Retaliatory Equilibria in a Japanese Ascending Auction for Multiple Objects

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Abstract

We construct a family of retaliatory equilibria for the Japanese ascending auction for multiple objects, thus showing that while it is immune to many of the tacitly collusive equilibria studied in the literature, it is not entirely immune.

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1 Introduction

Since 1994 the United Stated Federal Communication Commission has been using simultaneous ascending auctions for the sale of spectrum licenses. While they ensure a transparent bidding process that enables extensive information revelation of bidders' valuations, they also allow bidders to use their bids to effectively communicate among themselves and tacitly collude.[1] When few bidders compete for few objects, bidders may have important incentives to reduce demands and hence prices. Engelbrecht-Wiggans and Kahn (1999) and Brusco and Lopomo (2002) show that, for auction formats close to the FCC's, there are equilibria where bidders can coordinate a division of the available objects at very low prices. On the other hand, Albano *et al.* (2001, 2004) study a variant of the simultaneous ascending auction, namely the Japanese auction for multiple objects (JAMO), which differs from the FCC auction mechanisms in that (i) prices are exogenously raised by the auctioneer, and (ii) closing is object-by-object. Among other things, they show that because of these two features all the collusive "low-revenue" equilibria constructed by Engelbrecht-Wiggans and Kahn (1999) and Brusco and Lopomo (2002) are not possible in the JAMO.

This paper shows that collusive equilibria of a retaliatory type, similar to ones reported in Cramton and Schwartz (2000, 2002), nonetheless exist in the JAMO. The logic of such equilibria is as follows. Suppose two objects are put for sale to two bidders, one bundle bidder interested in both objects, and one unit bidder who wishes to buy only object 1. Assume this to be common knowledge. The two bidders have overlapping interests on object 1; in particular, the unit bidder wants the bundle bidder to exit early from object 1. In order to achieve this, the unit bidder actively bids on object 2, although he has zero value for the object. Such a strategy is potentially costly to both the unit and the bundle bidder; we refer to the unit bidder's behavior as a *retaliatory strategy.* The extent to which the unit bidder is successful in inducing the bundle bidder to drop out early from object 1 depends on whether he succeeds in making his threat credible. We show that the JAMO admits equilibria that effectively involve such strategies.

2 A Japanese Ascending Auction

2.1 Framework

Throughout the paper we work with the framework of Krishna and Rosenthal (1996). Two objects are auctioned to a set of participants of two types: $M \geq 1$ bundle bidders who are

[1] Cramton and Schwartz (2000, 2002) have a detailed analysis of the signaling that took place during the FCC's DEF auction; Salmon (2004) contains a survey of collusive equilibria in ascending auctions.

interested in both objects and one unit bidder interested in only one of the two objects, $k = 1, 2$. Bundle and unit bidders draw their values independently from some smooth distribution F with positive density f, both defined over $[0, 1]$. Let v_k and u_k denote the value of object $k = 1, 2$ to a bundle and to a unit bidder respectively. The value of the bundle v_B to a bundle bidder is greater or equal than the sum of stand-alone values, that is,

$$v_B = v_1 + v_2 + \alpha,$$

where $\alpha \geq 0$ is commonly known and coincides across all bundle bidders. The nature of bidders, bundle and unit, is also commonly known.We restrict the analysis to the following cases: (i) $v_1 = v_2 \in [0, 1]$ and $\alpha \geq 0$; (ii) $v_1, v_2 \in [0, 1]$ and $\alpha = 0$. Krishna and Rosenthal (1996) consider case (i); case (ii) is considered by Brusco and Lopomo (2002).

2.2 The Auction Mechanism

We consider a Japanese (or English clock) auction for multiple objects. Prices start from zero on all objects and are simultaneously and continuously increased until only one agent is left on a given object, in which case prices on that object stop and continue to rise on the remaining ones. Once an agent has dropped from a given object, the exit is irrevocable. The last agent receives the object at the price at which bidding on that object stopped. Whenever an agent exits an object, the clock (price) temporarily stops on all objects, giving the opportunity to other bidders to exit at the same price. If all active bidders exit simultaneously, then the object is allocated randomly among those bidders. The number and the identity of agents active on any object is publicly known at any given time. The overall auction ends when all agents but one have dropped out from all objects. We refer to this mechanism as the Japanese auction for multiple objects (JAMO); this auction is also studied in Albano *et al.* (2001, 2004).

3 Retaliatory Equilibria

Our results are summarized as follows.

Proposition 1 *There exist Perfect Bayesian Equilibria of the JAMO where bidders use retaliatory strategies effectively.*

We show this by means of the following three examples, presented in order of generality.

Example 1. Consider the JAMO with two objects and two bidders; one unit bidder interested in object 1 and one bundle bidder interested in both objects 1 and 2, with the same value for the two objects, $v_1 = v_2 = x$, and $\alpha = 0$; assume also all values are drawn according to the uniform distribution on $[0, 1]$. It is easy to see that the following is a PBE of the JAMO:

— all types of the unit bidder bid on *both* objects and stay on object 1 until u_1 and on object 2 until $\min(u_1, t_1^2)$, where t_1^2 is the bundle bidder's exiting time from object 1;

— all types of the bundle exit from object 1 at t if at t the unit bidder is on object 2; otherwise all types of the bundle bidder stay on both objects until x.

In equilibrium, the bundle bidder immediately drops out of object 1 inducing the unit bidder to also immediately drop out of object 2. As is often typical in such retaliatory equilibria, the retaliating bidder (here the unit bidder) obtains a higher ex ante payoff than in the "competitive" equilibrium [2] (1/2 versus 1/6), while the other agents (here the bundle bidder and the auctioneer) are both worse off (1/2 versus 2/3 and 0 versus 1/3 respectively). □

The above example relies on the fact that the unit bidder has some extra information about the bundle bidder's valuation of object 1 relative to object 2 (the values are perfectly correlated). Without this information he needs to resort to a more refined threat.

Example 2. Consider now case (ii) with two objects and two bidders; one unit bidder interested in object 1 and one bundle bidder interested in both objects 1 and 2; assume again all values are drawn according to the uniform distribution on $[0, 1]$. Then, for any $l \in (0, 1]$, the following is a PBE of the JAMO:

— all types of unit bidder with $u_1 \leq l$ bid only on object 1 and stay until u_1; all types of unit bidder with $u_1 > l$ bid on *both* objects and stay on object 1 until u_1 and on object 2 until $c = l(\sqrt{2} - 1) < l$;

— If the unit bidder is active on the two objects, then all types of bundle bidder with $v_1 < l$ bid on both objects and stay on object 1 until c and on object 2 until v_2; all types of bundle bidder with $v_1 \geq l$ bid on both objects always staying until v_1, v_2 respectively.

— If the unit bidder is active only on object 1 then all types of bundle bidder stay until v_1, v_2 on object 1 and 2 respectively;

This characterizes a family of retaliatory equilibria indexed by the parameter l that are PBE of the JAMO. Note that the equilibria are *not* in undominated strategies, since the unit bidder always has a (weakly) dominant strategy to drop from object 2 whenever it is the only object he is bidding on. If the unit bidder is active on both auctions this signals that his valuation is above the threshold l, i.e., $u_1 > l$; if he bids only on object 1, then $u_1 \leq l$, and both bidders bid up to their valuations and only on the objects they value.

[2]See Brusco and Lopomo (2002) and Albano et al. (2001, 2004).

When $l = 1$ we get the competitive equilibrium , since with probability one the unit bidder will not be active on object 1. When $l \to 0$ we almost get the competitive equilibrium, since $c \to 0$, i.e., the unit bidder enters both auctions but almost immediately exits object 2.

Unlike the equilibrium of Example 1, here, to ensure incentive compatibility for the unit bidder, the bidding threshold c is such that he only weakly prefers the retaliatory equilibrium, his ex ante payoff is the same as in the standard equilibrium, i.e., 1/6; the bundle bidder continues to be worse off than in the standard equilibrium, her ex ante expected payoff being

$$\frac{2}{3} + \frac{l}{2} - \sqrt{2}l + \frac{3l^2}{2} - \frac{13l^3}{6} + \sqrt{2}l^3 + \frac{l^4}{6} \leq 2/3 \ \ \forall l,$$

while due to the extra bidding on object 2, the auctioneer actually earns higher ex ante revenues than in the previous example and in the standard equilibrium

$$\frac{1}{3} - l + \sqrt{2}l - 2l^2 + \sqrt{2}l^2 + 3l^3 - 2\sqrt{2}l^3 \geq 1/3 \ \ \forall l.$$

Example 3. Example 2 can be extended to a unit bidder competing against an arbitrary number M of bundle bidders with preferences as in cases (i) and (ii), (in particular α may now be positive), and private values distributed according to a general distribution function F defined on $[0, 1]$. The PBE described above is still a PBE for any $l \in (\alpha, 1]$, where the parameter value $c(< l)$ is now chosen as the unique solution to the equation

$$\int_{c}^{l-\alpha} z \cdot G'(z)dz = (c - \alpha) \cdot G(l - \alpha), \tag{1}$$

where $G = F^M$ is the distribution function for the bundle bidders' highest valuation for object 1 (or 2). In presence of complementarities, the only difference is that a bundle bidder exits object 1 at c if $v_1 + \alpha < l$,(the bundle bidder's optimal strategy when $v_1 + \alpha > l$ is described in Proposition 2 in Albano *et al.* (2004)). In order to ensure incentive compatibility for the unit bidder, as M increases, the parameter c also increases.

All three examples are in some sense related to the collusive equilibria of Brusco and Lopomo (2002): bundle bidders and the unit bidder have overlapping interests on object 1; the unit bidder threatens to retaliate (i.e., to be active) on object 2 if the bundle bidders do not exit object 1. This signaling device is effective since it is common knowledge that the retaliatory bidder is interested in one object only. Thus by not "turning the light off" on object 2 when the price is zero, the unit bidder triggers the beliefs that sustain the collusive equilibrium.

Proof of Examples 2 and 3. We directly prove the general case of Example 3. There are M bundle bidders and one unit bidder on object 1; preferences are according to cases (i) or (ii); values are drawn from a smooth distribution F defined on $[0, 1]$.

We first check optimality for any bundle bidder, then we check it for the unit bidder. If the unit bidder is active on both objects, bundle bidders infer that $u_1 > l$. Hence, a bundle bidder with $v_1 + \alpha \leq l$ knows that he will not win object 1 even if he wins object 2 and then continues optimally until $v_1 + \alpha$ on object 1. Therefore, exiting object 1 at time c and exiting object 2 at v_2 is a (weak) best reply for such a bundle bidder. If, however, $v_1 + \alpha > l$, then a bundle bidder is better off remaining on each object so long as his expected continuation payoff remains strictly positive. Namely, if $v_1, v_2 \in [0, 1]$ and $\alpha = 0$, (case (ii)), then a bundle bidder remains on objects 1 and 2 until v_1 and v_2 respectively. On the other hand, if $v_1 = v_2 = x$ and $\alpha \geq 0$, (case (i)), then a bundle bidder's strategy will be of the type described in Section **??**: there is an optimal time τthat depends on v_B and H_t such that, if he does not win any object before τ he exits both objects, and he continues optimally until $x + \alpha$ otherwise. This proves that the bundle bidders' strategy is a best reply to the unit bidder's strategy.

To prove optimality for the unit bidder, we need to show that the unit bidder's strategy is a best reply and that it is profitable for the unit bidder to bid on both objects if and only if $u_1 > l$, i.e., that the equilibrium is incentive compatible, so that being active on both objects gives a credible signal that $u_1 > l$. Let $y_k \in [0, 1]$ denote the bundle bidders' highest valuation for object $k \in \{1, 2\}$, and let $G \equiv F^M$ denote the corresponding distribution function. Recall that the bundle bidder with the highest valuation for object 2 will exit auction 2 before c only if $y_2 < c$. When the unit bidder is not active on object 2 and still active on object 1, the bundle bidder with the highest valuation for object 1 will exit auction 1 at $y_1 + \alpha$. When the unit bidder is active on object 2, if $y_1 \leq l - \alpha$, then the bundle bidder with the highest valuation for object 1 will exit auction 1 at c; whereas if $y_1 > l - \alpha$ and the unit bidder is still active on object 1, then the bundle bidder with the highest valuation for object 1 will exit auction 1 at $y_1 + \alpha$.

If $u_1 < c$, then it is clearly not optimal for the unit bidder to bid on both objects since he will have to pay at least c for object 1. Hence we assume $u_1 \geq c$. Suppose that $u_1 \leq l$. If the unit bidder decides to implement the retaliatory strategy, then his expected payoff is $\int_0^{l-\alpha}(u_1 - c) \cdot G'(y_1)dy_1 - \int_0^c y_2 \cdot G'(y_2)dy_2$. The first integral is the unit bidder's payoff from object 1: the unit bidder wins object 1 only if $y_1 < l - \alpha$, (recall that $l \in (\alpha, 1]$) and he pays c. The second integral is the expected payoff from object 2: if $y_2 < c$, then he has to buy object 2 at a price y_2. If, however, at time 0 the unit bidder decides to bid only on object 1, his expected payoff is $\int_0^{u_1-\alpha}(u_1 - y_1 - \alpha)G'(y_1)dy_1$.

At equilibrium we want the unit bidder to bid only on object 1 when $u_1 \leq l$, i.e., the following needs to be satisfied

$$\int_0^{l-\alpha} (u_1 - c) \cdot G'(y_1)dy_1 - \int_0^c y_2 \cdot (y_2)dy_2 \leq \int_0^{u_1-\alpha} (u_1 - y_1 - \alpha) \cdot G'(y_1)dy_1,$$

which is satisfied for c solving Eq. (1).

Suppose now that $u_1 > l$. Then, at $t = 0$, the unit bidder's expected payoff from adopting the retaliatory strategy must be greater or equal than the payoff from bidding only on object 1, i.e.,

$$\begin{aligned} \int_0^{l-\alpha} (u_1 - c) \cdot G'(y_1) dy_1 \quad &+ \quad \int_{l-\alpha}^{u_1-\alpha} (u_1 - y_1 - \alpha) \cdot G'(y_1) dy_1 \\ - \int_t^c y_2 \cdot G'(y_2) dy_2 \quad &\geq \quad \int_0^{u_1-\alpha} (u_1 - y_1 - \alpha) \cdot G'(y_1) dy_1 \end{aligned}$$

It is easy to check that the above inequality is satisfied for any $t < c$ and for any $l \in (\alpha, 1]$. Finally an appropriate choice of out-of-equilibrium-path beliefs guarantees that at any $t < c$ the unit bidder's expected payoff by insisting with the retaliatory strategy is greater or equal than the payoff of exiting object 2 and continuing on object 1. ■

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References

Albano, G. L., Lovo S. and F. Germano (2001), "A Comparison of Standard Multi-Unit Auctions with Synergies," *Economics Letters*, 71, pp. 55-60.

Albano, G. L., Lovo S. and F. Germano (2004), "Ascending Auctions for Multiple Objects: The Case for the Japanese Design," University College London, downloadable from www.ucl.ac.uk/˜uctpgla/page2.htm

Brusco, S., and G. Lopomo (2002), "Collusion via Signaling in Open Ascending Auctions with Multiple Objects and Complementarities," *Review of Economic Studies*, 69, pp. 1-30.

Cramton, P., and J. Schwartz (2000), "Collusive Bidding: Lessons from the FCC Spectrum Auctions," *Journal of Regulatory Economics*, 17, pp. 229-252.

Cramton, P., and J. Schwartz (2002), "Collusive Bidding in the FCC Spectrum Auctions," *Contributions to Economic Analysis and Policy*, 1, pp. 1-11.

Engelbrecht-Wiggans, R., and C. Kahn (1998) "Low Revenue Equilibria in Ascending Price Multi-Object Auctions," Mimeo, University of Illinois.

Krishna, V., and R. Rosenthal (1996), "Simultaneous Auctions with Synergies," *Games and Economic Behavior*, 17, pp. 1-31.

Salmon, T. (2004), Preventing Collusion Between Firms in Auctions In: Auctioning Public Assets: Analysis and Alternatives, Edited by M.C.W. Janssen, Cambridge University Press.