No-Trade in Second-Price Auctions with Entry Costs and Secret Reserve Prices

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Abstract

We consider a second-price private-value auction in the presence of an exogenous participation cost and a secret reserve price endogenously set by the seller. We show that, if the entry cost is strictly positive, the only equilibrium outcome is that the seller chooses a reserve price that deters entry and no buyer enters.

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1 Introduction

In many auctions, if all bids remain below a fixed but undisclosed reserve price, the item remains unsold.[1] Another common feature of auctions is that bidders incur participation costs, even when they do not face uncertainty about their valuation.

This note analyzes the strategic interaction between a seller who sets a secret reserve price in a second-price auction and a group of buyers who know their valuations but need to pay a cost to enter the auction. We focus on a private-value framework in which the seller's valuation is commonly known to be zero. The seller chooses the reserve price that maximizes her expected revenue. Simultaneously, all buyers choose whether to participate in the auction and how much to bid. Because there is no uncertainty about the seller's problem, buyers perfectly anticipate the reserve price in equilibrium. Yet, the reserve is not necessarily set at the same level as when it is public. We show that in in all equilibria the reserve price is so high that no buyer enters the auction.

2 The Model

We consider the sale of an item by a risk-neutral seller who values it at 0. Let I be the finite set of $N > 2$ risk-neutral potential acquirers ("buyers"). Each buyer i privately observes his valuation $\tilde{v}_i$, where $\tilde{v}_i$ is independently distributed with a c.d.f. F_i that is associated with a continuous and strictly positive density on $[0, 1]$. The seller then chooses a reserve price $r \in [0, 1]$. Simultaneously, every buyer chooses whether to enter the auction. Participating buyers are called "bidders". Each bidder i has to pay an exogenously-determined and common-knowledge participation cost $c \in (0, 1)$ that is non-recoverable. The auctioneer runs a second-price sealed-bid auction with reserve price r. All bidders compete to purchase the item. Bidders do not observe the number of other participants, and r is kept secret until all bids have been submitted. If all bids are below r, there is a "buy-in": the seller keeps the item and receives no payment. If the highest bid is above r, the highest bidder

[1] Auctions with secret reserve prices have been analyzed in a number of settings (e.g., Elyakime et al. (1994), Vincent (1995), Rosar (2014), Jehiel and Lamy (2015)).

wins the item and pays the maximum of r and the second-highest bid.

As a preamble to our analysis, let us consider the entry and bidding game where an exogenous reserve price r is publicly announced after buyers learn their valuations but before they make their entry decision. Equilibria for such auctions have been studied in prior literature (e.g., Samuelson (1985), Stegeman (1996), Menezes and Monteiro (2000), Tan and Yilankaya (2006)). The equilibrium provides for each buyer the probability that he enters, given his type and the common-knowledge exogenous reserve price, as well as the bids he makes upon entry.

In a sub-game perfect equilibrium for an auction with secret reserve prices, buyers correctly anticipate the reserve. Therefore, if a buyer expects a reserve price r, she enters and bids as if a reserve price of r had been publicly announced. Let us fix r and an equilibrium of the entry and bidding game where buyers know r before entering. Denote by $R(x, r)$ the seller's ex-ante expected revenue from secretly setting a reserve price equal to x when buyers' entry and bidding strategies are based on the expectation that the reserve price is r. In equilibrium, the seller chooses the reserve price r^* that maximizes his expected revenue, while buyers correctly anticipate the seller's reserve price. Hence in equilibrium one must have:

$$r^* \in \arg\max_{x\in[0,1]} R(x, r^*). \tag{1}$$

We solve the game by backward induction. After the entry decision has been made, the bidding equilibrium must be of the type described by Blume and Heidhues (2004) (henceforth BH), who characterize all bidding equilibria of the second-price private-value auction for $N > 2$. In their Corollary 1 they show that in the presence of a positive reserve price r, the equilibrium is unique and such that a bidder with valuation $v_i < r$ does not bid, whereas any bidder with valuation $v_i \geq r$ bids his valuation.[2]

We can now make the following statement about the equilibrium of the whole game:

Proposition 1 *In all equilibria of the second-price auction with positive entry cost and secret reserve price the seller sets a reserve price larger than* $1 - c$ *and no buyer enters.*

[2]Uniqueness is up to changes in the bidding function on a set of types with zero measure.

Proof. Clearly setting $r^* > 1 - c$ is a best response for the seller if no buyer enters. Because buyer i's profit cannot exceed $v_i - r^* - c$, not entering when expecting a reserve price larger than $1-c$ is a best response for the buyers. Let us prove by contradiction that there are no equilibria with entry. Suppose that in equilibrium the reserve price is such that some buyer enters with positive probability. Then it must be that $r^* \leq 1 - c$ and that only bidders with a valuation larger than r^*+c enter the auction. Hence bidders' values are continuously distributed on the interval $[r^*+c, 1]$. Then take the equilibrium of the bidding game following entry. It must be as described by BH, and hence all bidders bid their value that is at least $r^* + c$. Consider the seller's expected revenue from deviating to a reserve price $x \in (r^*, r^* + c)$. Let Q_2 be the probability that at least two bidders enter the auction. In this case the item sells for the second-highest bid, which is at least $r^*+c > r^*$. Let $\hat{R}$ denote the seller's expected revenue in this event. Note that $\hat{R}$ is not affected by deviating to reserve price $x \in (r^*, r^* + c)$. Let $Q_1 \geq \min_i[(1 - F_i(r^* + c))\Pi_{j\neq i}F_j(r^* + c)] > 0$ be the probability that exactly one bidder enters the auction. In this case the item sells for the reserve price x. Thus, the seller's expected revenue $R(x, r^*)$ from deviating to a reserve price $x \in (r^*, r^* + c)$ is:

$$R(x, r^*) = Q_1 x + Q_2 \hat{R}, \tag{2}$$

which is strictly increasing in x because $Q_1 > 0$. Given that choosing $x \in (r^*, r^* + c)$ is a profitable deviation for the seller, we have a contradiction. ■

3 Discussion

Two-bidder case. BH characterize all bidding equilibria of the second-price private-value auction for $N > 2$. However, for the case of two bidders there are more equilibria (see BH, p. 172, footnote 2). In the following example, we show that our no-trade result also applies to the two-bidder case. We focus on bidding equilibria where one of the bidders i is expected to bid the reserve price for a non-trivial measure of his valuations. In this case, by deviating to a reserve price larger than what bidders expect, the seller triggers a buy-in when the only bidder present is precisely the one who bids the reserve price. Thus, in these bidding equilibria the seller has the least incentive to secretly change the reserve price. Below we show that even if the seller expects a bidding equilibrium of this type, he still has an incentive to deviate, implying that our no-trade result extends to the case $N = 2$.

Example 1 *There are 2 buyers. Fix $r^* < 1 - c$ and consider three thresholds w_1, w_2, and W with $r^* + c < w_1, w_2 < W < 1$ and $F_{-i}(w_{-i})(w_i - r^*) - c = 0$. Let buyers' strategies be as follows. Buyer i enters only if his valuation of the item is at least w_i. Bidder 1 bids r^* if $v_1 \leq W$ and v_1 if $v_1 > W$. Bidder 2 bids W if $v_2 \leq W$ and v_2 if $v_2 > W$.*

Let us first show that this is an equilibrium entry and bidding strategy when r^* is public. Because of $F_{-i}(w_{-i})(w_i - r^*) - c = 0$, buyer i enters only if $v_i \geq w_i$. Bidding their valuation is optimal for bidders with a valuation above W.

Consider bidder i with $w_i \leq v_i \leq W$. The equilibrium payoff is $F_2(w_2)(v_1 - r^*) - c \geq 0$ for bidder 1 and $F_1(W)(v_2 - r^*) - c \geq 0$ for bidder 2. If i bids less than r^*, he gets $-c$. If he bids between r^* and W, he gets his equilibrium payoff. If he bids more than W, he gets his equilibrium payoff minus the expected loss in case he wins against a bidder who bids more than W but less than his bid. So bidding r^* (resp. W) is optimal for bidder 1 (resp. 2).

Let us now show that in the game with a secret reserve price, if buyers expect r^* and adopt the entry and bidding strategy described above, then the seller can deviate profitably by setting a reserve price $x = W$. Let $\overline{R}$ be the seller's expected revenue if both buyers' valuation exceeds W. Then

$$\begin{aligned} R(r^*, r^*) &= (1 - F_1(W))(1 - F_2(W))\overline{R} + (1 - F_1(w_1))F_2(w_2)r^* \\ &+ F_1(W)(1 - F_2(w_2))r^* + (1 - F_1(W))(F_2(W) - F_2(w_2))W, \end{aligned} \tag{3}$$

whereas

$$\begin{aligned} R(W, r^*) &\geq (1 - F_1(W))(1 - F_2(W))\overline{R} + 0(1 - F_1(w_1))F_2(w_2) \\ &+ F_1(W)(1 - F_2(w_2))W + (1 - F_1(W))(F_2(W) - F_2(w_2))W \end{aligned} \tag{4}$$

We therefore have that:

$$\begin{aligned} R(W,r^*) - R(r^*,r^*) &\geq F_1(W)(1-F_2(w_2))(W-r^*) - (1-F_1(w_1))F_2(w_2)r^* \\ &\geq F_1(W)(1-F_2(w_2))c - (1-F_1(w_1))F_2(w_2)r^* \\ &= w_1(1-F_1(w_1))F_2(w_2) > 0 \end{aligned} \tag{5}$$

where the first inequality follows from (3) and (4), the second inequality follows from $W > w_1$ *and* $w_1 > r^* + c$*, and the equality comes from* $F_2(w_2)(w_1 - r^*) - c = 0$*. Thus the seller has a profitable deviation.*

Uncertainty about reserve price. So far we have focused on the situation where the seller's reserve price is perfectly anticipated by buyers. However, if the seller's valuation is unknown to the buyers and/or the seller uses a mixed strategy when choosing his reserve price, in equilibrium buyers will face some uncertainty regarding the reserve price. We can extend the no-trade result to these cases. Namely, suppose that in equilibrium buyers face uncertainy about the seller's actual reserve price and let $\underline{r} \geq 0$ be the inf of the possible equilibrium values of the reserve price. In an equilibrium with entry it must be that only bidders with a valuation of at least $\underline{r} + c$ enter. Upon entry, the bidding equilibrium must take the form as described by BH and our argument would apply. Namely, all bids must exceed some $\underline{b} > \underline{r}$, but then the seller strictly prefers a reserve price in $x \in (\underline{r}, \underline{b})$ to $\underline{r}$ because x would increase his revenue in the positive-probability event that only one bidder enters.

Common-value auction. Our no-trade result does not rely on the private-value framework. Consider an item that has the same value $\tilde{v} \in [0,1]$ to all buyers. Each buyer i observes a conditionally independent signal $s_i \in [0,1]$ that is positively correlated with $\tilde{v}$. Let $v(s_i) \in [0,1]$ denote the expected value of $\tilde{v}$ conditional on s_i being the highest of the N buyers' signals. A buyer i expecting a reserve price $r^* < 1$ will then enter only if $v(s_i) \geq r^* + c$ and bid $v(s_i)$. The argument underpinning the proof of Proposition 1 can then be applied to show that the seller can deviate profitably by setting $x > r^*$.

Entry cost. However, for the impossibility of trade, it is crucial to have a strictly positive entry cost. If $c = 0$, then in addition to our no-trade equilibrium there is an equilibrium where all buyers enter and the seller sets the 'textbook' optimal reserve price.

First-price auction. Finally, our result also depends on the second-price format. For example, in a first-price auction a secret reserve price will generate indeterminacy of equilibrium. By deviating from the expected reserve price, a seller can only increase the probability of a buy-in, but not the price paid by the winning bidder. In other words, by choosing a reserve price different from what buyers expect, he can only reduce his revenue. Hence, in a first-price auction with entry cost and secret reserve price any reserve price can be sustained in equilibrium.

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