategy consisting in investing in many different risky assets allows to

- Obtain a risk less portfolio.
- Eliminate the systemic risk of the portfolio.
- Minimize the idiosyncratic risk of the portfolio. ←
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Portfolio D is positively correlated with the market portfolio, whereas asset E is negatively correlated with market portfolio. Then, according to CAPM,

- Portfolio D is efficient
- The expected return of portfolio E is negative.
- The risk premium on portfolio D is larger than the risk premium on portfolio E. ←
- Portfolio E is below the SML.

Question 2 [40 points]:

You are asked to evaluate an investment project that will last exactly 10 years. Undertaking this project requires an investment of 30 million in equipment at date t=0. The sales projections are based on a market-demand assessment study that was conducted a year ago and for which 500 thousand was paid previously. The annual operating expenses, which the company will start to incur with date t=1, are 6 million per year, and will remain constant through the years. The company uses straight-line depreciation (i.e., the company depreciates the equipment that was bought at a rate of 3 million per year). The company operates in a part of the country where there are no taxes for 10 years for any new investment made. The opportunity cost of capital for this project is 12% per year.

a) If the sales revenue is 10 million per year, what are the annual cash flows for this project?

* 0.5 million paid last year: sunk cost and should not be taken into account
* Depreciation does not provide any tax-shield as there are no taxes to pay.

At t=0: $CF_0 = -30$

From t=1 to t=10: $CF_t = 10 - 6 = 4$ million per year

b) Should you undertake this project? Why? Clearly show all of your calculations.

$$NPV_0 = -30 + \frac{4}{0.12}\left(1 - \frac{1}{(1+0.12)^{10}}\right) = -30 + 22.60 = -7.40 \text{ million} < 0$$

do not undertake this project with negative NPV. Alternatively:

$$NPV_0 = -30 + \frac{10}{0.12}\left(1 - \frac{1}{(1+0.12)^{10}}\right) - \frac{6}{0.12}\left(1 - \frac{1}{(1+0.12)^{10}}\right) = -30 + 56.50 - 33.90 = -7.40 \text{ million}$$

c) Suppose that the sales at the end of the first-year (at t=1) will be 10 million, and that the sales in subsequent years will grow at a constant rate of 5% per year. Would your answer to part b) change? Clearly show all of your calculations.

$$NPV_0 = -30 + \frac{10}{0.12 - 0.05}\left(1 - \left(\frac{1+0.05}{1+0.12}\right)^{10}\right) - \frac{6}{0.12}\left(1 - \frac{1}{(1+0.12)^{10}}\right)$$

$$NPV_0 = -30 + 67.93 - 33.90 = +4.03 \text{ million} > 0$$

Yes, you would undertake the project since NPV is positive.

d) Suppose now that the sales at the end of the first-year (at t=1) will be 10 million, that the sales in subsequent years will grow at a constant rate of 5% per year for the first 4 years (i.e., from t=1 to t=5), but that the sales thereafter will <u>decrease</u> at a constant rate of 5% until t=10. Would your answer change: would you still undertake this project? Clearly show all of your calculations.

$$NPV_0 = -30 + \frac{10}{0.12 - 0.05}\left(1 - \left(\frac{1+0.05}{1+0.12}\right)^{5}\right) + \frac{\frac{10 \times 1.05^4 \times (1-0.05)}{0.12 - (-0.05)}\left(1 - \left(\frac{1+(-0.05)}{1+0.12}\right)^{5}\right)}{(1+0.12)^5}$$

$$- \frac{6}{0.12}\left(1 - \frac{1}{(1+0.12)^{10}}\right)$$

$$NPV_0 = -30 + 39.40 + 21.62 - 33.90 = -2.88 \text{ million} < 0$$

Alternatively:

$$NPV_0 = -30 + \frac{10}{0.12-0.05}\left(1-\left(\frac{1+0.05}{1+0.12}\right)^4\right) + \frac{\frac{10\times 1.05^4}{0.12-(-0.05)}\left(1-\left(\frac{1+(-0.05)}{1+0.12}\right)^6\right)}{(1+0.12)^4}$$

$$-\frac{6}{0.12}\left(1-\frac{1}{(1+0.12)^{10}}\right)$$

$$NPV_0 = -30 + 32.50 + 28.52 - 33.90 = -2.88 \text{ million} < 0$$

Question 4 [45 points]:

Consider an economy with two risky assets s_1 and s_2. Suppose that the expected rate of return of asset 1 is 10% and that σ_1=10%. The expected rate of return of asset 2 is 5% and σ_2=20%. The correlation coefficient between s_1 and s_2 is $\rho_{1,2} = 0.875$.

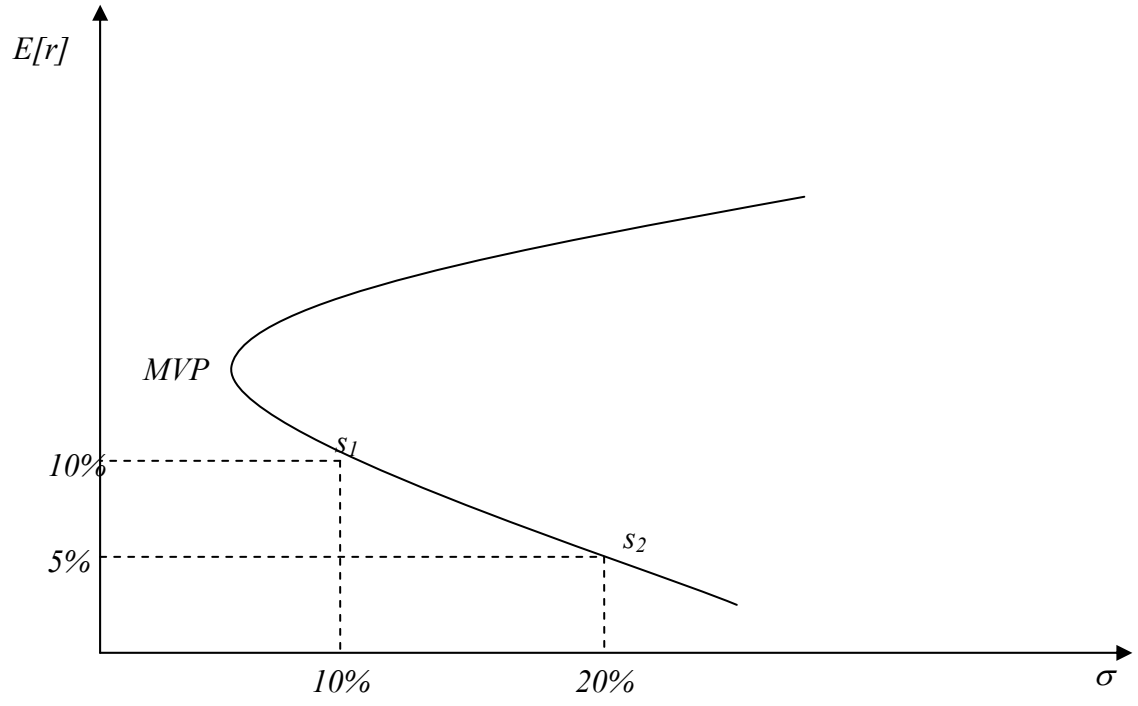
a) What is the composition, the expected return and the standard deviation of the minimum variance portfolio?

$x_{1,MVP} = 0.2(0.2 - 0.875 * 0.1)/(0.2\text{^}2+0.1\text{^}2- 2*0.875*0.1*0.2) = 1.5$
$x_{2,MVP} = -0.5$

$E[r_{MVP}] = 1.5*10\%-0.5*5\% = 12.5\%$

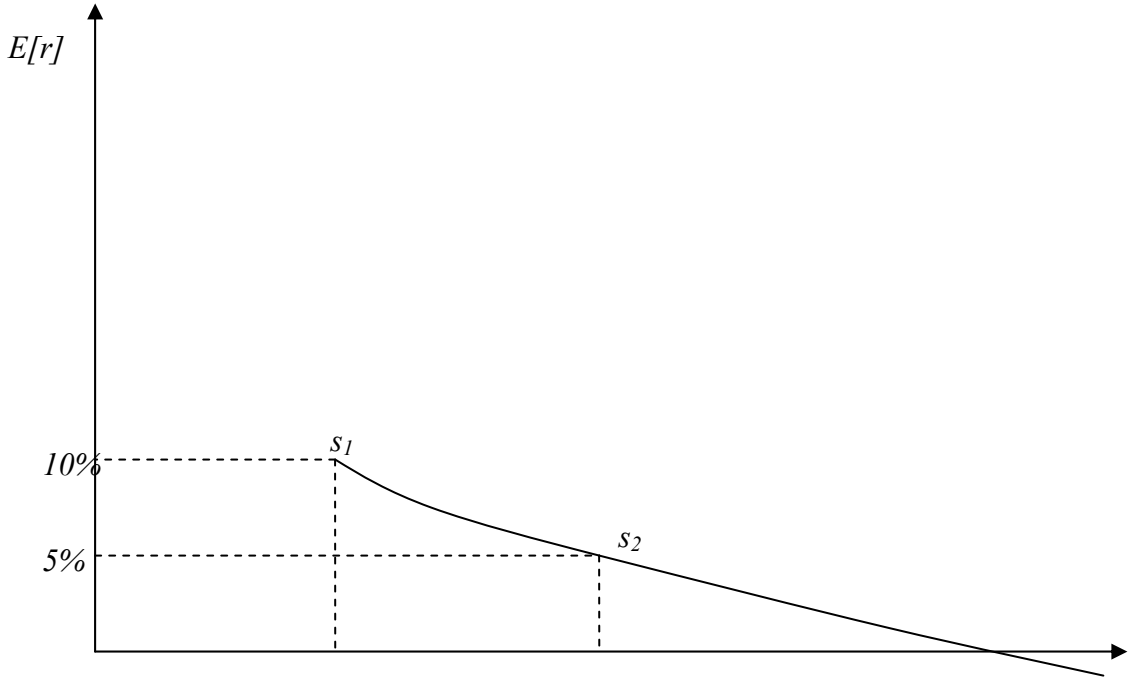
$\sigma_{MVP} =((1.5*.1)\text{^}2+(-0.5*0.2)\text{^}2-2*1.5*0.5*0.875*0.2*0.1)\text{^}(1/2) =7.9\%$

b) Represent in a graphic all the combination risk/return that can be obtained by combining s_1 and s_2. Clearly identify s_1, s_2 and the minimum variance portfolio.



c) Harry is a mean-variance investor with risk aversion A=3. Harry is not allowed to short sell asset s_2 while he can short sell asset s_1.

c.1) In a new graph, represent the combinations risk/return that are available to Harry.

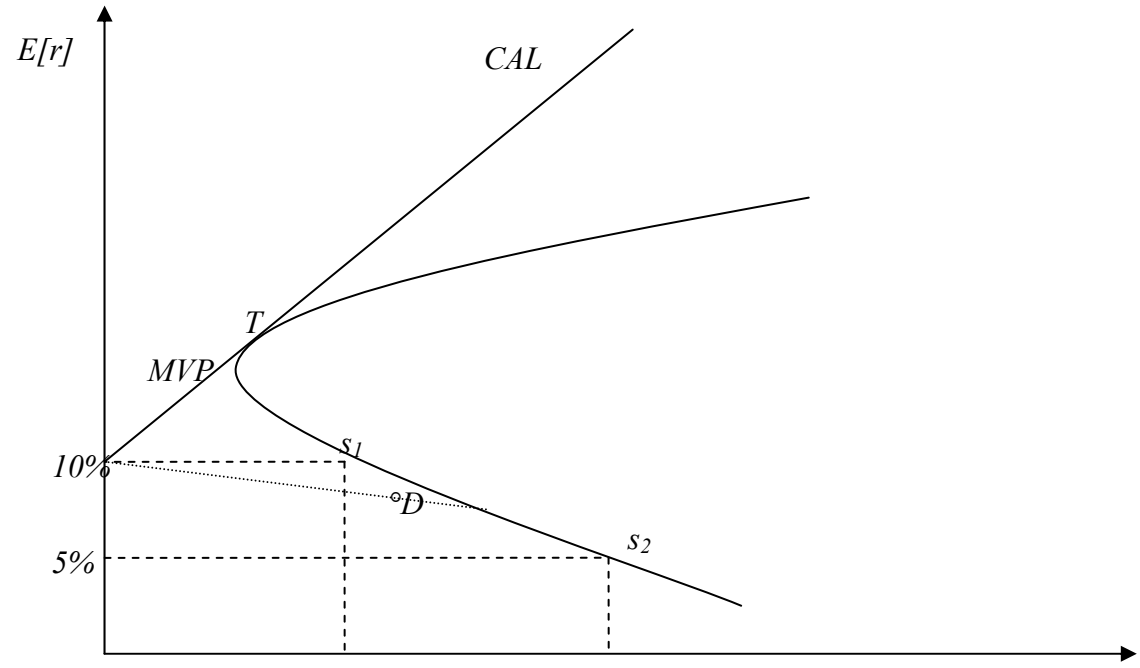


c.2) What is the composition of Harry optimal portfolio?

Harry will invest all his wealth in asset 1 as this is the only efficient portfolio given his short-selling constraint.

d) Let introduce a risk free asset s_f with return is r_f =10%. All short sales are possible.

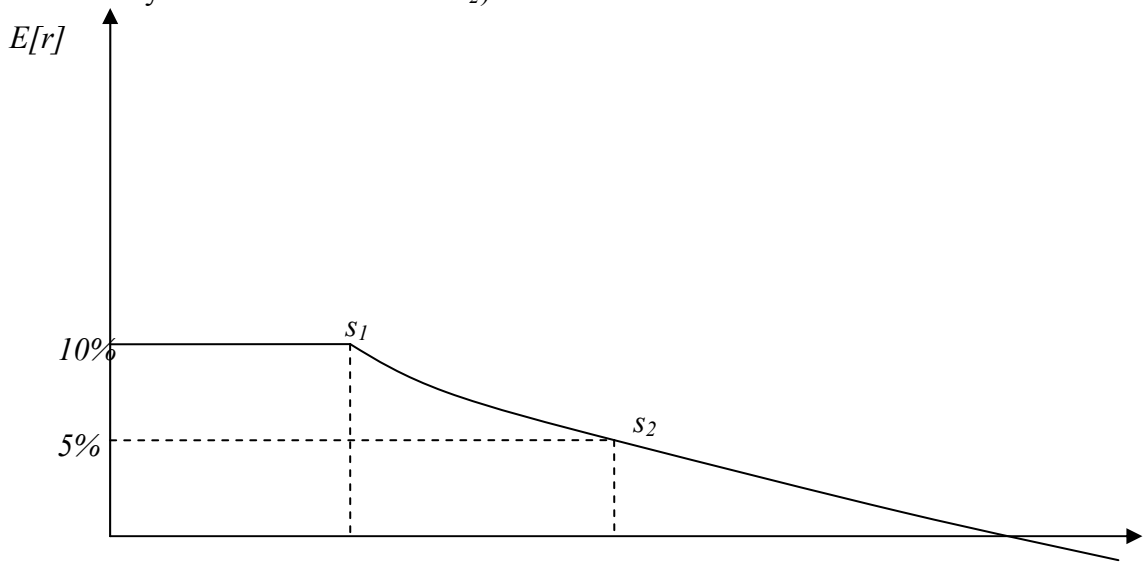
d.1) In a third graph represents, s_1, s_2 and s_f , the set of risk/return combination that can be obtained with portfolios composed only of s_1 and s_2 , the tangency portfolio, and the capital allocation line.



d.2) Let portfolio D has the composition $X_D = \{x_1=1/3,\ x_2=1/3,\ x_f=1/3,\}$. Is portfolio D efficient? Explain carefully but concisely why it is or why it is not.

The tangency portfolio is obtained bu short selling 2 and to buy 1. An efficient portfolio is a combination of the risk free asset and the tangency portfolio. Hence it must have a negative weight of asset 2 if it has a positive weight of asset 1. Portfolio D has positive weight of asset 1 ans 2 hence it is not combination of T and s_f, hence it is not efficient. (See picture)

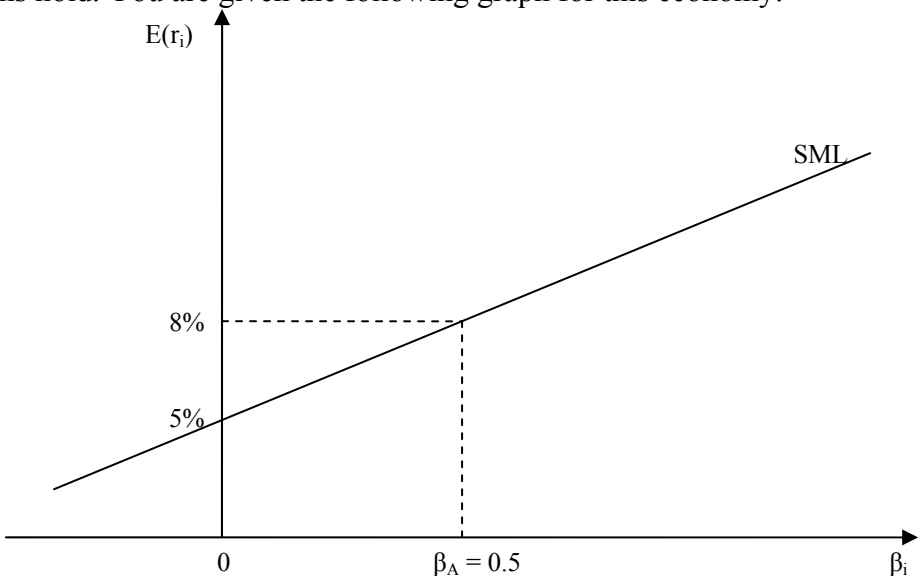
d.3) What is the composition of Harry optimal portfolio in this case?. (recall that Harry cannot short sell asset s_2)



Now Harry can reach all points on the the line and the only efficient portfolio is obtained by investing all his wealth in s_f.

Question 4 [45 points]:

Consider an economy with many risky assets and one risk-free asset. Suppose that CAPM assumptions hold. You are given the following graph for this economy:



a) What is the risk-free asset's return in this economy?

$\beta_f = 0$ ➔ $r_f = 5\%$ (the intercept of the SML)

b) What is the expected return of the market portfolio in this economy?

$$E(r_A) = r_f + \beta_A(E(r_M) - r_f) \rightarrow E(r_M) = \frac{E(r_A) - r_f}{\beta_A} + r_f = \frac{0.08 - 0.05}{0.5} + 0.05 = 0.11 = 11\%$$

c) What must be the composition of the efficient portfolio B that has a beta of 1.5?

An efficient portfolio must be on the CML if CAPM assumptions apply. A portfolio with a beta of 1.5 that is an efficient portfolio must be a combination of the risk-free asset and the market portfolio:

$\beta_B = w_f\beta_f + (1 - w_f)\beta_M$ ➔ $1.5 = w_f \times 0 + (1 - w_f) \times 1$ ➔ $w_f = -0.5$ ➔ $w_M = +1.5$

d) Knowing that $\rho_{A,M} = 0.25$ and $\sigma_A = 40\%$, clearly indicate the portfolio B's position in the total risk – expected return space (i.e., in the σ – E(r) space). Mark your graph's horizontal and vertical axes; clearly label all the important curves, lines, line-segments; indicate the numerical values that correspond to portfolio B's position along the horizontal and vertical axes.

$$\beta_A = \frac{\text{cov}(r_A, r_M)}{\sigma_M^2} = \frac{\rho_{A,M} \times \sigma_A \times \sigma_M}{\sigma_M^2} = \frac{\rho_{A,M} \times \sigma_A}{\sigma_M}$$

➔ $$\sigma_M = \frac{\rho_{A,M} \times \sigma_A}{\beta_A} = \frac{0.25 \times 0.40}{0.5} = 0.20 = 20\%$$

➔ the equation of the CML:

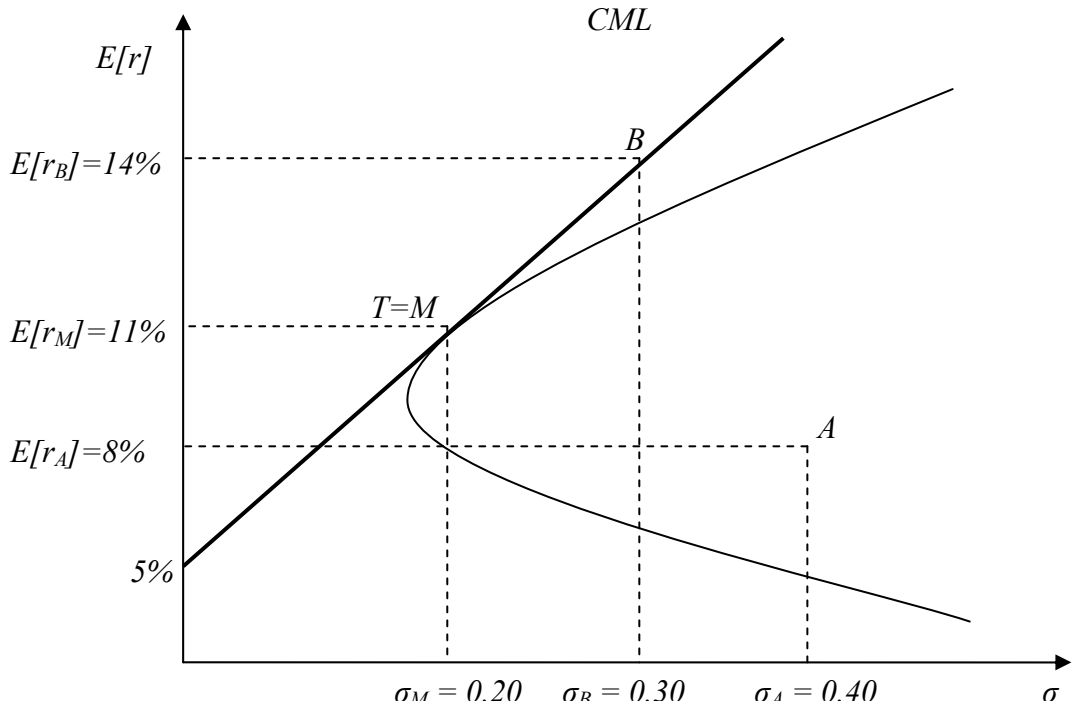
$$E(r_p) = r_f + \lambda_M\sigma_p = r_f + \left(\frac{E(r_M) - r_f}{\sigma_M}\right) \times \sigma_p = 0.05 + \frac{0.11 - 0.05}{0.20} \times \sigma_p = 0.05 + 0.3\sigma_p$$

➔ $E(r_B) = r_f + \lambda_M\sigma_B = 0.05 + 0.3\sigma_B$

➔ But CAPM suggests that:

$$E(r_B) = r_f + \beta_B(E(r_M) - r_f) = 0.05 + 1.5 \times (0.11 - 0.05) = 0.14$$

➔ $$\sigma_B = \frac{E(r_B) - r_f}{\lambda_M} = \frac{0.14 - 0.05}{0.3} = 0.30 = 30\%$$



e) Can you also show portfolio A in the graph you drew in part (d)? If yes, clearly indicate point A on the same graph the same way you did portfolio B. If no, briefly explain why.

Yes, Portfolio A has an expected return of 8% and a standard deviation of 40%. It is below the CML since the expected return of an efficient portfolio with risk 40% is equal to

$$r_f + \lambda\sigma_A = 5\%+(6\%/20\%)*40\%= 17\% >8\%=E[r_A]$$

Also since $E[r_A]< E[r_M]$ and $\sigma_A >\sigma_M$, Portfolio A must be below the efficient portfolio frontier.