Financial Economics, **HEC, Paris**

Problem Set 2

Exercise 1

1.7 Your firm spent \$2 million in R&D to conceive a new generation computer. The computer will be built in a new factory. The cost of this factory is \$10 million paid today. In five years from now, the computer will be outdated and the factory will be shut down. The resale value of the factory and its equipment is expected to be \$11 million. The cost associated with the project will be entirely financed by shareholders.

For this project, the financial planning division of the firm forecasts the following Income Statement (in million dollars) for the first year of operations:

Sales revenues	50
Cost of goods sold	35
Depreciation of the factory	2
General Selling and Administrative Expenses	3
Operating Income	10

Sales revenues are expected to grow at a rate of 10% per year while the cost of goods sold and the other expenses are expected to remain constant.

Today, the working Capital requirement (*) is \$10 million. It is then expected to grow at a rate of 3% per year.

The tax rate is 40% (taxes are paid at the end of each year).

The rate of return on a one year Treasury bill is 6%.

a) Compute the net cash flow of the project for each year of operation.
b) The chief Financial Officer decides to use a discount rate of 6% to compute the NPV of the project. Does she decide to undertake the project or not?
c) Is the choice of 6% discount rate a good idea (discuss)?

(*) The Working Capital requirement is defined for the purpose of this exercise as:
Inventories + Receivables - Accounts Payable.

Exercise 2

AB is a conglomerate that has €1.2 billion in common stock. Its capital is invested in two subsidiaries: A and B. The two subsidiaries are expected to perform differently, depending on the economic environment.

Subsidiary	*Investment In € millions*	*Poor Economy*	*Average Economy*	*Good Economy*
A	700	20 %	-5 %	-8 %
B	500	15 %	10 %	-20 %

Assuming that the good economy is twice as likely to take place as the other two, calculate implicit portfolio weights for each subsidiary and an expected return and standard deviation of return for the equity in the AB conglomerate.

Exercise 3

Consider the following four assets:
The S&P 500 ("asset SP" henceforth) with $E[r_{SP}]= 20\%$, $\sigma_{SP}=25\%$.
Future on gold ("asset G" henceforth) with $E[r_G] = 3\ \%$, $\sigma_G=35\%$.
Future on crude oil ("asset O" henceforth) with $E[r_O] = 15\ \%$, $\sigma_O=18\%$.
Risk-free Treasury Bill ("asset *f*" henceforth) with $E[r_f] = 2\ \%$.

Portfolio P1 is composed for $x^{P1}_{SP} = 2/3$ of asset SP and for $x^{P1}_{O} = 1/3$ of asset O. Portfolio P2 is composed for $x^{P2}_{G} = ½$ of asset G and for $x^{P2}_{f} = ½$ of the asset *f*.

a) Adam invests €600 Euros in portfolio P1 and €200 in portfolio P2. What are the composition $X^A=\{ x^A_{SP}, x^A_G, x^A_O, x^A_f\}$ and the expected return $E[r^A]$ of Adam's portfolio ?
b) Bob short sells portfolio P2 for €200 and invests €1000 in portfolio P1. What are the composition $X^B=\{ x^B_{SP}, x^B_G, x^B_O, x^B_f\}$ and the expected return $E[r^B]$ of Bob's portfolio ?
c) Suppose that the standard deviation of portfolio P1 is 22.22%. What are the covariance and the correlation coefficient between r_{SP} and r_O.
d) Suppose that asset G is negatively correlated with asset SP. Then which one between Adam's and Bob's portfolios do you think is the less risky.

Elements of answer PS2

Exercise 1

Remarks

i) The **cost of the factory** (-10 million $) is a negative cash flow that occurs in time 0. Indeed the decision of building the factory has not been taken at time 0.

ii) The cost in R&D should **not** be considered as it incurred before time 0 (see rule 4 page 18)

iii) I should subtract to the net income the **variation of the working capital** (flow variable) as it is a negative cash flow; the working capital is stock variable and not a cash flow.

In particular we will make the following additional assumption not specified in the text (other assumptions are ok as long as they are not in contradiction with the text and that your solution is consistent with your assumptions):

- Working capital for the first year of activity is created at time 0 so that at the end of the first year 1 (t=1) working capital will be of 10.3.
- At the end of year 5 (t= 5), working capital is $10*1.03^5$ and will be liquidated in year 6 (i.e. at the end of the 6^{th} year (t=6), working capital is 0)
- The factory will be in activity until the end of year 5, hence from date t=1 until date t=5 included.
- The factory is sold at the end of year 5 (t=5) at a price that is higher than its book value (Book value = purchase value – depreciation = 10-2*5 = 0). This affects the period 5 cash flow in two ways: a positive cash flow of $11 million and a negative cash flow of $11*0.4 millions = $4.4 millions of taxes.

Given these assumptions, we have:

	t=0	t=1	t=2	t=3	t=4	t=5	t=6
Sale of factory						11	
Sales		50	55	60,5	66,55	73,205	
Costs of goods		35	35	35	35	35	
Depreciation		2	2	2	2	2	
General cost		3	3	3	3	3	
Operating income		10	15	20.5	26.55	44.205	
tax		4	6	8.2	10.62	17.682	
net income		**6**	**9**	**12.3**	**15.93**	**26.523**	

	t=-1	t=0	t=1	t=2	t=3	t=4	t=5	t=6
Working capital	0	10	10.3	10.609	10.927	11.255	11.593	0
delta w.c.		10	0.3	0.309	0.318	0.328	0.338	-11.593

Cash Flows= Net income +Depreciation - delta WC

	t=0	t=1	t=2	t=3	t=4	t=5	t=6
Cost of the factory	-10						
Cash flow	**-20**	**7.7**	**10.691**	**13.982**	**17.602**	**28.185**	**11.593**
Present values	-20	7.264	9.515	11.74	13.942	21.062	8.173

NPV= 51,696

b) the NPV is positive. Hence we shall undertake the project.

c) The text states that the rate of 6% corresponds to treasury bills return rate that is a risk free investment. A discount rate of 6% is reasonable for our project as long as the cash flows generated by the project are risk free. Note that in general cash flows are not perfectly known in advance and we typically face uncertainty regarding a business future revenues and costs. Hence it would be more appropriate to use an OCC that is adequate to the uncertainty of the project's cash-flows.

Exercise 2

Weight of A = 700/1200 = 0.5833
Weight of B = 500/1200 = 0.4167

$E(R_A) = 0.25\times(0.2) + 0.25\times(-0.05) + 0.5\times(-0.08) = -0.0025$ (i.e. –0.25 %)
$E(R_B) = 0.25\times(0.15) + 0.25\times(0.1) + 0.5\times(-0.2) = -0.0375$ (i.e. –3.75 %)
$E(R_P) = 0.5833 \times (-0.0025) + 0.4167 \times (-0.0375) = -0.0171$ (i.e. –1.71 %)

$Var(R_A)=0.25\times(0.2-(-0.0025))^2+ 0.25\times(-0.05-(-0.0025))^2 + 0.5\times(-0.08-(-0.0025))^2 = 0.0138$

$Var(R_B) = 0.25\times(0.5-(-0.0375))^2 + 0.25\times(0,1-(-0.0375))^2 + 0.5\times(-0,2-(-0.0375))^2 =0.0267$

$Cov(R_A, R_B) = 0.25\times(0.\ 2-(-0.0025))(0.15-(-0.0375)) + 0.25\times(-0.05-(-0.0025))(0.1-(-0.0375)) + 0.5\times(-0.08-(-0.0025))(-0.2-(-0.0375)) = 0.0142$

$Var(R_P) = 0.5833^2 \times 0.0138 + 0.4167^2 \times 0.0267 + 2 \times 0.5833 \times 0.4167 \times 0.0142 =0.0162$
$\sigma(R_P) = \sqrt{0.0162} = 0.1274$

Exercise 3

a) Adam invests €600/€(200+600)=3/4 into Portfolio 1 and €200/€800=1/4 into Portfolio 2. Hence:

$X^A=\{ x^A_{SP}, x^A_G, x^A_O, x^A_f\}=\{2/3, 0, 1/3, 0\}\times3/4 + \{0, ½, 0, ½ \}\times1/4 = \{1/2, 1/8, 1/4, 1/8\}$

$E[r^A]= 20\%\times1/2 +3\%\times1/8 + 15\%\times1/4 + 2\% \times1/8 = 14.375\%$

b) Bob's Invests €1000/(-€200+€1000)=5/4 into Portfolio 1 and -€200/€800 =-1/4 into Portfolio 2. Hence :

$X^B=\{ x^B_{SP}, x^B_G, x^B_O, x^B_f\}=\{2/3, 0, 1/3, 0\}\times5/4 + \{0, ½, 0, ½ \}\times(-1/4) =$
$=\{5/6, -(1/8), 5/12, -(1/8)\}$

$E[r^B]= 20\%\times5/6 -3\%\times1/8 + 15\%\times5/12 - 2\% \times1/8 =22.292\%$

c) Note that the variance of Portfolio 1 is:

$\sigma_{P1}{}^2 = 0.2222^2 = (2/3)^2\times0.25^2 +(1/3)^2\times0.18^2 +2\times2/3\times1/3\times Cov(r_{SP}, r_O)$

$\Rightarrow Cov(r_{SP}, r_O) = 0.04049$

$\Rightarrow \rho_{SP,O} = Cov(r_{SP}, r_O)/(\sigma_{SP}\times\sigma_O) = 0.9$.

e) Note first that asset SP and asset O are strongly positively correlated as $\rho_{SP,O} = 0.9$. Hence asset G is negatively correlated with both SP and O. Thus portfolio P1 and portfolio P2 are negatively correlated. Hence Adam's portfolio is less risky than Bob's portfolio as the former contains positive quantities of two negatively correlated portfolios, whereas the latter contains a long and a short position into two negatively correlated portfolios.

More formally let us compare the variances of returns of Bob's and Adam's portfolios.
The variance of Portfolio 2 is
$\sigma_{P2}^2 = (1/2)^2\times 0.35^2 = 3.0625\%$, therefore $\sigma_{P2} = 3.0625\%^{1/2} = 17.5\%$.
The variance of Adam's portfolio is
$\sigma_A^2 = (3/4)^2\times\sigma_{P1}^2 + (1/4)^2\times\sigma_{P2}^2 + 2\times 3/4\times 1/4\times Cov(r_{P1}, r_{P2})$
$= (3/4)^2\times 0.2222^2 + (1/4)^2\times 0.175^2 + 2\times 3/4\times 1/4\times Cov(r_{P1}, r_{P2}) = 2.97\% + 3/8\times Cov(r_{P1}, r_{P2})$
The variance of Bob's portfolio is
$\sigma_B^2 = (5/4)^2\times\sigma_{P1}^2 + (-1/4)^2\times\sigma_{P2}^2 - 2\times 5/4\times 1/4\times Cov(r_{P1}, r_{P2})$
$= (5/4)^2\times 0.2222^2 + (-1/4)^2\times 0.175^2 - 2\times 5/4\times 1/4\times Cov(r_{P1}, r_{P2}) = 7.91\% - 5/8\times Cov(r_{P1}, r_{P2})$

$Cov(r_{P1}, r_{P2}) = cov(2/3r_{SP}+1/3r_O, 1/2r_G+1/2r_F)$
$= 1/3cov(r_{SP},r_G) + 1/6cov(r_0,r_G) = 1/3.\rho_{SP,G}.25\%.35\% + 1/6.\rho_{0,G}.18\%.35\%$
$= 2.92\%\rho_{SP,G} + 1.05\%\rho_{0,G}$.
Since $\rho_{SP,G} < 0$, $cov(r_{P1}, r_{P2}) < 1.05\%$. Therefore, $\sigma_A^2 < 2.97\% + 3/8\times 1.05\% = 3.36\%$, and $\sigma_B^2 > 7.91\% - 5/8\times 1.05\% = 7.25\%$. Conclusion: $\sigma_A^2 < \sigma_B^2$.