Infinitely lived representative agent exchange economy with myopia*

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Abstract

We consider a family of infinitely long lived representative agent economy where, each period, agents can only decide consumption plan of finite dimension n. It is shown that myopia generates indeterminacy and monetary equilibrium in infinitely lived representative economy. Any invertible dynamics with at most one monetary steady state that is increasing in the quantity of money can represent the set of equilibria of an appropriate myopic economy.

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1 Introduction

Two paradigms play an important role in macroeconomics and in monetary theory: following Ramsey [11], the paradigm of an infinitely lived, representative agent (ILRA henceforth); and, following Samuelson [13], the paradigm of overlapping generations (OLG henceforth).

Overlapping generations economies with finitely lived individuals are quite different from those with a finite number of infinitely lived individuals. The main differences concern the determinacy and the Pareto optimality of competitive equilibria and the existence of monetary equilibria. In infinitely lived individual economies, competitive equilibrium allocations are Pareto optimal and locally unique and the equilibrium price of fiat money is zero. On the contrary, in overlapping generation economies, competitive equilibrium allocations are not necessarily Pareto optimal or locally unique, while fiat money may maintain a positive price at equilibrium. These differences have important implications concerning on one hand the optimality of a decentralized, competitive market as a mechanism for the intertemporal allocation of resources; on the other hand the possibility and desirability of active macroeconomic and monetary policy.

The fact that the two paradigms used in dynamic macroeconomics lead to such different result is a weakness for macroeconomic theory that can be solved providing a theory able to generate the two feature as particular cases.

It is possible to distinguish two branches in the literature that try to explain the differences between ILRA economies and OLG economies. One approach considers the ILRA economy as an extreme case of the OLG economy. Aiyagari [1] and [3] shows as a ILRA economy can be obtained introducing bequest motive in a OLG economy.

The other approach starts from ILRA economy and it proves that introducing cash in advance constraint (Huo [10]) or finance constraints (Woodford [14]) the equilibrium dynamics is equivalent to the one of a two period life span OLG economy.

Both these approaches make the link between two extreme cases: the two period life span OLG economy and the ILRA economy. None explain what are the relationships between an OLG economy where agent's life span is greater than two and ILRA economy.

This paper has two main purposes: on one hand we want to test the robustness of some results concerning ILRA equilibrium through the introduction of some degree of bounded rationality, and on the other hand we want to provide an alternative way to link the OLG literature with the ILRA, that allows to generate "long lived" OLG economy starting from ILRA economy.

We study what are the implication on the equilibrium price dynamics of an ILRA exchange economies when introducing bounded rationality. Furthermore we try to understand under what conditions the equilibrium set of an OLG economy can be obtained as the equilibrium set of an associated economy of infinitely lived bounded rational individuals.

We model bounded rationality in a simple way: at each date, individuals optimize as if they were facing a finite horizon of n periods. Even if agents derive utility from the consumption in each period from 0 to infinity, they are able to choose only a finite dimension consumption plan. The dimension of this plan measures the level of myopia in the economy. It follows that in each date t the agent revises the consumption decision he made in $t-1$. Indeed in $t-1$ he did not consider the price and the endowment of period $t+n$ while these variables enter in the maximization problem solved in period t.

The first result (Theorem 2) concerns the properties of the equilibrium prices dynamics when the representative agent is myopic. Using a constructive proof that applies some of the technics used in Boldrin and Montrucchio [4] to the theory of integrability of incomplete system of demand function (Epstein [6]), we prove that there are few restrictions on the equilibrium price dynamics. Namely given any function $G : R^n_+ \times R \to R_+$ homogenous of degree 1 and increasing in its last argument, there exist a myopic economy whose backward monetary equilibrium price dynamic is $p_t = G(p_{t+1}, ...p_{t+n}, M)$ when the aggregate level of money is M.

Thus even very irregular price dynamics can be observed at equilibrium and, more striking is that introducing myopia is sufficient to generate indeterminacy of the equilibrium and monetary equilibria. Relaxing, even slightly, the assumption of perfect rationality, the equilibrium loses its nice properties.

To understand what drives the result notice that the equilibrium of a infinitely lived representative agent exchange economy is the sequence of prices that induces agents to demand his endowment in each period. When agents are perfectly rational it follows that: 1) the equilibrium is unique as there exist only one infinite sequence of price that support the endowment of the representative agent (see also Dana [5]); 2) fiat money has no value in the economy as the value of the consumption is equal to the value of endowment plus the value of money.

When the representative agent is myopic at each date he revise his demand and his actual consumption need not to be equal to what he planned he would have consumed. This allows to weaken the link between the budget constraints, that appears in the maximization problems, and the equilibrium condition allowing for monetary equilibria and indeterminacy.

We show that when the utility function is separable and strictly concave the equilibrium dynamics of interest rate has always two steady states: one associated with a positive amount of money and constant price and one where the interest rate is constant and different from zero but money has no value. This properties recall the dynamics of overlapping generation economies. This paper shows that the similarities between myopia and OLG are even stronger: given some overlapping generation exchange economy, if the resulting dynamic of equilibrium prices is invertible, then there exists a myopic economy generating the same equilibrium dynamics. In particular, if the agents life span in the OLG economy is l, there is a corresponding myopic economy where the degree of myopia is $n = 2(l - 1)$.

This result is stronger than those provided by the existing literature as here the equivalence between myopia and OLG extends to "long lived" OLG economy.

Nevertheless, as most of the literature on OLG concerns the case where agents live two periods and have separable utility function, it is interesting to analyze the equivalence between OLG and myopia in this particular case. Under these assumption we provide a direct method for obtaining a myopic model equivalent to a given OLG economy, and vice-versa we show how to build an OLG economy equivalent to some given myopic economy. Unfortunately this simple method applies only to extreme myopia and overlapping generations with a life span of two dates.

The paper is organized as follows: section 2 describes the family of economies we consider, the assumption on individual behavior and the concept of equilibrium. Section 3 studies the property of the equilibrium dynamics and include the main result of the paper. Section 4 analyzes the equivalence between myopic and OLG economies, and section 5 concludes. Some of the proofs are in the appendix.

2 The model

We consider a family of economies parametrized by the natural number n. A typical element $EM(n)$ of this family has the following properties: there is a continuum of identical infinitely long lived agents and one consumption good per period. At beginning of time the representative agent possesses an amount M of fiat money and at any given date t he receives a constant endowment ω and he consumes an amount x_t. Let $u(x_t, x_{t+1}, \ ..., \ x_t, ...)$ be the utility derived by the consumption of path $\{x_t, x_{t+1}, \ ..., \ x_t, ...\}$, u is increasing, quasi concave and C^2 in its first n arguments. Let $Y = \{y, y, ..., y, ...\}$, where $y \in R_+$.

The agent's rationality is bounded in some sense: even if he lives an infinity of periods and he derives utility from consumption in every period, he can choose just a consumption plan of dimension n. In each period t the agent solves the following problem

$$MP_t(n) = \begin{cases} \max_{\{x(t,n)\}} u(x_t^t, x_{t+1}^t, ..., x_{t+n}^t, Y) \\ s.t. \ \ \sum_{j=t}^{t+n} p_j \left(x_j^t - \widetilde{\omega}_{j-t} \right) \leq M_t \\ \quad M_t = given \end{cases} \tag{1}$$

where $x(t,n) = \{x_t^t, \ x_{t+1}^t, \ ...x_{t+n}\}$, x_i^j ($j \leq i$) is the level of time i consumption planned in period j, p_t is the monetary price of the good in period t and M_i is the quantity of money that the agent holds at beginning of time i.

The natural number n represents the consumer's maximization horizon at any period t, and $1/n$ can be interpreted as the level of myopia. Indeed when, at beginning of period t, agent solves problem $MP_t(n)$, he plans consumption from t to $t+n$ and he does not consider consumption level in the periods after $t+n$.

Bounded rationality can also concern the expectation about future endowment. We assume $\widetilde{\omega}_\tau = \omega\delta(n,\tau)$, where $\delta : N \times N \rightarrow R_+$ satisfies $\delta(n,0) = 1$, and $\delta(n,\tau) = 0$ for any $\tau > n$. $\widetilde{\omega}_i$ represents the endowment that the representative agent expects to perceive after i periods. The condition $\delta(n,0) = 1$ means that the agent knows his present endowment. Notice that as the real endowment is constant, agent's expectation about future n endowments is correct if and only if $\delta(k,i) = 1$, $i = 1, 2, ..., n$. Finally $\delta(n,\tau) = 0$ for any $\tau > n$ means that the agent does not consider the endowments that are too far in the future. Thus, the heptoses of perfect foresight of future n endowments is a particular case of this specification of the model. Note that the expectation about future endowment are stationary in the sense that $\widetilde{\omega}_i$ do not depends on t.

As in each period t the agent plans consumption form t to $t+n$, each time $t+1$ he solves a new maximization problem where he considers consumption, endowment and prices from $t+1$ to $t+n+1$. Indeed prices and endowment of period $t+n+1$ were not considered in the previous period maximization phases. This implies that agent's consumption in some period t is not necessarily equal to what, some period before t, he planned he would have consumed in t. Thus it will be often the case that $x_t^t \neq x_t^j$ for $j < t$.

There are many explanations for this myopic behavior, one possible is that even if the agent has perfect foresight on the price level in the short run (from today to n periods after today), he has no idea about the long run level of prices (after $n+1$ periods on) and therefore he cannot plan consumption that are too far in the future. Nevertheless after each period he knows one more price he did not know the pervious period so that he maximizes again his objective function taking into account this new information. Therefore as time pass by he revises his consumption decision on the basis of the new information.

Let $p(t) = \{p_i\}_{i=t}^{\infty}$, let $p(t,n) = \{p_i\}_{i=t}^{t+n}$ and let $x(p(t,n),\ M_t, n)$ be the demand function of good at time t expressed in time t as a function of the prices from t to $t+n$, the endowment vector $\widetilde{\omega}$ and the quantity of money held at beginning of t. Formally $x(p(t,n), M_t, n)$ is the first element of the vector $\arg\max MP_t(n)$.

Notice that if myopia disappear (i.e. n goes to infinity) and the agent has perfect foresight of endowments, the representative agent behaves as a perfectly rational infinitely lived individual and so we obtain the standard ILRA economy.

We are ready now to define the concept of equilibrium:

Definition 1 *A perfect foresight equilibrium of the economy $EM(n)$ is a sequence of prices $\widehat{p}(0) = \{\widehat{p}_i\}_{i=0}^{\infty}$ such that spot markets of good and money are in equilibrium:*

$$x(\widehat{p}(t,n), M, n) = \omega, \quad \forall t \tag{2}$$

where $\widehat{p}(t,n) = \{\widehat{p}_i\}_{i=t}^{t+n}$.

As in the standard ILRA model, at equilibrium the representative agent consumes his endowment in each period. Nevertheless when agents are perfectly rational, the level of consumption they plan to sustain in the future is equal to the consumption they will actually sustain, this implies that spot market are at equilibrium only if future markets clear. When agent are myopic, they revise their consumption decision after each period, so that the minimal equilibrium condition is that what agents consume in each period is equal to their present demand, that is only spot market need to clear.

The sequence of equilibrium prices is such that in any period, when the agent chooses his present consumption considering the next n ones, he decides to consume his present endowment. Notice that in any period t, the agent has perfect foresight about the price level in the next n periods. Nevertheless, as agents change their mind after every period, the equilibrium does not require any condition on planned future demand.

If myopia disappear (i.e. n is infinity) and the agent has perfect foresight of endowment, the representative agent behaves as a perfectly rational infinitely lived individual. If this is the case, the equilibrium is unique as there exist only one infinite sequence of prices that supports the endowment. Furthermore money has zero value.

Intuitively, when n is finite, the sequence of equilibrium price $\widehat{p}(0)$ can be built as follows: fix arbitrarily the first n prices and find $\widehat{p}_n$, the price of period n, such that the spot market in period 0 is in equilibrium: $x(\widehat{p}(0,n), M, n) = \omega$; then, given prices from period 1 to n, find p_{n+1} so that $x(\widehat{p}(1,n), M, n) = \omega$ and so on . Therefore, given some n the equilibrium prices dynamics is given by the n-th order difference equation that is implicitly defined by (2).

3 Equilibrium dynamics

Consider some myopic economy where the degree of myopia is n. Let $Z : R_+^n \times R \to R_+$ be

$$Z(p_t, ...p_{t+n}, M_t) = x(p(t,n), M_t, n) - \omega$$

Z is the excess demand for time t consumption as it is expressed in time t. This demand is function of the monetary prices from t to $t+n$ and the quantity of money held at beginning of period t. Given that the amount of money is M in any period, the equilibrium condition for the myopic economy is

$$Z(p_t, ..., p_{t+n}, M) = 0, \ \forall t \tag{3}$$

The equilibrium condition (3) defines implicitly a n-th order dynamics of equilibrium prices parametrized by M.

What can be said about the dynamics of equilibrium price in myopic economies?

Notice that Z is homogenous of degree 0 and therefore the equilibrium dynamics of price will be homogenous of degree 1 in prices and money. Furthermore from Slutzky equation, we know that when the excess demand is zero the income effect vanishes; therefore the derivative of Z with respect to its first argument evaluated at some point satisfying (3) is always negative. This implies that the backward dynamics of prices is always well defined, or in other word, for any level of future prices $(p_{t+1}, \ ...p_{t+n-1})$ and money M there exist at most one present price p_t that induces the agent to demand exactly his endowment. Thus the equilibrium price dynamics generated by myopic representative agents economy is always invertible[1].

These are necessary condition that the equilibrium dynamics must satisfy. The following theorem provides sufficient condition on the backward dynamics of prices in order to be generated by some myopic economy

Theorem 1 *Let $D_{-0} \subset R^n_+$, $D_M \subset R$ and $D_0 \subset R_+$ be convex and compact sets satisfying $D_{-0} \supseteq D^n_0$; let $G : D_{-0} \times D_M \rightarrow D_0$ be a twice continuously differentiable function satisfying the following conditions: i) G is homogeneous of degree one, ii) $\partial G/\partial M > 0$ on $D_{-0} \times D_M$; then there exists a myopic economy whose degree of myopia is n and whose backward equilibrium dynamics is represented by $p_0 = G(p_1...p_n, M)$ for any initial condition and quantity of money in $D_{-0} \times D_M$.*

From theorem 2 it follows that there are few restrictions on the dynamics of equilibrium prices that can be generated by a myopic economy. Nevertheless from the homogeneity of G it follows that G has at most one monetary equilibrium. That means that there exist at most one level of real amount of money M/p that is compatible with constant equilibrium prices.

Theorem 2 may some how recall the indeterminacy result of the policy function obtained by Boldrin and Montrucchio [4], nevertheless their result concern the optimal solution of dynamic programming problem that is exacerbated from a general equilibrium context. Here we are considering the set of equilibria that can arise from a simple general equilibrium economy.

To prove theorem 2 means to find a representative agent myopic economy whose equilibrium dynamics can be described by G, where G is a given function that satisfies the hypothesis of the theorem. A myopic economy is identified with the representative agent's maximization problem (1) and

[1]This property follows from the representative agent assumption and can be weakened assuming that agents are heterogenous.

its equilibrium dynamics is implicitly defined by equation (3), given that Z comes from the solution of (1). Therefore we need to find an appropriate utility function $U(x_0, x_1, ..., x_{n-1})$ and endowment level ω such that, calling Z the excess demand for present good, it results:

$$Z(p_0, ..., p_n, M) = 0 \Leftrightarrow p_0 = G(p_1, ..., p_n, M) \tag{4}$$

for all $(p_0, p_1, ..., p_n, M) \in D_0 \times D_{-0} \times D_M$.

Note that there is no loss of generality in studying the equilibrium condition at time 0 as the fundamentals of the economy are stationary and the function Z does not depend on time. Indeed the dynamic properties of the equilibrium depends only on the shape of $Z(.)$.

It is useful to normalize prices and to express Z and G as functions of relative prices and real amount of money. Let $q_0 = p_0/p_n$, $q_{-0} = (p_1/p_n, ...p_{n-1}/p_n)$ and $m = M/p_n$. Define the functions z and g as follows:

$$z(q_0, q_{-0}, m) \stackrel{def}{=} Z(p_0/p_n, ..., p_{n-1}/p_n, 1, M/p_n)$$
$$g(q_{-0}, m) \stackrel{def}{=} G(p_1/p_n ..., p_{n-1}/p_n, 1, M/p_n)$$

From the homogeneity of degree 1 of G, it results that $q_0 = g(q_{-0}, m)$. Notice that g maps $d_{-0} \times d_m$ into d_0 where $d_{-0} \times d_m$ and d_0 are respectively the relevant domain and co-domain for g. Furthermore, from the hypothesis on G it follows that $\partial g/\partial m = \frac{1}{p_n}\partial G/\partial M > 0$.

Consider finally the function $\widehat{m} : d_0 \times d_{-0} \to d_m$ defined as follows:

$$\widehat{m}(q_0, q_{-0}) = m \Leftrightarrow q_0 = g(q_{-0}, m)$$

for all $(q_0, q_{-0}) \in d_0 \times d_{-0}$. $\widehat{m}$ is obtained inverting g with respect to m. If G represents the equilibrium dynamics, then $\widehat{m}(q_0, q_{-0})$ is the real quantity of money that is compatible with relative prices (q_0, q_{-0}) at equilibrium. Notice that from implicit function theorem and the hypothesis on G, $\widehat{m}$ is C^1 and there exist a $\mu > 0$ satisfying $\partial\widehat{m}/\partial q_0 \geq \mu$ on $d_0 \times d_{-0}$.

Restating condition (4) in real terms, it results:

$$z(q_0, q_{-0}, m) = 0 \Leftrightarrow q_0 = g(q_{-0}, m) \quad \forall\ (q_0, q_{-0}, m) \in d_0 \times d_{-0} \times d_m$$

It is now possible to define the set of excess demand function that originate the equilibrium dynamics G:

$$\Omega \stackrel{def}{=} \{z \mid \forall (q_0, q_{-0}) \in d_0 \times d_{-0},\ z(q_0, q_{-0}, m) = 0 \Leftrightarrow m = \widehat{m}(q_0, q_{-0})\}$$

I need to show that whenever $\widehat{m}$ comes from some G that satisfies the hypothesis of the theorem, there exist a myopic economy whose excess demand z belongs to Ω. The theorem is proved if there exist a utility maximization problem of the form (1) whose solution provides an excess demand for good 0, $z(q_0, q_{-0}, m)$, that belongs to Ω.

This problem is somehow closed to that of the integrability of an incomplete system of demand functions (see Epstein [6]): given the demand z for one good as function of the price of all the goods and the quantity of money, we want to know if this demand can be generated by a utility maximization problem. The difference with Epstein's problem is firstly that here instead of having only one demand function we can choose it in a class of functions (the set Ω). Secondly we are dealing with excess demand function instead of demand functions.

To integrate back from a demand function X to an utility function is equivalent to integrate back from X to an expenditure function $E(p_0, ..., p_n, u)$. Where the expenditure function $E(p_0, ..., p_n, u)$ is the minimum income required to reach utility level u when prices are $(p_0, ..., p_n)$. Indeed given the expenditure function it is always possible to recover the indifference curves.

Since here we are dealing with excess demand as functions of both monetary wealth and endowments, it is useful to define the *modified expenditure function* F obtained subtracting to the expenditure function the value of the endowment. If the agent receives an endowment equal to ω for all goods, the modified expenditure function associated with the maximization problem (1) is:

$$F(p_0, p_{-0}, u) = E(p_0, p_{-0}, u) - \omega \sum_{i=0}^{n} p_i$$

The modified expenditure function represents the minimal transfer of money required to reach exactly the utility level u when the level of price is (p_0, p_{-0}) and the agent have an endowment equal to ω for all n goods. Clearly while E is non-negative, F can be negative; this because for small u it could be necessary to "tax" the consumer in order to force him to reach just the utility level u. Furthermore as we are dealing with relative prices and real quantity of money it is useful to define the *normalized expenditure function* and the *normalized modified expenditure function* (NMEF henceforth) as

$$e(q_0, q_{-0}, u) \quad = \quad \frac{E(p_0, ..., p_n, u)}{p_n} = E(q_0, q_{-0}, 1, u)$$

$$f(q_0, q_{-0}, u) \quad = \quad \frac{F(p_0, ..., p_n, u)}{p_n} = F(q_0, q_{-0}, 1, u)$$

Where the right hand side equalities follows form the homogeneity of degree 1 of the expenditure function.

Let $f(q_0, q_{-0}, u)$ be the NMEF deriving from some maximization problem of the form (1) and let $z_0(q_0, q_{-0}, m)$ be the corresponding excess demand for good zero. From classical consumer theory it results that

$$\frac{\partial f(q_0, q_{-0}, u)}{\partial q_0} = z_0(q_0, q_{-0}, f(q_0, q_{-0}, u)) \tag{5}$$

Equation (5)[2] allows to characterize the set of modified expenditure function that are associated with an excess demand function that belongs to Ω.

Lemma 1 $z_0(q_0, q_{-0}, m) \in \Omega$ *if and only if*

$$\frac{\partial f(q_0, q_{-0}, u)}{\partial q_0} = 0 \Leftrightarrow u \textit{ is such that } f(q_0, q_{-0}, u) = \widehat{m}(q_0, q_{-0}) \tag{6}$$

Proof: Note first that from expression (5), $z_0(q_0, q_{-0}, f(q_0, q_{-0}, u)) = 0$ if and only if $\partial f(q_0, q_{-0}, u)/\partial q_0 = 0$.

Suppose that $z_0(q_0, q_{-0}, m) \in \Omega$, this means that $z_0(q_0, q_{-0}, m) = 0 \Leftrightarrow m = \widehat{m}(q_0, q_{-0})$ but then $z_0(q_0, q_{-0}, f(q_0, q_{-0}, u)) = 0 \Leftrightarrow u$ is such that $f(q_0, q_{-0}, u) = \widehat{m}(q_0, q_{-0})$ that implies that $\frac{\partial f(q_0, q_{-0}, u)}{\partial q_0} = 0 \Leftrightarrow u$ is such that $f(q_0, q_{-0}, u) = \widehat{m}(q_0, q_{-0})$ for expression (5).

Suppose now that $\frac{\partial f(q_0, q_{-0}, u)}{\partial q_0} = 0 \Leftrightarrow u$ is such that $f(q_0, q_{-0}, u) = \widehat{m}(q_0, q_{-0})$, but as $z_0(q_0, q_{-0}, f(q_0, q_{-0}, u)) = 0 \Leftrightarrow \partial f(q_0, q_{-0}, u)/\partial q_0 = 0$, this means that $z_0(q_0, q_{-0}, m) = 0 \Leftrightarrow m = \widehat{m}(q_0, q_{-0})$ and thus $z_0 \in \Omega$.$\square$

Proof of theorem 2: We proceed as follows: firstly we find the a modified expenditure functions that is associated with the excess demand functions that are in Ω. Choosing an appropriate endowment level ω we derive the expenditure function from the modified expenditure function. Finally we show that the expenditure function we derived satisfies all the integrability condition so that the associated utility function $U(x_0, x_1, ..x_n)$ is increasing and concave. Thus we have found a consumer maximization problem

[2] Let $\widetilde{x}(q_0, q_{-0})$ be the demand as function of income and price. From classical consumer theory we know that $\partial e/\partial q_0 = \widetilde{x}_0(q_0, q_{-0}, e)$; considering that $x_0(., m) = \widetilde{x}_0(., q\omega + m)$ it follows that $\partial e/\partial q_0 = x_0(q_0, q_{-0}, e - q\omega)$, and so the equation (5).

that generate an excess demand for present consumption that belongs to Ω. Therefore, the equilibrium dynamics of the myopic economy associated with this consumer maximization problem can be represented by the given function G.

Step 1) Construction of the modified expenditure function.

The following lemma provides some of the ingredients that are used to build an appropriate NMEF.

Lemma 2 *If G satisfies the hypothesis of theorem 2 then there exist a couple (γ, θ) of twice continuously differentiable functions, $\gamma : d_{-0} \times R \to d_m$ and $\theta : d_{-0} \times R \to d_0$, satisfying:*

i) $\gamma(q_{-0}, u)$ is bounded, strictly increasing in u, and strictly concave in q_{-0} for any u in $d_u \subset R$;

ii) $\widehat{m}\,(\theta(q_{-0}, u), q_{-0}) = \gamma(q_{-0}, u)$;

iii) $\theta(q_{-0}, u)$ is strictly increasing u;

iv) the derivative of θ with respect to q_{-0} is bounded.

Proof: Chose some $\gamma(q_{-0}, u) : d_{-0} \times d_u \to d_m$ satisfying property i) in the lemma. Note that for any $x \in d_m$ and any $q_{-0} \in d_{-0}$ there always exist some q_0 such that $\widehat{m}\,(q_0, q_{-0}) = x$ and namely $q_0 = g(q_{-0}, x)$; thus defining $\theta(q_{-0}, u) = g(q_{-0}, \gamma(q_{-0}, u))$, it results that the function γ and θ satisfy properties i) and ii) by construction. Property iii) follows from the definition of θ and γ and from $\partial G/\partial M > 0$. Property iv) comes from the boundedness of the partial derivative of $\widehat{m}$.□

In the following lemma we provide a candidate for the NMEF.

Lemma 3 *Let γ and θ be as defined in lemma 4, and let f defined as follows:*

$$f(q_0, q_{-0}, u) = \alpha \left(-q_0^2 + 2q_0\theta(q_{-0}, u) - \theta(q_{-0}, u)^2\right) + \gamma(q_{-0}, u)$$

if

$$0 < \alpha < \min \left[\frac{\mu}{2\delta}, \frac{\sigma}{2\delta|\Sigma|}\right]$$

then the there exist a real number k such that function $f(q_0, q_{-0}, u)$ satisfies the following conditions:

i) $f_u > 0$

ii) f is concave in (q_0, q_{-0}) for each u

iii) $f_{q_i} > k$, for $i = 0, 1, ...n - 2$

iv) $f - \sum_{i=0}^{n-2} q_i f_{q_i} > k$

where $\delta = diam\ d_0 = \max\{||x-y||, x, y \in d_0\}$, $\Sigma = \max\{||D^2_{q-0}\theta(q_{-0}, u)||, (q_{-0}, u) \in d_{-0} \times d_u\}$, $\sigma = -\max\{||D^2_{q-0}\gamma(q_{-0}, u)||,\ (q_{-0}, u) \in d_{-0} \times d_u\}$.

Proof: Deriving f with respect to u it results: $f_u = \theta_u 2\alpha(q_0 - \theta(q_{-0}, u)) + \gamma_u$; considering that $\gamma_u = \widehat{m}_{q_0}\theta_u$, it follows that $f_u = \theta_u(2\alpha(q_0 - \theta(q_{-0}, u)) + \widehat{m}_{q_0})$. As $\theta_u > 0$, $q_0 - \theta(q_{-0}, u) \geq -\delta$ and $m_{q_0} \geq \mu$, it follows that f_u is not smaller than $\theta_u(-2\alpha\delta + \mu)$ that is strictly positive for $\alpha < \mu/2\delta$, thus property i).

The concavity of f in (q_0, q_{-0}, u) is equivalent to the concavity of the function $h : R \to R$ defined as

$$h(t) = f(x_0 + tx_1, y_0 + ty_1, u)$$

with x_0 and y_0 fixed and $(x_0 + tx_1, y_0 + ty_1) \in d_0 \times d_{-0}$. Without loss of generality set $||y_1|| = 1$ and x_1 bounded. As f is C^2 it results

$$\begin{aligned} h''(t) &= \alpha\left(-2(x_1 - y_1'\theta_{q-0})^2 + 2\left(x_0 + tx_1 - \theta(y_0 + ty_1, u)\right)\right. \\ &\quad \left.(y_1' D^2\theta(y_0 + ty_1, u)y_1)\right) + y_1' D^2\gamma(y_0 + ty_1, u)y_1 \end{aligned}$$

if $\Sigma > 0$, then $h''(t)$ is less or equal to $\alpha 2\delta\Sigma - \sigma$ that is strictly negative for $\alpha < \sigma/2\delta\Sigma$; if $\Sigma < 0$, then $h''(t)$ is less or equal to $-\alpha 2\delta\Sigma - \sigma$ that is strictly negative for $\alpha < -\sigma/2\delta\Sigma$. Thus f is concave in (q_0, q_{-0}) for $\alpha < \sigma/2\delta|\Sigma|$. Note that as γ is strictly concave in q_{-0}, σ is strictly positive.

Properties iii), iv) and v) follow from the boundedness of the derivative of θ and γ and the boundedness of f. □

The following lemma shows that if f as defined in lemma 5 is a modified expenditure function, then the associated excess demand for good 0 will belong to Ω.

Lemma 4 *Let γ, θ and f be as defined in lemma 4 and 5, then*

$$\partial f(q_0, q_{-0}, u)/\partial q_0 = 0 \Leftrightarrow u \text{ is such that } f(q_0, q_{-0}, u) = \widehat{m}(q_0, q_{-0})$$

Proof: Note that $\partial f(q_0, q_{-0}, u)/\partial q_0 = 0 \Leftrightarrow q_0 = \theta(q_{-0}, u)$; if $q_0 = \theta(q_{-0}, u)$, then $f(q_0, q_{-0}, u) = \gamma(q_{-0}, u)$ that, by lemma 4, is equal to $\widehat{m}\,(\theta(q_{-0}, u)p_{-0}) = \widehat{m}(q_0, q_{-0})u$ the only u^* such that $f(q_0, q_{-0}, u^*) = \widehat{m}(q_0, q_{-0})$ is the one that satisfies $\theta(q_{-0}, u^*) = q_0$; but this implies that if $f(q_0, q_{-0}, u) = \widehat{m}(q_0, q_{-0})$, then $\partial f(q_0, q_{-0}, u)/\partial q_0 = 0$.□

Step 2) From the NMEF to the expenditure function.

The function f as defined in lemma 5 is a candidate for the normalized modified expenditure function, but in order for f to be a NMEF coming from a maximization problem of the form (1), it must satisfy some regularity conditions. In particular there must exist some endowment ω such that calling

$$e(q_0, q_{-0}, u) = f(q_0, q_{-0}, u) + \omega \left(\sum_{i=1}^{n} \frac{p_i}{p_n} + 1 \right) \tag{7}$$

the function $e(.)$ is a normalized expenditure function.

Form classical consumer theory we know that a function e is a normalized expenditure function if it satisfies the following conditions on its domain:

1)$\partial e/\partial u > 0$;

2)$\partial e/\partial q_i > 0$, $i = 0, ..., n-1$;

3)$e - \sum_{i=0}^{n-1} q_i \partial e/\partial q_i > 0$;

4) e is concave in (q_0, q_{-0}) for each u;

5) $e > 0$

Condition 3 comes from $\partial E/\partial p_n > 0$, where $E(p_0, \cdots, p_n, u) = p_n e(p_0/p_n, \cdots, p_{n-1}/p_n)$ is the expenditure function.

Now it is possible to show that for α small, the f defined in lemma 5 is actually a normalized modified expenditure function coming from a utility maximization problem of the form 1.

Choosing $\omega > \max[0, -k]$ and defining the function e as in expression (7), it results that e is actually a normalized expenditure function. Indeed e satisfies all the conditions 1-5: 1) $\partial e/\partial u = \partial f/\partial u > 0$ from lemma 5; 2) considering that $\partial e/\partial q_i = \partial f/\partial q_i + \omega > 0$ and that $\partial f/\partial q_i \geq k$ (lemma 5), property 2 follows trivially if $k > 0$, while if $k < 0$ it follows from $\omega > -k$; 3) considering that $e - \sum_{i=0}^{n-1} q_i \partial e/\partial q_i = f - \sum_{i=0}^{n-1} q_i \partial f/\partial q_i + \omega$, and that $f - \sum_{i=0}^{n-1} q_i \partial f/\partial q_i > k$ (lemma 5), property 3 follows trivially if $k > 0$,while if $k < 0$ it follows from $\omega > -k$; 4) e is concave in (q_0, q_{-0}) as it is the sum of a strictly concave function f and a linear function; 5) considering that $e \geq f + \omega$, and that $f > k$, property 5 follows trivially if $k > 0$,while if $k < 0$ it follows from $\omega > -k$.

Thus the maximization problem where the utility function is that recovered from the expenditure function e as defined in (**??**) and the endowment is $\omega > \max[0, -k]$ originates a demand function for good zero that belongs to Ω. Thus the myopic economy defined by such maximization problem generates an inverse price dynamics equal to G on $d_{-0} \times d_m$.■

Notice that from the technics used in the proof there exist more then one maximization problem that is compatible with a given dynamics G, thus when agents are myopic, knowing the equilibrium dynamics is not sufficient for recovering their utility.

3.1 Separable utility and steady states

Until now we have not done any assumption on the shape of the utility function u rather than monotonicity and concavity. Most of the problem studied in this literature deal with additive utility function. In this paragraph I show that when representative agent utility is additive and concave then the equilibrium dynamics of interest rate has generically two steady states: one monetary and one non monetary.

Assumption 1: $u(x_1, x_2, ..., x_t, ...) = \sum_{t=0}^{\infty} u_t(x_t)$ where for all t, u_t is increasing, strictly quasi concave and satisfies the inada conditions.

Let $\rho_t = p_{t+1}/p_t$ be the factor of interest at time t, in a steady state equilibrium it must results $\rho_t = \rho$ and $x(p(t,n), M, n) = \omega$ for any t. Let $\{x_i(n,\rho)\}_{i=t}^{n} = \arg\max MP_t(n)$ given $\rho_t = \rho$, $\forall t$, then, under assumption 1 it results that at any steady state equilibrium the individual budget constraint will take the form:

$$\sum_{i=1}^{n} \rho^i \left[x_i\left(n,\rho\right) - \widetilde{\omega}_i\right] = \frac{M}{p_t} \tag{8}$$

for any t, where $x_i(n,\rho) = u_t'^{-1}\left(u_0'\left(\omega\right)\right)$. It follows that any ρ that satisfies equation (8) represents a steady state equilibrium of the economy.

Proposition 1 *Under assumption 1, for any finite n there always exists at most one monetary steady state and at least one non monetary steady state. In the monetary steady state the level of prices is constant and the real value of money is equal to $\sum_{i=1}^{\infty}\left[x_i\left(n,1\right) - \widetilde{\omega}_i\right]$. In the non monetary steady state the rate of interest is constant and the real value of money is equal to zero.*

Proof: Left hand side of equation (8) is finite as long as n is finite, furthermore it does not depends on t. Therefore equation (8) can be satisfied only in two cases: when p_t is constant and equal to $M/\sum_{i=1}^{\infty}\left[x_i\left(n,1\right) - \widetilde{\omega}_i\right]$, and when the real value of money is zero. The first case corresponds to the monetary steady state: prices are constant and money has positive value provided that the amount of money M has the same sign of $\sum_{i=1}^{\infty}\left[x_i\left(n,1\right) - \widetilde{\omega}_i\right]$.

The non monetary steady state exists when right hand side of (8) is zero. The existence of at least one non monetary steady state is derived observing that $\sum_{i=1}^{\infty} \rho^i \left[x_i\left(\rho^i\right) - \widetilde{\omega}_i\right]$ is continuous in ρ, it goes to $-\infty$ as ρ goes to $+\infty$ and it is positive for ρ small enough.■

Proposition 7 states that given any ILRA exchange economy with additive concave utility function, it is sufficient to introduce a small degree of preference incompleteness in order to have money with positive value at equilibrium.

We call $m(n) = \sum_{i=1}^{\infty} \left[x_i\left(n,1\right) - \widetilde{\omega}_i\right]$ the real amount of money at the monetary steady state. Using the overlapping generation convention (Gale [7]) one could denote with "classical" an economy where $m(n)$ is negative and Samuelson an economy where $m(n)$ is positive.

As it has been shown by Aiyagari [2] and Reichlin [12] for overlapping generations models, the existence of steady states where fiat money has positive value depends crucially on the weight that future consumption have in the utility function with respect to the expected value of endowment available in the future. Intuitively whenever the intertemporal discount factor is less than the expected factor of growth of the value of individual endowment, positive monetary steady state does not exist. In this model it happens exactly the same if we assume that $u_t(x) = \beta^t u_0(x)$. Given the intertemporal preferences and the expected future endowments (that depends on the shape of $\delta(t,n)$), the real amount of money is positive for δ small enough and it is negative for δ large.

4 Equivalence between myopia and OLG

In this section we study the observational equivalence between a myopic economy $EM(k)$ and an OLG exchange economy.

Definition 2 *An economy $E1$ is equivalent to an economy $E2$ if any equilibrium $\widehat{p}(0)$ of $E2$ is also an equilibrium of $E1$ and vice-versa.*

In the following we provide sufficient conditions on the equilibrium dynamics of an overlapping generation economy for the existence of an equivalent myopic economy, i.e. a myopic economy whose dynamics of equilibrium prices is the same of those generated by the given overlapping generation economy.

Consider an overlapping generation economy where time goes from -∞ to $+\infty$, there is one good per period and the representative agent live for $l \geq 2$ periods. The agent born in period t solves the following utility maximization problem

$$PO_t(l) : \begin{cases} \max_{x_t, x_{t+1}} V(x_t, x_{t+1}, ..., x_{t+l-1}) \\ s.t. \;\; p_{t+j}(x^t_{t+j} - \widehat{\omega}_i) + M^t_{t+1+j} \leq M^t_{t+j} \\ \text{for} \quad j = 0, 1, ...l-1 \\ \text{with } M^t_t = M_{t+l} = 0 \end{cases}$$

The solution to $PO_t(l)$ provide the excess demand function $\widehat{Z} = (\widehat{Z}^1, \widehat{Z}^2, ...\widehat{Z}^l)$, Where $\widehat{Z}^i(p_t,\ p_{t+1},\ ..., p_{t+l-1})$ is the excess demand for consumption in period $t+i$ coming from an agent that is born in period t, or in other words it is the excess demand of the agent during his i-th period of life. $\widehat{Z}^i(.)$ is function of the monetary prices that the agent faces during his life. As the economy is invariant the excess demand function $\widehat{Z}$ is the same for every generation.

Let $M^i(p_t, p_{t+1}, ...,\ p_{t+l-1})$ be the amount of money that an agent born in t want to hold at beginning of his i-th period of life. Rearranging the budget constraint it results that $M^i(p_t, ...,\ p_{t+l-1})$ is equal to the monetary value of the sum of present and future excess demands:

$$M^i(p_t, p_{t+1}, ...,\ p_{t+l-1}) = \sum_{j=i}^{l} p_{t+j-1} \widehat{Z}^j(p_t, p_{t+1}, ...,\ p_{t+l-1}) \tag{9}$$

An equilibrium of this economy is a sequence of prices such that the aggregate excess demand for good and money is equal to zero in all periods.

Let M be the global amount of money in the economy, then a sequence of monetary prices $p(0)$ is an equilibrium if and only if it satisfies the following condition:

$$\sum_{i=1}^{l} M^i(p_{t-i}, ...,\ p_{t-i+l-1}) = M \quad \forall t$$

that means that the sum of quantity of money that agents hold at beginning of each period is equal to the aggregate quantity of money in the economy. Defining the aggregate demand for money as

$$\widehat{M}(p_{t-l+1}, ..p_{t-2+l}) \equiv \sum_{i=1}^{l} M^i(p_{t-i+1}, ..., p_{t-i+l})$$

the equilibrium condition can be written as

$$\widehat{M}(p_{t-l+1}, ..., p_{t-2+l}) - M = 0, \ \forall t \tag{10}$$

As $\widehat{Z}^i$ are the same functions for each generations, the equilibrium condition (10) defines implicitly a $(2l-3)$-th order dynamics of prices that depends on the global amount of money in the economy. Therefore it is sufficient to study the shape of the function $\widehat{M}(p_0, ...p_{2l-3})$ in order to analyze the dynamic properties of the equilibrium. Proposition 11 follows from theorem 2 and show that when the equilibrium backward dynamics of an overlapping generation economy is well defined it is always possible to build an equivalent myopic economy

Proposition 2 *Consider an overlapping generation economy where agents life span is l and the equilibrium backward dynamics can be represented by a function $\widehat{G}$ that satisfies hypothesis ii) of theorem 2, then there exists a myopic economy that is equivalent to the given OLG economy where the level of myopia is $n = 2l - 3$.*

Recalling that hypothesis ii) of theorem 2 is $\partial G/\partial M > 0$, using the definition of $\widehat{M}$ and applying implicit function theorem on (**??**), it results that $\partial G/\partial M > 0$ only if $\widehat{Z}_1^l > 0$, that means that the excess demand in the last period of life is increasing in the prices in the first period of life.

Proposition 9 implies that all the well known properties about indeterminacy of the equilibrium, existence of steady states and deterministic cycles, complex behavior and sunspot equilibria in OLG model can be directly extended to economies where agents are myopic.

Note that the present equivalence result is in some sense stronger than those obtained introducing cash in advance or finance constraint. Indeed the existing results restrict to the two periods life span OLG economies while here we are able to reproduce any invertible price dynamics resulting from an OLG economy with no restriction an agent's life span.

Hence, it is possible to move from the standard OLG model world to that of infinite life span agents model by just moving the value of the parameter n to infinity. In other words, some OLG model can be interpreted as an infinite lived myopic agent model.

4.1 High level of myopia

As most of the overlapping generations model used in macroeconomics involve agents that live two periods and that have a separable utility function, it is interesting to study the equivalence between myopia and OLG with this set up.

In this section we show that thanks to these simplifying assumptions on the OLG economy there exist a straight way to obtain the myopic economy equivalent to the OLG one, without making use of the integrability theory.

Furthermore we show a converse to Theorem 2 when $n = 2$: given any myopic economy where the level of myopia is $n = 2$ there always exist an equivalent OLG economy.

I start with the latter result.

Consider the economy $EM(n)$ where $n = 2$. In each period the agent chooses his present consumption and he plans his demand of the next period. His maximization problem in period t can be written as:

$$PM_t(2) : \begin{cases} \max_{x_t, x_{t+1}} U(x_t, x_{t+1}) \\ s.t.\ \ p_t x_t + M_{t+1} \leq p_t \omega + M_t \\ p_{t+1} x_{t+1} \leq p_{t+1} \widetilde{\omega}_1 + M_{t+1} \\ M_t \ \ \text{given} \end{cases}$$

First order condition leads to

$$U_1(x_t, x_{t+1}) = \frac{p_t}{p_{t+1}} U_2(x_t, x_{t+1})$$

Where U_i is the derivative of U with respect to the i-th argument. At equilibrium it results $x_t = \omega$ and, for the budget constraint, $x_{t+1} = \widetilde{\omega}_1 + m_{t+1}/p_{t+1}$; since this must hold for every period it results $M_t = M\ \forall t$: as at equilibrium the agent consume his endowment the quantity of money he holds is constant. Substituting these conditions in the f.o.c. it follows

$$U_1(\omega, \widetilde{\omega}_1 + \frac{M}{p_{t+1}}) = \frac{p_t}{p_{t+1}} U_2 \left(\omega,\ \widetilde{\omega}_1 + \frac{M}{p_{t+1}} \right) \tag{11}$$

Let $m_t = M/p_t$ be the time t real amount of money in the economy. Multiplying both side of the expression (11) by m_t we obtain the expression which links two consecutive equilibrium levels of the real amount of money:

$$U_1(\omega, \widetilde{\omega}_1 + m_{t+1}) m_t = U_2 \left(\omega,\ \widetilde{\omega}_1 + m_{t+1} \right) \tag{12}$$

The following proposition show how to built an OLG economy whose set of equilibria coincides with those of the given myopic economy.

Proposition 3 *$EM(2)$ is equivalent to an OLG exchange economy with aggregate amount of money equal to M, where representative agent lives two periods, he receives an endowment equal to ω and $\widetilde{\omega}_1$ in the first and second period of life respectively and whose utility function $V(x_t, x_{t+1})$ is such that $V_1(x_t^t, x_{t+1}^t) = U_1(\omega, x_{t+1}^t)$ and $V_2(x_t^t, x_{t+1}^t) = U_2\left(\omega,\ x_{t+1}^t\right)$, where x_j^i is time j consumption of agent born in period i.*

Proof: It will be sufficient to show that equation (12) characterizes the sequence of equilibrium real quantity of money in the specified OLG model. When considering the OLG economy, the maximization problem of individual born in period t is

$$PO_t(2): \begin{cases} \max_{x_t, x_{t+1}} V\left(x_t^t,\ x_{t+1}^t\right) \\ s.t.\ \ p_t x_t^t + M_{t+1}^t \leq p_t \omega \\ p_{t+1} x_{t+1}^t \leq p_{t+1} \widetilde{\omega}_1 + M_{t+1}^t \\ M_t \quad \text{given} \end{cases}$$

Where M_j^i is the quantity of money held by the agent born in i at beginning of period j . First order conditions give

$$V_1(x_t, x_{t+1}) = \frac{p_t}{p_{t+1}} V_2(x_t, x_{t+1})$$

Assuming that at time 0 there is just one old agent that holds the total amount of money M, the equilibrium condition can be written as

$$\begin{aligned} x_t^t &= \omega - \frac{M}{p_t} \quad \forall t \\ x_{t+1}^t &= \omega_1 + \frac{M}{p_{t+1}} \quad \forall t \end{aligned}$$

That is young agents consume their endowment minus the amount of the good they have to sell to the old for buying money, and old agents consume their endowment plus the quantity of good they buy with their money. Substituting equilibrium condition in the f.o.c. and multiplying both side for M/p_t and considering the expression of V_1 and V_2 given in the proposition, it follows that at equilibrium

$$U_1\left(\omega, \widetilde{\omega}_1 + \frac{M}{p_{t+1}}\right) \frac{M}{p_t} = U_2\left(\omega,\ \widetilde{\omega}_1 + \frac{M}{p_{t+1}}\right)$$

which is exactly equation (12). ■

It is easy to check that this result extends to the case where there are $l > 1$ goods per period when U is intertemporal additive. In particular let $u_i(x) : R_+^l \to R$, $i = 1, 2$ with $x \in R_+^l$, the instantaneous utility today and tomorrow respectively, $\omega \in R_+^l$ the vector of endowment of time t goods as perceived in period t, $\widetilde{\omega}_1 \in R_+^l$ the vector of endowment of time $t + 1$ goods as perceived in period t and M the quantity of money held by the representative agent in period 0. The equilibrium condition in this myopic economy is $x_t^t = \omega$, $\forall t$. This implies that relative prices between goods of the same period are constant, namely the equilibrium relative price between two goods i and j of period t is equal to marginal rate of substitution between these two goods at the endowment.

In the corresponding OLG economy, representative agent's endowment is equal to $\omega \in R_+^l$ and $\widetilde{\omega}_1 \in R_+^l$ respectively in the first and second period of life, representative agent's utility function is: $u_1'(x)|_\omega x_t^t + u_2(x_{t+1}^t)$, where $x_j^i \in R_+^l$ and $u_1'(x)|_\omega$ is the gradient of u_1 computed at the young's endowment. Since in each time t there are agents whose utility is linear in the goods of that period, then relative prices within these goods do not depend on time t. Therefore the equilibrium dynamic is completely resumed by the inflation rate.

Proposition 10 shows that for any myopic economy when $n = 2$ there always exist an equivalent overlapping generation economy.

Proposition 11 states the relation between 2 periods overlapping generation economies "a la Grandmont" and myopic economy with $n = 2$.

Let $EO(2)$ be an OLG exchange economy where the aggregate amount of money is equal to M, agents live two periods, they receives an endowment equal to ω_0 and ω_1 in their first and second period of life respectively, and their utility function is of the form $v\left(x_t^t\right) + w\left(x_{t+1}^t\right)$. Let $x_t^{t-1} = \theta\left(x_{t+1}^t\right)$ be the equilibrium dynamic of old agents' consumption and let $R_v(x) = -\frac{v''(x)x}{v'(x)}$ and $R_w(x) = -\frac{w''(x)x}{w'(x)}$.

Proposition 4 *If $R_v(x)$ and R_w are non-decreasing function of x for every $x > 0$, $R_v(x) < x/(x - \omega_0)$, $\forall x > 0$, $v'''(x) > 0$ for $x < \omega_0$, $w'''(x) > 0$ for $x < \omega_1$ and $\lim_{x\to 0} w'(x) = +\infty$, then $EO(2)$ is equivalent to an economy of type EM(2) where $\widetilde{\omega}_1 = \omega_1$, the aggregate quantity of money is M and the* *utility function is* $U(x_t, x_{t+1}) = u(x_t) + \beta u(x_{t+1})$*, with*

$$u'(x) = \frac{(\omega - \omega_1)(\theta(x) - \omega_1)}{(\theta(\omega) - \omega_1)(x - \omega_1)} \tag{13}$$

$$\beta = \frac{\theta(\omega) - \omega_1}{(\omega - \omega_1)} \tag{14}$$

The hypothesis of proposition 11 state that the demand in the second period of life is non increasing in the rate of interest, and the offer curve is concave as long as it is downward slopping. This implies that the backward equilibrium dynamics of the overlapping generation economy is well defined. Note that the hypothesis of proposition 11 with the exception of the assumption on v''' and w''' are the same used by Grandmont [8]. Furthermore most of the specification of the utility function used in macroeconomic models satisfy these hypothesis.

Lemma 5 *If* $R_v(x)$ *and* R_w *are non-decreasing function of* x *for every* $x > 0$, $R_v(x) < x/(x-\omega_0)$, $\forall x > 0$, $v'''(x) > 0$ *for* $x < \omega_0$ *and* $w'''(x) > 0$ *for* $x < \omega_1$, *and* $\theta(.)$ *is the resulting equilibrium dynamic of old agents' consumption, then*

$$\theta'(x) < \frac{\theta(x) - \omega_1}{x - \omega_1} \quad \forall x > \omega_1 \tag{15}$$

and

$$\theta'(x) > \frac{\theta(x) - \omega_1}{x - \omega_1} \quad \forall x < \omega_1 \tag{16}$$

Furthermore if $\lim_{x\to 0} w'(x) = +\infty$ *then* $\lim_{x\to 0} u'(x) = +\infty$*, where* u *is defined as in proposition 11.*

Proof: See the appendix.

Proof of proposition 11: Notice that for the equilibrium condition in the OLG economy

$$x_t^{t-1} = \omega_1 + m_t$$

where $m_t = M/p_t$ is the real amount of money in period t. From the hypothesis it follows that

$$m_t = \theta(m_{t+1} + \omega_1) - \omega_1 \tag{17}$$

is the equilibrium dynamics of real amount of money in the OLG economy. From equation (12) and considering that $U(x_t, x_{t+1}) = u(x_t) + \beta u(x_{t+1})$, we have that the equilibrium sequence of real asset in EM(2) is

$$m_t = \frac{\beta u'(\widetilde{\omega}_1 + y_{t+1}) y_{t+1}}{u'(\omega)}$$

Substituting the expression of $u'(x)$ and β we obtain exactly (17) so the equilibrium dynamics of monetary price is the same.

It remains to be proved that the utility function defined in (5) and (14) respects the conditions: $\beta > 0$, $u'(.) > 0$, $u''(.) < 0$. Notice first that $x > \omega_1$ if and only if $\theta(x) > \omega_1$: indeed if old agents of some period $t+1$ are consuming more (resp. less) than their endowment, $x^t_{t+1} > \omega_1$ (resp. $x^t_{t+1} < \omega_1$), than aggregate quantity of money is positive (resp. negative). This implies that it was positive (resp. negative) also in the previous period and hence $x^{t-1}_t > \omega_1$ or $\theta(x^t_{t+1}) > \omega_1$ (resp. $\theta(x^t_{t+1}) < \omega_1$). It follows that $(\theta(\omega) - \omega_1)$ has the same sign of $(\omega - \omega_1)$ and hence it results $\beta > 0$ and $u'(.) > 0$. Deriving expression (5) we have

$$u''(x) = \frac{(\omega - \omega_1)\left[\theta'(x)(x - \omega_1) - \theta(x) + \omega_1\right]}{(\theta(\omega) - \omega_1)(x - \omega_1)^2}$$

which is negative if and only if $\theta'(x) < \frac{\theta(x) - \omega_1}{x - \omega_1}$ $\forall x > \omega_1$ and $\theta'(x) > \frac{\theta(x) - \omega_1}{x - \omega_1}$ $\forall x < \omega_1$ that follow from first part of lemma 12. The second part of lemma 12 guaranties that the first order condition we used to characterize the equilibrium dynamics are sufficient condition for the optimum in the consumer maximization problem.■

5 Conclusion

We have studied infinite lived representative agents economy where agents are myopic. We have shown that there are few restrictions on the dynamics of equilibrium prices that can be generate by such economy. In particular any invertible and monetary price dynamics homogenous of degree 1 and increasing in the quantity of money is the equilibrium price dynamics of an appropriate myopic economy. Furthermore we studied the robustness of some properties of the equilibrium in infinite lived representative agent exchange economy, introducing a myopic behavior. In particular we showed that the indeterminacy of equilibria and the monetary equilibria appear when a small level of myopia is introduced.

Finally we show how to obtain a myopic economy equivalent to some given OLG economy and the converse result is provided for high level of myopia. Thus the model allows to move from the OLG world into the ILRA one just changing the degree of myopia in the economy.

If representative agent utility function is additive, introducing myopia in an infinitely lived representative agent exchange economy the qualitative properties of the equilibrium are those of an OLG economy.

6 Appendix

Proof of lemma 12: First order condition for the OLG economy is

$$\frac{v'(x_t^t)}{p_t} = \frac{w'(x_{t+1}^t)}{p_{t+1}}$$

Multiplying both sides for m and substituting the equilibrium condition $y_t = m/p_t = \omega_0 - x_t^t = x_t^{t-1} - \omega_1$ it results that the dynamics of real amount of is implicitly defined by

$$v'(\omega_0 - y_t)y_t = w'(\omega_1 + y_{t+1})y_{t+1} \tag{18}$$

deriving left hand side of this equality we have

$$D\left[v'(\omega_0 - y_t)y_t\right] > 0 \Leftrightarrow -\frac{v''(\omega_0 - y_t)y_t}{v'(\omega_0 - y_t)} > -1$$

substituting the equilibrium condition one obtain that for any y_t, $D\left[v'(\omega_0 - y_t)\right.$ $\left.y_t\right] > 0 \Leftrightarrow R_v(x) < x/(x - \omega_0)$, $\forall x > 0$. Hence from hypothesis it follows that left hand side of (18) is strictly increasing and hence invertible and for any y_{t+1} one can write

$$y_t = \varphi\left(w'(\omega_1 + y_{t+1})y_{t+1}\right)$$

where φ is the inverse function of left hand side of (18). Using the equilibrium condition and the definition of $\theta(.)$ it results

$$\theta\left(x_{t+1}^t\right) = \varphi\left(w'\left(x_{t+1}^t\right)\left(x_t^{t+1} - \omega_1\right)\right) + \omega_1$$

whose derivative is

$$\theta'(x) = \varphi'\left(w'\left(x\right)\left(x-\omega_1\right)\right)\left[w''(x)(x-\omega_1)+w'(x)\right]$$

which is always positive for $x < \omega_1$ and it can be negative only for $x > x^* > \omega_1$ if it exists for instance some x^{**} such that $R_w(x^{**}) > 1$.[3] If this is the case then equation (15) is satisfied for any $x > x^*$. When $\theta'(x)$ is positive (i.e. when $x < x^*$ or $x = \Re^+$ if x^* does not exist) inequalities (15) and (16) imply the concavity of θ, or in other term that the offer curve is convex when $\partial x_{t+1}^t / \partial x_t^t < 0$. $\theta''(x) < 0$ follows from the condition on R_v, R_w, w''' and v'''.

In order to prove that $\lim_{x\to 0} u'(x) = +\infty$ consider that, from the hypothesis, $\lim_{x\to 0} w'\left(x_{t+1}^t\right)\left(x_t^{t+1}-\omega_1\right) = -\infty$. From equation (18) it follows that $\lim_{x\to -\infty}\varphi(x) = -\infty$ substituting in expression (5) we obtain the desired result $\Box$.

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[3] For the prof of this statement see Grandmont (1985) lemma 1.3 p.1002.

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