Sample Quiz Exercises

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Remark: The following exercises are from two past quizzes.

a) Which one of the following cannot be a good reason for short-selling an asset?

- ☐ You think the price of the asset will increase.
- ☐ You think the price of the asset will decrease.
- ☐ You want to reduce the risk of your portfolio.
- ☐ You want to increase the money you can invest in another asset.

b) At date 0, you invest in a portfolio that contains the same number of shares of stock X and stock Y. The prices of stocks X and Y at date 0 are P_X^0 = €10 and P_Y^0 = €20, respectively. At date 1, the composition of your portfolio is unchanged, that is, it still contains the same number of shares of stock X and stock Y. The price of stock X, P_X^1, is higher at date 1 than it was at date 0, while the price of stock Y, P_Y^1, is still equal to €20. How did the weight of stock X in your portfolio change between dates 0 and 1?

- ☐ It went up
- ☐ It went down
- ☐ It did not change
- ☐ We cannot tell. It depends on the dividends paid by the two stocks between dates 0 and 1

c) The financial market consists of two stocks, A and B. Stock A has an expected return of 8%. Stock B has an expected return of 12%. How can you construct a portfolio containing stocks A and B and with an expected return of 15%?

- ☐ Such a portfolio does not exist
- ☐ You should sell short stock A and buy stock B
- ☐ You should sell short stock B and buy stock A
- ☐ You should sell short both stock A and stock B

d) Suppose that half of your portfolio is invested in the risk-free asset F, the remaining half is invested equally in stocks A and B. Stocks A and B have the same standard deviation of returns and their returns are uncorrelated. What is the standard deviation of returns of stocks A and B if your portfolio has a standard deviation of 8%?

- ☐ 5.12%
- ☐ 22.63%
- ☐ 64%
- ☐ 38.52%

The following applies to questions e) and f): You own a portfolio, which contains equal amounts of three assets: security A, security B, and a risk-free asset F. The coefficient of correlation of security A's returns with the returns of the two other assets is zero. Securities A and B have standard deviations of returns equal to 10% each.

e) What is the standard deviation of returns of your portfolio?

- ☐ 0.22%
- ☐ 2.22%
- ☐ 3.33%
- ☐ 4.71%
- ☐ 6.67%

f) What combination of the three assets above has the smallest standard deviation of returns?

- ☐ 50% in A, 50% in B
- ☐ 1% in A, 99% in the risk-free asset
- ☐ 1% in B, 99% in the risk-free asset
- ☐ 100% in the risk-free asset

Solution Key:

a) "You think the price of the asset will increase"
Explanation: With short-selling, you gain if the stock price decreases and you lose if the stock price increases.

b) "It went up"
Explanation: Let *n* be the number of shares that you hold of both stocks X and Y. The old weights are W_X^0 = n*10/(n*10+n*20) = 1/3 and W_Y^0 = n*20/(n*10+n*20) = 2/3. The price of X goes up (P_X^1 >10), the price of Y stays constant (P_Y^1 =20). The new weight becomes
W_X^0 = n* P_X^1/(n*P_X^1+n*20) = P_X^1/(P_X^1+20) > 1/3.

c) "You should sell short stock A and buy stock B"
Explanation: As we saw in the class example, we can achieve a higher return than the return of any individual asset by short-selling the lower-return asset in order to "lever up" (invest more into) the higher-return asset.
In fact, you can calculate the exact portfolio weights that achieve this return:
$E(r_P) = w_A*E(r_A) + w_B*E(r_B)$
Here: 15% = w_A*8% + w_B*12%
We also know that: $w_A + w_B = 1$
Therefore: 15% = w_A*8% + (1-w_A)*12%
Solving for w_A delivers: w_A = -75% and w_B = 175%

d) 22.63%
Explanation: Portfolio weights of stocks A and B are $w_A = w_B$ = ¼. Let σ be the standard deviation of stocks A and B.
Portfolio variance is given by: Var(r_P) = 0.08^2 = (1/4)^2 * σ^2 + (1/4)^2 * σ^2
[the other terms drop out because stocks A and B are uncorrelated and the risk-free asset has zero variance.]
Solving this for σ^2 yields σ^2 = 0.0512. Taking the square-root gives: σ = 22.63%.

e) 4.71%
Explanation: Portfolio weights are $w_A = w_B = w_F$ = 1/3. Assets A, B and F are uncorrelated (all the covariances between them are zero). [Remember that the covariance/correlation between any any asset and the risk-free asset is always zero. Remember also that the variance of the risk-free asset is zero.] Applying our formula for the portfolio variance, we get
Var(r_P) = (1/3)^2 * (0.1)^2 + (1/3)^2 * (0.1)^2 = 0.002222
Taking the square root in order to find the standard deviation, leads to 4.71%.

f) "100% in the risk-free asset"
Explanation: If you invest 100% in the risk-free asset, the standard deviation of your portfolio will be zero. It cannot be lower.