

Problem Set 1Question 1

You would like to buy a new car selling for 40,000. You are wondering whether it would be better (i) to lease the car for three years or (ii) to get a three-year loan to buy the car. The lease would cost 600 per month starting immediately for a period of three years (36 payments). The loan would have consistent payments to be paid back in three years starting with the next month with an annual effective rate of 10%. The opportunity cost of capital is 10%.

- If you think that in three years from now you could sell the car for 30,000, should you buy it today? Explain clearly why. Show all calculations.
- Suppose now that road tax plus insurance, that need to be paid at the beginning of each year, cost 1,000 per year. You should pay these only if you buy the car. Does this information change your decision as to whether buy or lease? Show all calculations.

Question 2:

You would like to evaluate an investment strategy for retirement savings. Today you are 27 years old and you have exactly 40 years to your retirement. The effective annual interest rate is 6% per year for this strategy.

You should represent all interest rates with 4 decimal places (for ex., 0.0600), all cash flows with 2 decimal places (for ex., € 123.45).

- What must be the quarterly (per three-month) interest rate that corresponds to the effective annual rate of 6% (that is, 0.0600) per year?
- You invest € 3,000 at the end of each quarter for the next 40 years. What is the present value of this investment?
- How much money would you have in your investment account on the day of your retirement?
- The day you begin your retirement, your savings in the investment account are turned into an annuity contract that makes quarterly payments of € 46,810. The effective annual interest rate on the annuity contract is 6% per year. The payments are made at the end of the quarter, starting with the first quarter of your retirement. What is the number of quarters during which this annuity contract will make you a payment?

Question 3:

Three months ago, the ABC transportation company asked a consulting firm to analyze the prospect for operating a shuttle bus service between the HEC campus and the Châtillon Subway station (line 13). The consulting firm, which was paid €18,000 a month ago, produced the following analysis of the project for ABC Corp:

At date $t=0$, ABC would have to purchase a shuttle bus estimated at € 100,000. The service would be operating for 10 years. The following annual forecasts are made for each of the 10 years during which the shuttle would operate:

- Annual Ticket sales received at the end of each year: € 35,000
- Annual Operating costs (gas, drivers' salaries, etc.) paid at the end of each year: € 10,000

Depreciation will be linear over the life of the project (that is, € 10,000 per year). The shuttle bus will have no scrap (resale) value at the end of the 10th year. The corporate tax rate is 40% and taxes are paid in the same year in which the revenues are generated. The opportunity cost of capital is 10 % per year for this project.

- What are the cash flows for time $t = 0, 1, \dots, 10$?
- Should ABC Corp. realise the project? Justify your answer.

Solution key to Problem Set 1

Question 1:

- a) PV of the loan + resale value :
$$-40,000 + \frac{30,000}{1.1^3} = -€17,460.6$$

 Monthly effective rate : $r_{month} = (1 + r_e)^{1/12} - 1 = (1 + 0.1)^{1/12} - 1 = 0.00797 = 0.797\%$
 PV lease

$$-\frac{600}{0.00797} \times \left(1 - \frac{1}{(1 + 0.00797)^{12 \times 3}}\right) 1.00797 = -€18,862.4$$

Borrowing is less costly than leasing.

- b) The cost of buying in this case is

$$-17,460.6 - 1,000 - \frac{1,000}{1.1} - \frac{1,000}{1.1^2} = -€20,196.1$$

It is cheaper to lease the car.

Question 2:

- a) $r_{Quarterly} = (1 + r_e)^{1/4} - 1 = (1 + 0.06)^{1/4} - 1 = 0.0147 = 1.47\%$
 b)

$$\frac{3000}{0.0147} \times \left(1 - \frac{1}{(1 + 0.0147)^{40 \times 4}}\right) = €184,322.03$$

- c) How much money would you have saved in your account the day of your retirement?
 $€184,322.03 \times (1 + 0.06)^{40} = €1,895,884.41$

d)

$$\frac{46,810}{0.0147} \times \left(1 - \frac{1}{(1 + 0.0147)^T}\right) = €1,895,884.41 \quad \rightarrow$$

$$T = \frac{\ln\left(\frac{1}{1 - \frac{1,895,884.41 * 0.0147}{46,810}}\right)}{\ln(1 + 0.0147)} = 62 \text{ quarters}$$

Question 3:

- a) What are the cash flows for time t = 0, ..., 10 ?

Solution :

Sales	35	
- cost	-10	
- dep	-10	
=	=15	
- tax	-6	
=Net Income		9

Annual CF = 9 + 10 = 19

b) $NPV = -100 + \sum_{t=1}^{10} \frac{19}{(1.1)^t} = -100 + \frac{19}{0.1} \left(1 - \frac{1}{1.1^{10}}\right) = 16.75 > 0$

The project should be done. NB the consulting fee is a sunk cost.