HEC, Paris   Financial Economics   S. Lovo

Formulae

1. **Time Value of Money**

   r_e: Effective annual rate (EAR)

   r_a: Percentage annual rate (APR)

   k: frequency of compounding.

   r_{ek}: Effective rate corresponding to a fraction k of a year.

   t: duration of the investment in years.

   F.V. of 1 Euros invested for t years:

   $$(1+r_e)^t = \left(1+\frac{r_a}{k}\right)^{k\times t} = (1+r_{ek})^{k\times t}$$

   P.V. of annuity of n payments growing at rate g, with the first payment occurring at time t and being equal to C :

   $$\frac{C}{r-g}\left(1-\left(\frac{1+g}{1+r}\right)^n\right)(1+r)^{1-t} \text{ if } g \neq r$$

   $$n \times \frac{C}{(1+r)^t} \text{ if } g = r$$

   P.V. of a Perpetuity increasing at rate g with the first payment equal to C and occurring at time t:

   $$\frac{C}{r-g}(1+r)^{1-t} \quad \text{if} \quad r > g$$

   $$\infty \times C \quad \text{otherwise}$$

2. **Capital Budgeting**

   Net Income = (Revenues - Costs - Depreciation)(1-Tax Rate)

   Working Capital = Inventories + Receivable - Accounts Payable

   $$\begin{aligned}\text{Cash-Flow in year } t &= \text{Net Income}_t + \text{Depreciation}_t - \Delta\text{Working Capital}_t \\ &- \text{Cost of factory (if } t = \text{the year you start the project)} \\ &+ \text{Book value of the factory (if } t = \text{the year you sell the factory)}\end{aligned}$$

3. **Uncertainty**

$$
\begin{aligned}
E[\widetilde{r}] &= \pi_1 r_1 + \pi_2 r_2 + ... + \pi_m r_m \\
Var[\widetilde{r}] &= \sigma_r^2 = \pi_1 (r_1 - E[\widetilde{r}])^2 + \pi_2 (r_2 - E[\widetilde{r}])^2 + ... + \pi_m (r_m - E[\widetilde{r}])^2 \\
Cov[\widetilde{r}_A, \widetilde{r}_B] &= E[(\widetilde{r}_A - E[\widetilde{r}_A])(\widetilde{r}_B - E[\widetilde{r}_B])] = \\
&= \pi_1 (r_{A1} - E[\widetilde{r}_A]) (r_{B1} - E[\widetilde{r}_B]) + \pi_2 (r_{A2} - E[\widetilde{r}_A]) (r_{B2} - E[\widetilde{r}_B]) + \\
&\quad + ... + \pi_m (r_{Am} - E[\widetilde{r}_A]) (r_{Bm} - E[\widetilde{r}_B]) \\
Cov(x\tilde{r}_A + (1-x)\tilde{r}_B, \tilde{r}_C) &= xCov(\tilde{r}_A, \tilde{r}_C) + (1-x)Cov(\tilde{r}_B, \tilde{r}_C) \\
E[\alpha\widetilde{r}_A + \beta\widetilde{r}_A + \gamma] &= \alpha E[\widetilde{r}_A] + \beta E[\widetilde{r}_A] + \gamma \\
\rho_{AB} &= \frac{Cov[\widetilde{r}_A, \widetilde{r}_B]}{\sigma_A \sigma_B}
\end{aligned}
$$

4. **Portfolio Theory**

Portfolio $X_P = \{x_1, x_2, ..., x_{n-1}, x_f\}$

$$
\begin{aligned}
E[\widetilde{r}_P] &= x_1 E[\widetilde{r}_1] + x_2 E[\widetilde{r}_2] + ... + x_f r_f \\
Var[\widetilde{r}_P] &= \sigma_P^2 = \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} x_i x_j \sigma_{ij}
\end{aligned}
$$

Portfolio $X_P = \{x_1, x_2\} = \{x, 1-x\}$

$$
\begin{aligned}
E[\widetilde{r}_P] &= xE[\widetilde{r}_1] + (1-x)E[\widetilde{r}_2] \\
\sigma_P^2 &= x^2\sigma_1^2 + (1-x)^2\sigma_2^2 + 2x(1-x)\rho_{12}\sigma_1\sigma_2
\end{aligned}
$$

Minimum Variance portfolio

$$
\begin{aligned}
x_1^{\min} &= \frac{\sigma_2 (\sigma_2 - \rho_{12}\sigma_1)}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2} \\
\sigma_{\min} &= \left(\frac{\sigma_1^2\sigma_2^2 (1 - \rho_{12}^2)}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2} \right)^{1/2} \\
E[\widetilde{r}_{\min}] &= x_1^{\min} E[\widetilde{r}_1] + (1 - x_1^{\min}) E[\widetilde{r}_2]
\end{aligned}
$$

Composition of the tangent portfolio when $S = \{s_1, s_2, s_f\}$:

$$x_1^T = \frac{(E[\widetilde{r}_2] - r_f)\rho_{1,2}\sigma_1\sigma_2 - (E[\widetilde{r}_1] - r_f)\sigma_2^2}{(E[\widetilde{r}_1] + E[\widetilde{r}_2] - 2r_f)\rho_{1,2}\sigma_1\sigma_2 - (E[\widetilde{r}_1] - r_f)\sigma_2^2 - (E[\widetilde{r}_2] - r_f)\sigma_1^2}$$

Capital Allocation Line

$$\begin{aligned} E[\widetilde{r}_P] &= r_f + \lambda\sigma_P \\ \lambda &= \frac{E[\widetilde{r}_T] - r_f}{\sigma_T} \end{aligned}$$

Optimal portfolio:

$$x_T^* = \frac{E[\widetilde{r}_T] - r_f}{A\sigma_T^2}$$

5. **CAPM**

Market portfolio = Tangency portfolio.

Efficient portfolios are on the Capital Market Line

$$\begin{aligned} E[\widetilde{r}_P] &= r_f + \lambda\sigma_P \\ \lambda &= \frac{E[\widetilde{r}_M] - r_f}{\sigma_M} \end{aligned}$$

All portfolios and assets are on the Security Market Line:

$$\begin{aligned} E[\widetilde{r}_i] - r_f &= \beta_i\,(E[\widetilde{r}_M] - r_f) \\ \beta_i &= \frac{Cov[\widetilde{r}_i, \widetilde{r}_M]}{\sigma_M^2} \end{aligned}$$

Beta of a portfolio $X_P = \{x_1, x_2, ..., x_{n-1}, x_f\}$:

$$\beta_P = x_1\beta_1 + x_2\beta_2 + \ldots x_{n-1}\beta_{n-1}$$