Financial Economics **HEC, Paris**

Problem Set 4

Problem 1:

Consider an economy with two risky assets $S=\{s_1, s_2\}$. Asset s_1 expected return rate is $E(r_1)=20\%$ and the standard deviation of its rate of return is $\sigma_1=30\%$. Asset s_2 expected return rate is $E(r_2)=10$ % and the standard deviation of its rate of return is $\sigma_2=10\%$. The correlation coefficient between the return rates of s_1 and s_2 is $\rho_{1,2} = 0.6$.

a) What is the composition of the minimum variance portfolio, its expected return and the standard deviation of its rate of return?
b) Represent in the Risk/Expected-Return plane the two risky assets, the minimum variance portfolio and the set of portfolios you can obtain with the two risky assets.
c) On the same graph clearly indicate the region where you have to short sell asset s_1 and the region where you have to short sell asset s_2.

Now, suppose that a risk-free asset F with return of 3% is introduced in this economy, i.e. set of assets S available is now $S=\{s_1, s_2, s_F\}$. Let the expected return of the tangency portfolio be $E[r_T] = 11.20\%$ and its standard deviation be $\sigma_T = 11.33\%$.

d) What is the composition of the tangency portfolio?
e) Represent the Capital Allocation Line and the tangency portfolio in your graph.
f) Would it ever be efficient to short sell one of the two risky assets?

Your aunt Anna is a mean-variance investor with utility function:

$$U\left(E[r_P],\sigma_P^2\right) = E[r_P] - \frac{A}{2}\sigma_P^2$$

where A= 10. Her financial advisor proposes to buy portfolio D whose composition is
$X_D=\{x_1^D=0.3, x_2^D=0.2, x_F^D=0.5\}$.

g) Is portfolio D efficient? Explain briefly (no explanation, no points).
h) What is the exact composition $\{x_1, x_2, x_F\}$ of the portfolio that maximizes Anna's utility function?

Problem 2:

Consider an economy E with two risky assets A and B. The characteristics of the risky assets are provided below:

Asset	**E(r)**	**σ**	**$\rho_{A,B}$**
A	20%	10%	
B	25%	20%	+1

a. If short-sales are allowed in economy E, draw the feasible set of portfolios. Draw your graph to scale.
* Indicate the minimum-variance portfolio MV1 on your graph.
* What is the composition of MV1?
* What are the expected return and standard deviation of MV1?

b. Suppose now that short-sales are no longer allowed in the economy E. In a new graph, draw the feasible set of portfolios (again, please draw the graph to scale).

* Now, what is the composition of the minimum-variance portfolio MV2?
* Indicate MV2 on your graph.

c. Now, we introduce the risk-free asset F with $r_F = 5\%$ into the economy E. Short-sales are still **not** allowed in E. On the graph of part (b) above, draw the set of efficient portfolios.

d. In a new graph depict how your answer to part (c) would change, if short-sales are allowed, but only up to 30% (that is, you cannot short-sell more than 30% of any asset). What is the composition of the minimum variance portfolio MV3 that is composed of only risky assets if short-sales are allowed up to 30%?

Problem Set 4 – SOLUTION KEY

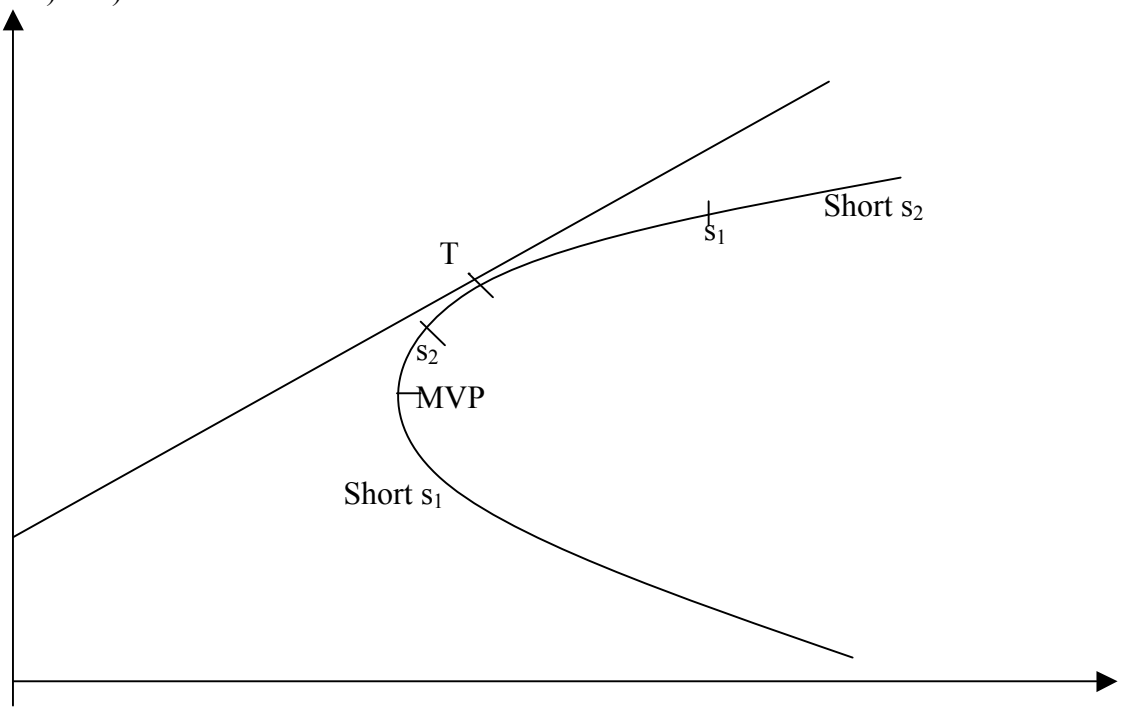
Problem 1:

a) The minimum variance portfolio composition is

$x_1^{MVP} = (0.1^2 - 0.6*0.1*0.3)/ (0.1^2 + 0.3^2 - 2*0.6*0.1*0.3) = -0.125$

$X^{MVP} = \{-0.125, 1.125\}$

$E[r^{MVP}] = 8.75\%$
$\sigma^{MVP} = 9.49\%$
b) & c) & e)



d) $E[r_T] = \mathbf{0.1120} = x_1^T\ 0.20 + (1 - x_1^T)\ 0.10$ ➔ $x_1^T = 0.1200$, $x_2^T = 0.8800$ & $x_F^T = 0$

f) N0, because in this case I would not be combining the risk-free asset with the tangency portfolio.

g) portfolio D is <u>not</u> efficient as the weights of the risky assets are not multiples of the weights of the tangency portfolio.

h) The weight of the tangency portfolio in aunt Anna's optimal portfolio is :

$x_T^* = (0.1120 - 0.0300) / [10 * 0.1133^2] = 0.6388$

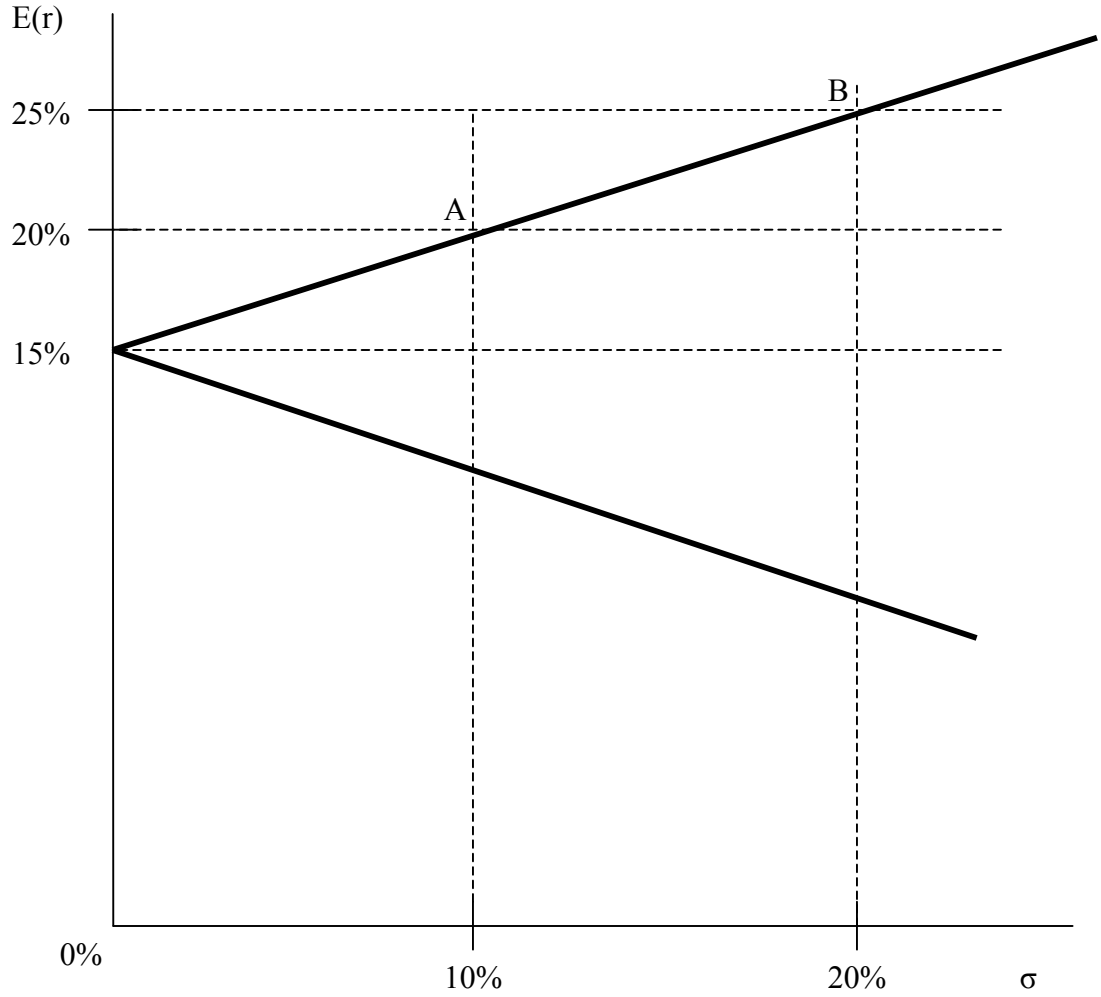
Hence her optimal portfolio will be:

$\{x_1^*=0.6388 * 0.1200;$ $x_2^*=0.6388 * 0.8800;$ $x_F^* = 1 - 0.6388\}$, that is:
$\{x_1^*=0.0767;$ $x_2^*=0.5621;$ $x_F^* = 0.3612\}$

Problem 3:

a. If short-sales are allowed in economy E, draw the feasible set of portfolios. Please draw your graph to scale.

* Indicate the minimum-variance portfolio MV1 on your graph.
* What is the composition of MV1?
* What are the expected return and standard deviation of MV1?



$$x_A^{MV1} = \frac{\sigma_B^2 - \rho_{A,B}\sigma_A\sigma_B}{\sigma_A^2 + \sigma_B^2 - 2\rho_{A,B}\sigma_A\sigma_B} = \frac{0.2^2 - 1\times 0.1\times 0.2}{0.1^2 + 0.2^2 - 2\times 1\times 0.1\times 0.2} = \frac{0.02}{0.01} = +2 = 200\%$$

$$x_B^{MV1} = 1 - x_A^{\text{min.-var.}} = -1 = -100\%$$

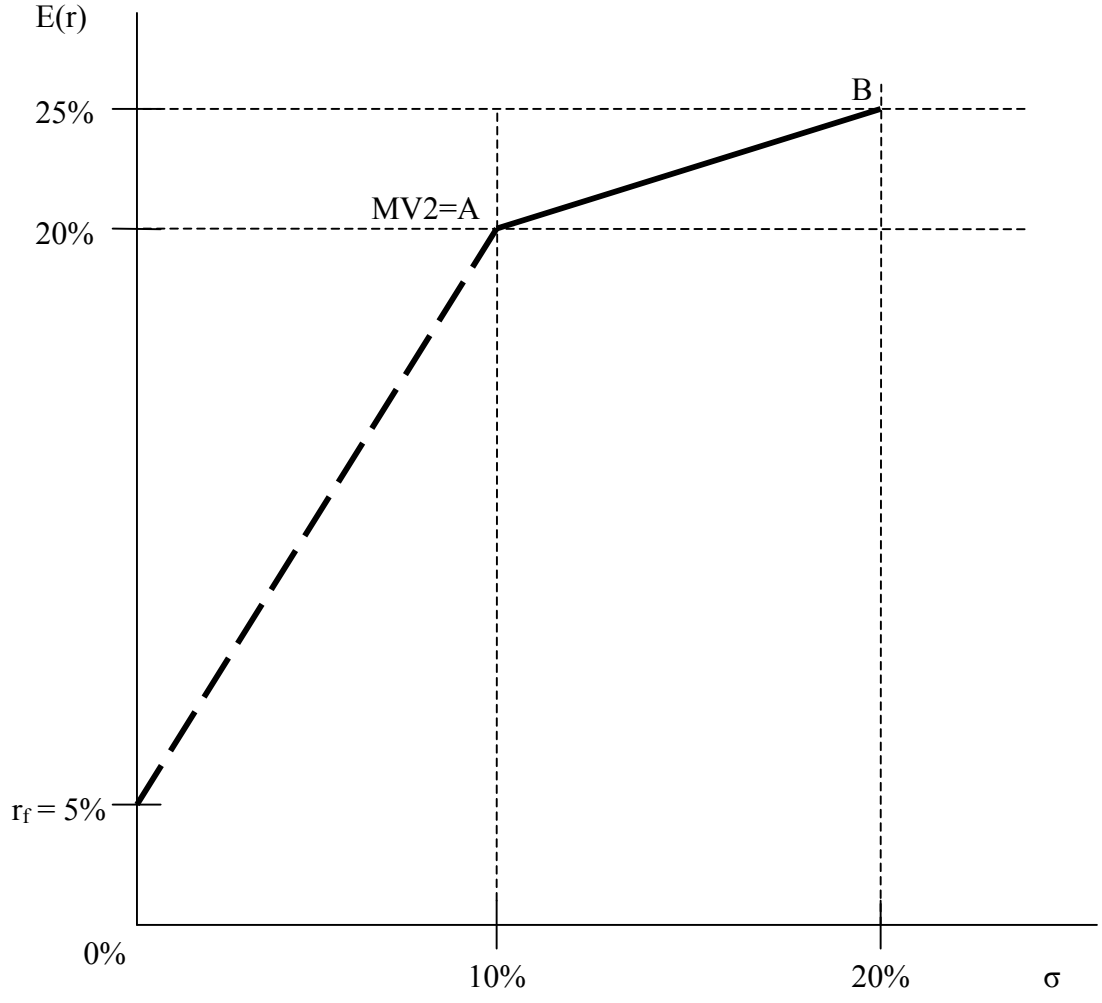
$$E(r_{MV1}) = 2\times E(r_A) - 1\times E(r_B) = 2\times 0.20 - 0.25 = 0.15 = 15\%$$

$$\sigma_{MV1} = 0 = 2\times 0.10 - 1\times 0.20$$

Note : Many of you forget to draw the portion of the line with a negative slope. This is necessary since the question is "draw the **feasible** set" . The feasible set is the set of **all** risk/return combinations that can be obtained with a portfolio combining A and B (not just the efficient portfolios).

b. Suppose now that short-sales are no longer allowed in the economy E. In a new graph, draw the feasible set of portfolios (again, please draw the graph to scale).

* Now, what is the composition of the minimum-variance portfolio MV2?
* Indicate MV2 on your graph.



The feasible set is simply the line in between A and B. Other risk return combinations require to short-sale either A or B. Hence, they are not feasible when short-sales are forbidden. Obviously A is the minimum variance portfolio in this case. Hence:

$$x_A^{MV2} = 1 = 100\% \qquad \textit{and} \qquad x_B^{MV2} = 1 - x_A^{MV2} = 0$$

c) Now, we introduce the risk-free asset F with $r_F = 5\%$ into the economy E. Short-sales are still **not** allowed in E. On the graph of part (b) above, draw the set of efficient portfolios. Answer : The efficient set is the dotted line + the portion of line in between points A and B (see graphic). Obviously, this portion of line remains feasible when there is a riskless security. Moreover it is efficient as short-sales of the riskless security would be required to obtain a larger expected rate of return than the expected rate of return of a portfolio on the segment [A,B]. The dotted line are efficient portfolios obtained by combining the riskless security (without short-sales) and the tangency portfolio, which obviously here is A.

d. In a new graph depict how your answer to part (c) would change, if short-sales are allowed, but only up to 30% (that is, you cannot short-sell more than 30% of any asset). What is the composition of the minimum variance portfolio MV3 that is composed of only risky assets if short-sales are allowed up to 30%?

This was the toughest part of question 4. The correct answer is as follows: