Bid-Ask Price Competition with Asymmetric Information between Market Makers*

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Abstract

This paper studies the effect of asymmetric information on the price formation process in a quote-driven market. One market maker receives private information on the value of the quoted asset, and repeatedly competes with market makers who are uninformed. We show that despite the fact that the informed market maker's quotes are public, the market is never strong-form efficient with certainty until the last stage. We characterize a reputational equilibrium in which the informed market-maker influences and possibly misleads the uninformed market makers' beliefs. At this equilibrium, a price leadership effect arises, the informed market maker's expected payoff is positive and the rate of price discovery increases in the last stages of trade.

Keywords: market efficiency; quote-driven markets; reputational equilibrium.

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1 Introduction

Ever since Kyle (1985) and Glosten and Milgrom (1985) a common assumption in market microstructure literature on asymmetric information is that while market makers do not have superior information on market fundamentals, some traders do have private information. However, several empirical studies have reported stylized facts that are difficult to reconcile with the assumption that market makers are equally uninformed. In this paper we study the dynamic interaction between market makers when one of them does, in fact, possess superior information and we characterize the equilibrium dynamic quoting strategy that results.

The first of the stylized facts from the empirical literature suggesting that dealers are asymmetrically informed is the price leadership effect. In the Foreign Exchange market (FX market henceforth), Peiers (1997) examines the quoting behavior of dealers in the DM-US$ market around Bundesbank interventions and finds evidence of price leadership by Deutsche Bank before the announcement of intervention. This conclusion is confirmed by de Jong et al. (1999). In his analysis of the same market, Sapp (2002) observes that certain banks consistently incorporate new information into prices before other banks do so. Ito et al. (1998) study the change in the pattern of returns volatility in the Tokyo FX market: they conclude that their empirical observations are consistent with the assumption of privately informed dealers (where private information is considered common knowledge). Studying the relative contribution of electronic communication networks and market makers in providing informative quotes on the Nasdaq market, Huang (2002) finds that, among the Nasdaq market makers, some provide more timely information, which suggests that they are likely to possess superior information. Moreover, he shows that being a price leader is not associated with posting the best quotes. Heidle and Li (2003) study the quoting behavior of the market makers affiliated to analysts' brokerage firms. They find strong evidence that these market makers systematically change their quoting behavior well before the analysts publicly announce the reports containing their investment recommendations. Finally, in the secondary market for Italian sovereign bonds, Albanesi and Rindi (2000) and Massa and Simonov (2001) found evidence of imitative pricing behavior and attribute it to the fact that some market makers are reputed to be better informed.

A second stylized fact that is difficult to explain using classical market microstructure models is the separate role of bid and ask quotes for the transmission of private information. Sapp (2002) finds evidence that only one side of the price leaders' quotes, i.e., ask or bid, provides additional information contributing to price discovery. Also, Heidle and Li (2003) find evidence on Nasdaq-listed stocks that informed market makers use only one price to signal their private information about the analysts' reports (they quote more aggressive bids when the report is positive, and more aggressive asks when it is negative). Finally, Naranjo and Nimalendran (2000) observe that the bid-ask spread changes more around the Bundesbank's unexpected interventions than around its expected interventions, suggesting that the width of the spread may contain some information. This empirical evidence seems to suggest that a dealer with superior private information uses bids and asks separately to signal (or to conceal) information to the market.

These papers support two facts that seem to be common to these markets. First, that there is a small group of market makers who have superior information on fundamentals. Second, that the identities of these market makers are known by other market makers. These hypotheses are also confirmed by Goodhart (1988) who concludes from interviews with London based specialists that some dealers are perceived as being better informed than others. Lyons (1997), (2001) backs this view, concluding that banks with larger customer share likely have better information.

In fact, an important common feature of these markets is that market makers' bid and ask quotes are not anonymous. Consequently, quotes posted by the better informed dealer have a role in both influencing the market participants' beliefs on fundamentals, and in determining the transaction prices.

Theoretical models such as Kyle (1985), Glosten and Milgrom (1985), Easley and O' Hara (1987), and Holden and Subrahmanyam (1992), among many others, cannot account for price leadership or for signaling through quotes. This is mainly because their analyses are based on the assumption that market makers are equally uninformed.

A third stylized fact concerns the evolution of the spread before the announcement date. There is rich empirical evidence (Koski and Michaely (2000), Krinsky and Lee (1996) , and Venkatesh and Chiang (1986), among others) that shows that the average spread widens as the announcement period approaches, implying that asymmetric information should be the greatest just before the public release of information. This pattern is in sharp contrast with the classical market microstructure prediction that spreads steadily decrease as information is gradually incorporated into the price (Glosten and

Milgrom (1985), Easley and O'Hara (1987)).

In order to address these points, we study a model in which market makers and liquidity traders exchange a risky asset for a risk-less asset during T periods. In each period, market makers simultaneously set quotes and automatically execute market orders submitted by liquidity traders. We assume that one of the market makers has superior information about the fundamental value of the risky asset. In our model, we will only consider the case in which all floor traders are liquidity investors who do not possess any private information. This assumption is admittedly strong, but it is needed both for analytical tractability, and to clearly disentangle the effects of asymmetric information among dealers from those coming from informed floor traders. The identity of the informed dealer is commonly known and the posted quotes are not anonymous. Therefore, the uninformed market makers extract information on the value of the asset by observing past quotes posted by the informed market maker. The latter takes into account the impact that his current quotes will have on the quoting strategy of uninformed dealers in the future.

The trading mechanism we consider is a close representation of existing trading mechanisms. For example, in Nasdaq's screen-based order routing and execution systems, such as SelectNet and the Small Order Execution System (SOES), clients' orders are automatically executed against market makers at the best prices. We quote from a document of NASD Department of Economic Research:

> "Nasdaq market makers have also been subject to an increasing level of mostly affirmative obligations.
>
> Market makers must continuously post firm two-sided quotes, good for 1000 shares [...]; they must report trades promptly; they must be subject to automatic execution against their quotes via SOES; [...]" (J. W. Smith, J. P. Selway III, D. Timothy McCormick, 1998-01, page 2).

The model also fits FX markets as, on the one hand, traders execute their orders against the market makers who post the best quotes while, on the other hand, the identity of quotes issuers is observable. Finally, the proposed trading mechanism is a stylized representation of "pit" trading.

We show that in such a highly transparent quote-driven market, a privately informed market maker gradually reveals his information.

More precisely, we first study whether the market is strong-form efficient, in the sense that prices convey all available private and public information. We prove that in the last trading period, the informed market maker's quotes fully reveal his private information, but the probability that this revelation would occur earlier in time is less than one. In other words, the market is strong form efficient with certainty only seconds before the public announcement.

Second, we analyze market makers' quoting strategies and show that the informed market maker generates some "noise" in his quoting activity, which precludes other market participants from immediately inferring his private information, allowing him to exploit his informational advantage over several trading rounds. The distribution of noise corresponds to the equilibrium mixed strategy used by the informed dealer. The intuition of our result is based on two observations: (i) if the value of the asset is high it is worth buying it by setting high bid quotes, whereas if its value is low it is worth selling it by setting low ask quotes; and (ii) the more accurate the uninformed dealers' belief, the smaller the profit will be for the informed market maker. On the one hand, when the informed market maker chooses the quotes that maximize his current payoff, he reveals part of his information and decreases his future payoff. On the other hand, if he chooses quotes that cause a loss in current trade, he misleads the uninformed market makers, thereby increasing his future payoff. We will show that, as long as there are impending trading rounds, it is optimal for the informed market maker to randomize between revealing information and misleading the market by trading against his signal.

Finally, we provide empirically testable implications that run contrary to the results of the existing models of informed trading and are in line with the stylized empirical facts mentioned above. First, we find that the equilibrium presents a positive serial correlation between the quotes set by the informed dealer at time t, and the quotes set by the uninformed market makers at time $t+1$. In view of the fact that with equally informed market makers there is no specific dealer that leads the price discovery process, our result suggests that the empirical evidence in which some dealers appear to be price leaders is indeed compatible with the presence of asymmetrically informed market makers.

Second, we prove that at equilibrium the informed market maker uses the bid and the ask price differently in order to strategically signal his type. In fact, in the mixed strategy equilibrium we characterize, the informed market maker either posts aggressive (i.e., high) bids in tandem with very high asks, or aggressive (i.e., low) asks together with very low bids. Thus, in each trading round only the quote on one side of the market incorporates new information. This is consistent with the empirical findings in Sapp (2002). It is clear that one does not necessarily have to post the best quotes to signal information, as empirically observed by Huang (2002). Finally, informed dealers set the spread more frequently on the profitable side, but they also participate in the unprofitable side of the market, which corresponds to the empirical findings in Heidle and Li (2003).

Third, we find that the revelation of information increases as the public announcement approaches. The adverse selection is stronger at the last stages of the trading game because the opportunity cost of concealing private information is at its greatest at this time. Thus, the informed market maker will mainly participate in the profitable side of the market. This increases the winner's curse and results in more conservative quotes. The overall effect is to widen the inside spread as the end of the game approaches.

1.1 Related literature

In existing markets where dealers compete in prices, their interaction can be well represented by a first-price auction. In fact, the incoming orders can always be executed at the best possible price [1]. This is the generic approach taken by the theoretical market microstructure literature. However, this literature widely assumes that market makers are equally uninformed and that the best informed traders are floor traders. Given these common assumptions, price competition among market makers is simple Bertrand competition and, consequently authors have focused on the information content of the volume of trade rather than quotes. Biais (1993) is an exception. He considers a static model in which market makers are risk averse and privately informed about their own inventory of the risky asset. Thus, competition among market makers turns out to be a first price, independent private value auction. In our model, private information concerns the fundamental value of the asset and the resulting one-stage game is a common value, first price bid-ask auction. Roell (1988), Bloomfield and O'Hara (2000) and de Frutos and Marzano (2005) analyze a market in which dealers have asymmetric information on the asset fundamentals. The authors study the

[1] See for the example the "Order Handling Rule" valid on the Nasdaq.

effects of market transparency on dealer behavior, in particular. Contrary to our paper, these models analyze only one period of trade during which market makers are asymmetrically informed and therefore cannot address the issue of strategic transmission of information through time. De Meyer and Moussa-Saley (2002) study a repeated zero-sum game where two dealers reciprocally exchange a risky asset. They show that the resulting price dynamics is related to a Brownian motion. There are two assumptions that make it difficult to apply their appealing result directly to financial markets. First, it is assumed that in each period the two dealers mutually exchange the asset i.e., no third party participates in the market. Second, the zero-sum format does not fit financial markets as, in fact, every market maker can guarantee a zero payoff simply by quoting a sufficiently large spread. Finally, Gould and Verecchia (1985) study the pricing strategy of a specialist who has unique private information on market fundamentals. In a static set up, they show that a rational expectations equilibrium with noisy prices exists. Still, their result requires that the specialist be able to commit himself ahead of time to adding an exogenous noise to his price. As the actual price at which the specialist trades does not necessarily maximize his payoff function, it is unclear whether the same equilibrium would exist in case the specialist is unable to commit himself in advance to a noisy pricing rule.

Our paper also contributes to the literature on auctions as our market model corresponds to a sequential first-price bid-ask auction for identical objects with common value. The value of proprietary information in one-shot auctions has been studied by Engelbrecht-Wiggans, Milgrom and Weber (1983). Proposition 1 extends this result to an auction with an ask (selling) side.

The existing literature on sequential auctions analyzes situations in which several objects are put up for sale consecutively to the same set of bidders. The fundamental difference between these models and the problem we study here is that in our set up, bidders can buy and sell the objects simultaneously.

The first paper on sequential auctions is by Ortega-Reichert (1968), who studies a two-person, two-stage (i.e. two objects), first-price sealed-bid auction. The Ortega-Reichert result is innovative in that the author first recognizes the incentive for bidders to deceive their opponents in the first auction in order to reap an expected gain in the second auction. The result differs from ours, in that there is no real deception at equilibrium, since each bidding strategy is invertible and each player can infer his opponent's information from his bid.

Engelbrecht-Wiggans and Weber (1983) (EWW henceforth) is closer to our framework since they study a pure common value, sequential auction of identical objects, where one bidder learns the true value of the objects prior to the first sale, while the other bidders are aware that he is perfectly informed. The authors show that, at equilibrium, the uninformed player may have a higher expected profit than the informed one, on condition that the number of objects for sale is high enough. A similar result is obtained in Horner and Jamison (2004). Here, the authors extend the analysis of EWW to an infinitely repeated game between two bidders and to a more general discrete distribution of the value of the object. The main difference with our set up is that bidders can buy the objects but do not sell them. In a bid auction, when the value of the object is low, the informed bidder reaps no advantage from deceiving the uninformed bidder as the object is of no worth to him. By contrast, in our bid-ask auction, the informed market maker has an incentive to mislead the bidder who is uninformed because his action will encourage the sale of the low-value asset at a higher price and increase his future profit. This leads to a different type of manipulation activity by the informed player and to different conclusions on the value of information.

Finally, Bikhchandani (1988) studies a finite series of n second-price auctions where the value of the objects are independently distributed across different auctions. Here, different objects have different values, in contrast to the case presented in our study.

The remainder of this paper is organized as follows. Section 2 presents the formal model. In Section 3 we collect the construction of the equilibrium, and prove the short run information inefficiency of the equilibrium. In Section 4 we develop some empirical predictions of the model. Section 5 we discuss the case where the asset fundamentals are continuously distributed and in Section 6 we conclude. All proofs are in the Appendix.

2 The model

Consider a market with N risk-neutral market-makers (MMs in the following) who trade a single security over T periods against liquidity floor traders. The liquidation value of the security is a random variable $\widetilde{V}$ which can, for simplicity, take two values, $\{\overline{V}, \underline{V}\}$, with $\overline{V} > \underline{V}$, according to a probability distribution $(p, 1-p)$ commonly known by all MMs, where $p = \Pr(\widetilde{V} = \overline{V})$.

We denote by $v = p\overline{V} + (1-p)\underline{V}$ the expected value of the asset for any given p. The realization of $\widetilde{V}$ occurs at time 0 and at time $T+1$ a public report will announce it to all market participants. Time is discrete and T is finite.

Information structure

At the beginning of the first period of trade, one[2] of the MMs, $MM1$, is privately informed about of the risky asset's realized liquidation value. When $\widetilde{V} = \overline{V}$ (resp. $\widetilde{V} = \underline{V}$), we will refer to the informed market maker as "type $\overline{V}$" (resp. "type $\underline{V}$") denoted $MM1(\overline{V})$ (resp. $MM1(\underline{V})$). The other $N-1$ market makers do not observe any private signals but they do know that $MM1$ has received a superior information; we will treat these market makers as a unique dealer called $MM2$[3]. $MM2$ updates his belief by observing $MM1$'s past quotes. We use p_t to denote the uninformed dealer's belief at beginning of period t, that is, after he has observed $MM1$'s quotes during the preceding $t-1$ periods. The expected value of the asset at the beginning of period t is denoted by $v_t = p_t\overline{V} + (1-p_t)\underline{V}$.

Market Rules

In each period, the two MMs simultaneously[4] announce their ask and bid quotes, which are firm for one unit of the asset[5]. Then, transactions take place between liquidity traders and the market makers. We assume that, each time, liquidity traders sell one unit of the asset to the market maker who sets the highest bid quote, and buy one unit of the asset from the market maker who sets the lowest ask quote (i.e., price priority is enforced)[6]. If both

[2] As in Kyle (1985), we assume that there is only one agent who receives private information on the realization of $\widetilde{V}$.

[3] To the extent that $MM2$ equilibrium payoff is zero, this assumption is made without loss of generality because the informed market maker only considers the probability of winning the auctions at a given price, whether this probability is the outcome of the strategy of one uninformed player or n equally uninformed players (see also Engelbrecht-Wiggans et al.(1983) and section 3.1).

[4] We do not consider the timing problem that arises when the bidding process is sequential, as in Cordella and Foucault (1998).

[5] In the literature it is standard procedure to fix the tradable quantity at each step (see O'Hara (1995)), and, as mentioned before, this assumption corresponds quite closely to the rules of a number of markets.

[6] This is isomorphic to a situation where, for each period, the expected volume of buy orders is constant and equal to the expected volume of sell orders. As market makers are risk neutral and the volume of trade incorporates no information on $\widetilde{V}$, this would correspond to multiplying MMs' stage payoffs by a factor equal to the expected volume of

market makers set the same quotes, then liquidity traders will be indifferent between $MM2$ or $MM1$. The game then has a continuum of strategies and discontinuous payoffs. In order to guarantee the existence of equilibrium, we follow Simon and Zame (1990) and endogenously determine the tie-break rule in case of identical (bid or ask) quotes. More precisely, let us denote by q the probability that liquidity traders will trade with $MM1$ in the event of a tie. Instead of specifying an exogenous level for q as a characteristic of the model, and then solve for the equilibrium, q will be determined as part of the equilibrium[7]. We require the probability q to be independent on the realization of $\widetilde{V}$, as it is supposed to be unknown to liquidity traders, but in equilibrium, q will be affected by other factors that are common knowledge at the time of a tie.

Each MM can observe the past quotes of all market makers. Finally, we assume that market makers cannot trade with each other and that short sales are permitted.[8]

Behavior of market participants and equilibrium concept

In each period, a buy market order and a sell market order are proposed by floor traders trading for liquidity reasons. In order to focus on the role of quotes as a mechanism for the strategic transmission of information, we exclude the presence of other sources of information such as informative floor traders' orders. In our model, traders do not act for informational purposes, and so the order flow neither incorporates nor depends on any information about the value of the asset. As price priority is enforced in all periods, each market maker knows that he will buy (resp. sell) one asset if he proposes the best bid (resp. ask) quote. We denote by $a_{i,t}$ and $b_{i,t}$ the ask and bid price respectively set by market maker i in period t. We denote by q_t the probability that $MM1$ executes the order in the case of a tie in period t. We can write the single period expected payoff functions for our risk-neutral

buy (or sell) orders.

[7] The procedure suggested by Simon and Zame (1990) consists in defining a payoff correspondence, which is interpreted as the union of all possible tie-break rules when the prices posted by $MM1$ and $MM2$ are identical. An equilibrium for the game will be a selection from the payoff correspondence of a particular rule together with the (Perfect Bayesian) equilibrium for the resulting game.

[8] Allowing a market maker to submit anonymous market orders to the other market makers would improve $MM1$'s payoff but would not rule out the signaling role of his quotes, which is our main concern here.

market makers as follows:

$$E[\Pi_{1,t}\,|\overline{V}\,] = (a_{1,t}-\overline{V})\left(\Pr(a_{2,t}>a_{1,t})+q_t\Pr(a_{2,t}=a_{1,t})\right)+ \\ (\overline{V}-b_{1,t})\left(\Pr(b_{2,t}<b_{1,t})+q_t\Pr(b_{2,t}=b_{1,t})\right) \tag{1}$$

$$E[\Pi_{1,t}\,|\underline{V}\,] = (a_{1,t}-\underline{V})\left(\Pr(a_{2,t}>a_{1,t})+q_t\Pr(a_{2,t}=a_{1,t})\right)+ \\ (\underline{V}-b_{1,t})\left(\Pr(b_{2,t}<b_{1,t})+q_t\Pr(b_{2,t}=b_{1,t})\right) \tag{2}$$

for $MM1(\overline{V})$ and $MM1(\underline{V})$ respectively, and for $MM2$

$$\begin{aligned} E[\Pi_{2,t}] = \; & p_t(a_{2,t}-\overline{V})\left(\Pr(a_{1,t}>a_{2,t}|\overline{V})+(1-q_t)\Pr(a_{1,t}=a_{2,t}|\overline{V})\right)+ \\ & (1-p_t)(a_{2,t}-\underline{V})\left(\Pr(a_{1,t}>a_{2,t}|\underline{V})+(1-q_t)\Pr(a_{1,t}=a_{2,t}|\underline{V})\right)+ \\ & p_t(\overline{V}-b_{2,t})\left(\Pr(b_{1,t}<b_{2,t}|\overline{V})+(1-q_t)\Pr(b_{1,t}=b_{2,t}|\overline{V})\right)+ \\ & (1-p_t)(\underline{V}-b_{2,t})\left(\Pr(b_{1,t}<b_{2,t}|\underline{V})+(1-q_t)\Pr(b_{1,t}=b_{2,t}|\underline{V})\right) \end{aligned} \tag{3}$$

The overall payoff of each MM is the (non-discounted) sum for $t=1,...,T$ of these payoffs:

$$\pi_1(V,T,p) = \sum_{t=1}^{T} E[\Pi_{1,t}\,|V\,] \text{ for } V\in\{\overline{V},\underline{V}\}$$

$$\pi_2(T,p) = \sum_{t=1}^{T} E[\Pi_{2,t}]$$

We focus on equilibria where MMs' strategies at each round depend on the number of rounds before the public report and on the overall information that past quotes provide about the true value of $\widetilde{V}$. Namely, we denote with $\boldsymbol{\gamma}_t=(\tau,p_t)$ the *state of the game* at time t, where $\tau=T-1+t$ is the remaining number of trading rounds before the public report. In consequence a mixed strategy for $MM2$ in period t is a function σ_2 that maps the state of the game γ_t into a probability distribution over all couples of bid-ask quotes. As $MM1$'s strategy depends also on his private information, a mixed strategy for $MM1$ in period t is a function σ_1 that maps the realized value of the asset and the state of the game γ_t into a probability distribution over all couples of bid-ask quotes. Finally, the liquidity trader's tie-break strategy is a function q that maps the state of the game γ_t into the probability of trading with $MM1$ in case of a tie in quotes. For a given state of the game $\gamma=(\tau,p)$ we denote $\pi_1^*(\widetilde{V},\tau,p)$ and $\pi_2^*(\tau,p)$ the expected equilibrium payoff for $MM1(\widetilde{V})$ and for $MM2$ respectively.

We characterize the Perfect Bayesian Equilibrium strategies σ_1^*, σ_2^* and q^* by solving the game by backward induction: at any time t the MMs solve the following problems:

$$\begin{aligned}
\sigma_1^*(\overline{V},\tau,p_t) &= \arg\max_{\sigma_1(\overline{V})} E[\Pi_{1,t}\left|\overline{V}\right.] + \pi_1^*(\overline{V},\tau-1,p_{t+1})\text{ , given }\sigma_2^*,q^* \\
\sigma_1^*(\underline{V},\tau,p_t) &= \arg\max_{\sigma_1(\underline{V})} E[\Pi_{1,t}\left|\underline{V}\right.] + \pi_1^*(\underline{V},\tau-1,p_{t+1})\text{ , given }\sigma_2^*,q^* \\
\sigma_2^*(\tau,p_t) &= \arg\max_{\sigma_2} E[\Pi_{2,t}] + \pi_2^*(\tau-1,p_{t+1})\text{ , given }\sigma_1^*,q^* \\
q^*(\tau,p_t) &\in [0,1]
\end{aligned}$$

where $\tau = T+1-t$ and $p_{t+1} = \Pr(\widetilde{V} = \overline{V}|a_{1,t},b_{1,t})$ is determined by Bayes' rule when this is possible, otherwise it is chosen arbitrarily.

We denote with $\Gamma(T,p)$ the game representing the strategic interaction among MMs when there are T finite rounds of trade and $\Pr(\widetilde{V} = \overline{V}) = p$ at the beginning of the game ($t=0$).

It is worth emphasizing that, as market makers can alternatively buy or sell the security without inventory considerations, whatever the true value of the asset, one of the two auctions will always be profitable and the other one not. This suggests that what really matters for the equilibrium of the game is not the actual value of the asset, $\overline{V}$ or $\underline{V}$, but how close to the truth $MM2$'s belief p is: intuitively, the more correct the belief of $MM2$, the smaller $MM1$'s profit. In the appendix we formally state the game's symmetry property.

3 Equilibrium characterization

3.1 One trading round

In this section, we analyze the dealers' price competition when $T=1$,which can also be interpreted as the last trading round. The bid auction alone has been solved by Engelbrecht-Wiggans, Milgrom and Weber (1983) (EMW henceforth) for an arbitrary distribution of the value of the object for sale. Proposition 1 extends the authors' result to the ask auction in the case of a binomial distribution of $\widetilde{V}$. It also provides the equilibrium distribution of bid and ask quotes and market makers' equilibrium payoffs.

Proposition 1 *The equilibrium of the one-shot game $\Gamma(1,p)$ is unique and is such that:*

$MM2$ randomizes ask and bid prices according to

$$\Pr(a_{2,1} < x) = F^*(x) = \begin{cases} 0 & \text{for } x \in]-\infty, v] \\ \frac{x-v}{x-\underline{V}} & \text{for } x \in]v, \overline{V}] \\ 1 & \text{for } x \in]\overline{V}, +\infty] \end{cases}$$

$$\Pr(b_{2,1} < x) = G^*(x) = \begin{cases} 0 & \text{for } x \in]-\infty, \underline{V}] \\ \frac{\overline{V}-v}{\overline{V}-x} & \text{for } x \in]\underline{V}, v] \\ 1 & \text{for } x \in]v, +\infty] \end{cases}$$

If the value of the asset is $\overline{V}$, then $MM1$ sets $a_{1,1} = \overline{V}$ and he randomizes the bid price according to

$$\Pr(b_{1,1} \leq x \left|\overline{V}\right.) = \overline{G}^*(x) = \begin{cases} 0 & \text{for } x \in]-\infty, \underline{V}] \\ \frac{(1-p)(x-\underline{V})}{p(\overline{V}-x)} & \text{for } x \in]\underline{V}, v] \\ 1 & \text{for } x \in]v, +\infty] \end{cases}$$

If the value of the asset is $\underline{V}$, then $MM1$ sets $b_{1,1} = \underline{V}$ and he randomizes the ask price according to

$$\Pr(a_{1,1} \leq x \left|\underline{V}\right.) = \underline{F}^*(x) = \begin{cases} 0 & \text{for } x \in]-\infty, v] \\ \frac{x-v}{(1-p)(x-\underline{V})} & \text{for } x \in]v, \overline{V}] \\ 1 & \text{for } x \in]\overline{V}, +\infty] \end{cases}$$

The equilibrium payoffs are $\pi_2^*(1,p) = 0, \pi_1^*(\overline{V},1,p) = (1-p)(\overline{V}-\underline{V})$ and $\pi_1^*(\underline{V},1,p) = p(\overline{V}-\underline{V})$.

In case of tie in quotes the probability of trading with $MM1$ is $q^* \in [0,1]$.

Just before the public report, the informed MM has a last opportunity to make a profit from his private information. Therefore, if the liquidation value of the asset is $\overline{V}$, he will try to buy the asset by winning the bid auction, whereas if the liquidation value of the asset is $\underline{V}$, he will try to sell it by winning the ask auction. Because the uninformed MM does not know whether it is profitable to buy or to sell the asset, he will bid more conservatively in both auctions, taking into account the "winner's curse" resulting from the competition with a better informed MM.

In short, in a static game, the asymmetry of information between market makers leads to three important implications. First, the full revelation of information by $MM1$ makes the market strong-form efficient at the last stage of trade. This follows from the fact that $MM1$'s quotes are observable[9]. Second, contrary to the case with symmetric information, bid and ask market prices are different from the expected liquidation value of the asset, given the public information. In fact, the market spread is strictly positive generically, and bid and ask quotes straddle v. However, there is no restriction over the spread's width (up to $\overline{V} - \underline{V}$) which depends on the outcome of the mixed strategies. Third, although $MM2$'s expected equilibrium payoff is zero, $MM1$ obtains a positive expected payoff. Namely, the more erroneous $MM2$'s belief, the larger $MM1$'s informational rent, as he will be able to win the profitable auction at a more lucrative price.

3.2 Informational efficiency of the quote-driven market

In the last trading period $MM1$ reveals his private information to the market through his posted quotes.

At first glance, given that the informed dealer's quotes are observable by other market makers, it would seem likely that he would lose his informational advantage at the first trading round. However, this is not true of any period prior to the last one. More precisely, we show that before the last trading round the probability that private information is fully conveyed into prices is less than one.

Theorem 2 *There exists no Bayesian-Nash equilibrium where $MM1$'s private information is revealed with certainty before T.*

Theorem 2 states that private information is never revealed with probability one before the final round T. Hence, in the short run, it is not always possible to infer $MM1$'s private information unambiguously, despite the fact that his quotes are perfectly observable. This informational inefficiency recalls results obtained in other market microstructure models. However, in models "*à la*" Kyle or Glosten and Milgrom, the market is not strong-form efficient because the insider traders conceal their actions within the exogenous

[9]With unobservability of the insider's actions (ex. Kyle (1985)) the market is not strong-form efficient even at the last stage.

random demand that comes from noise traders. Uninformed agents cannot directly observe the informed traders' action and are therefore unable to infer the informed trader's private information. By contrast, our result does not rely on the existence of exogenous noise due to anonymous orders. Theorem 2 shows that when an informed dealer cannot hide behind noise traders, he will endogenously generate some noise. The rationale of the Theorem 2 proof is that before the last trading round, a phase in which information is fully revealed is simply not credible. More precisely, if at some $t < T$, $MM1$'s private information was fully revealed for certain, then in period t market makers would optimally play the unique equilibrium of the one shot game. However, in this case, $MM1$ has at least one profitable deviation that consists in misleading $MM2$ in period t and then profiting from $MM2$'s totally wrong beliefs in the following trading period. Hence, he cannot be committed to truthfully announcing his inside information until the last stage.

3.3 Equilibrium in manipulating strategies

Theorem 2 states that in the short run the market is not strong-form efficient, but does not specify how, in equilibrium, $MM1$ manages to conceal and exploit his information. In this section, we characterize a Perfect Bayesian equilibrium of the dynamic bid-ask auction in which $MM1$ generates endogenous noise in his quotes. By doing so, he can profit from his informational advantage during several periods. The interest of the particular equilibrium we study here[10] is that it is consistent with several empirical observations described in the literature, such as: the "price leadership" effect; the fact that the informed market maker participates in the unprofitable side of the market less frequently than in the profitable side; the fact that in each step only one of the two sides of the market incorporates new information; the increase of the quoted spread and quotes' volatility as the date of a public announcement gets closer.

First, we explain the leading economic forces that produce our result.

From the analysis of the one-period case, it results that in the last trading stage, $MM1$ only competes in the profitable side of the market, i.e., he tries to sell the asset if $\widetilde{V} = \underline{V}$ or to buy it if $\widetilde{V} = \overline{V}$. In the following we prove that during the trading periods prior to the final one, $MM1$ conceals his

[10]Note that we are looking for an equilibrium of a particular kind, leaving the question of the existence of other equilibria unresolved.

information by participating in the unprofitable side of the market with positive probability. In doing so, $MM2$ cannot unambiguously deduce $MM1$'s information by observing whether $MM1$ tried to buy or to sell the asset in the previous period. More precisely, after observing that $MM1$ tried to buy the asset (to sell the asset), $MM2$ will increase (resp. decrease) the probability he attaches to the event $\widetilde{V} = \overline{V}$, but his posterior belief will not be necessarily equal to 1 (resp. 0). We define these strategies as *manipulating strategies* since there is a positive probability that $MM1$ will take an action with the aim of turning $MM2$'s belief in the wrong direction.

$MM1$'s incentive to mislead $MM2$ by trying to win the unprofitable auction depends on two factors. First, the benefit that a misleading action will have on the future payoff. Second, the current cost of misleading. Intuitively, the greater the number of remaining trading periods, the higher $MM1$'s benefit from misleading $MM2$ in the current period. For example, in the last trading round, as the future payoff is zero, it is never optimal to mislead.[11] By contrast, a misleading action in the early rounds can be turned into profit in the following trading rounds. Thus, whenever there are trading rounds still to be conducted, it can be optimal for $MM1$ to mislead $MM2$. Nevertheless, misleading is optimal only if the expected cost of winning the unprofitable auction is small. The cost decreases with the correctness of $MM2$'s belief which can be measured by $|v_t - \widetilde{V}|$, i.e., the distance between the expectation of $\widetilde{V}$ and the realization of $\widetilde{V}$. Take the case $\widetilde{V} = \overline{V}$, for example. Roughly speaking, if $MM1$ wants to mislead $MM2$ in period t, he must try to sell the asset by posting an ask price close to the current expected value v_t[12]. The cost of misleading is given by the risk of selling the asset at a price close to v_t lower than its actual value, $\overline{V}$. When p_t is close to 1, v_t approaches the actual value of the asset $\overline{V}$ and the cost of misleading is low. Therefore, when $MM2$'s belief is sufficiently correct, misleading is cheap and $MM1$ will bid in the unprofitable side of the market with positive probability. However, when $MM2$'s belief is sufficiently wrong, the cost of misleading becomes too large and $MM1$ will only participate in the profitable side of the auction.

Let $\tau = T - t + 1$ be the number of trading stages before the public report. The following proposition provides a qualitative description of the equilibrium:

[11] See Proposition 1.

[12] Intuitively, $MM2$ will never accept to sell the asset at a price $a_{2,t} < v_t$, so $MM1$ is sure to win the ask auction with an $a_{1,t}$ sufficiently close to v_t.

Proposition 3: *There exists an equilibrium of the game* $\Gamma(T,p)$ *that satisfies the following features:*

(1) In any given trading round t*, whenever a market maker tries to buy the asset (to sell the asset), he randomizes his current bid (resp. current ask) on the support* $[b_{\min}, v_t]$ *(resp.* $[v_t, a_{\max}]$*); where* $b_{\min}$ *(resp.* $a_{\max}$*) depends on the state of the game* $\gamma_t = (\tau, p_t)$*.*

(2) In each trading round, $MM2$ *tries both to buy and to sell the asset simultaneously.*

(3) $MM1$ *never tries to buy and sell the asset simultaneously. Namely, in trading round* t*:*

- *If* $p_t < 2^{1-\tau}$*, then* $MM1(\overline{V})$ *tries to buy the asset and stays out of the ask auction setting* $a_{1,t} = a_{\max}$*. If* $p_t > 2^{1-\tau}$*, then* $MM1(\overline{V})$ *randomizes between trying to buy the asset and trying to sell it.*
- *If* $p_t > 1-2^{1-\tau}$*, then* $MM1(\underline{V})$ *tries to sell the asset and stays out of the bid auction setting* $b_{1,t} = b_{\min}$*. If* $p_t < 1 - 2^{1-\tau}$*, then* $MM1(\underline{V})$ *randomizes between trying to buy the asset and trying to sell it.*

(4) A market maker's equilibrium expected payoff is zero if he is uninformed and positive if he is informed.

Regarding features (1), (2) and (4) the equilibrium of Proposition 3 is similar to the equilibrium of the static game described in Proposition 1: (1) quotes are generated from mixed strategies, and bid and ask prices straddle v_t; (2) $MM2$ always tries to win both the bid and the ask auctions; (4) a market maker's payoff is strictly positive only if he has some private information. By contrast, feature (3) is specific to the dynamic game. According to this property, if $MM2$'s belief is sufficiently wrong, the informed market maker tries to win only the profitable auction, given his information (the bid auction when $\overline{V}$ realized and the ask auction when $\underline{V}$ realized, respectively). On the contrary, if $MM2$'s belief is sufficiently correct, then $MM1$ misleads the market with positive probability. He will do this by randomizing between two actions: competing only in the profitable auction given his information, and competing only in the other auction. Take the case $\widetilde{V} = \overline{V}$, for example: the closer p_t is to 1, the closer $MM2$'s belief to the truth. Feature (3) states that $MM1(\overline{V})$ misleads $MM2$ by trying to sell the asset with positive probability only if p_t is sufficiently close to 1, namely $p_t > 2^{1-\tau}$. However, if $MM2$'s belief is substantially wrong (i.e., $p_t < 2^{1-\tau}$), then $MM1(\overline{V})$ will only try to buy the asset, as misleading will prove too costly. A symmetric reasoning applies to $MM1(\underline{V})$ who will find it profitable to mislead $MM2$, by trying to buy the asset, only if $MM2$'s belief is sufficiently correct, i.e., if $p_t < 1 - 2^{1-\tau}$.

Note that when misleading occurs with positive probability, $MM1$'s strategy can be seen as a two-step lottery. First, he flips a (biased) coin to determine whether he will compete in the bid or in the ask auction. Second, he randomly fixes the level of the quote (either bid or ask) that he will submit in the auction in which he competes. The other quote will be set at a level that will make him certain to lose.

There are two implications in the fact that the threshold $2^{1-\tau}$ and $1-2^{1-\tau}$ decrease and increase, respectively, with the length of the game. First, for any given belief and any realization of $\widetilde{V}$, misleading occurs with positive probability, provided that there are enough trading rounds before the public report. Second, a misleading action is more likely to occur in the early stages of trade as it can be turned into profit during a longer period. Thus, during the initial trading rounds, the sign of $MM1$'s information affects his quoting strategy only slightly. However, as the date of the public report approaches, the incentive to mislead decreases and private information strongly affects $MM1$'s strategies. In other words, at the beginning of the game, the winner's curse is weak since observing whether $MM1$ buys or sells does not reveal much about the true value of the asset. However, when the value-relevant announcement is drawing near, $MM1$'s strategy will depend significantly on his private information and winner's curse heavily affects competition between market makers. In the following section we show that this has clear empirical implications on the informational content of $MM1$'s quotes and the expected market spread that shall increase as T approaches.

4 Equilibrium properties and empirical implications

In this section we assess some equilibrium properties in terms of informational efficiency and liquidity. We also provide some empirical predictions that can be used to detect the presence of asymmetric information among dealers in quote-driven markets. Despite the fact that it is possible to obtain the closed form expressions for the equilibrium quotes distribution for any repetition of the game recursively[13], these expressions are not always tractable. Therefore, we obtain some of the empirical implications of the model by computing the expected quotes numerically, using $\overline{V} = 1$ and $\underline{V} = 0$ and varying the initial belief p and the length of the game T.

4.1 Price leadership

The standard market microstructure theory in which market makers are equally uninformed does not explain the price leadership effect that has been documented in the empirical literature on FX markets, over-the-counter markets and Nasdaq[14]. The manipulating equilibrium of Proposition 3 shows this characteristics as it exhibits a positive correlation between the quotes posted by the uninformed market maker ($MM2$) at a given period t and the quotes that the informed one ($MM1$) posted in $t-1$. The explanation is simple. $MM1$ is more likely to post relatively high quotes when he knows $\widetilde{V} = \overline{V}$ rather than when $\widetilde{V} = \underline{V}$. Thus, high $MM1$'s quotes induce $MM2$ to believe that $\widetilde{V} = \overline{V}$ is more likely. As a consequence, in the following period $MM2$'s expected quotes will increase. More precisely, in equilibrium, $MM2$'s posterior belief on the event $\widetilde{V} = \overline{V}$ is an increasing function of $MM1$'s last quotes, and $MM2$'s expected quotes are increasing functions of his belief.

Restating the equilibrium of Proposition 3 for $T = 2$ it is possible to explicitly quantify the price-leadership effect[15]. Let $Post(a, b)$ be $MM2$'s posterior probability of $\widetilde{V} = \overline{V}$ after observing $(a_{1,1}, b_{1,1}) = (a, b)$. We obtain:

Lemma 4: *In the equilibrium of the game $\Gamma(2, p)$, $MM2$'s expected quotes in the second period increase with the first period $MM1$'s quotes, whereas $MM2$'s first period quotes do not affect $MM1$'s second period quotes. More precisely,*

(i) *if* $p > 1/2$,

[13] See Appendix.

[14] See introduction for a complete list of references.

[15] When $T > 2$ we can still show that

$$\frac{\partial E[a_{2,t+1}]}{\partial a_{1,t}} > 0 \quad \frac{\partial E[a_{2,t+1}]}{\partial b_{1,t}} > 0$$
$$\frac{\partial E[b_{2,t+1}]}{\partial a_{1,t}} > 0 \quad \frac{\partial E[b_{2,t+1}]}{\partial b_{1,t}} > 0$$

$$\frac{\partial E[a_{2,2}]}{\partial a_{1,1}} = -\ln(Post(a_{1,1}, b_{\inf}))\frac{(2p-1)(\overline{V}-\underline{V})^2}{(2a_{1,1}-\overline{V}-\underline{V})^2} > 0$$

$$\frac{\partial E[b_{2,2}]}{\partial a_{1,1}} = -\ln(1-Post(a_{1,1}, b_{\inf}))\frac{(2p-1)(\overline{V}-\underline{V})^2}{(2a_{1,1}-\overline{V}-\underline{V})^2} > 0$$

(ii) *if* $p < 1/2$,

$$\frac{\partial E[a_{2,2}]}{\partial b_{1,1}} = -\ln(Post(a_{\max}, b_{1,1}))\frac{(1-2p)(\overline{V}-\underline{V})^2}{(2b_{1,1}-\overline{V}-\underline{V})^2} > 0$$

$$\frac{\partial E[b_{2,2}]}{\partial b_{1,1}} = -\ln(1-Post(a_{\max}, b_{1,1}))\frac{(1-2p)(\overline{V}-\underline{V})^2}{(2b_{1,1}-\overline{V}-\underline{V})^2} > 0$$

(iii) *and for all* p :

$$\frac{\partial E[a_{1,2}]}{\partial a_{2,1}} = \frac{\partial E[b_{1,2}]}{\partial a_{2,1}} = \frac{\partial E[a_{1,2}]}{\partial b_{2,1}} = \frac{\partial E[b_{1,2}]}{\partial b_{2,1}} = 0$$

Lemma 4 also shows that $MM1$'s quote revisions remain unexplained by $MM2$'s quote adjustments. This allows to run empirical tests on the Granger-causality of the observed market makers quotes.

Simulations for $p > 1/2$ suggest that the covariance between $MM1$ and $MM2$'s two successive ask quotes is roughly 15% of $(\overline{V}-\underline{V})$, which represents a significative price effect of $MM1$ over $MM2$.

Moreover, as $MM1$'s quotes become more informative as the date of the public report approaches, the price-leadership effect will increase as well.

4.2 Informational efficiency

One of the appealing properties of auction mechanisms is that it is possible to extract the bidders' private information on the value of the auctioned object by observing the bidders' bids. Not surprisingly, this observation is confirmed by the analysis of our one shot auction. Indeed, in the last period, $MM1$ fully reveals his private information through his quotes. However, Theorem 2 shows that this is not always the case when identical assets are traded sequentially.

As is standard in market microstructure literature, we measure the weak-form efficiency of the market using the evolution of the variance of $\widetilde{V}$ conditioned on all relevant public information, $\Sigma_t = Var[\widetilde{V}\,|H_t]$. The faster the convergence of Σ_t to zero (i.e., the higher the rate at which Σ_t decreases), the better the properties of the market in terms of efficiency. In models of order-driven markets (Kyle (1985), Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), Huddart, Hughes and Levine (2001)) Σ_t either decrease at a constant or at a dwindling rate, implying that most of the private information is conveyed into the prices relatively early on in the game.

Contrary to what occurs in order-driven markets, in our framework, the first stages of the game are "waiting" stages with a relatively low signaling activity, while most of the information is released in the very last stages of trading. This is shown in Figure 1 which plots the expected rate of change of Σ_t for a game repeated 5 times. The two lines correspond to two different levels of the initial prior belief. The variance of the risky asset's value decreases at a rate that depends on the level of the initial prior belief. When this prior belief is close to 1 or 0 (thick line), the initial variance of $\widetilde{V}$ decreases more slowly than when the prior is close to 1/2 (dotted line). In both cases, however, Σ_t reduces at an increasing rate, which means that less information is revealed at the early stages and that $MM1$'s quotes reveal more information during the last rounds of trade.

4.3 The expected cost of trading

Some empirical and experimental evidence (Venkatesh and Chiang (1986), Krinsky and Lee (1996) and Koski and Michaely (2000) has shown that the inside spread usually widens as the moment of public release of information draws nearer. This can be verified along the equilibrium of Proposition 3 as well. As a measure of liquidity we consider the expected inside spread. Figure 2 shows that for a fixed level of p, the expected inside spread increases as the date of public report approaches. In the last stages of the game, the spread is maximum.

This finding is easy to explain. In the early trading rounds, the winner's curse is weak, hence bid-ask quotes are concentrated on average around the ex-ante expected value of the asset. The winner's curse increases when T draws near and this effect forces the uninformed MM to quote more "conservatively", so that on average the spread increases.

4.4 The value of private information

Finding the value of private information has been a central issue in financial economics. In most of the market microstructure literature, the existence of equilibria in which the information has a positive value appears to be related to the presence of exogenous noise in the economy. For example, in Kyle (1985), the profit of the insider trader is proportional to the volatility of noise traders' demand. We show that this is not the case in a quote-driven market, as a market maker can derive a positive profit from superior information even without exogenous noise in the market. There are two factors that affect the value of the private information: the ex-ante volatility of $\widetilde{V}$ and the number of repetitions T.

The volatility of the asset fundamental is measured by the unconditional variance of $\widetilde{V}$, which is equal to $p(1-p)(\overline{V}-\underline{V})^2$. Figure 3 plots $MM1$'s ex-ante equilibrium payoff as a function of p when the game is repeated once (thin curve), 15 times, and 30 times (thick curve). The ex-ante payoff is maximum when the uncertainty in the market is high, which corresponds to a p close to $1/2$. Not surprisingly, private information is more valuable in markets in which little is known about large shocks on the fundamentals.

Figure 3 also shows that the informed MM's payoff increases with the number of trading rounds available before the public report occurs. The increment in $MM1$'s payoff from one additional trading round decreases with T. Figure 4 plots the marginal increase in $MM1$'s ex-ante expected profit from adding two additional trading rounds when p is around 0.5. The increase in $MM1$'s profit is low for high T since an additional round of trading would not provide him with substantial additional profits because $MM2$ will bid quite aggressively in these periods owing to the low winner's curse effect.

Finally, please note that in our model the uninformed bidder earns a lower expected profit than he would in one sided auctions with asymmetric information (cfr. Engelbrecht-Wiggans and Weber (1983) and Horner and Jamison (2004) for example). In these studies the high type informed bidder is able to make profits on one object at the most, as after that his true information is revealed. If he wants to conceal his information during some stages, he has to constantly underbid the uninformed bidder. The uninformed bidder thereby obtains several objects at sufficiently low prices. This scenario does not apply to our bid-ask auction. The equilibrium of Proposition 3 shows that $MM1$ of *each* type can mimic the behavior of a different type during many stages. In doing so, he can force the payoff of $MM2$ down to zero, exactly as in a one-shot auction: $MM2$ is "squeezed" between $MM1$ of low type who tries to sell the asset and $MM1$ of high type who tries to buy it. Consequently, $MM2$ is never sure to win either auction at a convenient price.

5 Extension: Continuous distribution of $\widetilde{\mathbf{V}}$

In Section 3 we show that if the fundamentals of the asset can only take two values, $\overline{V}$ or $\underline{V}$, then in equilibrium MMs select their quotes using mixed strategies and the probability of observing informational efficient quotes before T is less than 1. In this section we will discuss the robustness of this result when $\widetilde{V}$ is continuously distributed over a closed interval $[\underline{V}, \overline{V}]$.

In the case of a continuous distribution, $MM2$ faces an informed market maker with a continuum of types (representing the information about the realized $\widetilde{V}$). EMW study what can be reinterpreted as a one-side, one-shot version of this model and show that when the value of the asset is continuously distributed, the informed bidder uses a pure strategy that is monotonic in his type. Crawford and Sobel (1982) consider the problem of strategic information transmission when there is a continuum of types for the informed player. They show that the information is partially revealed with a "semi-pooling" pure strategy equilibrium where the informed player's strategy is a step function of his information. Neither one of these results, however, extends to the dynamic auction that we are considering in this paper. In fact, it is possible to show that there exists no sub-game perfect equilibrium in which $MM1$ uses a pure strategy before the last repetition of the game. Namely, $MM1$ pooling pure strategies are dominated strategies while semi-pooling or fully revealing pure strategies equilibria contain some not credible action. An equilibrium in which $MM1$ strategy is pure and not pooling at $t < T$, would imply that at $t+1$, $MM2$ will attach 0 probability to all realization of $\widetilde{V}$ that are inconsistent with $MM1$'s quotes in period t. But in this case there would always exist a $V \in [\underline{V}, \overline{V}]$ such that $MM1$ of type V finds it profitable at time t to post quotes that mislead $MM2$ and then gain in the following period from $MM2$'s completely wrong belief.[16] In general, at equilibrium, $MM1$ never reveals his private signal with probability one before

[16] The complete proof of this statement is available from the authors upon request.

the last stage.[17] Therefore a quote-driven market where the posted quotes are not anonymous is not strong-form efficient with certainty until actual public release of information, and this is true independently of the modelling assumption on the fundamental $\widetilde{V}$. Moreover, if an equilibrium exists, it is in mixed strategies in the early rounds of trade. The question of the existence of subgame-perfect equilibria when $\widetilde{V}$ is continuously distributed remains open.

6 Conclusion

We have studied a quote-driven market with asymmetric information between market makers and shown that an informed market maker strategically releases his private information using mixed strategies. This generates an endogenous noise that allows the informed market maker to exploit his informational advantage over several periods. Despite the highest possible level of market transparency, which allows all dealers to observe the best informed agent's actions (i.e., his bid and ask quotes), the market is not strong-form efficient in the short run with positive probability. In fact, it is only in the very last trading round, immediately before an informational event, that quotes will fully incorporate private information with certainty. This equilibrium behavior has several empirical implications. First, there is a positive correlation between the informed market makers' quotes at time t and the uninformed market maker's quotes at $t+1$. Second, the information content of the best informed market maker's quotes increases as the date of the public report draws near, and in consequence the expected market spread increases as well. Third, trading prices are different from the expected value of the risky asset given the public information. Fourth, even if no new shocks hit the fundamentals, quotes are volatile. Fifth, the private information has a positive value even in such a highly transparent market, which justifies the costly activity of information collection by institutional dealers.

One possible direction for further research would be to study a more complex situation in which floor traders also have private information. In this case in point, the incentive for the informed market maker to mislead the market would probably diminish. However, this would probably not change the main economic trade-off the market maker faces in deciding his

[17] For a formal proof of this statement, we refer the reader to the following website: www.restud.com/supplements.htm

optimal strategies. Hence, we can expect that the "strategic" noise in the informed market maker 's quotes would persist.

7 Appendix

Symmetry Property (SP) : *The game* $\Gamma(T,p)$ *is symmetric with respect to the following transformation:*

$$\widetilde{V}' = \overline{V} + \underline{V} - \widetilde{V} \tag{4}$$
$$a'_{i,t} = \overline{V} + \underline{V} - b_{i,t} \tag{5}$$
$$b'_{i,t} = \overline{V} + \underline{V} - a_{i,t} \tag{6}$$
$$p' = 1 - p \tag{7}$$

Proof : It is sufficient to write MMs' payoffs substituting to $a_{i,t}$ the expression $\overline{V} + \underline{V} - b'_{i,t}$ and to $b_{i,t}$ the expression $\overline{V} + \underline{V} - a'_{i,t}$, $i = 1, 2$. Once MMs types are changed following (4), we obtain payoffs that differ from the original ones only for the use of the new variables ($a'_{i,t}$, $b'_{i,t}$, p') and types ($\underline{V}'$, $\overline{V}'$). Thus, using this symmetry we can deduce the equilibrium of the game $\Gamma(T, 1-p)$ from the equilibrium strategies of the game $\Gamma(T, p)$.

Proof of Proposition 1: The bid auction has been studied in EMW. Considering that the ask auction can be rewritten into a bid auction using the symmetry property (SP), this proposition follows from the authors result. For expositional completeness, we show that the described strategy profile is an equilibrium while we leave its uniqueness as a consequence of EMW.

Substituting the expression $\underline{F}^*(.)$ and $\overline{G}^*(.)$ in expression (3), it results that $MM2$'s payoff is equal to 0 for any $b_2 \leq v$ and any $a_2 \geq v$. If $MM2$ sets $b_2 > v$, then he is certain to win the bid auction with an expected profit of $v - b_2 < 0$. Similarly, any $a_2 < v$ would lead to a loss in the ask auction. Therefore, there is no profitable deviation for $MM2$. Substituting the $G^*(.)$ in (1), it follows that $MM1(\overline{V})$'s payoff is equal to $(1-p)(\overline{V} - \underline{V})$ for any $b_1 \in]\underline{V}, v]$; if $b_1 \leq \underline{V}$, then $MM1(\overline{V})$ does not win the bid auction and his payoff is 0; if $b_1 > v$, then $MM1(\overline{V})$ wins the bid auction and his payoff is $\overline{V} - b_1 < \overline{V} - v = (1-p)(\overline{V} - \underline{V})$. This means that $MM1(\overline{V})$ does not have a profitable deviation on the bid auction. On the ask auction any $a_1 < \overline{V}$ (resp. $a_1 > \overline{V}$) would lead to negative profit (resp. 0 profit), so that $a_1 = \overline{V}$ is a best reply. A symmetric argument applies for $MM1(\underline{V})$. ■

Proof of Theorem 2. The proof contains one lemma.

Lemma 7: *If, in equilibrium, private information is revealed with probability one at $t \le T$, then time t equilibrium strategies are those of the one shot game equilibrium described in Proposition 1.*

Proof : Let us assume that $(\sigma_1(\overline{V}),\ \sigma_1(\underline{V}),\ \sigma_2)$ is some fully revealing equilibrium strategy profile played in t. After time t there is no asymmetry of information and each player will set bid and ask prices equal to the true value of the asset. Hence, by backward induction, the players' equilibrium payoff after t is equal to zero. Thus, the players' total payoff from time t to T is equal to the stage t payoff whose unique equilibrium is described in Proposition 1. □

Suppose that an equilibrium exists in a period $t < T$ where the probability of full revelation is one. In that case, after time t, there will be no information asymmetry, and each MM will set bid and ask prices equal to the true value of the asset and MMs will make no profit.

From lemma 7, at time t all agents behave as if they were in the last repetition of the game whose unique equilibrium is described in Proposition 1. From proposition 1, $MM1(\overline{V})$'s equilibrium payoff is equal to $(1-p_t)(\overline{V}-\underline{V})$.

Now consider the following deviation for $MM1(\overline{V})$:

$$
\begin{aligned}
b_{1,t} &= \underline{V} \\
a_{1,t} &= \overline{V} - \varepsilon
\end{aligned}
$$

with $\varepsilon > 0$. $MM1(\overline{V})$'s stage t deviation payoff is equal to $-\varepsilon \Pr(a_2 > \overline{V} - \varepsilon)$ that can be set arbitrarily close to 0 by choosing a small enough ε. In the one shot equilibrium of Proposition 1, the quotes $b_{1,t} = \underline{V}$ and $a_{1,t} = \overline{V} - \varepsilon$ are played with positive probability only when the state of nature is $\underline{V}$. Therefore, when $MM2$ observes $b_{1,t} = \underline{V}$ and $a_{1,t} = \overline{V} - \varepsilon$, he believes that the value of the asset is $\underline{V}$ and his posterior belief in $t+1$ will be $p_{t+1} = 0$. Thus, in $t+1$ the uninformed market maker will set $a_{2,t+1} = b_{2,t+1} = \underline{V}$. Consequently, in $t+1$, $MM1(\overline{V})$ can reach a payoff arbitrarily close to $(\overline{V}-\underline{V})$ by playing $a_{1,t+1} = \overline{V}$ and $b_{1,t+1} = \underline{V} + \varepsilon$. It follows that $MM1(\overline{V})$'s overall deviation payoff can be arbitrarily close to $(\overline{V} - \underline{V})$ that is greater than his equilibrium payoff $(1 - p_t)(\overline{V} - \underline{V})$. Thus, a contradiction. ■

Proof of Proposition 3

For expositional clarity we provide a complete proof of the proposition for the game with $T = 2$. This restriction does not affect the main economic intuition of the proof for the game with $T > 2$, outlined at the end of this subsection.

Take the game $\Gamma(2, p)$. From Proposition 1, we know that the unique equilibrium of the second (and last) trading round satisfies properties (1)-(4) described in Proposition 3. Thus, we only need to prove the result for the first round of trade. To this purpose, we will distinguish between three cases: $p > 1/2$, $p < 1/2$ and $p = 1/2$. We first prove the proposition for $p > 1/2$. We then use the symmetric properties (SP) to study the case $p < 1/2$ and finally we provide the equilibrium for $p = 1/2$.

Case $p > 1/2$

To begin with, it is useful to graphically represent the set of first round bid-ask quotes that, according to Proposition 3, are played with positive probability in equilibrium when $p > 1/2$.

Features (1) and (2) imply that $MM2$ randomizes his bid and ask quotes on the intervals $[b_{\min}, v]$ and $[v, a_{\max}]$, respectively. Thus, the rectangle $ABCD = [v, a_{\max}] \times [b_{\min}, v]$ in Figure 5 represents $MM2$'s equilibrium support in the plane of bid and ask prices. Let us denote this region by S_2.

In the first round of trading, τ is 2. Thus, according to Feature (3), $MM1$'s equilibrium strategy in the first round can be described as follows. If the value of the asset is $\underline{V}$, the informed market maker competes only in the profitable auction (the ask side): he posts a bid price equal to $b_{\min}$ and randomizes the ask price in the interval $[v, a_{\max}[$. Thus $MM1(\underline{V})$'s equilibrium support is represented in Figure 5 by the line AB. Let us denote this region by $S_1(\underline{V})$. If $\widetilde{V} = \overline{V}$, then $MM1(\overline{V})$ randomizes between trying to buy the asset, and misleading $MM2$ by trying to sell the asset. If he tries to buy the asset, he randomizes the bid price in $]b_{\min}, v]$ and posts the ask price equal to $a_{\max}$. If he misleads, he will mimic the strategy of $MM1(\underline{V})$ by posting a bid equal to $b_{\min}$ and randomizing the ask in $[v, a_{\max}[$. Thus, $MM1(\overline{V})$'s equilibrium support is represented by the two lines AB and BC. Let us denote this region by $S_1(\overline{V})$.

The following Lemma provides the equilibrium distribution of MMs' quotes on the equilibrium supports S_2, $S_1(\overline{V})$ and $S_1(\underline{V})$. This equilibrium satisfies features (1)-(4) of Proposition 3.

Lemma 8: *If $p > 1/2$, then in the first round of the game $\Gamma(2, p)$ a* *perfect Bayesian equilibrium exists and has the following properties:*

(i) $MM2$ randomizes ask and bid prices on the support S_2 according to the marginal distributions:

$$\Pr(a_{2,1} < x) = F_2(x) = \begin{cases} 0 & \text{for } x \in]-\infty, v] \\ \frac{x-v}{x-(\overline{V}+\underline{V})/2} & \text{for } x \in]v, a_{\max}] \\ 1 & \text{for } x \in]a_{\max}, +\infty] \end{cases} \tag{8}$$

$$\Pr(b_{2,1} < x) = G_2(x) = \begin{cases} 0 & \text{for } x \in]-\infty, b_{\min}] \\ \frac{\overline{V}-v}{\overline{V}-x} & \text{for } x \in]b_{\min}, v] \\ 1 & \text{for } x \in]v, +\infty] \end{cases} \tag{9}$$

(ii) if the value of the asset is $\overline{V}$, then $MM1$ randomizes his bid and ask quotes on the support $S_1(\overline{V})$ according to the marginal distributions:

$$\Pr(b_{1,1} < x \,|\overline{V}) = \overline{G}(x) = \begin{cases} 0 & \text{for } x \in]-\infty, b_{\min}] \\ \frac{(1-p)(x-\underline{V})}{p(\overline{V}-x)} & \text{for } x \in]b_{\min}, v] \\ 1 & \text{for } x \in]v, +\infty] \end{cases} \tag{10}$$

$$\Pr(a_{1,1} < x \,|\overline{V}) = \overline{F}(x) = \begin{cases} 0 & \text{for } x \in]-\infty, v] \\ \overline{F}^{**}(x) & \text{for } x \in]v, a_{\max}] \\ 1 & \text{for } x \in]a_{\max}, +\infty] \end{cases} \tag{11}$$

(iii) if the value of the asset is $\underline{V}$, then $MM1$ randomizes his quotes on the support $S_1(\underline{V})$ according to the marginal distributions:

$$\Pr(a_{1,1} < x|\underline{V}) = \underline{F}(x) = \begin{cases} 0 & \text{for } x \in]-\infty, v] \\ \frac{x-v+p(\overline{V}-x)\overline{F}^{**}(x)}{(1-p)(x-\underline{V})} & \text{for } x \in]v, a_{\max}] \\ 1 & \text{for } x \in]a_{\max}, +\infty] \end{cases} \tag{12}$$

$$\Pr(b_{1,1} = b_{\min}) = 1$$

where $\overline{F}^{**}$ is the solution of the differential equation

$$F'(x) = \frac{(x-\underline{V}+(\overline{V}-x)F_2(x)-(1-q^*)(1-p)(\overline{V}-\underline{V}))(1-F(x))}{(x-\underline{V})((1-q^*)(1-p)(\overline{V}-\underline{V}))-(\overline{V}-x)F_2(x))} \tag{13}$$

with the boundary condition $F(v) = 0$. Moreover, $a_{\max} = \overline{V}$, $b_{\min}$ solves $\overline{F}^{**}(\overline{V}) = \overline{G}(b_{\min})$ and $q^* \in [0,1]$ is chosen so that the equation $\overline{F}^{**}(\overline{V}) = \overline{G}(b_{\min})$ has a solution for $b_{\min} \in [(\overline{V}+\underline{V})/2, v]$.

(iv) The equilibrium payoffs for $MM2$, $MM1(\overline{V})$ and $MM1(\underline{V})$ are respectively: $\pi_2^*(2,p) = 0$, $\pi_1^*(\overline{V},2,p) = (1-p)(\overline{V}-\underline{V})$ *and* $\pi_1^*(\underline{V},2,p) = (3p-1)(\overline{V}-\underline{V})$.

Proof of Lemma 8: The proof is divided into two main steps.

Step 1: As the equilibrium is in mixed strategies, in order to prove Lemma 8, we have to show that each market maker, given his competitor's strategy, is indifferent among all couples of bid ask quotes that belong to his equilibrium support.

Step 2: we will prove that for each market maker there is no couple of bid and ask quotes outside his equilibrium support that provides a higher expected payoff.

Step 1: construction of the mixed strategies

In Lemma 9, we show that if $MM1$ plays according to the strategies in *(ii)* and *(iii)*, then MM2 obtains an expected profit equal to zero by playing any bid and ask in S_2.

Lemma 9: *If $MM1$ randomizes his ask and bid quotes according to (ii) and (iii), then $MM2$ is indifferent among any ask $a_{2,1} \in [v, a_{\max}]$ and any bid $b_{2,1} \in [b_{\min}, v]$. His expected profit is zero.*

Proof: Since $MM2$ plays the bid and the ask auctions independently, we first show that any bid quote $b_{2,1} \in [b_{\min}, v]$ gives him a zero expected payoff.

By Proposition 1, $MM2$'s expected payoff in the second round is equal to zero. If, in the first round, $MM2$ sets any $b_{2,1} \in]b_{\min}, v]$, then $\Pr(b_{1,1} = b_{2,1}) = 0$, and his expected payoff will be

$$p(\overline{V} - b_{2,1}) \Pr(b_{1,1} < b_{2,1}|\overline{V}) + (1-p)(\underline{V} - b_{1,2}) \Pr(b_{1,1} < b_{2,1}|\underline{V}) = 0$$

where the equality follows from (10) and $\Pr(b_{1,1} = b_{\min}|\underline{V}) = 1$. Similarly, if he sets $b_{2,1} = b_{\min}$, then $\Pr(b_{1,1} < b_{2,1}) = 0$, and his expected payoff will be equal to

$$(1-q^*)\left(p(\overline{V} - b_{\min}) \Pr(b_{1,1} = b_{\min}|\overline{V}) + (1-p)(\underline{V} - b_{\min})\right) = 0$$

as $\Pr(b_{1,1} = b_{\min}|\overline{V}) = (1-p)(b_{\min} - \underline{V})/p(\overline{V} - b_{\min})$ for (10). A similar argument applies to the ask auction. □

Now we want to show that $MM1(\overline{V})$'s (resp. $MM1(\underline{V})$ expected payoff is constant for all bid and an ask quotes belonging to his equilibrium support $S_1(\overline{V})$ (resp. $S_1(\underline{V})$). As $MM1$'s continuation payoff in the second round depends on $MM2$'s posterior belief after observing $(a_{1,1}, b_{1,1})$, we first have to determine how $MM1$'s first round quotes affect $MM2$'s posterior belief. Lemma 10 determines $MM2$'s posterior belief after observing couple of quotes $(a_{1,1}, b_{1,1})$ included into $MM1$'s equilibrium support.

Lemma 10: *Let $Post(a_{1,1}, b_{1,1}) = \Pr(\widetilde{V} = \overline{V}|a_{1,1}, b_{1,1})$ be the $MM2$'s posterior belief after observing $(a_{1,1}, b_{1,1})$. If $MM2$ expects $MM1$ to use the mixed strategies in (ii) and (iii), then:*

$$Post(a_{1,1}, b_{1,1}) = 1 \quad \text{for } (a_{1,1}, b_{1,1}) \in \{a_{\max}\}\times]b_{\min}, v] \tag{14}$$

$$Post(a_{1,1}, b_{1,1}) = \frac{\overline{f}(a_{1,1})(a_{1,1}-\underline{V})^2}{(\overline{V}-\underline{V})(\overline{f}(a_{1,1})(a_{1,1}-\underline{V})+1-\overline{F}(a_{1,1}))} \quad \text{for } (a_{1,1}, b_{1,1}) \in [v, a_{\max}[\times\{b_{\min}\} \tag{15}$$

where$\overline{f}(.)$ is the derivative of $\overline{F}(.)$.

Proof: First note that quotes $(a_{1,1}, b_{1,1}) \in \{a_{\max}\}\times]b_{\min}, v]$ belong to $MM1(\overline{V})$'s equilibrium support $S_1(\overline{V})$ while they do not belong to $S_1(\underline{V})$, the equilibrium support of $MM1(\underline{V})$. Indeed, $MM1$ competes on the bid side only if $\widetilde{V} = \overline{V}$. Consequently, after observing $(a_{1,1}, b_{1,1}) \in \{a_{\max}\}\times]b_{\min}, v]$, $MM2$ unambiguously deduces that $\widetilde{V} = \overline{V}$, and so his posterior belief jumps to 1. Thus (14). By contrast quotes $(a_{1,1}, b_{1,1}) \in [v, a_{\max}[\times\{b_{\min}\}$ belong to the equilibrium support of both $MM1(\overline{V})$ and $MM1(\underline{V})$ and, as a result, $MM2$'s posterior belief will depend on the density distribution used by $MM1(\overline{V})$ and $MM1(\underline{V})$ to select quotes in this region. Expressions *(ii)* and *(iii)* imply that for $x \in [v, a_{\max}[$, we obtain a $\Pr(a_{1,1} < x \text{ and } b_{1,1} = b_{\min}|\underline{V})$ that is equal to :

$$\underline{F}(x) = \frac{x - v + p(\overline{V} - x)\overline{F}(x)}{(1-p)(x - \underline{V})}$$

where $\overline{F}(x) = \Pr(a_{1,1} < x \text{ and } b_{1,1} = b_{\min}|\overline{V})$. By differentiating both sides of this equality with respect to x, we have

$$\underline{f}(x) = \frac{p\left((x - \underline{V})(\overline{V} - x)\overline{f}(x) + (1 - \overline{F}(x)(\overline{V} - \underline{V})\right)}{(1-p)(x - \underline{V})^2} \tag{16}$$

where $\underline{f}(.) = \underline{F}'(.)$. If $MM1$ randomizes the ask prices according to the lotteries with densities $\overline{f}(.)$, $\underline{f}(.)$, then by Bayes' rule:

$$\Pr(\widetilde{V} = \overline{V}|a_{1,1}, b_{\min}) = \frac{p\overline{f}(a_{1,1})}{p\overline{f}(a_{1,1}) + (1-p)\underline{f}(a_{1,1})}$$

By substituting $\underline{f}(a_{1,1})$ with the right hand side of (16) evaluated for $x = a_{1,1}$, we obtain equation (15). □

Now we can study $MM1(\overline{V})$'s equilibrium payoff. In Lemma 11 we prove that if $MM2$ plays the strategies *(i)* and revises his beliefs according to Lemma 10, then $MM1(\overline{V})$'s payoff from setting any bid ask quotes $(a_{1,1}, b_{1,1}) \in S_1(\overline{V})$ is equal to $(1-p)(\overline{V} - \underline{V})$.

Lemma 11: *If $MM2$ randomizes his ask and bid quotes according to (i) and updates his belief according to Lemma 10, then $MM1(\overline{V})$ 'expected payoff from setting $(a_{1,1}, b_{1,1}) \in \{a_{\max}\}\times]b_{\min}, v]$ or $(a_{1,1}, b_{1,1}) \in [v, a_{\max}[\times\{b_{\min}\}$ is equal to $\pi_1^*(\overline{V}, 2, p) = (1-p)(\overline{V} - \underline{V})$.*

Proof: Suppose $MM1(\overline{V})$ sets $(a_{1,1}, b_{1,1}) \in \{a_{\max}\}\times]b_{\min}, v]$: by Lemma 10, we have $Post(a_{\max}, b_{1,1}) = 1$. Moreover, as $a_{\max} = \overline{V}$, then we have $(a_{\max} - \overline{V})\Pr(a_{2,1} \geq a_{\max}) = 0$. Thus, $MM1(\overline{V})$'s expected payoff from posting $(a_{1,1}, b_{1,1}) \in \{a_{\max}\}\times]b_{\min}, v]$ reduces to

$$\pi_1^*(\overline{V}, 2, p) = (\overline{V} - b_{1,1})\Pr(b_{2,1} < b_{1,1})$$

and substituting $\Pr(b_{2,1} < b_{1,1})$ with $G_2(.)$ given in (9), we get $\pi_1^*(\overline{V}, 2, p) = (1-p)(\overline{V} - \underline{V})$.

$MM1(\overline{V})$ must obtain the same payoff from mimicking $MM1(\underline{V})$: his expected payoff from setting $(a_{1,1}, b_{1,1}) \in [v, a_{\max}[\times\{b_{\min}\}$ is equal to

$$q^*(\overline{V} - b_{\min})\Pr(b_{2,1} = b_{\min}) + (a_{1,1} - \overline{V})(1 - F_2(a_{1,1})) + (1 - Post(a_{1,1}, b_{\min}))(\overline{V} - \underline{V})$$

where the first term is the expected payoff from the bid side in the first round in case of a tie, i.e., if $b_{2,1} = b_{\min}$, the second term is the expected payoff from the ask side in the first round, and the last term is the expected continuation payoff. From expression (9) it results $\Pr(b_{2,1} = b_{\min}) = \frac{(1-p)(\overline{V}-\underline{V})}{(\overline{V}-b_{\min})}$. By substituting the expression of $Post(.)$ stated in (15), we obtain that this payoff is equal to $\pi_1^*(\overline{V}, 2, p) = (1-p)(\overline{V} - \underline{V})$ only if

$$\overline{f}(a_{1,1}) = \frac{(a_{1,1} - \underline{V} + (\overline{V} - a_{1,1})F_2(a_{1,1}) - (1-q^*)(1-p)(\overline{V} - \underline{V}))(1 - \overline{F}(a_{1,1}))}{(a_{1,1} - \underline{V})((1-q^*)(1-p)(\overline{V} - \underline{V})) - (\overline{V} - x)F_2(a_{1,1}))} \tag{17}$$

Note that expression (11) states that for $a_{1,1} \in [v, a_{\max}[$, the function $\overline{f}(a_{1,1})$ is equal to $\overline{f}^{**}(a_{1,1})$ that is defined as the solution of the differential equation (13) and identical to (17). Thus condition (17) is met thereby concluding the proof. In Lemma 15, we will prove that a closed-form solution for (13) exists.□

Finally, in Lemma 12 we show that if $MM2$ plays according to *(i)* and revises his beliefs according to Lemma 10, then $MM1(\underline{V})$ is indifferent among the quotes in the support $S_1(\underline{V})$, and his payoff will be $(3p-1)(\overline{V}-\underline{V})$.

Lemma 12: *Suppose* $q^* = 0$ *whenever* $b_{\min} \neq (\overline{V}+\underline{V})/2$. *If* $MM2$ *randomizes his ask and bid quotes according to (i) and updates his belief according to Lemma 10, then* $MM1(\underline{V})$*'s expected payoff from setting* $(a_{1,1}, b_{1,1}) \in [v, a_{\max}[\times\{b_{\min}\}$ *is equal to* $\pi_1^*(\underline{V}, 2, p) = (3p-1)(\overline{V}-\underline{V})$.

Proof: From Lemma 11 we know that if $(a_{1,1}, b_{1,1}) \in [v, a_{\max}[\times\{b_{\min}\}$, then

$$\begin{aligned}\pi_1^*(\overline{V}, 2, p) &= (1-p)(\overline{V}-\underline{V}) = \\ &\quad q^*(\overline{V}-b_{\min})\Pr(b_{2,1} = b_{\min}) + \\ &\quad (a_{1,1}-\overline{V})(1-F_2(a_{1,1})) + (1-Post(a_{1,1}, b_{1,1}))(\overline{V}-\underline{V})\end{aligned}$$

This means that for $(a_{1,1}, b_{1,1}) \in [v, a_{\max}[\times\{b_{\min}\}$, it results

$$Post(a_{1,1}, b_{1,1})(\overline{V}-\underline{V}) =$$

$$p(\overline{V}-\underline{V}) + q^*(\overline{V}-b_{\min})\Pr(b_{2,1} = b_{\min}) + (a_{1,1}-\overline{V})(1-F_2(a_{1,1})) \tag{18}$$

Now, $MM1(\underline{V})$'s overall expected payoff from setting quotes $(a_{1,1}, b_{1,1}) \in [v, a_{\max}[\times\{b_{\min}\}$ is equal to

$$\pi_1^*(\underline{V}, 2, p) =$$

$$q^*(\underline{V}-b_{\min})\Pr(b_{2,1} = b_{\min}) + (a_{1,1}-\underline{V})(1-F_2(a_{1,1})) + Post(a_{1,1}, b_{1,1}))(\overline{V}-\underline{V})$$

By substituting $Post(a_{1,1}, b_{1,1}))(\overline{V}-\underline{V})$ and $F_2(a_{1,1})$ from the expressions (18) and (8) respectively, we obtain:

$$\pi_1^*(\underline{V}, 2, p) = 2q^*((\underline{V}+\underline{V})/2 - b_{\min})\Pr(b_{2,1} = b_{\min}) + (3p-1)(\overline{V}-\underline{V})$$

This implies that $\pi_1^*(\underline{V}, 2, p) = (3p-1)(\overline{V}-\underline{V})$ provided that $q^* = 0$ for $b_{\min} \neq (\overline{V}+\underline{V})/2$. If $b_{\min} = (\underline{V}+\underline{V})/2$, then the result holds for any $q^* \in [0, 1]$. □

Step 2: No profitable deviations for MMs

Now we show that the strategies illustrated in Lemma 8 are best replies to each other and form a PBE of the game $\Gamma(2,p)$. This will require three Lemma.

Lemma 13: *If $MM1$ randomizes his ask and bid quotes according to (ii) and (iii), then for $MM2$ it is optimal to set his ask quotes and bid quotes in the intervals $[v, a_{\max}]$ and $[b_{\min}, v]$, respectively.*

Proof: By Lemma 9, we have to show that $MM2$ cannot get a payoff higher than zero given the strategies *(i)-(ii)* of $MM1$. Let us check this for the bid auction. If $MM2$ sets $b_{2,1} > v$, then he is certain to win the bid auction and his payoff will be $v - b_{2,1} < 0$. If he sets $b_{2,1} < b_{\min}$, then he is certain to lose the bid auction and his payoff will be zero. Thus, $b_{2,1} \in [b_{\min}, v]$ is optimal and $MM2$ has no profitable deviation in the bid auction. A similar argument applied to the ask auction proves that $MM2$ has no profitable deviations. $\square$

In the following Lemma, we show the conditions in which neither $MM1(\overline{V})$ or $MM1(\underline{V})$ have any profitable unilateral deviations.

Lemma 14: *If $b_{\min} \geq (\overline{V}+\underline{V})/2$ and $q^* = 0$ whenever $b_{\min} > (\overline{V}+\underline{V})/2$, then it is optimal for $MM1$ to randomize his quotes according to (ii) and (iii).*

Proof: From Lemma 11 and 12, we know that if $q^* = 0$ whenever $b_{\min} \neq (\overline{V}+\underline{V})/2$, then $MM1(\underline{V})$'s expected payoff from setting $(a_{1,1}, b_{1,1}) \in S_1(\underline{V})$ is equal to $\pi_1^*(\underline{V}, 2, p) = (3p-1)(\overline{V} - \underline{V})$ and $MM1(\overline{V})$'s expected payoff from setting $(a_{1,1}, b_{1,1}) \in S_1(\overline{V})$ is equal $\pi_1^*(\overline{V}, 2, p) = (1-p)(\overline{V} - \underline{V})$. We need to show that there is no $(a_{1,1}, b_{1,1}) \notin S_1(\underline{V})$ (resp.$(a_{1,1}, b_{1,1}) \notin S_1(\overline{V})$) that provides $MM1(\underline{V})$ (resp. $MM1(\overline{V})$) with a payoff strictly greater than $(3p-1)(\overline{V} - \underline{V})$ (resp. $(1-p)(\overline{V} - \underline{V})$).

First, consider $MM1(\underline{V})$: a possible deviation would be to mimic $MM1(\overline{V})$ at $t = 1$, by setting $b_{1,1} = b_{\min} + \varepsilon$, $a_{1,1} = a_{\max}$, and at $t = 2$, $b_{1,2} = \underline{V}$ $a_{1,2} = \overline{V} - \varepsilon$. After observing $MM1$'s quotes in the first stage, $MM2$ will believe that $\widetilde{V} = \overline{V}$ and he will set $a_{2,2} = b_{2,2} = \overline{V}$. Thus, $MM1(\underline{V})$'s expected payoff from this deviation can be, at maximum, arbitrarily close to

$$(\underline{V} - b_{\min})G_2(b_{\min}) + (\overline{V} - \underline{V})$$

where the first term is the loss in the first period and the second term is the gain in the second period. Considering (9), it results that this expression is not greater than $\pi_1^*(\underline{V}, 2, p) = (3p-1)(\overline{V} - \underline{V})$ if $b_{\min} \geq (\overline{V} + \underline{V})/2$.

Another possibility would be that $MM1(\overline{V})$ or $MM1(\underline{V})$ post $(a_{1,1}, b_{1,1}) \notin S_1(\underline{V}) \cup S_1(\overline{V})$. Namely, they could post bid and ask prices that have a positive probabilityof winning both bid and ask auctions , i.e., $b_{1,1} > b_{\min}$ and $a_{1,1} < a_{\max}$. This is not profitable if an out-of-equilibrium-path belief $\widehat{P}(a_{1,1}, b_{1,1})$ exists, so that

$$\begin{aligned}
(1-p)(\overline{V} - \underline{V}) &\geq (a_{1,1} - \overline{V})(1 - F_2(a_{1,1})) + (\overline{V} - b_{1,1})G_2(b_{1,1}) + \\
&\quad +(1 - \widehat{P}(a_{1,1}, b_{1,1}))(\overline{V} - \underline{V}) \\
(3p-1)(\overline{V} - \underline{V}) &\geq (a_{1,1} - \underline{V})(1 - F_2(a_{1,1})) + (\underline{V} - b_{1,1})G_2(b_{1,1}) + \\
&\quad +\widehat{P}(a_{1,1}, b_{1,1})(\overline{V} - \underline{V})
\end{aligned}$$

Where $F_2(.)$ and $G_2(.)$ are given by (8) and (9). In other words, out-of-equilibrium-path belief must be such that this deviation is not profitable for either $MM1(\overline{V})$ or $MM1(\underline{V})$. Easy computation shows that such a belief exists whenever $b_{\min} \geq (\overline{V} + \underline{V})/2$. A second possible deviation for $MM1(\overline{V})$ or $MM1(\underline{V})$ might be to propose an ask price that has a positive probability of winning and a bid price smaller than $b_{\min}$(i.e., $b_{1,1} < b_{\min}$ and $a_{1,1} < a_{\max}$). This is not profitable if an out-of-equilibrium-path belief $\widehat{\widehat{P}}(a_1, b_1)$ exists so that

$$\begin{aligned}
(1-p)(\overline{V} - \underline{V}) &\geq (a_{1,1} - \overline{V})(1 - F_2(a_{1,1})) + (1 - \widehat{\widehat{P}}(a_{1,1}, b_{1,1}))(\overline{V} - \underline{V}) \\
(3p-1)(\overline{V} - \underline{V}) &\geq (a_{1,1} - \underline{V})(1 - F_2(a_{1,1})) + \widehat{\widehat{P}}(a_{1,1}, b_{1,1})(\overline{V} - \underline{V})
\end{aligned}$$

that are both satisfied for $\widehat{\widehat{P}}(a_1, b_1)(\overline{V} - \underline{V}) = (3p-1)(\overline{V} - \underline{V}) - (a_1 - \underline{V})(1 - F_2(a_1))$. A third possible deviation could be to post a bid price that has a positive probability of winning and an ask price larger than $a_{\max}$ (i.e., $b_{1,1} > b_{\min}$ and $a_{1,1} > a_{\max}$). This clearly leads to the same payoff of posting $b_{1,1} > b_{\min}$ and $a_{1,1} = a_{\max} = \overline{V}$, that is the equilibrium payoff. Finally, since cross quotes and posting very large spreads are clearly dominated strategies, we can conclude that if $b_{\min} \geq (\overline{V} + \underline{V})/2$ and $q^* = 0$ whenever $b_{\min} > (\overline{V} + \underline{V})/2$, then $MM1$ has no profitable deviations. □

In order to end the proof of Lemma 8, we still have to show that the conditions of Lemma 14 and 12 are always met, i.e., $b_{\min} \geq (\overline{V} + \underline{V})/2$, and $q^* = 0$ whenever $b_{\min} > (\overline{V} + \underline{V})/2$. This last result is provided in the following Lemma.

Lemma 15: *A $q^* \in [0,1]$ such that $b_{\min} \in [(\overline{V} + \underline{V})/2, v]$ always exists. Moreover, if $b_{\min} > (\overline{V} + \underline{V})/2$ then $q^* = 0$.*

Proof: From the description of the equilibrium, observe that $MM1$ never tries to buy and sell the asset simultaneously. Then the probability with which $MM1$ tries to sell, and sets $a_{1,1} < a_{\max}$, must be equal to the probability in which he stays out of the bid auction and sets $b_{1,1} = b_{\min}$. This implies that

$$\begin{aligned} \Pr(a_{1,1} &< a_{\max}|\underline{V}) = \Pr(b_{1,1} = b_{\min}|\underline{V}) = 1 && (19)\\ \Pr(a_{1,1} &< a_{\max}|\overline{V}) = \Pr(b_{1,1} = b_{\min}|\overline{V}) && (20)\end{aligned}$$

As $1 = \Pr(b_{1,1} = b_{\min}|\underline{V}) = \underline{F}(a_{\max})$, the condition (19) and expression (12) lead to $a_{\max} = \overline{V}$. Note also that, from (10), $\Pr(b_{1,1} = b_{\min}|\overline{V}) = \frac{(1-p)(b_{\min}-\underline{V})}{p(\overline{V}-b_{\min})}$ and from $a_{\max} = \overline{V}$, we have $\Pr(a_{1,1} < a_{\max}|\overline{V}) = \overline{F}(\overline{V})$. Thus, $b_{\min}$ is characterized by the equation (20) that becomes:

$$\overline{F}(\overline{V}) = \frac{(1-p)(b_{\min} - \underline{V})}{p(\overline{V} - b_{\min})} \tag{21}$$

where $\overline{F}(.)$ is the solution of the differential equation (13). Namely $\overline{F}(.)$ is:

$$\overline{F}(x) = 1 - \frac{(x-\underline{V})\sqrt{2(1-p)(p-1/2)(1-q)}\exp[\theta(x)]}{p\sqrt{(1-q)(1-p)(\overline{V}-\underline{V})(2x-\overline{V}-\underline{V}) - 2(x-v)(\overline{V}-x)}} \tag{22}$$

with

$$\begin{aligned}\theta(x) \quad = \quad & \\ & \frac{1-q-(2-q)p}{\sqrt{(1-p)(p(2-q)^2+(2-q)q-2)}}\left(\arctan\left[\frac{q(1-p)}{\sqrt{(1-p)(p(2-q)^2+(2-q)q-2)}}\right] + \right. \\ & \left. + \arctan\left[\frac{2(x-\underline{V})-(q+p(2-q))(\overline{V}-\underline{V})}{(\overline{V}-\underline{V})\sqrt{(1-p)(p(2-q)^2+(2-q)q-2)}}\right]\right)\end{aligned}$$

Consider the expression of $\theta(x)$. Note that the argument in the square-roots $(1-p)(p(2-q)^2+(2-q)q-2)$ is zero for $q = \frac{\sqrt{2p-1}\left(1-\sqrt{2p-1}\right)}{1-p}$ and positive for

$q \in \left[0, \frac{\sqrt{2p-1}\left(1-\sqrt{2p-1}\right)}{1-p}\right[$. Consequently, $\overline{F}(x)$ is well defined and continuous for all q in this interval.

We are interested in the properties of the expression (22) evaluated at $x = \overline{V}$. This gives $\overline{F}(\overline{V})$ that is a continuous function of q and p and does not depend on $\overline{V}$ and $\underline{V}$. In fact, it results:

$$\lim_{q \to \left(\frac{\sqrt{2p-1}(1-\sqrt{2p-1})}{1-p}\right)^-} \overline{F}(\overline{V}) = 1$$

$$\begin{aligned}
\lim_{q\to 0} \overline{F}(\overline{V}) &= 1 - \frac{\sqrt{2p-1}}{p} \exp\left[\frac{(1/2-p)}{\sqrt{(1-p)(p-1/2)}} \arctan\left[\frac{1-p}{\sqrt{(1-p)(p-1/2)}}\right]\right] \leq 1 \\
\lim_{q\to 0} \overline{F}(\overline{V}) &\leq \frac{1-p}{p} \text{ iff } p < p^*
\end{aligned}$$

where the first limit is taken from the left and $p^* \simeq 0.64087$ is the level of p that solves:

$$\lim_{q\to 0} \overline{F}(\overline{V}) = \frac{1-p}{p}$$

Our objective is to show that there exists always a couple $(q, b_{\min})$ with $q \in \left[0, \left(\frac{\sqrt{2p-1}(1-\sqrt{2p-1})}{1-p}\right)\right[$ and $b_{\min} \in [(\overline{V}+\underline{V})/2, v]$ thus ensuring that condition (21) is met. Note that $\frac{(1-p)}{p} = \left.\frac{(1-p)(b_{\min}-\underline{V})}{p(\overline{V}-b_{\min})}\right|_{b_{\min}=(\overline{V}+\underline{V})/2}$.

Suppose $p \in]1/2, p^*]$, then it results

$$\lim_{q\to 0} \overline{F}(\overline{V}) \leq \frac{1-p}{p} < 1 = \lim_{q \to \left(\frac{\sqrt{2p-1}(1-\sqrt{2p-1})}{1-p}\right)^-} \overline{F}(\overline{V})$$

By continuity of $\overline{F}(\overline{V})$ in q, these two inequalities imply that that condition (21) is met for $b_{\min} = (\overline{V}+\underline{V})/2$ and some $q^* \in \left[0, \frac{\sqrt{2p-1}(1-\sqrt{2p-1})}{1-p}\right]$.

Now consider the case $p \geq p^*$, then we have

$$\begin{aligned}
\left.\frac{(1-p)(\underline{V}-b_{\min})}{p(\overline{V}-b_{\min})}\right|_{b_{\min=(\overline{V}+\underline{V})/2}} &= \frac{(1-p)}{p} < \lim_{q\to 0} \overline{F}(\overline{V}) \\
&\leq 1 = \left.\frac{(1-p)(b_{\min}-\underline{V})}{p(\overline{V}-b_{\min})}\right|_{b_{\min=v}}
\end{aligned}$$

that implies that by continuity of $\frac{(1-p)(\underline{V}-b_{\min})}{p(\overline{V}-b_{\min})}$ in $b_{\min}$, there is a $b_{\min} \in](\overline{V}+\underline{V})/2, v[$ so that condition (21) is met for $q = 0$. In other words for each level of $p > 1/2$, there exists an appropriate q^* and $b_{\min}$ that satisfies equation (21) and the conditions in Lemma 14 and 12. □

Steps 1 and 2 complete the proof of Lemma 8. □

In order to conclude the proof of Proposition 3 in the case of two trading rounds, we must now consider the cases in which $p < 1/2$ and $p = 1/2$.

Case $p < 1/2$.
The equilibrium in the last round of the game is known, so we analyze the first round of trade. By using the symmetry property (SP) it is easy to characterize the equilibrium strategy in the first round of trade when $p < 1/2$. In this case, $MM1(\overline{V})$ always tries to buy the asset, while $MM1(\underline{V})$ randomizes between trying to sell it and mimicking $MM1(\overline{V})$. The ex-interim equilibrium payoffs are equal to $(2-3p)(\overline{V}-\underline{V})$ for $MM1(\overline{V})$ and $p(\overline{V}-\underline{V})$ for $MM1(\underline{V})$, while $MM2$ has an expected payoff equal to zero.

Case $p = 1/2$.
Finally, if $p = 1/2$, then at the first stage of the game, all market makers set bid and ask quotes equal to $v = (\overline{V}+\underline{V})/2$ and the posterior belief does not change. Such a pure strategy, pooling equilibrium exists only for $p = 1/2$ and is sustained by the following out of equilibrium path belief:

$$\Pr(V = \overline{V}|a_{1,1}, b_{1,1}) = \begin{cases} 1 \text{ for } b_{1,1} > 1/2 \\ 0 \text{ for } a_{1,1} < 1/2 \end{cases}$$

This ends the Proof of Proposition 3 when $T = 2$. □

Before considering the case of a general T, note that $MM1$'s ex-interim total equilibrium payoffs for the game $\Gamma(2,p)$ are continuous piecewise-linear monotone function in p.[18]

$$\begin{aligned} \pi_1^*(\overline{V}, 2, p) &= \begin{cases} (2-3p)(\overline{V}-\underline{V}) \text{ if } p \leq 1/2 \\ (1-p)(\overline{V}-\underline{V}) \text{ if } p > 1/2 \end{cases} \\ \pi^*(\underline{V}, 2, p) &= \begin{cases} p(\overline{V}-\underline{V}) \text{ if } p \leq 1/2 \\ (3p-1)(\overline{V}-\underline{V}) \text{ if } p > 1/2 \end{cases} \end{aligned}$$

[18] The same can be seen in the equilibrium payoff of the one shot game.

This suggests that we can apply the same method used in this section recursively to obtain the equilibrium when the market makers interactions continue for an arbitrary number of periods T.

Construction of an equilibrium for $T > 2$: sketch

The equilibrium can be characterized recursively applying the method used for the two period case.[19]

By fixing a number of repetitions T for all natural numbers $j \leq T$ and all $t \leq T$, we generate the numbers $r_{j,T}$ recursively starting from $r_{0,T} = 0$ and $r_{1,T} = 1$ as follows:

$$r_{j,t} = \begin{cases} 0 \text{ if } j \leq 0 \\ 1 \text{ if } j \geq t \\ \dfrac{r_{j-1,t-1} + r_{j,t-1}}{2} \text{ elsewhere} \end{cases}$$

In this way, for a fixed τ we partition the interval $[0, 1]$ in successively τ number of sub-intervals: $[r_{0,\tau}, r_{1,\tau}]$, $[r_{1,\tau}, r_{2,\tau}]$, ..., $[r_{\tau-1,\tau}, r_{\tau,\tau}]$. Take the game at time t, and let $\tau + 1$ be the number of trading rounds that remain to be played. Suppose that $MM1(\overline{V})$ and $MM1(\underline{V})$'s equilibrium continuation payoff is continuous and linear in the level of posterior belief p_{t+1}within each sub-interval $[r_{0,\tau}, r_{1,\tau}]$, ..., $[r_{\tau-1,\tau}, r_{\tau,\tau}]$, as is the case, for example, in the one shot game and in the twice repeated game. This allows us to construct the equilibrium strategies in exactly the same way that we constructed the equilibrium for the twice repeated game. It turns out that the resulting equilibrium payoff is still continuous and piecewise linear in the level of beliefs, so that we can use the argument recursively for any T. The only difference with the twice repeated game is that now the belief p_t follows a process that makes it jump into different sub-intervals at each stage. Namely, if $p \in [r_{i-1,T}, r_{i,T}]$ and $MM1$ tries to buy (resp. to sell) the asset, then the posterior belief will belong to the interval $[r_{i-1,T-1}, r_{i,T-1}]$ (resp. $[r_{i-2,T-1}, r_{i-1,T-1}]$). Therefore one has to take the piecewise linearity of $MM1$'s continuation payoff into account when writing the differential equations that define the informed market maker's quotes distribution. Apart from this, the characterization of MMs' equilibrium strategies is analogous to that in the case of the twice repeated game.

End of the proof of Proposition 3. ■

[19]The complete proof is available from the authors upon request.

Proof of lemma 4:
Let $p_2 = \Pr(V = \overline{V} | a_{1,1}, b_{1,1} = b_{\inf})$ and let $v_2 = p_2\overline{V} + (1-p_2)\underline{V}$

$$\begin{aligned} E[a_{2,2}] &= \int_{v_2}^{\overline{V}} x dF^*(x) + \overline{V}(1 - F^*(\overline{V})) = v_2 - p_2 \ln(p_2)(\overline{V} - \underline{V}) \\ E[b_{2,2}] &= \int_{\underline{V}}^{v_2} x dG^*(x) + \underline{V}G^*(\underline{V}) = v_2 - (1-p_2)\ln(1-p_2)(\overline{V} - \underline{V}) \end{aligned}$$

where $F^*(.)$ and $G^*(.)$ are given in Proposition 1. Differentiating this expression with respect to p_2 we obtain

$$\begin{aligned} \frac{\partial E[a_{2,2}]}{\partial p_2} &= -(\overline{V} - \underline{V})\ln(p_2) > 0 \\ \frac{\partial E[b_{2,2}]}{\partial p_2} &= -(\overline{V} - \underline{V})\ln(1-p_2) > 0 \end{aligned}$$

If $p > 1/2$:

$$p_2 = Post(a_{1,1}, b_{\min}) = \frac{\pi_1^*(\underline{V}, 2, p) - (a_{1,1} - \underline{V})(1 - F_2(a_{1,1}))}{(\overline{V} - \underline{V})}$$

Using the expression of $F_2(a)$ provided by (8), and differentiating with respect to $a_{1,1}$ we obtain:

$$\frac{\partial p_2}{\partial a_{1,1}} = \frac{(2p-1)(\overline{V} - \underline{V})}{(2a_{1,1} - \overline{V} - \underline{V})^2}$$

that is positive because $p > 1/2$. The result follows from $\frac{\partial E[a_{2,2}]}{\partial a_{1,1}} = \frac{\partial E[a_{2,2}]}{\partial p_2}\frac{\partial p_2}{\partial a_{1,1}}$ and $\frac{\partial E[b_{2,2}]}{\partial a_{1,1}} = \frac{\partial E[b_{2,2}]}{\partial p_2}\frac{\partial p_2}{\partial a_{1,1}}$. The result for $p < 1/2$ follows from the symmetry of the model. Finally, in order to prove that $MM1$'s quotes in the second period do not depend on $MM2$'s quotes in the first period, it is sufficient to observe that the distribution of $(a_{1,2}, b_{1,2})$ is only affected by p_2, which does not change with $MM2$'s quotes. ■

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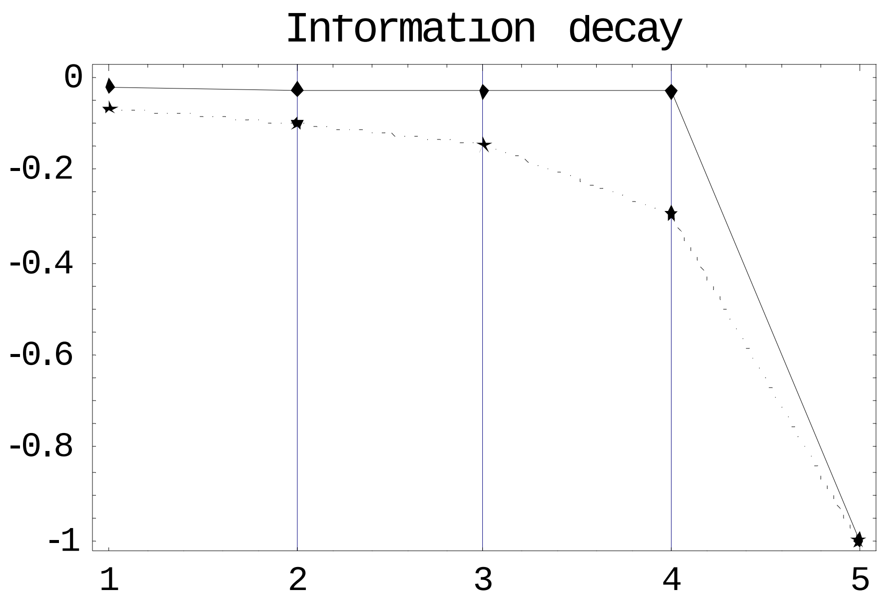

Figure 1: This Figure shows the expected rate of change of Σ_t, the variance of $\widetilde{V}$ conditional on public information. For both p close to 1/2 (- - -) and p close to 1 or 0 (—), Σ_t decreases at an increasing rate.

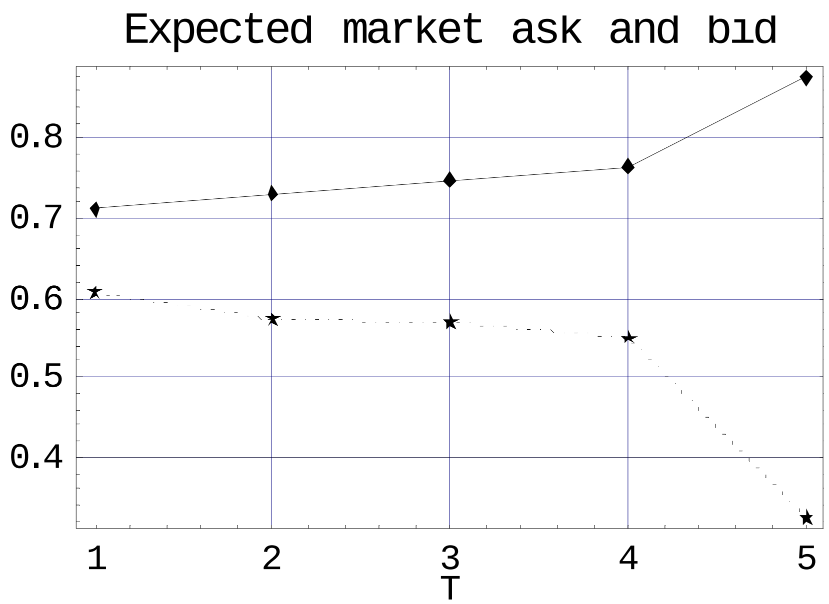

Figure 2: This Figure shows the expected minimum ask (—) and maximum bid (- - -) as a function of time. The parameter set is $p = 0.65$ and $T = 5$, $\underline{V} = 0$ and $\overline{V} = 1$.

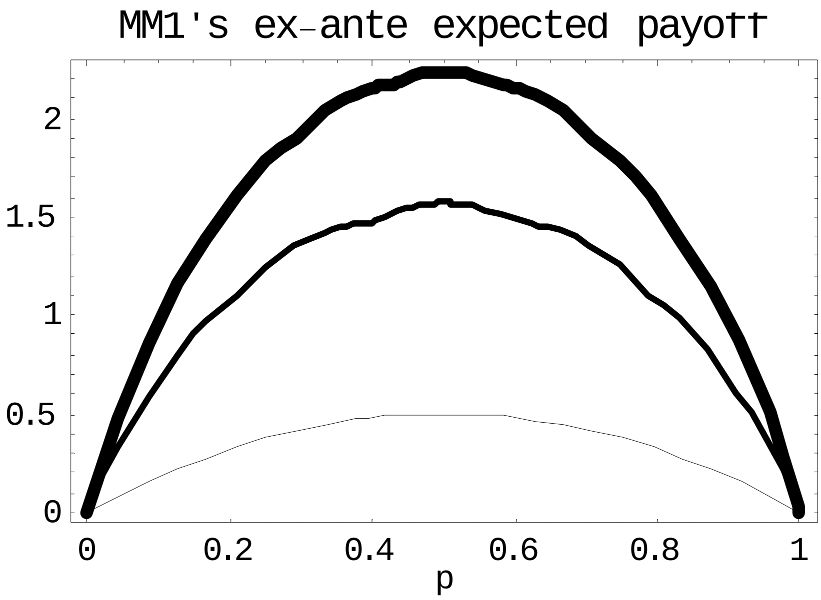

Figure 3: This Figure shows $MM1$'s ex-ante expected payoff as a function of p for $T = 1$ (thin line), $T = 15$ and $T = 30$ (thick line) and $\underline{V} = 0$, $\overline{V} = 1$.

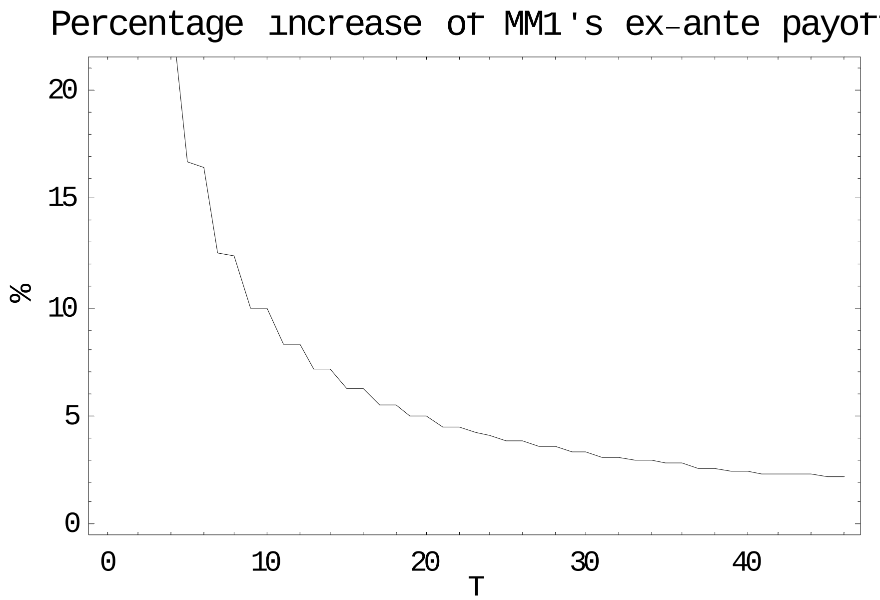

Figure 4: This Figure represents the increase in $MM1$'s expected equilibrium payoff when T increases. The parameters set is $p = 0,51$, $\underline{V} = 0$, $\overline{V} = 1$.

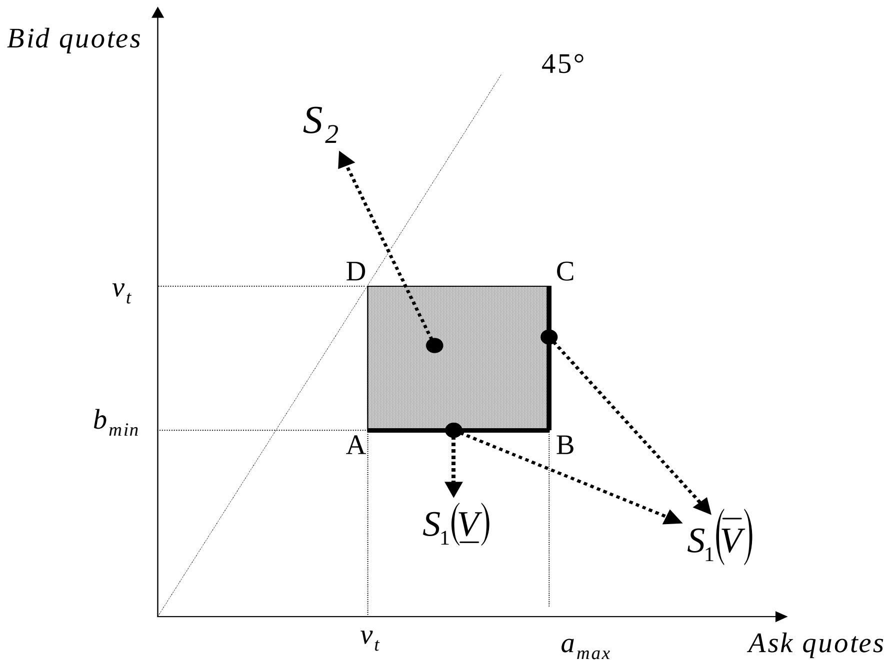

Figure 5: This Figure displays MMs' equilibrium supports in the first round of the game $\Gamma(p,2)$ when $p > 1/2$: $MM2$ randomizes his quotes on S_2 that is the shaded rectangle$ABCD$; $MM1(\underline{V})$ randomizes his quotes on $S_1(\underline{V})$ that is the line AB; $MM1(\overline{V})$ randomizes his quotes on $S_1(\overline{V})$ that is the union of the lines AB and BC.