Financial Economics
2: Capital Budgeting

Stefano Lovo

HEC, Paris

Ostrich breeding

Ostrich breeding

Time 0	Year 1
Eu −10	*Eu* +15

Spot price of one ostrich egg: *Eu* 10. After 1 year, the egg will be transformed into an adult ostrich whose market price on the livestock market is *Eu* 15. The discount rate is 10%.

Should I buy the egg?

Time 0	Year 1
$Eu\ -10$	$Eu\ +15$

	Time 0	Year 1
Borrow $Eu\ \frac{15}{1.1} = 13.64$	$+13.64$	$-13.64 * 1.1 = -15$
Buy the egg	-10	
Sell the ostrich		$+15$
Total	$+3.64$	0

Net Present Value

Consider a project that costs C today and that will pay cash flows F_{t_1} , $F_{t_2}, \ldots, F_{t_n}$ after $t_1, t_2, \ldots t_n$ years, respectively.

Time 0	t_1 years	t_2 years	...	t_n years
$-C$	F_{t_1}	F_{t_2}	...	F_{t_n}

Definition

The **net present value** of this project is the present value of all the cash flows generated by the project including the initial cost:

$$NPV := -C + \sum_{i=1}^{n} \frac{F_{t_i}}{(1+r)^{t_i}} = -C + \frac{F_{t_1}}{(1+r)^{t_1}} + \cdots + \frac{F_{t_n}}{(1+r)^{t_n}}$$

NPV example

Example

Consider the following investment project:

time 0	6 months	1 year
$-10,000$	$4,000$	$7,000$

If the discount rate is 12%, the NPV of the project is

$$NPV = -10,000 + \frac{4,000}{1.12^{0.5}} + \frac{7,000}{1.12} = 29.64$$

NPV Interpretation

The NPV of a project represents the variation in my current wealth resulting from the implementation of the project. Indeed, it is possible to **implement** the project and **sell the future cash-flows**.

Example

time 0	6 months	1 year
$-10,000$	$4,000$	$7,000$

$r = 12\% \Rightarrow NPV = 29.64$

	Today	6 months	1 year
Implement the project	-10,000	4,000	7,000
Borrow during 6 months $4,000/1.12^{0.5}$	3,779.64	-4,000	
Borrow during 1 years 7,000/1.12	6250		-7,000
Total	29.64	0	0

The NPV Criterion

- All the projects whose NPV is positive should be implemented.
- If projects are mutually exclusive, then choose the project with the greatest positive NPV.

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Why?
Because implementing a project is equivalent to changing the current wealth of an amount equal to the NPV of the project.

Problems:

1. Which discount rate should one use to compute the NPV?
2. How can one estimate the cash flows of a project?

Discount rate choice

Definition

The **opportunity cost of capital** (OCC) for a given project is the interest rate one can gain from an alternative investment with the same risk factors of the project.

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Example

Investment project A is risk-free (future cash flows will be received with certainty).

- The return rate from investing in the stock market is 20%;
- The return rate from investing in a risk-free treasury bill is 1%;

What is the OCC for project A?

Rule:The discount rate in NPV should correspond to the opportunity cost of capital.

Example

You can implement the following risk-free project:

time 0	Year 1	Year 2
-40,000	30,000	12,000

Your wealth is currently invested in a bank account at interest rate of 4%. How much should you invest in your bank account in order to have $30,000$ in year 1 and 12,000 in year 2 ?

$$\frac{30{,}000}{1.04} + \frac{12{,}000}{1.04^2} = 39,940.83$$

The NPV of the project is

$$-40,000 + \frac{30{,}000}{1.04} + \frac{12{,}000}{1.04^2} = -59.17 < 0$$

Cash-flows estimation

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improve the things you can change;
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General rule: I should take into account **all and only** the monetary consequences that the implementation of the project has on my wealth:

- Do not consider cash-flows that would occur independently of the implementation of the project.
- Consider all direct and indirect cash-flows generated with the implementation of the projects.

Cash-flows estimation: Sunk costs

Definition

A Sunk cost is a cost related to the project, that has been paid in the past and is not recoverable.

Example

- R&D investments.
- Cost related to feasibility studies.
- Past salaries related to the project.
- ...

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Rule: Sunk costs do not matter as my current decision of undertaking or not the project cannot change the sunk costs.

Sunk costs: examples

Example

During the last 5 years PWC Inc. has invested *Eu* $3,000,000$ to develop low-cost fuel-cell engines. Today, in order to start mass production PWC has to invest *Eu* $4,000,000$ into a new division that will generate annual net revenue of *Eu* $900,000$ for the next 30 years. Discount rate is (OCC) is $r = 15\%$.

Should PWC start mass production?

Sunk costs: examples

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Should PWC start mass production?

$$NPV = -4,000,000 + \frac{900,000}{0.15}\left(1 - \frac{1}{1.15^{30}}\right) = 1,909,382$$

$$NPV \neq -3,000,000 - 4,000,000 + \frac{900,000}{0.15}\left(1 - \frac{1}{1.15^{30}}\right) < 0$$

Cash-flows estimation: Incremental approach

Definition

The incremental approach consists in comparing the firm's future cash-flows with and without the project implementation.

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Currently PWC inc. annual revenue from its traditional production of power-cells is *Eu* $5,000,000$. If the fuel-cell project starts, then during the next two years the revenue from the power-cell division will drop by 2% due to reorganization costs.
Should PWC start mass production of fuel-cells?

$$\begin{aligned} NPV &= 1,909,382 - \frac{0.02 \times 5,000,0000}{1.15} - \frac{0.02 \times 5,000,0000}{1.15^2} \\ &= 1,746,811 > 0 \end{aligned}$$

Rule: Take into account taxes and the time at which they are payed.

Example

Today, in order to start mass production PWC has to invest *Eu* $4,000,000$. This will lead to an increase in PWC annual taxable income of *Eu* $900,000$ for the next 30 years. Discount rate (OCC) is $r = 15\%$. Annual taxes are 36% of the annual income and are payed with one year lag.
Should PWC start mass production of fuel-cells?

$$\begin{aligned} NPV &= 1,909,382 - \frac{0.36 \times 900,000}{0.15}\left(1 - \frac{1}{1.15^{30}}\right)\frac{1}{1.15} \\ &= 59.488 \end{aligned}$$

Cash-flows estimation: Inflation

We shall distinguish nominal cash-flows from real cash-flows.

- $\pi :=$ inflation rate.
- $r_n :=$ nominal discount rate
- $r :=$ real discount rate

$$r = \frac{1 + r_n}{1 + \pi} - 1 \simeq r_n - \pi$$

Example

The annual rate on a livret A is $r_n = 0.75\%$. The annual inflation rate is $\pi = 0.4\%$. The real annual rate on a 'livret A' is $r \simeq 0.75\% - 0.4\% = 0.35\%$

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Rule:

- Discount nominal cashflows with nominal discount rate.
- Discount real cashflows with the real discount rate.

Example

Today, in order to start mass production PWC has to invest *Eu* $4,000,000$. During the next 30 years the demand for fuel-cells is estimated to be constant at 900 units per year. Today, the net margin (cash-flow) on one fuel-cell is *Eu* $1,000$. In real terms, this margin is expected to remain constant for the next 30 years. The annual inflation rate is expected to be $\pi = 2\%$ for the next 30 years. The nominal OCC is $r_n = 17.3\%$. Should PWC start mass production of fuel-cells?

$$\text{Real OCC} = \frac{1+r_n}{1+\pi} - 1 = \frac{1.173}{1.02} - 1 = 15\%$$

$$\begin{aligned} NPV_{\text{Real}} &= -4,000,000 + \frac{900{,}000}{0.15}\left(1 - \frac{1}{1.15^{30}}\right) \\ &= 1,909,382 \\ NPV_{\text{Nom.}} &= -4,000,000 + \frac{900{,}000 \times 1.02}{0.173 - 0.02}\left(1 - \left(\frac{1.02}{1.173}\right)^{30}\right) = \\ &= 1,909,382 \end{aligned}$$

Finance $\neq$ accounting: (1) Time

- In **accounting**, costs are subtracted to revenues occurring at different periods of the year.
- In **finance**, we discount cash-flows before summing or subtracting them.

Example

Supermarket ABC's annual sales revenue are *Eu* $500,000$, the annual cost of goods sold is *Eu* $501,000$. Suppliers are payed at the end of the year. Customers pay at the beginning of the year. The OCC is 5%. What is the annual net income of ABC?

Sales revenue	$500,000$
Cost of goods sold	$501,000$
Net income	$-1,000$

What is the NPV of one year activity of ABC?

$$500,000 - \frac{501,000}{1.05} = 22,857 > 0$$

(2) Cash-flows $\neq$ accounting flows

How to deduce cash flows from the income statement and the balance sheet?

$$
\begin{array}{c}
\text{Net Income} \\
+ \text{ Depreciation} \\
+\Delta \text{ Accounts Payable} \\
-\Delta \text{ Receivable} \\
-\Delta \text{ Inventory} \\
\hline\hline
\textbf{Cash flow}
\end{array}
$$

Definition

Working capital: = Inventories + Receivable - Accounts payable

(2) Cash-flows $\neq$ accounting flows

How to deduce cash flows from the income statement and the balance sheet?

$$
\begin{array}{c}
\text{Net Income at } t \\
+ \text{ Depreciation at } t \\
-\Delta \text{ Working Capital at } t \\
- \text{ cost of the factory (if } t\text{= year you start the project)} \\
+ \text{ Book value of the factory (if } t\text{= year the project ends)} \\
\hline\hline
\textbf{Cash flow}
\end{array}
$$

Example

Calculate the NPV of the following investment project given an OCC of 12%.

Balance Sheet	t=0	year 1	year 2	year 3	year 4	year 5	year 6
Investment	-15,000						
Working capital	1,500	3,000	4,500	5,000	4,000	2,500	0

Income Statement	t=0	year 1	year 2	year 3	year 4	year 5	year 6
Revenues		16,000	16,750	17,500	18,250	19,000	
Expenses		10,000	10,500	11,000	11,500	12,000	
Depreciation		3,000	3,000	3,000	3,000	3,000	
Pre-tax Income		3,000	3,250	3,500	3,750	4,000	
Tax 35%		1,050	1,138	1,225	1,313	1,400	
Net income		1,950	2,112	2,275	2,437	2,600	

Example

Answer:

	t=0	year 1	year 2	year 3	year 4	year 5	year 6
Investment	-15,000						
Net Income		1,950	2,112	2,275	2,437	2,600	
Depreciation		3,000	3,000	3,000	3,000	3,000	
$-\Delta$ working capital	-1,500	-1,500	-1,500	-500	1,000	1,500	2,500
Cash Flows	-16,500	3,450	3,612	4,775	6,437	7,100	2,500

$$\begin{aligned} NPV &= -16,500 + \frac{3,450}{1.12} + \frac{3,612}{1.12^2} + \frac{4,775}{1.12^3} + \frac{6,437}{1.12^4} + \frac{7,100}{1.12^5} + \frac{2,500}{1.12^6} \\ &= 2,244.71 \end{aligned}$$

Internal Rate of Return

Internal Rate of Return

Definition

The **internal rate of return** (IRR) of an investment project is the discount rate y such that the NPV of the project equals zero:

$$-C + \sum_{i=1}^{N} \frac{F_i}{(1+y)^{t_i}} = 0$$

Definition

IRR Criterion: All projects whose IRR are greater than the opportunity cost of capital should be implemented.

Using the IRR criterion: examples

Example

OCC =8%

	Today	year 1	year 2
Project A	-100	115	0
Project B	-100	9	109
Project C	-100	40	70

$$
\begin{aligned}
-100 + \frac{115}{1+IRR_A} &= 0 \Rightarrow IRR_A = 15\% > 8\% \\
-100 + \frac{9}{1+IRR_B} + \frac{109}{(1+IRR_B)^2} &= 0 \Rightarrow IRR_B = 9\% > 8\% \\
-100 + \frac{40}{1+IRR_C} + \frac{70}{(1+IRR_C)^2} &= 0 \Rightarrow IRR_C = 6\% < 8\%
\end{aligned}
$$

Implement project A and B but not project C

Why IRR can be hazardous to your wealth: 1

Caveat 1: Projects that last more than one period may have more than one IRR.

Example

OCC=10%

	Today	year 1	year 2
Project D	-200	500	-300

$$IRR_D = 0\%, 50\%$$

What shall we do according to the IRR criterion?

And according to the NPV criterion?

$$NPV_D = -200 + \frac{500}{1.1} - \frac{300}{1.1^2} = 6.6 > 0$$

Why IRR can be hazardous to your wealth: 2

Caveat 2: Projects that last more than one period may have no IRR.

Example

OCC=10%

	Today	year 1	year 2
Project E	-200	500	-320

$$IRR_E = \varnothing$$

What shall we do according to the IRR criterion?

And according to the NPV criterion?

$$NPV_E = -200 + \frac{500}{1.1} - \frac{320}{1.1^2} = -9.9 < 0$$

Why IRR can be hazardous to your wealth: 3

Caveat 3: IRR and NPV criterion can lead to different solutions.

Example

OCC=10%

	Today	year 1	year 2
Project F	100	-150	50

$$IRR_F = 0\%, -50\% < 10\%$$

What shall we do according to the IRR criterion?

And according to the NPV criterion?

$$NPV_F = 100 - \frac{150}{1.1} + \frac{50}{1.1^2} = 4.96 > 0$$

Why IRR can be hazardous to your wealth: 3

Caveat 3: IRR and NPV criterion can lead to different solutions.

Example

OCC=2%

	Today	4 years	5 years
Project 1	-100	0	200
Project 2	-100	190	0

What is best between 1 and 2?

$$IRR_1 = 14.87\%; NPV_1 = 81.14$$

$$IRR_2 = 17.41\% : NPV_2 = 75.5$$

What shall we choose?

Other hazardous criteria

Definition

A project's **payback** is the time required to recover the initial investment.
Payback criterion: Choose the project with the shorter payback.

Definition

A project's **normalized NPV** is the project NPV divided by the initial investment.
Normalized NPV criterion: Choose the project with larger normalized NPV.

CONCLUSION

Projects should be selected using the NPV criterion.

http://www.youtube.com/watch?v=YUhb0XII93I