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Bid–Ask Price Competition with Asymmetric Information between Market-Makers

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This paper studies the effect of asymmetric information on the price formation process in a quote-driven market. One market-maker receives private information on the value of the quoted asset and repeatedly competes with market-makers who are uninformed. We show that despite the fact that the informed market-maker's quotes are public, the market is never strong-form efficient with certainty until the last stage. We characterize a reputational equilibrium in which the informed market-maker influences and possibly misleads the uninformed market-makers' beliefs. At this equilibrium, a price leadership effect arises, the informed market-maker's expected pay-off is positive and the rate of price discovery increases in the last stages of trade.

1. INTRODUCTION

Ever since Kyle (1985) and Glosten and Milgrom (1985), a common assumption in market microstructure literature on asymmetric information is that while market-makers do not have superior information on market fundamentals, some traders do have private information. However, several empirical studies have reported stylized facts that are difficult to reconcile with the assumption that market-makers are equally uninformed. In this paper, we study the dynamic interaction between market-makers when one of them does, in fact, possess superior information, and we characterize the equilibrium dynamic quoting strategy that results.

The first of the stylized facts from the empirical literature suggesting that dealers are asymmetrically informed is the price leadership effect. In the Foreign Exchange market (FX market, henceforth), Peiers (1997) examines the quoting behaviour of dealers in the DM–US$ market around Bundesbank interventions and finds evidence of price leadership by Deutsche Bank before the announcement of intervention. This conclusion is confirmed by de Jong, Mahieu, Schotman and van Leeuwen (1999). In his analysis of the same market, Sapp (2002) observes that certain banks consistently incorporate new information into prices before other banks do so. Ito, Lyons and Melvin (1998) study the change in the pattern of returns volatility in the Tokyo FX market: they conclude that their empirical observations are consistent with the assumption of privately informed dealers (where private information is considered common knowledge). Studying the relative contribution of electronic communication networks and market-makers in providing informative quotes on the Nasdaq market, Huang (2002) finds that, among the Nasdaq market-makers, some provide more timely information, which suggests that they are likely to possess superior information. Moreover, he shows that being a price leader is not associated with posting the best quotes. Heidle and Li (2004) study the quoting behaviour of the market-makers affiliated to analysts' brokerage firms. They find strong evidence that these market-makers systematically change their quoting behaviour well before the analysts publicly announce the reports containing their investment recommendations. Finally, in the secondary market for Italian sovereign bonds, Albanesi and Rindi (2000) and Massa and Simonov (2003) found evidence of imitative pricing behaviour and attribute it to the fact that some market-makers are reputed to be better informed.

A second stylized fact that is difficult to explain using classical market microstructure models is the separate role of bid and ask quotes for the transmission of private information. Sapp (2002) finds evidence that only one side of the price leaders' quotes, that is, ask or bid, provides additional information contributing to price discovery. Also, Heidle and Li (2004) find evidence on Nasdaq-listed stocks that informed market-makers use only one price to signal their private information about the analysts' reports (they quote more aggressive bids when the report is positive, and more aggressive asks when it is negative). Finally, Naranjo and Nimalendran (2000) observe that the bid–ask spread changes more around the Bundesbank's unexpected interventions than around its expected interventions, suggesting that the width of the spread may contain some information. This empirical evidence seems to suggest that a dealer with superior private information uses bids and asks separately to signal (or to conceal) information to the market.

These papers support two facts that seem to be common to these markets. First, that there is a small group of market-makers who have superior information on fundamentals. Second, that the identities of these market-makers are known by other market-makers. These hypotheses are also confirmed by Goodhart (1988) who concludes from interviews with London-based specialists that some dealers are perceived as being better informed than others. Lyons (1997, 2001) backs this view, concluding that banks with larger customer share likely have better information.

In fact, an important common feature of these markets is that market-makers' bid and ask quotes are not anonymous. Consequently, quotes posted by the better-informed dealer have a role in both, influencing the market participants' beliefs on fundamentals and in determining the transaction prices.

Theoretical models such as Glosten and Milgrom (1985), Kyle (1985), Easley and O' Hara (1987), and Holden and Subrahmanyam (1992), among many others, cannot account for price leadership or for signalling through quotes. This is mainly because their analyses are based on the assumption that market-makers are equally uninformed.

A third stylized fact concerns the evolution of the spread before the announcement date. There is rich empirical evidence (Venkatesh and Chiang, 1986; Krinsky and Lee, 1996; Koski and Michaely, 2000 among others) that shows that the average spread widens as the announcement period approaches, implying that asymmetric information should be the greatest just before the public release of information. This pattern is in sharp contrast with the classical market microstructure prediction that spreads steadily decrease as information is gradually incorporated into the price (Glosten and Milgrom, 1985; Easley and O'Hara, 1987).

In order to address these points, we study a model in which market-makers and liquidity traders exchange a risky asset for a riskless asset during T periods. In each period, market-makers simultaneously set quotes and automatically execute market orders submitted by liquidity traders. We assume that one of the market-makers has superior information about the fundamental value of the risky asset. In our model, we will only consider the case in which all floor traders are liquidity investors who do not possess any private information. This assumption is admittedly strong, but it is needed both for analytical tractability and to clearly disentangle the effects of asymmetric information among dealers from those coming from informed floor traders. The identity of the informed dealer is commonly known and the posted quotes are not anonymous. Therefore, the uninformed market-makers extract information on the value of the asset by observing past quotes posted by the informed market-maker. The latter takes into account the impact that his or her current quotes will have on the quoting strategy of uninformed dealers in the future.

The trading mechanism we consider is a close representation of existing trading mechanisms. For example, in Nasdaq's screen-based order routing and execution systems, such as SelectNet and the Small Order Execution System (SOES), clients' orders are automatically executed against market-makers at the best prices. We quote from a document of the National Association of Securities Dealers Department of Economic Research:

> Nasdaq market-makers have also been subject to an increasing level of mostly affirmative obligations.
>
> Market-makers must continuously post firm two-sided quotes, good for 1000 shares [...]; they must report trades promptly; they must be subject to automatic execution against their quotes via SOES; [...]. (Smith, Selway, III and McCormick, 1998–2001, p. 2)

The model also fits FX markets as, on the one hand, traders execute their orders against the market-makers who post the best quotes while, on the other hand, the identity of quotes issuers is observable. Finally, the proposed trading mechanism is a stylized representation of "pit" trading.

We show that in such a highly transparent quote-driven market, a privately informed market-maker gradually reveals information.

More precisely, we first study whether the market is strong-form efficient, in the sense that prices convey all available private and public information. We prove that in the last trading period, the informed market-maker's quotes fully reveal his or her private information, but the probability that this revelation would occur earlier in time is less than 1. In other words, the market is strong-form efficient with certainty only seconds before the public announcement.

Second, we analyse market-makers' quoting strategies and show that the informed market-maker generates some "noise" in his or her quoting activity, which precludes other market participants from immediately inferring his or her private information, allowing the market-maker to exploit informational advantage over several trading rounds. The distribution of noise corresponds to the equilibrium mixed strategy used by the informed dealer. The intuition of our result is based on two observations: (i) if the value of the asset is high it is worth buying it by setting high bid quotes, whereas if its value is low it is worth selling it by setting low ask quotes and (ii) the more accurate the uninformed dealers' belief, the smaller the profit will be for the informed market-maker. On the one hand, when the informed market-maker chooses the quotes that maximize his or her current pay-off, part of the information is revealed and the future pay-off decreases. On the other hand, if he or she chooses quotes that cause a loss in current trade, he or she misleads the uninformed market-makers, thereby increasing his or her future pay-off. We will show that, as long as there are impending trading rounds, it is optimal for the informed market-maker to randomize between revealing information and misleading the market by trading against his or her signal.

Finally, we provide empirically testable implications that run contrary to the results of the existing models of informed trading and are in line with the stylized empirical facts mentioned above. First, we find that the equilibrium presents a positive serial correlation between the quotes set by the informed market-maker at time t and the quotes set by the uninformed market-makers at time $t+1$. In view of the fact that with equally informed market-makers there is no specific dealer that leads the price discovery process, our result suggests that the empirical evidence in which some dealers appear to be price leaders is indeed compatible with the presence of asymmetrically informed market-makers.

Second, we prove that at equilibrium the informed market-maker uses the bid and the ask price differently in order to strategically signal his or her type. In fact, in the mixed strategy equilibrium we characterize, the informed market-maker either posts aggressive (*i.e.* high) bids in tandem with very high asks or aggressive (*i.e.* low) asks together with very low bids. Thus, in each trading round only the quote on one side of the market incorporates new information. This is consistent with the empirical findings in Sapp (2002). It is clear that one does not necessarily have to post the best quotes to signal information, as empirically observed by Huang (2002). Finally, informed dealers set the spread more frequently on the profitable side, but they also participate in the unprofitable side of the market, which corresponds to the empirical findings in Heidle and Li (2004).

Third, we find that the revelation of information increases as the public announcement approaches. The adverse selection is stronger at the last stages of the trading game because the opportunity cost of concealing private information is at its greatest at this time. Thus, the informed market-maker will mainly participate in the profitable side of the market. This increases the winner's curse and results in more conservative quotes. The overall effect is to widen the inside spread as the end of the game approaches.

1.1. *Related literature*

In existing markets where dealers compete in prices, their interaction can be well represented by a first-price auction. In fact, the incoming orders can always be executed at the best possible price.[1] This is the generic approach taken by the theoretical market microstructure literature. However, this literature widely assumes that market-makers are equally uninformed, and that the best informed traders are floor traders. Given these common assumptions, price competition among market-makers is simple Bertrand competition and, consequently authors have focused on the information content of the volume of trade rather than quotes. Biais (1993) is an exception. He considers a static model in which market-makers are risk averse and are privately informed about their own inventory of the risky asset. Thus, competition among market-makers turns out to be a first-price, independent private value auction. In our model, private information concerns the fundamental value of the asset, and the resulting one-stage game is a common value, first-price bid–ask auction. Roell (1988), Bloomfield and O'Hara (2000), and de Frutos and Marzano (2005) analyse a market in which dealers have asymmetric information on the asset fundamentals. The authors study the effects of market transparency on dealer behaviour, in particular. Contrary to our paper, these models analyse only one period of trade during which market-makers are asymmetrically informed and therefore cannot address the issue of strategic transmission of information through time. De Meyer and Moussa Saley (2002) study a repeated zero-sum game where two dealers reciprocally exchange a risky asset. They show that the resulting price dynamics is related to a Brownian motion. There are two assumptions that make it difficult to apply their appealing result directly to financial markets. First, it is assumed that in each period the two dealers mutually exchange the asset, that is, no third party participates in the market. Second, the zero-sum format does not fit financial markets as, in fact, every market-maker can guarantee a zero pay-off simply by quoting a sufficiently large spread. Finally, Gould and Verecchia (1985) study the pricing strategy of a specialist who has unique private information on market fundamentals. In a static set-up, they show that a rational expectations equilibrium with noisy prices exists. Still, their result requires that the specialist be able to commit himself or herself ahead of time to adding an exogenous noise to his or her price. As the actual price at which the specialist trades does not necessarily maximize his or her pay-off function, it is unclear whether the same equilibrium would exist in case the specialist is unable to commit in advance to a noisy pricing rule.

1. See for the example the "Order Handling Rule" valid on the Nasdaq.

Our paper also contributes to the literature on auctions as our market model corresponds to a sequential first-price bid–ask auction for identical objects with common value. The value of proprietary information in one-shot auctions has been studied by Engelbrecht-Wiggans, Milgrom and Weber (1983, EMW henceforth). Proposition 1 extends this result to an auction with an ask (selling) side.

The existing literature on sequential auctions analyses situations in which several objects are put up for sale consecutively to the same set of bidders. The fundamental difference between these models and the problem we study here is that in our set-up, bidders can buy and sell the objects simultaneously.

The first paper on sequential auctions is by Ortega-Reichert (1968), who studies a two-person, two-stage (*i.e.* two objects), first-price sealed-bid auction. The Ortega-Reichert result is innovative in that the author first recognizes the incentive for bidders to deceive their opponents in the first auction in order to reap an expected gain in the second auction. The result differs from ours, in that there is no real deception at equilibrium, since each bidding strategy is invertible and each player can infer the opponent's information from his or her bid.

Engelbrecht-Wiggans and Weber (1983) (EWW henceforth) is closer to our framework since they study a pure common value, sequential auction of identical objects, where one bidder learns the true value of the objects prior to the first sale, while the other bidders are aware that he or she is perfectly informed. The authors show that, at equilibrium, the uninformed bidder may have a higher expected profit than the informed one, on condition that the number of objects for sale is high enough. A similar result is obtained in Hörner and Jamison (2004). Here, the authors extend the analysis of EWW to an infinitely repeated game between two bidders and to a more general discrete distribution of the value of the object. The main difference with our set-up is that bidders can buy the objects but do not sell them. In a bid auction, when the value of the object is low, the informed bidder reaps no advantage from deceiving the uninformed bidder as the object is of no worth to him or her. By contrast, in our bid–ask auction, the informed market-maker has an incentive to mislead the bidder who is uninformed because the action will encourage the sale of the low-value asset at a higher price and increase his or her future profit. This leads to a different type of manipulation activity by the informed player and to different conclusions on the value of information.

Finally, Bikhchandani (1988) studies a finite series of n second-price auctions where different objects have different values, in contrast to the case presented in our study.

The remainder of this paper is organized as follows. Section 2 presents the formal model. In Section 3 we collect the construction of the equilibrium, and prove the short-run information inefficiency of the equilibrium. In Section 4 we develop some empirical predictions of the model. In Section 5 we discuss the case where the asset fundamentals are continuously distributed and in Section 6 we conclude. All proofs are in the Appendix.

2. THE MODEL

Consider a market with N risk-neutral market-makers who trade a single security over T periods against liquidity floor traders. The liquidation value of the security is a random variable $\widetilde{V}$ which can, for simplicity, take two values, $\{\overline{V}, \underline{V}\}$, with $\overline{V} > \underline{V}$, according to a probability distribution $(p, 1-p)$ commonly known by all market-makers, where $p = \Pr(\widetilde{V} = \overline{V})$. We denote by $v = p\overline{V} + (1-p)\underline{V}$ the expected value of the asset for any given p. The realization of $\widetilde{V}$ occurs at time 0 and at time $T+1$ a public report will announce it to all market participants. Time is discrete and T is finite.

2.1. *Information structure*

At the beginning of the first period of trade, one[2] of the market-makers, MM1, is privately informed about the risky asset's realized liquidation value. When $\widetilde{V}=\overline{V}$ (respectively $\widetilde{V}=\underline{V}$), we will refer to the informed market-maker as "type $\overline{V}$" (respectively "type $\underline{V}$") denoted MM1$(\overline{V})$ (respectively MM1$(\underline{V})$). The other $N-1$ market-makers do not observe any private signals, but they do know that MM1 has received superior information; we will treat these market-makers as a unique dealer called MM2.[3] MM2 updates his or her belief by observing MM1's past quotes. We use p_t to denote the uninformed dealer's belief at the beginning of period t, that is, after observing MM1's quotes during the preceding $t-1$ periods. The expected value of the asset at the beginning of period t is denoted by $v_t = p_t\overline{V}+(1-p_t)\underline{V}$.

2.2. *Market rules*

In each period, the two market-makers simultaneously[4] announce their ask and bid quotes, which are firm for one unit of the asset.[5] Then, transactions take place between liquidity traders and the market-makers. We assume that, each time, liquidity traders sell one unit of the asset to the market-maker who sets the highest bid quote and buy one unit of the asset from the market-maker who sets the lowest ask quote (*i.e.* price priority is enforced).[6] If both market-makers set the same quotes, then liquidity traders will be indifferent between MM2 or MM1. The game then has a continuum of strategies and discontinuous pay-offs. In order to guarantee the existence of equilibrium, we follow Simon and Zame (1990) and endogenously determine the tie-break rule in case of identical (bid or ask) quotes. More precisely, let us denote by q the probability that liquidity traders will trade with MM1 in the event of a tie. Instead of specifying an exogenous level for q as a characteristic of the model, and then solve for the equilibrium, q will be determined as part of the equilibrium.[7] We require the probability q to be independent on the realization of $\widetilde{V}$, as it is supposed to be unknown to liquidity traders, but in equilibrium, q will be affected by other factors that are common knowledge at the time of a tie.

Each market-maker can observe the past quotes of all market-makers. Finally, we assume that market-makers cannot trade with each other and that short sales are permitted.[8]

2. As in Kyle (1985), we assume that there is only one agent who receives private information on the realization of $\widetilde{V}$.

3. To the extent that MM2 equilibrium pay-off is 0, this assumption is made without loss of generality because the informed market-maker only considers the probability of winning the auctions at a given price, whether this probability is the outcome of the strategy of one uninformed player or n equally uninformed players (see also EMW; and Section 3.1).

4. We do not consider the timing problem that arises when the bidding process is sequential, as in Cordella and Foucault (1999).

5. In the literature it is standard procedure to fix the tradable quantity at each step (see O'Hara, 1995), and, as mentioned before, this assumption corresponds quite closely to the rules of a number of markets.

6. This is isomorphic to a situation where, for each period, the expected volume of buy orders is constant and equal to the expected volume of sell orders. As market-makers are risk neutral and the volume of trade incorporates no information on $\widetilde{V}$, this would correspond to multiplying market-makers' stage pay-offs by a factor equal to the expected volume of buy (or sell) orders.

7. The procedure suggested by Simon and Zame (1990) consists in defining a pay-off correspondence, which is interpreted as the union of all possible tie-break rules when the prices posted by MM1 and MM2 are identical. An equilibrium for the game will be a selection from the pay-off correspondence of a particular rule together with the (perfect Bayesian) equilibrium for the resulting game.

8. Allowing a market-maker to submit anonymous market orders to the other market-makers would improve MM1's pay-off but would not rule out the signalling role of his or her quotes, which is our main concern here.

2.3. *Behaviour of market participants and equilibrium concept*

In each period, a buy market order and a sell market order are proposed by floor traders trading for liquidity reasons. In order to focus on the role of quotes as a mechanism for the strategic transmission of information, we exclude the presence of other sources of information such as informative floor traders' orders. In our model, traders do not act for informational purposes, and so the order flow neither incorporates nor depends on any information about the value of the asset. As price priority is enforced in all periods, each market-maker knows that the one who proposes the best bid (respectively ask) quote will buy (respectively sell) one asset. We denote by $a_{i,t}$ and $b_{i,t}$ the ask and bid price respectively set by market-maker i in period t. We denote by q_t the probability that MM1 executes the order in the case of a tie in period t. We can write the single period expected pay-off functions for our risk-neutral market-makers as follows:

$$\begin{aligned}E[\Pi_{1,t}\,|\overline{V}\,] &= (a_{1,t}-\overline{V})(\Pr(a_{2,t}>a_{1,t})+q_t\Pr(a_{2,t}=a_{1,t}))\\&\quad+(\overline{V}-b_{1,t})(\Pr(b_{2,t}<b_{1,t})+q_t\Pr(b_{2,t}=b_{1,t})) \qquad (1)\\E[\Pi_{1,t}\,|\underline{V}\,] &= (a_{1,t}-\underline{V})(\Pr(a_{2,t}>a_{1,t})+q_t\Pr(a_{2,t}=a_{1,t}))\\&\quad+(\underline{V}-b_{1,t})(\Pr(b_{2,t}<b_{1,t})+q_t\Pr(b_{2,t}=b_{1,t})) \qquad (2)\end{aligned}$$

for MM1$(\overline{V})$ and MM1$(\underline{V})$ respectively, and for MM2

$$\begin{aligned}E[\Pi_{2,t}] = {} & p_t(a_{2,t}-\overline{V})(\Pr(a_{1,t}>a_{2,t}|\overline{V})+(1-q_t)\Pr(a_{1,t}=a_{2,t}|\overline{V}))\\&+(1-p_t)(a_{2,t}-\underline{V})(\Pr(a_{1,t}>a_{2,t}|\underline{V})+(1-q_t)\Pr(a_{1,t}=a_{2,t}|\underline{V}))\\&+p_t(\overline{V}-b_{2,t})(\Pr(b_{1,t}<b_{2,t}|\overline{V})+(1-q_t)\Pr(b_{1,t}=b_{2,t}|\overline{V}))\\&+(1-p_t)(\underline{V}-b_{2,t})(\Pr(b_{1,t}<b_{2,t}|\underline{V})+(1-q_t)\Pr(b_{1,t}=b_{2,t}|\underline{V})). \qquad (3)\end{aligned}$$

The overall pay-off of each market-maker is the (non-discounted) sum for $t = 1,\dots,T$ of these pay-offs:

$$\pi_1(V,T,p)=\sum_{t=1}^{T}E[\Pi_{1,t}\,|V\,]\ \text{ for } V\in\{\overline{V},\underline{V}\}$$

$$\pi_2(T,p)=\sum_{t=1}^{T}E[\Pi_{2,t}].$$

We focus on equilibria where market-makers' strategies at each round depend on the number of rounds before the public report and on the overall information that past quotes provide about the true value of $\widetilde{V}$. Namely, we denote with $\gamma_t = (\tau, p_t)$ the *state of the game* at time t, where $\tau = T-1+t$ is the remaining number of trading rounds before the public report. In consequence a mixed strategy for MM2 in period t is a function σ_2 that maps the state of the game γ_t into a probability distribution over all couples of bid–ask quotes. As MM1's strategy depends also on his or her private information, a mixed strategy for MM1 in period t is a function σ_1 that maps the realized value of the asset and the state of the game γ_t into a probability distribution over all couples of bid–ask quotes. Finally, the liquidity trader's tie-break strategy is a function q that maps the state of the game γ_t into the probability of trading with MM1 in case of a tie in quotes. For a given state of the game $\gamma = (\tau, p)$ we denote $\pi_1^*(\widetilde{V}, \tau, p)$ and $\pi_2^*(\tau, p)$ the expected equilibrium pay-off for MM1$(\widetilde{V})$ and for MM2, respectively.

We characterize the perfect Bayesian equilibrium strategies σ_1^*, σ_2^*, and q^* by solving the game by backward induction. At any time t the market-makers solve the following problems:

$$\begin{aligned}
\sigma_1^*(\overline{V}, \tau, p_t) &= \underset{\sigma_1(\overline{V})}{\arg\max}\ E[\Pi_{1,t} \,|\overline{V}\,] + \pi_1^*(\overline{V}, \tau-1, p_{t+1}), \quad \text{given } \sigma_2^*, q^* \\
\sigma_1^*(\underline{V}, \tau, p_t) &= \underset{\sigma_1(\underline{V})}{\arg\max}\, E[\Pi_{1,t} \,|\underline{V}\,] + \pi_1^*(\underline{V}, \tau-1, p_{t+1}), \quad \text{given } \sigma_2^*, q^* \\
\sigma_2^*(\tau, p_t) &= \underset{\sigma_2}{\arg\max}\, E[\Pi_{2,t}] + \pi_2^*(\tau-1, p_{t+1}), \quad \text{given } \sigma_1^*, q^* \\
q^*(\tau, p_t) &\in [0,1],
\end{aligned}$$

where $\tau = T+1-t$ and $p_{t+1} = \Pr(\widetilde{V} = \overline{V} | a_{1,t}, b_{1,t})$ is determined by Bayes' rule when this is possible, otherwise it is chosen arbitrarily.

We denote with $\Gamma(T, p)$ the game representing the strategic interaction among market-makers when there are T finite rounds of trade and $\Pr(\widetilde{V} = \overline{V}) = p$ at the beginning of the game ($t = 0$).

It is worth emphasizing that, as market-makers can alternatively buy or sell the security without inventory considerations, whatever the true value of the asset, one of the two auctions will always be profitable and the other one not. This suggests that what really matters for the equilibrium of the game is not the actual value of the asset, $\overline{V}$ or $\underline{V}$, but how close to the truth MM2's belief p is: intuitively, the more correct the belief of MM2, the smaller MM1's profit. In the Appendix we formally state this symmetry property of the game.

3. EQUILIBRIUM CHARACTERIZATION

3.1. *One trading round*

In this section, we analyse the dealers' price competition when $T = 1$, which can also be interpreted as the last trading round. The bid auction alone has been solved by EMW for an arbitrary distribution of the value of the object for sale. Proposition 1 extends the authors' result to the ask auction in the case of a binomial distribution of $\widetilde{V}$. It also provides the equilibrium distribution of bid and ask quotes and market-makers' equilibrium pay-offs.

Proposition 1. *The equilibrium of the one-shot game $\Gamma(1, p)$ is unique and is such that* MM2 *randomizes ask and bid prices according to*

$$\Pr(a_{2,1} < x) = F^*(x) = \begin{cases} 0 & \textit{for } x \in]-\infty, v] \\ \frac{x-v}{x-\underline{V}} & \textit{for } x \in]v, \overline{V}] \\ 1 & \textit{for } x \in]\overline{V}, +\infty] \end{cases}$$

$$\Pr(b_{2,1} < x) = G^*(x) = \begin{cases} 0 & \textit{for } x \in]-\infty, \underline{V}] \\ \frac{\overline{V}-v}{\overline{V}-x} & \textit{for } x \in]\underline{V}, v] \\ 1 & \textit{for } x \in]v, +\infty]. \end{cases}$$

If the value of the asset is $\overline{V}$, then MM1 *sets $a_{1,1} = \overline{V}$ and randomizes the bid price according to*

$$\Pr(b_{1,1} \le x \,|\overline{V}\,) = \overline{G}^*(x) = \begin{cases} 0 & \textit{for } x \in]-\infty, \underline{V}] \\ \frac{(1-p)(x-\underline{V})}{p(\overline{V}-x)} & \textit{for } x \in]\underline{V}, v] \\ 1 & \textit{for } x \in]v, +\infty]. \end{cases}$$

If the value of the asset is $\underline{V}$, *then* MM1 *sets* $b_{1,1} = \underline{V}$ *and randomizes the ask price according to*

$$\Pr(a_{1,1} \leq x \,|\underline{V}) = \underline{F}^*(x) = \begin{cases} 0 & \text{for } x \in]-\infty, v] \\ \frac{x-v}{(1-p)(x-\underline{V})} & \text{for } x \in]v, \overline{V}] \\ 1 & \text{for } x \in]\overline{V}, +\infty]. \end{cases}$$

The equilibrium pay-offs are $\pi_2^*(1, p) = 0$, $\pi_1^*(\overline{V}, 1, p) = (1-p)(\overline{V} - \underline{V})$, *and* $\pi_1^*(\underline{V}, 1, p) = p(\overline{V} - \underline{V})$.

In case of a tie in quotes the probability of trading with MM1 *is* $q^* \in [0, 1]$.

Just before the public report, the informed market-maker has a last opportunity to make a profit from his or her private information. Therefore, if the liquidation value of the asset is $\overline{V}$, MM1 will try to buy the asset by winning the bid auction, whereas if the liquidation value of the asset is $\underline{V}$, the market-maker will try to sell it by winning the ask auction. Because the uninformed market-maker does not know whether it is profitable to buy or to sell the asset, he or she will bid more conservatively in both auctions, taking into account the "winner's curse" resulting from the competition with a better-informed market-maker.

In short, in a static game, the asymmetry of information between market-makers leads to three important implications. First, the full revelation of information by MM1 makes the market strong-form efficient at the last stage of trade. This follows from the fact that MM1's quotes are observable.[9] Second, contrary to the case with symmetric information, bid and ask market prices are different from the expected liquidation value of the asset, given the public information. In fact, the market spread is strictly positive generically, and bid and ask quotes straddle v. However, there is no restriction over the spread's width (up to $\overline{V} - \underline{V}$), which depends on the outcome of the mixed strategies. Third, although MM2's expected equilibrium pay-off is 0, MM1 obtains a positive expected pay-off. Namely, the more erroneous MM2's belief, the larger MM1's informational rent, as he or she will be able to win the profitable auction at a more lucrative price.

3.2. *Informational efficiency of the quote-driven market*

In the last trading period MM1 reveals his or her private information to the market through posted quotes.

At first glance, given that the informed dealer's quotes are observable by other market-makers, it would seem likely that MM1 would lose the informational advantage at the first trading round. However, this is not true of any period prior to the last one. More precisely, we show that before the last trading round the probability that private information is fully conveyed into prices is less than 1.

Theorem 2. *There exists no Bayesian–Nash equilibrium where* MM1*'s private information is revealed with certainty before* T.

Theorem 2 states that private information is never revealed with probability 1 before the final round T. Hence, in the short run, it is not always possible to infer MM1's private information unambiguously, despite the fact that his or her quotes are perfectly observable. This informational inefficiency recalls results obtained in other market microstructure models. However, in models "*à la*" Kyle or Glosten and Milgrom, the market is not strong-form efficient because the insider traders conceal their actions within the exogenous random demand that comes from

9. With unobservability of the insider's actions (*e.g.* Kyle, 1985) the market is not strong-form efficient even at the last stage.

noise traders. Uninformed agents cannot directly observe the informed traders' action and are therefore unable to infer the informed trader's private information. By contrast, our result does not rely on the existence of exogenous noise due to anonymous orders. Theorem 2 shows that when an informed dealer cannot hide behind noise traders, he or she will *endogenously* generate some noise. The rationale of the Theorem 2 proof is that before the last trading round, a phase in which information is fully revealed is simply not credible. More precisely, if at some $t < T$, MM1's private information was fully revealed for certain, then in period t market-makers would optimally play the unique equilibrium of the one-shot game. However, in this case, MM1 has at least one profitable deviation that consists in misleading MM2 in period t and then profiting from MM2's totally wrong beliefs in the following trading period. Hence, MM1 cannot be committed to truthfully disclosing his or her inside information until the last stage.

3.3. *Equilibrium in manipulating strategies*

Theorem 2 states that in the short run the market is not strong-form efficient, but does not specify how, in equilibrium, MM1 manages to conceal and exploit his or her information. In this section, we characterize a perfect Bayesian equilibrium of the dynamic bid–ask auction in which MM1 generates endogenous noise in his or her quotes. By doing so, MM1 can profit from informational advantage during several periods. The interest of the particular equilibrium we study here[10] is that it is consistent with several empirical observations described in the literature, such as the "price leadership" effect; the fact that the informed market-maker participates in the unprofitable side of the market less frequently than in the profitable side; the fact that in each step only one of the two sides of the market incorporates new information; the increase of the quoted spread and quotes' volatility as the date of a public announcement gets closer.

First, we explain the leading economic forces that produce our result.

From the analysis of the one-period case, it results that in the last trading stage, MM1 only competes in the profitable side of the market, that is, MM1 tries to sell the asset if $\widetilde{V} = \underline{V}$ or to buy it if $\widetilde{V} = \overline{V}$. In the following, we prove that during the trading periods prior to the final one, MM1 conceals information by participating in the unprofitable side of the market with positive probability. In doing so, MM2 cannot unambiguously deduce MM1's information by observing whether MM1 tried to buy or to sell the asset in the previous period. More precisely, after observing that MM1 tried to buy the asset (to sell the asset), MM2 will attach a larger (respectively smaller) probability to the event $\widetilde{V} = \overline{V}$, but the posterior belief will not be necessarily equal to 1 (respectively 0). We define these strategies as *manipulating strategies* since there is a positive probability that MM1 will take an action with the aim of turning MM2's belief in the wrong direction.

MM1's incentive to mislead MM2 by trying to win the unprofitable auction depends on two factors. First, the benefit that a misleading action will have on the future pay-off. Second, the current cost of misleading. Intuitively, the greater the number of remaining trading periods, the higher MM1's benefit from misleading MM2 in the current period. For example, in the last trading round, as the future pay-off is 0, it is never optimal to mislead.[11] By contrast, a misleading action in the early rounds can be turned into profit in the following trading rounds. Thus, whenever there are trading rounds still to be conducted, it can be optimal for MM1 to mislead MM2. Nevertheless, misleading is optimal only if the expected cost of winning the unprofitable auction is small. The cost decreases with the correctness of MM2's belief, which can be measured by $|v_t - \widetilde{V}|$, that is, the distance between the expectation of $\widetilde{V}$ and the realization of $\widetilde{V}$. Take the

10. Note that we are looking for an equilibrium of a particular kind, leaving the question of the existence of other equilibria unresolved.

11. See Proposition 1.

case $\widetilde{V} = \overline{V}$, for example. Roughly speaking, if MM1 wants to mislead MM2 in period t, then MM1 must try to sell the asset by posting an ask price close to the current expected value v_t.[12] The cost of misleading is given by the risk of selling the asset at a price close to v_t lower than its actual value, $\overline{V}$. When p_t is close to 1, v_t approaches the actual value of the asset $\overline{V}$ and the cost of misleading is low. Therefore, when MM2's belief is sufficiently correct, misleading is cheap and MM1 will bid in the unprofitable side of the market with positive probability. However, when MM2's belief is sufficiently wrong, the cost of misleading becomes too large, and MM1 will only participate in the profitable side of the auction.

Let $\tau = T - t + 1$ be the number of trading stages before the public report. The following proposition provides a qualitative description of the equilibrium:

Proposition 3. *There exists an equilibrium of the game $\Gamma(T, p)$ that satisfies the following features:*

(1) *In any given trading round t, whenever a market-maker tries to buy the asset (to sell the asset), he or she randomizes his or her current bid (respectively current ask) on the support $[b_{\min}, v_t]$ (respectively $[v_t, a_{\max}]$); where $b_{\min}$ (respectively $a_{\max}$) depends on the state of the game $\gamma_t = (\tau, p_t)$.*
(2) *In each trading round,* MM2 *tries both to buy and to sell the asset simultaneously.*
(3) MM1 *never tries to buy and sell the asset simultaneously. Namely, in trading round t:*
   - *If $p_t < 2^{1-\tau}$, then* MM1$(\overline{V})$ *tries to buy the asset and stays out of the ask auction setting $a_{1,t} = a_{\max}$. If $p_t > 2^{1-\tau}$, then* MM1$(\overline{V})$ *randomizes between trying to buy the asset and trying to sell it.*
   - *If $p_t > 1 - 2^{1-\tau}$, then* MM1$(\underline{V})$ *tries to sell the asset and stays out of the bid auction setting $b_{1,t} = b_{\min}$. If $p_t < 1 - 2^{1-\tau}$, then* MM1$(\underline{V})$ *randomizes between trying to buy the asset and trying to sell it.*
(4) *A market-maker's equilibrium expected pay-off is* 0 *if he or she is uninformed and positive if he or she is informed.*

Regarding features (1), (2), and (4) the equilibrium of Proposition 3 is similar to the equilibrium of the static game described in Proposition 1: (1) quotes are generated from mixed strategies, and bid and ask prices straddle v_t; (2) MM2 always tries to win both the bid and the ask auctions; (4) a market-maker's pay-off is strictly positive only if he or she has some private information. By contrast, feature (3) is specific to the dynamic game. According to this property, if MM2's belief is sufficiently wrong, the informed market-maker tries to win only the profitable auction (*i.e.* the bid auction when $\overline{V}$ realized and the ask auction when $\underline{V}$ realized, respectively). On the contrary, if MM2's belief is sufficiently correct, then MM1 misleads the market with positive probability. MM1 will do this by randomizing between two actions: competing only in the profitable auction, and competing only in the other auction. Take the case $\widetilde{V} = \overline{V}$, for example: the closer p_t is to 1, the closer MM2's belief to the truth. Feature (3) states that MM1$(\overline{V})$ misleads MM2 by trying to sell the asset with positive probability only if p_t is sufficiently close to 1, namely, $p_t > 2^{1-\tau}$. However, if MM2's belief is substantially wrong (*i.e.* $p_t < 2^{1-\tau}$), then MM1$(\overline{V})$ will only try to buy the asset, as misleading will prove too costly. A symmetric reasoning applies to MM1$(\underline{V})$ who will find it profitable to mislead MM2, by trying to buy the asset, only if MM2's belief is sufficiently correct, that is, if $p_t < 1 - 2^{1-\tau}$.

12. Intuitively, MM2 will never accept to sell the asset at a price $a_{2,t} < v_t$, so MM1 is sure to win the ask auction with an $a_{1,t}$ sufficiently close to v_t.

Note that when misleading occurs with positive probability, MM1's strategy can be seen as a two-step lottery. First, MM1 flips a (biased) coin to determine whether to compete in the bid or in the ask auction. Second, he or she randomly fixes the level of the quote (either bid or ask) that he or she will submit in the auction in which he or she competes. The other quote will be set at a level that will make him or her certain to lose.

There are two implications in the fact that the threshold $2^{1-\tau}$ and $1-2^{1-\tau}$ decrease and increase, respectively, with the length of the game. First, for any given belief and any realization of $\widetilde{V}$, misleading occurs with positive probability, provided that there are enough trading rounds before the public report. Second, a misleading action is more likely to occur in the early stages of trade as it can be turned into profit during a longer period. Thus, during the initial trading rounds, the sign of MM1's information affects his or her quoting strategy only slightly. However, as the date of the public report approaches, the incentive to mislead decreases and private information strongly affects MM1's strategies. In other words, at the beginning of the game, the winner's curse is weak since observing whether MM1 buys or sells does not reveal much about the true value of the asset. However, when the value-relevant announcement is drawing near, MM1's strategy will depend significantly on his or her private information and winner's curse heavily affects competition between market-makers. In the following section we show that this has clear empirical implications on the informational content of MM1's quotes and the expected market spread that shall increase as T approaches.

4. EQUILIBRIUM PROPERTIES AND EMPIRICAL IMPLICATIONS

In this section we assess some equilibrium properties in terms of informational efficiency and liquidity. We also provide some empirical predictions that can be used to detect the presence of asymmetric information among dealers in quote-driven markets. Despite the fact that it is possible to obtain the closed-form expressions for the equilibrium quotes distribution for any repetition of the game recursively,[13] these expressions are not always tractable. Therefore, we obtain some of the empirical implications of the model by computing the expected quotes numerically, using $\overline{V}=1$ and $\underline{V}=0$ and varying the initial belief p and the length of the game T.

4.1. *Price leadership*

The standard market microstructure theory in which market-makers are equally uninformed does not explain the price leadership effect that has been documented in the empirical literature on FX markets, over-the-counter markets, and Nasdaq.[14] The manipulating equilibrium of Proposition 3 shows this characteristic as it exhibits a positive correlation between the quotes posted by the uninformed market-maker (MM2) at a given period t and the quotes that the informed one (MM1) posted in $t-1$. The explanation is simple. MM1 is more likely to post relatively high quotes when $\widetilde{V}=\overline{V}$ rather than when $\widetilde{V}=\underline{V}$. Thus, MM1's high quotes induce MM2 to believe that $\widetilde{V}=\overline{V}$ is more likely. As a consequence, in the following period MM2's expected quotes will increase. More precisely, in equilibrium, MM2's posterior belief on the event $\widetilde{V}=\overline{V}$ is an increasing function of MM1's last quotes, and MM2's expected quotes are increasing functions of his or her belief.

13. See Appendix.
14. See Introduction for a complete list of references.

Restating the equilibrium of Proposition 3 for $T = 2$ it is possible to explicitly quantify the price-leadership effect.[15] Let $\text{Post}(a, b)$ be MM2's posterior probability of $\widetilde{V} = \overline{V}$ after observing $(a_{1,1}, b_{1,1}) = (a, b)$. We obtain

Lemma 4. *In the equilibrium of the game* $\Gamma(2, p)$, MM2*'s expected quotes in the second period increase with* MM1*'s first-period quotes, whereas* MM2*'s first-period quotes do not affect* MM1*'s second-period quotes. More precisely,*

(i) *if* $p > 1/2$,

$$\frac{\partial E[a_{2,2}]}{\partial a_{1,1}} = -\ln(\text{Post}(a_{1,1}, b_{\inf}))\frac{(2p-1)(\overline{V}-\underline{V})^2}{(2a_{1,1}-\overline{V}-\underline{V})^2} > 0$$

$$\frac{\partial E[b_{2,2}]}{\partial a_{1,1}} = -\ln(1-\text{Post}(a_{1,1}, b_{\inf}))\frac{(2p-1)(\overline{V}-\underline{V})^2}{(2a_{1,1}-\overline{V}-\underline{V})^2} > 0,$$

(ii) *if* $p < 1/2$,

$$\frac{\partial E[a_{2,2}]}{\partial b_{1,1}} = -\ln(\text{Post}(a_{\max}, b_{1,1}))\frac{(1-2p)(\overline{V}-\underline{V})^2}{(2b_{1,1}-\overline{V}-\underline{V})^2} > 0$$

$$\frac{\partial E[b_{2,2}]}{\partial b_{1,1}} = -\ln(1-\text{Post}(a_{\max}, b_{1,1}))\frac{(1-2p)(\overline{V}-\underline{V})^2}{(2b_{1,1}-\overline{V}-\underline{V})^2} > 0, \quad \textit{and}$$

(iii) *for all* p,

$$\frac{\partial E[a_{1,2}]}{\partial a_{2,1}} = \frac{\partial E[b_{1,2}]}{\partial a_{2,1}} = \frac{\partial E[a_{1,2}]}{\partial b_{2,1}} = \frac{\partial E[b_{1,2}]}{\partial b_{2,1}} = 0.$$

Lemma 4 also shows that MM1's quote revisions remain unexplained by MM2's quote adjustments. This allows to run empirical tests on the Granger-causality of the observed market-makers' quotes.

Simulations for $p > 1/2$ suggest that the covariance between MM1 and MM2's two successive ask quotes is roughly 15% of $(\overline{V} - \underline{V})$, which represents a significative price effect of MM1 over MM2.

Moreover, as MM1's quotes become more informative as the date of the public report approaches, the price-leadership effect will increase as well.

4.2. *Informational efficiency*

One of the appealing properties of auction mechanisms is that it is possible to extract the bidders' private information on the value of the auctioned object by observing the bidders' bids. Not surprisingly, this observation is confirmed by the analysis of our one-shot auction. Indeed, in the last period, MM1 fully reveals his or her private information through his or her quotes. However, Theorem 2 shows that this is not always the case when identical assets are traded sequentially.

15. When $T > 2$ we can still show that

$$\frac{\partial E[a_{2,t+1}]}{\partial a_{1,t}} > 0 \quad \frac{\partial E[a_{2,t+1}]}{\partial b_{1,t}} > 0$$

$$\frac{\partial E[b_{2,t+1}]}{\partial a_{1,t}} > 0 \quad \frac{\partial E[b_{2,t+1}]}{\partial b_{1,t}} > 0.$$

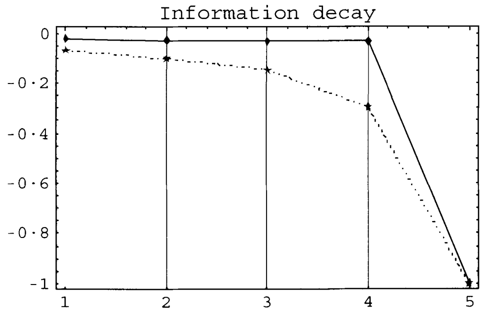

FIGURE 1

The expected rate of change of $\sum_t$, the variance of $\widetilde{V}$ conditional on public information is shown. For both p close to 1/2 (- - -) and p close to 1 or 0 (—), $\sum_t$ decreases at an increasing rate

As it is standard in market microstructure literature, we measure the weak-form efficiency of the market using the evolution of the variance of $\widetilde{V}$ conditioned on all relevant public information, $\sum_t = \text{Var}[\widetilde{V}\,|H_t]$. The faster the convergence of $\sum_t$ to 0 (*i.e.* the higher the rate at which $\sum_t$ decreases), the better the properties of the market in terms of efficiency. In models of order-driven markets (Kyle, 1985; Holden and Subrahmanyam, 1992; Foster and Viswanathan, 1996; Huddart, Hughes and Levine, 2001) $\sum_t$ either decreases at a constant or at a dwindling rate, implying that most of the private information is conveyed into the prices relatively early on in the game.

Contrary to what occurs in order-driven markets, in our framework, the first stages of the game are "waiting" stages with a relatively low signalling activity, while most of the information is released in the very last stages of trading. This is shown in Figure 1, which plots the expected rate of change of $\sum_t$ for a game repeated five times. The two lines correspond to two different levels of the initial prior belief. The variance of the risky asset's value decreases at a rate that depends on the level of the initial prior belief. When this prior belief is close to 1 or 0 (thick line), the initial variance of $\widetilde{V}$ decreases more slowly than when the prior is close to $1/2$ (dotted line). In both cases, however, $\sum_t$ reduces at an increasing rate, which means that less information is revealed at the early stages and that MM1's quotes reveal more information during the last rounds of trade.

4.3. *The expected cost of trading*

Some empirical and experimental evidence (Venkatesh and Chiang, 1986; Krinsky and Lee, 1996; Koski and Michaely, 2000) has shown that the inside spread usually widens as the moment of public release of information draws nearer. This can be verified along the equilibrium of Proposition 3 as well. As a measure of liquidity we consider the expected inside spread. Figure 2 shows that for a fixed level of p, the expected inside spread increases as the date of public report approaches. In the last stages of the game, the spread is maximum.

This finding is easy to explain. In the early trading rounds, the winner's curse is weak, hence bid–ask quotes are concentrated on average around the *ex ante* expected value of the asset. The winner's curse increases when T draws near, and this effect forces the uninformed market-maker to quote more "conservatively", so that on average the spread increases.

4.4. *The value of private information*

Finding the value of private information has been a central issue in financial economics. In most of the market microstructure literature, the existence of equilibria in which the information has

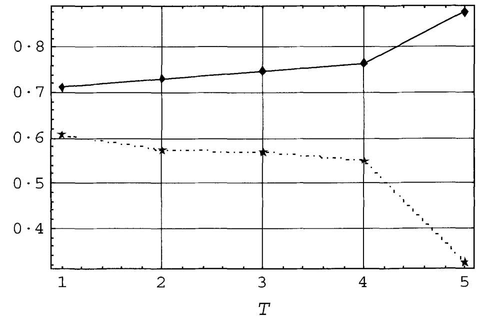

FIGURE 2

The expected minimum ask (—) and maximum bid (- - -) as a function of time is shown. The parameter set is $p = 0{\cdot}65$ and $T = 5$, $\underline{V} = 0$ and $\overline{V} = 1$

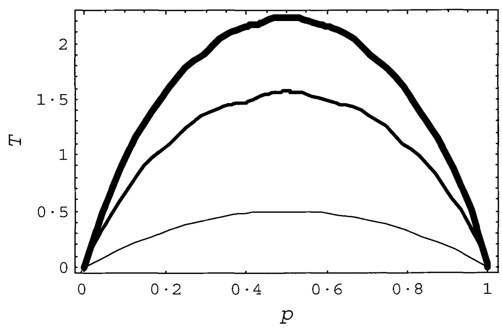

FIGURE 3

MM1's *ex ante* expected pay-off as a function of p for $T = 1$ (thin line), $T = 15$ and $T = 30$ (thick line) and $\underline{V} = 0$, $\overline{V} = 1$ is shown

a positive value appears to be related to the presence of exogenous noise in the economy. For example, in Kyle (1985), the profit of the insider trader is proportional to the volatility of noise traders demand. We show that this is not the case in a quote-driven market, as a market-maker can derive a positive profit from superior information even without exogenous noise in the market. There are two factors that affect the value of the private information: the *ex ante* volatility of $\widetilde{V}$ and the number of repetitions T.

The volatility of the asset fundamental is measured by the unconditional variance of $\widetilde{V}$, which is equal to $p(1-p)(\overline{V}-\underline{V})^2$. Figure 3 plots MM1's *ex ante* equilibrium pay-off as a function of p when the game is repeated once (thin curve), 15 times, and 30 times (thick curve). The *ex ante* pay-off is maximum when the uncertainty in the market is high, which corresponds to a p close to $1/2$. Not surprisingly, private information is more valuable in markets in which little is known about large shocks on the fundamentals.

Figure 3 also shows that the informed market-maker's pay-off increases with the number of trading rounds available before the public report occurs. The increment in MM1's pay-off from one additional trading round decreases with T. Figure 4 plots the marginal increase in MM1's *ex ante* expected profit from adding two additional trading rounds when p is around $0{\cdot}5$. The increase in MM1's profit is low for high T since an additional round of trading would not provide

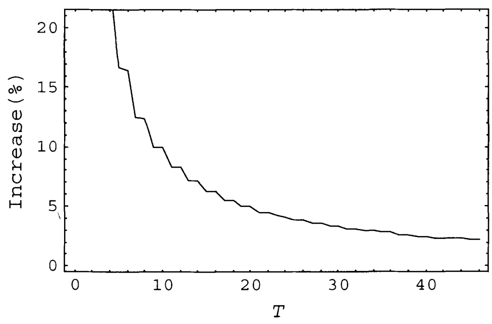

FIGURE 4

The increase in MM1's expected equilibrium pay-off when T increases is represented. The parameters set is $p = 0,51$, $\underline{V} = 0$, $\overline{V} = 1$

MM1 with substantial additional profits because MM2 will bid quite aggressively in these periods owing to the low winner's curse effect.

Finally, please note that in our model the uninformed bidder earns a lower expected profit than he or she would in one-sided auctions with asymmetric information (cf. EWW; Hörner and Jamison, 2004, for example). In these studies the high-type informed bidder is able to make profits on one object at the most, as after that his or her private information is revealed. In order to conceal information during some stages, the high type bidder has to constantly underbid the uninformed bidder. The uninformed bidder thereby obtains several objects at sufficiently low prices. This scenario does not apply to our bid–ask auction. The equilibrium of Proposition 3 shows that MM1 of *each* type can mimic the behaviour of a different type during many stages. In doing so, MM1 can force the pay-off of MM2 down to 0, exactly as in a one-shot auction: MM2 is "squeezed" between MM1 of low type who tries to sell the asset and MM1 of high type who tries to buy it. Consequently, MM2 is never sure to win either auction at a convenient price.

5. EXTENSION: CONTINUOUS DISTRIBUTION OF $\widetilde{V}$

In Section 3 we show that if the fundamentals of the asset can only take two values, $\overline{V}$ or $\underline{V}$, then in equilibrium market-makers select their quotes using mixed strategies and the probability of observing informational efficient quotes before T is less than 1. In this section we will discuss the robustness of this result when $\widetilde{V}$ is continuously distributed over a closed interval $[\underline{V}, \overline{V}]$.

In the case of a continuous distribution, MM2 faces an informed market-maker with a continuum of types (representing the information about the realized $\widetilde{V}$). EMW study what can be reinterpreted as a one-side, one-shot version of this model and show that when the value of the asset is continuously distributed, the informed bidder uses a pure strategy that is monotonic in the bidder's type. Crawford and Sobel (1982) consider the problem of strategic information transmission when there is a continuum of types for the informed player. They show that the information is partially revealed with a "semi-pooling" pure strategy equilibrium where the informed player's strategy is a step function of his or her information. Neither one of these results, however, extends to the dynamic auction that we are considering in this paper. In fact, it is possible to show that there exists no sub-game perfect equilibrium in which MM1 uses a pure strategy before the last repetition of the game. Namely, MM1 pooling pure strategies are dominated strategies while semi-pooling or fully revealing pure strategies equilibria contain some not credible action. An equilibrium in which MM1's strategy is pure and not pooling at $t < T$, would imply that at $t+1$, MM2 will attach zero probability to all realizations of $\widetilde{V}$ that are inconsistent with MM1's quotes in period t. But in this case there would always exist a $V \in [\underline{V}, \overline{V}]$ such that MM1 of type V finds it profitable at time t to post quotes that mislead MM2 and then gain in the following period from MM2's completely wrong belief.[16] In general, at equilibrium, MM1 never reveals his or her private signal with probability 1 before the last stage.[17] Therefore, a quote-driven market where the posted quotes are not anonymous is not strong-form efficient with certainty until actual public release of information, and this is true independently of the modelling assumption on the fundamental $\widetilde{V}$. Moreover, if an equilibrium exists, it is in mixed strategies in the early rounds of trade. The question of the existence of sub-game-perfect equilibria when $\widetilde{V}$ is continuously distributed remains open.

6. CONCLUSION

We have studied a quote-driven market with asymmetric information between market-makers and have shown that an informed market-maker strategically releases his or her private information using mixed strategies. This generates an endogenous noise that allows the informed market-maker to exploit his or her informational advantage over several periods. Despite the highest possible level of market transparency, which allows all dealers to observe the best informed agent's actions (*i.e.* their bid and ask quotes), the market is not strong-form efficient in the short run with positive probability. In fact, it is only in the very last trading round, immediately before an informational event, that quotes will fully incorporate private information with certainty. This equilibrium behaviour has several empirical implications. First, there is a positive correlation between the informed market-makers' quotes at time t and the uninformed market-maker's quotes at $t+1$. Second, the information content of the best-informed market-maker's quotes increases as the date of the public report draws near, and in consequence the expected market spread increases as well. Third, trading prices are different from the expected value of the risky asset given the public information. Fourth, even if no new shocks hit the fundamentals, quotes are volatile. Fifth, the private information has a positive value even in such a highly transparent market, which justifies the costly activity of information collection by institutional dealers.

One possible direction for further research would be to study a more complex situation in which floor traders also have private information. In this case in point, the incentive for the informed market-maker to mislead the market would probably diminish. However, this would probably not change the main economic trade-off the market-maker faces in deciding the optimal strategies. Hence, we can expect that the "strategic" noise in the informed market-maker's quotes would persist.

APPENDIX

Symmetry property (SP) : *The game $\Gamma(T, p)$ is symmetric with respect to the following transformation:*

$$\widetilde{V}' = \overline{V} + \underline{V} - \widetilde{V} \tag{A.1}$$

$$a'_{i,t} = \overline{V} + \underline{V} - b_{i,t} \tag{A.2}$$

$$b'_{i,t} = \overline{V} + \underline{V} - a_{i,t} \tag{A.3}$$

$$p' = 1 - p. \tag{A.4}$$

16. The complete proof of this statement is available from the authors upon request.
17. For a formal proof of this statement, we refer the reader to the following website: www.restud.com/supplements.htm.

Proof. It is sufficient to write market-makers' pay-offs substituting to $a_{i,t}$ the expression $\overline{V}+\underline{V}-b'_{i,t}$ and to $b_{i,t}$ the expression $\overline{V}+\underline{V}-a'_{i,t}$, $i=1,2$. Once market-makers' types are changed following (A.1), we obtain pay-offs that differ from the original ones only for the use of the new variables $(a'_{i,t}, b'_{i,t}, p')$ and types $(\underline{V}', \overline{V}')$. Thus, using this symmetry we can deduce the equilibrium of the game $\Gamma(T, 1-p)$ from the equilibrium strategies of the game $\Gamma(T, p)$. ||

Proof of Proposition 1. The bid auction has been studied in EMW. Considering that the ask auction can be rewritten into a bid auction using the SP, this proposition follows from the authors' result. For expositional completeness, we show that the described strategy profile is an equilibrium, while we leave its uniqueness as a consequence of EMW.

Substituting the expression $\underline{F}^*(\cdot)$ and $\overline{G}^*(\cdot)$ in expression (3), it results that MM2's pay-off is equal to 0 for any $b_2 \leq \upsilon$ and any $a_2 \geq \upsilon$. If MM2 sets $b_2 > \upsilon$, then he or she is certain to win the bid auction with an expected profit of $\upsilon - b_2 < 0$. Similarly, any $a_2 < \upsilon$ would lead to a loss in the ask auction. Therefore, there is no profitable deviation for MM2. Substituting the $\underline{G}^*(\cdot)$ in (1), it follows that MM1($\overline{V}$)'s pay-off is equal to $(1-p)(\overline{V}-\underline{V})$ for any $b_1 \in]\underline{V}, \upsilon]$; if $b_1 \leq \underline{V}$, then MM1($\overline{V}$) does not win the bid auction and his or her pay-off is 0; if $b_1 > \upsilon$, then MM1($\overline{V}$) wins the bid auction and his or her pay-off is $\overline{V}-b_1 < \overline{V}-\upsilon = (1-p)(\overline{V}-\underline{V})$. This means that MM1($\overline{V}$) does not have a profitable deviation on the bid auction. On the ask auction any $a_1 < \overline{V}$ (respectively $a_1 > \overline{V}$) would lead to negative profit (respectively 0 profit), so that $a_1 = \overline{V}$ is a best reply. A symmetric argument applies for MM1($\underline{V}$). ||

Proof of Theorem 2. The proof contains one lemma.

Lemma 5. *If, in equilibrium, private information is revealed with probability* 1 *at* $t \leq T$, *then time t equilibrium strategies are those of the one-shot game equilibrium described in Proposition* 1.

Proof. Let us assume that $(\sigma_1(\overline{V}), \sigma_1(\underline{V}), \sigma_2)$ is some fully revealing equilibrium strategy profile played in t. After time t there is no asymmetry of information, and each player will set bid and ask prices equal to the true value of the asset. Hence, by backward induction, the players' equilibrium pay-off after t is equal to 0. Thus, the players' total pay-off from time t to T is equal to the stage t pay-off whose unique equilibrium is described in Proposition 1. ||

Suppose that an equilibrium exists in a period $t < T$ where the probability of full revelation is 1. In that case, after time t, there will be no information asymmetry, and each market-maker will set bid and ask prices equal to the true value of the asset and market-makers will make no profit.

From Lemma 5, at time t all agents behave as if they were in the last repetition of the game whose unique equilibrium is described in Proposition 1. From Proposition 1, MM1($\overline{V}$)'s equilibrium pay-off is equal to $(1-p_t)(\overline{V}-\underline{V})$.

Now consider the following deviation for MM1($\overline{V}$) :

$$b_{1,t} = \underline{V}$$

$$a_{1,t} = \overline{V} - \varepsilon$$

with $\varepsilon > 0$. MM1($\overline{V}$)'s stage t deviation pay-off is equal to $-\varepsilon \Pr(a_2 > \overline{V} - \varepsilon)$ that can be set arbitrarily close to 0 by choosing a small enough ε. In the one-shot equilibrium of Proposition 1, the quotes $b_{1,t} = \underline{V}$ and $a_{1,t} = \overline{V} - \varepsilon$ are played with positive probability only when the state of nature is $\underline{V}$. Therefore, when MM2 observes $b_{1,t} = \underline{V}$ and $a_{1,t} = \overline{V} - \varepsilon$, he or she believes that the value of the asset is $\underline{V}$ and the posterior belief in $t+1$ will be $p_{t+1} = 0$. Thus, in $t+1$ the uninformed market-maker will set $a_{2,t+1} = b_{2,t+1} = \underline{V}$. Consequently, in $t+1$, MM1($\overline{V}$) can reach a pay-off arbitrarily close to $(\overline{V}-\underline{V})$ by playing $a_{1,t+1} = \overline{V}$ and $b_{1,t+1} = \underline{V} + \varepsilon$. It follows that MM1($\overline{V}$)'s overall deviation pay-off can be arbitrarily close to $(\overline{V}-\underline{V})$ that is greater than his or her equilibrium pay-off $(1-p_t)(\overline{V}-\underline{V})$. Thus, a contradiction. ||

Proof of Proposition 3. For expositional clarity we provide a complete proof of the proposition for the game with $T=2$. This restriction does not affect the main economic intuition of the proof for the game with $T>2$, outlined at the end of this subsection.

Take the game $\Gamma(2, p)$. From Proposition 1, we know that the unique equilibrium of the second (and last) trading round satisfies properties (1)–(4) described in Proposition 3. Thus, we only need to prove the result for the first round of trade. To this purpose, we will distinguish between three cases: $p > 1/2$, $p < 1/2$, and $p = 1/2$. We first prove the proposition for $p > 1/2$. We then use the SP to study the case $p < 1/2$, and finally we provide the equilibrium for $p = 1/2$. ||

Case 1. $p > 1/2$.

To begin with, it is useful to graphically represent the set of first round bid–ask quotes that, according to Proposition 3, are played with positive probability in equilibrium when $p > 1/2$.

Features (1) and (2) imply that MM2 randomizes bid and ask quotes on the intervals $[b_{\min}, v]$ and $[v, a_{\max}]$, respectively. Thus, the rectangle $ABCD = [v, a_{\max}] \times [b_{\min}, v]$ in Figure 5 represents MM2's equilibrium support in the plane of bid and ask prices. Let us denote this region by S_2.

In the first round of trading, τ is equal to 2. Thus, according to feature (3), MM1's equilibrium strategy in the first round can be described as follows. If the value of the asset is $\underline{V}$, the informed market-maker competes only in the profitable auction (the ask side): MM1 posts a bid price equal to $b_{\min}$ and randomizes the ask price in the interval $[v, a_{\max}[$. Thus MM1($\underline{V}$)'s equilibrium support is represented in Figure 5 by the line AB. Let us denote this region by $S_1(\underline{V})$. If $\widetilde{V} = \overline{V}$, then MM1($\overline{V}$) randomizes between trying to buy the asset, and misleading MM2 by trying to sell the asset. If MM1 ($\overline{V}$) tries to buy the asset, he or she randomizes the bid price in $]b_{\min}, v]$ and posts the ask price equal to $a_{\max}$. If MM1 ($\overline{V}$) misleads, he or she will mimic the strategy of MM1($\underline{V}$) by posting a bid equal to $b_{\min}$ and randomizing the ask in $[v, a_{\max}[$. Thus, MM1($\overline{V}$)'s equilibrium support is represented by the two lines AB and BC. Let us denote this region by $S_1(\overline{V})$.

The following lemma provides the equilibrium distribution of market-makers' quotes on the equilibrium supports S_2, $S_1(\overline{V})$, and $S_1(\underline{V})$. This equilibrium satisfies features (1)–(4) of Proposition 3.

Lemma 6. *If $p > 1/2$, then in the first round of the game $\Gamma(2, p)$ a perfect Bayesian equilibrium exists and has the following properties:*

(i) MM2 *randomizes ask and bid prices on the support S_2 according to the marginal distributions:*

$$\Pr(a_{2,1} < x) = F_2(x) = \begin{cases} 0 & \text{for } x \in]-\infty, v] \\ \frac{x-v}{x-(\overline{V}+\underline{V})/2} & \text{for } x \in]v, a_{\max}] \\ 1 & \text{for } x \in]a_{\max}, +\infty] \end{cases} \tag{A.5}$$

$$\Pr(b_{2,1} < x) = G_2(x) = \begin{cases} 0 & \text{for } x \in]-\infty, b_{\min}] \\ \frac{\overline{V}-v}{\overline{V}-x} & \text{for } x \in]b_{\min}, v] \\ 1 & \text{for } x \in]v, +\infty]; \end{cases} \tag{A.6}$$

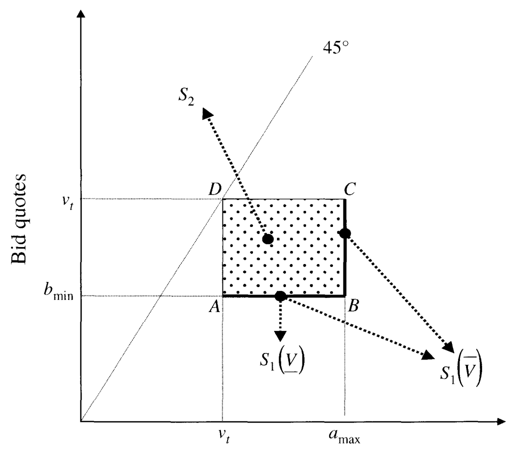

FIGURE 5

Market-makers' equilibrium supports in the first round of the game $\Gamma(p, 2)$ when $p > 1/2$: MM2 randomizes quotes on S_2 that is the shaded rectangle $ABCD$; MM1($\underline{V}$) randomizes quotes on $S_1(\underline{V})$ that is the line AB; MM1($\overline{V}$) randomizes the quotes on $S_1(\overline{V})$ that is the union of the lines AB and BC

(ii) *if the value of the asset is* $\overline{V}$, *then* MM1 *randomizes the bid and ask quotes on the support* $S_1(\overline{V})$ *according to the marginal distributions:*

$$\Pr(b_{1,1} < x \mid \overline{V}) = \overline{G}(x) = \begin{cases} 0 & \text{for } x \in]-\infty, b_{\min}] \\ \frac{(1-p)(x-\underline{V})}{p(\overline{V}-x)} & \text{for } x \in]b_{\min}, v] \\ 1 & \text{for } x \in]v, +\infty] \end{cases} \tag{A.7}$$

$$\Pr(a_{1,1} < x \mid \overline{V}) = \overline{F}(x) = \begin{cases} 0 & \text{for } x \in]-\infty, v] \\ \overline{F}^{**}(x) & \text{for } x \in]v, a_{\max}] \\ 1 & \text{for } x \in]a_{\max}, +\infty]; \end{cases} \tag{A.8}$$

(iii) *if the value of the asset is* $\underline{V}$, *then* MM1 *randomizes his or her quotes on the support* $S_1(\underline{V})$ *according to the marginal distributions:*

$$\Pr(a_{1,1} < x \mid \underline{V}) = \underline{F}(x) = \begin{cases} 0 & \text{for } x \in]-\infty, v] \\ \frac{x-v+p(\overline{V}-x)\overline{F}^{**}(x)}{(1-p)(x-\underline{V})} & \text{for } x \in]v, a_{\max}] \\ 1 & \text{for } x \in]a_{\max}, +\infty] \end{cases} \tag{A.9}$$

$$\Pr(b_{1,1} = b_{\min}) = 1$$

where $\overline{F}^{**}$ *is the solution of the differential equation*

$$F'(x) = \frac{(x - \underline{V} + (\overline{V} - x)F_2(x) - (1-q^*)(1-p)(\overline{V} - \underline{V}))(1 - F(x))}{(x - \underline{V})((1-q^*)(1-p)(\overline{V} - \underline{V}) - (\overline{V} - x)F_2(x))} \tag{A.10}$$

with the boundary condition $F(v) = 0$. *Moreover,* $a_{\max} = \overline{V}, b_{\min}$ *solves* $\overline{F}^{**}(\overline{V}) = \overline{G}(b_{\min})$ *and* $q^* \in [0,1]$ *is chosen so that the equation* $\overline{F}^{**}(\overline{V}) = \overline{G}(b_{\min})$ *has a solution for* $b_{\min} \in [(\overline{V} + \underline{V})/2, v]$;

(iv) *the equilibrium pay-offs for* MM2, MM1$(\overline{V})$, *and* MM1$(\underline{V})$ *are, respectively:* $\pi_2^*(2,p) = 0$, $\pi_1^*(\overline{V}, 2, p) = (1 - p)(\overline{V} - \underline{V})$, *and* $\pi_1^*(\underline{V}, 2, p) = (3p-1)(\overline{V} - \underline{V})$.

Proof of Lemma 6. The proof is divided into two main steps.

Step 1: As the equilibrium is in mixed strategies, in order to prove Lemma 6, we have to show that each market-maker, given the other market-maker's strategy, is indifferent among all couples of bid–ask quotes that belong to the corresponding equilibrium support.

Step 2: we will prove that for each market-maker there is no couple of bid and ask quotes outside the equilibrium supports that provides a higher expected pay-off.

Step 1: *Construction of the mixed strategies*

In Lemma 7, we show that if MM1 plays according to the strategies in (ii) and (iii), then MM2 obtains an expected profit equal to 0 by playing any bid and ask in S_2.

Lemma 7. *If* MM1 *randomizes ask and bid quotes according to* (ii) *and* (iii), *then* MM2 *is indifferent among any ask* $a_{2,1} \in [v, a_{\max}]$ *and any bid* $b_{2,1} \in [b_{\min}, v]$. *The expected profit is* 0.

Proof. Since MM2 plays the bid and the ask auctions independently, we first show that any bid quote $b_{2,1} \in [b_{\min}, v]$ gives a 0 expected pay-off.

By Proposition 1, MM2's expected pay-off in the second round is equal to 0. If, in the first round, MM2 sets any $b_{2,1} \in]b_{\min}, v]$, then $\Pr(b_{1,1} = b_{2,1}) = 0$, and the expected pay-off will be

$$p(\overline{V} - b_{2,1})\Pr(b_{1,1} < b_{2,1} \mid \overline{V}) + (1-p)(\underline{V} - b_{2,1})\Pr(b_{1,1} < b_{2,1} \mid \underline{V}) = 0$$

where the equality follows from (A.7) and $\Pr(b_{1,1} = b_{\min} \mid \underline{V}) = 1$. Similarly, if MM2 sets $b_{2,1} = b_{\min}$, then $\Pr(b_{1,1} < b_{2,1}) = 0$, and the expected pay-off will be equal to

$$(1-q^*)\left(p(\overline{V} - b_{\min})\Pr(b_{1,1} = b_{\min} \mid \overline{V}) + (1-p)(\underline{V} - b_{\min})\right) = 0$$

as $\Pr(b_{1,1} = b_{\min} \mid \overline{V}) = (1-p)(b_{\min} - \underline{V})/p(\overline{V} - b_{\min})$ for (A.7). A similar argument applies to the ask auction. ||

Now we want to show that MM1$(\overline{V})$'s (respectively MM1$(\underline{V})$) expected pay-off is constant for all bid and an ask quotes belonging to equilibrium support $S_1(\overline{V})$ (respectively $S_1(\underline{V})$). As MM1's continuation pay-off in the second round depends on MM2's posterior belief after observing $(a_{1,1}, b_{1,1})$, we first have to determine how MM1's first round quotes affect MM2's posterior belief. Lemma 8 determines MM2's posterior belief after observing couple of quotes $(a_{1,1}, b_{1,1})$ included into MM1's equilibrium support.

Lemma 8. *Let* $\text{Post}(a_{1,1}, b_{1,1}) = \Pr(\widetilde{V} = \overline{V}|a_{1,1}, b_{1,1})$ *be the* MM2*'s posterior belief after observing* $(a_{1,1}, b_{1,1})$. *If* MM2 *expects* MM1 *to use the mixed strategies in* (ii) *and* (iii)*, then:*

$$\text{Post}(a_{1,1}, b_{1,1}) = 1 \quad \text{for } (a_{1,1}, b_{1,1}) \in \{a_{\max}\} \times]b_{\min}, v] \tag{A.11}$$

$$\text{Post}(a_{1,1}, b_{1,1}) = \frac{\overline{f}(a_{1,1})(a_{1,1} - \underline{V})^2}{(\overline{V} - \underline{V})(\overline{f}(a_{1,1})(a_{1,1} - \underline{V}) + 1 - \overline{F}(a_{1,1}))} \quad \text{for } (a_{1,1}, b_{1,1}) \in [v, a_{\max}[\times\{b_{\min}\} \tag{A.12}$$

where $\overline{f}(\cdot)$ *is the derivative of* $\overline{F}(\cdot)$.

Proof. First note that quotes $(a_{1,1}, b_{1,1}) \in \{a_{\max}\} \times]b_{\min}, v]$ belong to MM1$(\overline{V})$'s equilibrium support $S_1(\overline{V})$ while they do not belong to $S_1(\underline{V})$, the equilibrium support of MM1$(\underline{V})$. Indeed, MM1 competes on the bid side only if $\widetilde{V} = \overline{V}$. Consequently, after observing $(a_{1,1}, b_{1,1}) \in \{a_{\max}\} \times]b_{\min}, v]$, MM2 unambiguously deduces that $\widetilde{V} = \overline{V}$, and so posterior belief jumps to 1. Thus (A.11). By contrast, quotes $(a_{1,1}, b_{1,1}) \in [v, a_{\max}[\times\{b_{\min}\}$ belong to the equilibrium support of both MM1$(\overline{V})$ and MM1$(\underline{V})$ and, as a result, MM2's posterior belief will depend on the density distribution used by MM1$(\overline{V})$ and MM1$(\underline{V})$ to select quotes in this region. Expressions (ii) and (iii) imply that for $x \in [v, a_{\max}[$, we obtain a $\Pr(a_{1,1} < x \text{ and } b_{1,1} = b_{\min}|\underline{V})$ that is equal to

$$\underline{F}(x) = \frac{x - v + p(\overline{V} - x)\overline{F}(x)}{(1-p)(x - \underline{V})},$$

where $\overline{F}(x) = \Pr(a_{1,1} < x \text{ and } b_{1,1} = b_{\min}|\overline{V})$. By differentiating both sides of this equality with respect to x, we have

$$\underline{f}(x) = \frac{p\left((x - \underline{V})(\overline{V} - x)\overline{f}(x) + (1 - \overline{F}(x)(\overline{V} - \underline{V})\right)}{(1-p)(x - \underline{V})^2}, \tag{A.13}$$

where $\underline{f}(\cdot) = \underline{F}'(\cdot)$. If MM1 randomizes the ask prices according to the lotteries with densities $\overline{f}(\cdot)$, $\underline{f}(\cdot)$, then by Bayes' rule:

$$\Pr(\widetilde{V} = \overline{V}|a_{1,1}, b_{\min}) = \frac{p\overline{f}(a_{1,1})}{p\overline{f}(a_{1,1}) + (1-p)\underline{f}(a_{1,1})}.$$

By substituting $\underline{f}(a_{1,1})$ with the R.H.S of (A.13) evaluated for $x = a_{1,1}$, we obtain equation (A.12). ||

Now we can study MM1$(\overline{V})$'s equilibrium pay-off. In Lemma 9 we prove that if MM2 plays the strategies (i) and revises his or her beliefs according to Lemma 8, then MM1$(\overline{V})$'s pay-off from setting any bid–ask quotes $(a_{1,1}, b_{1,1}) \in S_1(\overline{V})$ is equal to $(1-p)(\overline{V} - \underline{V})$.

Lemma 9. *If* MM2 *randomizes ask and bid quotes according to* (i) *and updates posterior belief according to Lemma 8, then* MM1$(\overline{V})$*'s expected pay-off from setting* $(a_{1,1}, b_{1,1}) \in \{a_{\max}\} \times]b_{\min}, v]$ *or* $(a_{1,1}, b_{1,1}) \in [v, a_{\max}[\times\{b_{\min}\}$ *is equal to* $\pi_1^*(\overline{V}, 2, p) = (1-p)(\overline{V} - \underline{V})$.

Proof. Suppose MM1$(\overline{V})$ sets $(a_{1,1}, b_{1,1}) \in \{a_{\max}\} \times]b_{\min}, v]$: by Lemma 8, we have $\text{Post}(a_{\max}, b_{1,1}) = 1$. Moreover, as $a_{\max} = \overline{V}$, then we have $(a_{\max} - \overline{V})\Pr(a_{2,1} \geq a_{\max}) = 0$. Thus, MM1$(\overline{V})$'s expected pay-off from posting $(a_{1,1}, b_{1,1}) \in \{a_{\max}\} \times]b_{\min}, v]$ reduces to

$$\pi_1^*(\overline{V}, 2, p) = (\overline{V} - b_{1,1})\Pr(b_{2,1} < b_{1,1})$$

and substituting $\Pr(b_{2,1} < b_{1,1})$ with $G_2(\cdot)$ given in (A.6), we get $\pi_1^*(\overline{V}, 2, p) = (1-p)(\overline{V} - \underline{V})$.

MM1$(\overline{V})$ must obtain the same pay-off from mimicking MM1$(\underline{V})$: the expected pay-off from setting $(a_{1,1}, b_{1,1}) \in [v, a_{\max}[\times\{b_{\min}\}$ is equal to

$$q^*(\overline{V} - b_{\min})\Pr(b_{2,1} = b_{\min}) + (a_{1,1} - \overline{V})(1 - F_2(a_{1,1})) + (1 - \text{Post}(a_{1,1}, b_{\min}))(\overline{V} - \underline{V})$$

where the first term is the expected pay-off from the bid side in the first round in case of a tie, that is, if $b_{2,1} = b_{\min}$, the second term is the expected pay-off from the ask side in the first round, and the last term is the expected continuation pay-off. From expression (A.6) it results $\Pr(b_{2,1}=b_{\min})=\frac{(1-p)(\overline{V}-\underline{V})}{(\overline{V}-b_{\min})}$. By substituting the expression of Post$(\cdot)$ stated in (A.12), we obtain that this pay-off is equal to $\pi_1^*(\overline{V},2,p)=(1-p)(\overline{V}-\underline{V})$ only if

$$\overline{f}(a_{1,1})=\frac{(a_{1,1}-\underline{V}+(\overline{V}-a_{1,1})F_2(a_{1,1})-(1-q^*)(1-p)(\overline{V}-\underline{V}))(1-\overline{F}(a_{1,1}))}{(a_{1,1}-\underline{V})((1-q^*)(1-p)(\overline{V}-\underline{V})-(\overline{V}-x)F_2(a_{1,1}))}. \tag{A.14}$$

Note that expression (A.8) states that for $a_{1,1}\in[\upsilon,a_{\max}[$, the function $\overline{f}(a_{1,1})$ is equal to $\overline{f}^{**}(a_{1,1})$ that is defined as the solution of the differential equation (A.10) and identical to (A.14). Thus, condition (A.14) is met thereby concluding the proof. In Lemma 13, we will prove that a closed-form solution for (A.10) exists. ||

Finally, in Lemma 10 we show that if MM2 plays according to (i) and revises his or her beliefs according to Lemma 8, then MM1$(\underline{V})$ is indifferent among the quotes in the support $S_1(\underline{V})$, and his or her pay-off will be $(3p-1)(\overline{V}-\underline{V})$.

Lemma 10. *Suppose $q^*=0$ whenever $b_{\min}\neq(\overline{V}+\underline{V})/2$. If* MM2 *randomizes ask and bid quotes according to* (i) *and updates posterior belief according to Lemma* 8, *then* MM1$(\underline{V})$*'s expected pay-off from setting* $(a_{1,1},b_{1,1})\in[\upsilon,a_{\max}[\times\{b_{\min}\}$ *is equal to* $\pi_1^*(\underline{V},2,p)=(3p-1)(\overline{V}-\underline{V})$.

Proof. From Lemma 9 we know that if $(a_{1,1},b_{1,1})\in[\upsilon,a_{\max}[\times\{b_{\min}\}$, then

$$\begin{aligned}\pi_1^*(\overline{V},2,p)&=(1-p)(\overline{V}-\underline{V})\\&=q^*(\overline{V}-b_{\min})\Pr(b_{2,1}=b_{\min})+(a_{1,1}-\overline{V})(1-F_2(a_{1,1}))+(1-\mathrm{Post}(a_{1,1},b_{1,1}))(\overline{V}-\underline{V}).\end{aligned}$$

This means that for $(a_{1,1},b_{1,1})\in[\upsilon,a_{\max}[\times\{b_{\min}\}$, it results

$$\mathrm{Post}(a_{1,1},b_{1,1})(\overline{V}-\underline{V})=p(\overline{V}-\underline{V})+q^*(\overline{V}-b_{\min})\Pr(b_{2,1}=b_{\min})+(a_{1,1}-\overline{V})(1-F_2(a_{1,1})). \tag{A.15}$$

Now, MM1$(\underline{V})$'s overall expected pay-off from setting quotes $(a_{1,1},b_{1,1})\in[\upsilon,a_{\max}[\times\{b_{\min}\}$ is equal to

$$\pi_1^*(\underline{V},2,p)=q^*(\underline{V}-b_{\min})\Pr(b_{2,1}=b_{\min})+(a_{1,1}-\underline{V})(1-F_2(a_{1,1}))+\mathrm{Post}(a_{1,1},b_{1,1})(\overline{V}-\underline{V}).$$

By substituting $\mathrm{Post}(a_{1,1},b_{1,1})(\overline{V}-\underline{V})$ and $F_2(a_{1,1})$ from the expressions (A.15) and (A.5) respectively, we obtain:

$$\pi_1^*(\underline{V},2,p)=2q^*((\underline{V}+\underline{V})/2-b_{\min})\Pr(b_{2,1}=b_{\min})+(3p-1)(\overline{V}-\underline{V}).$$

This implies that $\pi_1^*(\underline{V},2,p)=(3p-1)(\overline{V}-\underline{V})$ provided that $q^*=0$ for $b_{\min}\neq(\overline{V}+\underline{V})/2$. If $b_{\min}=(\underline{V}+\underline{V})/2$, then the result holds for any $q^*\in[0,1]$. ||

Step 2: *No profitable deviations for* MM*s*

Now we show that the strategies illustrated in Lemma 6 are best replies to each other and form a perfect Bayesian equilibrium of the game $\Gamma(2,p)$. This will require three lemmas.

Lemma 11. *If* MM1 *randomizes ask and bid quotes according to* (ii) *and* (iii), *then for* MM2 *it is optimal to set the ask quotes and bid quotes in the intervals* $[\upsilon,a_{\max}]$ *and* $[b_{\min},\upsilon]$, *respectively.*

Proof. By Lemma 7, we have to show that MM2 cannot get a pay-off higher than 0 given the strategies (i)–(ii) of MM1. Let us check this for the bid auction. If MM2 sets $b_{2,1}>\upsilon$, then he or she is certain to win the bid auction and the pay-off will be $\upsilon-b_{2,1}<0$. If MM2 sets $b_{2,1}<b_{\min}$, then he or she is certain to lose the bid auction and the pay-off will be 0. Thus, $b_{2,1}\in[b_{\min},\upsilon]$ is optimal and MM2 has no profitable deviation in the bid auction. A similar argument applied to the ask auction proves that MM2 has no profitable deviations. ||

In the following lemma, we show the conditions in which neither MM1$(\overline{V})$ nor MM1$(\underline{V})$ have any profitable unilateral deviations.

Lemma 12. *If $b_{\min}\geq(\overline{V}+\underline{V})/2$ and $q^*=0$ whenever $b_{\min}>(\overline{V}+\underline{V})/2$, then it is optimal for* MM1 *to randomize quotes according to* (ii) *and* (iii).

Proof. From Lemmas 9 and 10, we know that if $q^*=0$ whenever $b_{\min}\neq(\overline{V}+\underline{V})/2$, then MM1$(\underline{V})$'s expected pay-off from setting $(a_{1,1},b_{1,1})\in S_1(\underline{V})$ is equal to $\pi_1^*(\underline{V},2,p)=(3p-1)(\overline{V}-\underline{V})$ and MM1$(\overline{V})$'s expected pay-off from setting $(a_{1,1}, b_{1,1}) \in S_1(\overline{V})$ is equal to $\pi_1^*(\overline{V}, 2, p) = (1-p)(\overline{V} - \underline{V})$. We need to show that there is no $(a_{1,1}, b_{1,1}) \notin S_1(\underline{V})$ (respectively $(a_{1,1}, b_{1,1}) \notin S_1(\overline{V})$) that provides MM1$(\underline{V})$ (respectively MM1$(\overline{V})$) with a pay-off strictly greater than $(3p-1)(\overline{V} - \underline{V})$ (respectively $(1-p)(\overline{V} - \underline{V})$).

First, consider MM1$(\underline{V})$: a possible deviation would be to mimic MM1$(\overline{V})$ at $t = 1$, by setting $b_{1,1} = b_{\min} + \varepsilon$, $a_{1,1} = a_{\max}$, and at $t = 2$, $b_{1,2} = \underline{V}$, $a_{1,2} = \overline{V} - \varepsilon$. After observing MM1's quotes in the first stage, MM2 will believe that $\tilde{V} = \overline{V}$ and will set $a_{2,2} = b_{2,2} = \overline{V}$. Thus, MM1$(\underline{V})$'s expected pay-off from this deviation can be, at maximum, arbitrarily close to

$$(\underline{V} - b_{\min}) G_2(b_{\min}) + (\overline{V} - \underline{V})$$

where the first term is the loss in the first period and the second term is the gain in the second period. Considering (A.6), it results that this expression is not greater than $\pi_1^*(\underline{V}, 2, p) = (3p-1)(\overline{V} - \underline{V})$ if $b_{\min} \geq (\overline{V} + \underline{V})/2$.

Another possibility would be that MM1$(\overline{V})$ or MM1$(\underline{V})$ post $(a_{1,1}, b_{1,1}) \notin S_1(\underline{V}) \cup S_1(\overline{V})$. Namely, they could post bid and ask prices that have a positive probability of winning both bid and ask auctions, that is, $b_{1,1} > b_{\min}$ and $a_{1,1} < a_{\max}$. This is not profitable if an out-of-equilibrium-path belief $\widehat{P}(a_{1,1}, b_{1,1})$ exists, so that

$$(1-p)(\overline{V} - \underline{V}) \geq (a_{1,1} - \overline{V})(1 - F_2(a_{1,1})) + (\overline{V} - b_{1,1}) G_2(b_{1,1}) + (1 - \widehat{P}(a_{1,1}, b_{1,1}))(\overline{V} - \underline{V})$$

$$(3p-1)(\overline{V} - \underline{V}) \geq (a_{1,1} - \underline{V})(1 - F_2(a_{1,1})) + (\underline{V} - b_{1,1}) G_2(b_{1,1}) + \widehat{P}(a_{1,1}, b_{1,1})(\overline{V} - \underline{V})$$

where $F_2(\cdot)$ and $G_2(\cdot)$ are given by (A.5) and (A.6). In other words, out-of-equilibrium-path belief must be such that this deviation is not profitable for either MM1$(\overline{V})$ or MM1$(\underline{V})$. Easy computation shows that such a belief exists whenever $b_{\min} \geq (\overline{V} + \underline{V})/2$. A second possible deviation for MM1$(\overline{V})$ or MM1$(\underline{V})$ might be to propose an ask price that has a positive probability of winning and a bid price smaller than $b_{\min}$(*i.e.* $b_{1,1} < b_{\min}$ and $a_{1,1} < a_{\max}$). This is not profitable if an out-of-equilibrium-path belief $\widehat{\widehat{P}}(a_1, b_1)$ exists so that

$$(1-p)(\overline{V} - \underline{V}) \geq (a_{1,1} - \overline{V})(1 - F_2(a_{1,1})) + (1 - \widehat{\widehat{P}}(a_{1,1}, b_{1,1}))(\overline{V} - \underline{V})$$

$$(3p-1)(\overline{V} - \underline{V}) \geq (a_{1,1} - \underline{V})(1 - F_2(a_{1,1})) + \widehat{\widehat{P}}(a_{1,1}, b_{1,1})(\overline{V} - \underline{V})$$

that are both satisfied for $\widehat{\widehat{P}}(a_1, b_1)(\overline{V} - \underline{V}) = (3p-1)(\overline{V} - \underline{V}) - (a_1 - \underline{V})(1 - F_2(a_1))$. A third possible deviation could be to post a bid price that has a positive probability of winning and an ask price larger than $a_{\max}$ (*i.e.* $b_{1,1} > b_{\min}$ and $a_{1,1} > a_{\max}$). This clearly leads to the same pay-off of posting $b_{1,1} > b_{\min}$ and $a_{1,1} = a_{\max} = \overline{V}$, that is, the equilibrium pay-off. Finally, since cross-quotes and posting very large spreads are clearly dominated strategies, we can conclude that if $b_{\min} \geq (\overline{V} + \underline{V})/2$ and $q^* = 0$ whenever $b_{\min} > (\overline{V} + \underline{V})/2$, then MM1 has no profitable deviations. ||

In order to end the proof of Lemma 6, we still have to show that the conditions of Lemmas 12 and 10 are always met, that is, $b_{\min} \geq (\overline{V} + \underline{V})/2$, and $q^* = 0$ whenever $b_{\min} > (\overline{V} + \underline{V})/2$. This last result is provided in the following Lemma 13.

Lemma 13. *A* $q^* \in [0, 1]$ *such that* $b_{\min} \in [(\overline{V} + \underline{V})/2, v]$ *always exists. Moreover, if* $b_{\min} > (\overline{V} + \underline{V})/2$ *then* $q^* = 0$.

Proof. From the description of the equilibrium, observe that MM1 never tries to buy and sell the asset simultaneously. Then the probability with which MM1 tries to sell, and sets $a_{1,1} < a_{\max}$, must be equal to the probability in which he or she stays out of the bid auction and sets $b_{1,1} = b_{\min}$. This implies that

$$\Pr(a_{1,1} < a_{\max} | \underline{V}) = \Pr(b_{1,1} = b_{\min} | \underline{V}) = 1 \tag{A.16}$$

$$\Pr(a_{1,1} < a_{\max} | \overline{V}) = \Pr(b_{1,1} = b_{\min} | \overline{V}). \tag{A.17}$$

As $1 = \Pr(b_{1,1} = b_{\min} | \underline{V}) = \underline{F}(a_{\max})$, the condition (A.16) and expression (A.9) lead to $a_{\max} = \overline{V}$. Note also that, from (A.7), $\Pr(b_{1,1} = b_{\min} | \overline{V}) = \frac{(1-p)(b_{\min} - \underline{V})}{p(\overline{V} - b_{\min})}$ and from $a_{\max} = \overline{V}$, we have $\Pr(a_{1,1} < a_{\max} | \overline{V}) = \overline{F}(\overline{V})$. Thus, $b_{\min}$ is characterized by the equation (A.17) that becomes

$$\overline{F}(\overline{V}) = \frac{(1-p)(b_{\min} - \underline{V})}{p(\overline{V} - b_{\min})} \tag{A.18}$$

where $\overline{F}(\cdot)$ is the solution of the differential equation (A.10). Namely $\overline{F}(\cdot)$ is

$$\overline{F}(x) = 1 - \frac{(x - \underline{V})\sqrt{2(1-p)(p-1/2)(1-q)} \exp[\theta(x)]}{p\sqrt{(1-q)(1-p)(\overline{V} - \underline{V})(2x - \overline{V} - \underline{V}) - 2(x - v)(\overline{V} - x)}} \tag{A.19}$$

with

$$\theta(x) = \frac{1-q-(2-q)p}{\sqrt{(1-p)(p(2-q)^2+(2-q)q-2)}}\left(\arctan\left[\frac{q(1-p)}{\sqrt{(1-p)(p(2-q)^2+(2-q)q-2)}}\right]\right.$$
$$\left. + \arctan\left[\frac{2(x-\underline{V})-(q+p(2-q))(\overline{V}-\underline{V})}{(\overline{V}-\underline{V})\sqrt{(1-p)(p(2-q)^2+(2-q)q-2)}}\right]\right).$$

Consider the expression of $\theta(x)$. Note that the argument in the square roots $(1-p)(p(2-q)^2+(2-q)q-2)$ is 0 for $q = \frac{\sqrt{2p-1}(1-\sqrt{2p-1})}{1-p}$ and positive for $q \in \left[0, \frac{\sqrt{2p-1}(1-\sqrt{2p-1})}{1-p}\right[$. Consequently, $\overline{F}(x)$ is well defined and continuous for all q in this interval.

We are interested in the properties of the expression (A.19) evaluated at $x = \overline{V}$. It results that $\overline{F}(\overline{V})$ is a continuous function of q and p and does not depend on $\overline{V}$ and $\underline{V}$. In fact, it results in

$$\lim_{q \to \left(\frac{\sqrt{2p-1}(1-\sqrt{2p-1})}{1-p}\right)^-} \overline{F}(\overline{V}) = 1$$

$$\lim_{q\to 0} \overline{F}(\overline{V}) = 1 - \frac{\sqrt{2p-1}}{p}\exp\left[\frac{(1/2-p)}{\sqrt{(1-p)(p-1/2)}}\arctan\left[\frac{1-p}{\sqrt{(1-p)(p-1/2)}}\right]\right] \leq 1$$
$$\lim_{q\to 0} \overline{F}(\overline{V}) \leq \frac{1-p}{p} \text{ iff } p < p^*$$

where the first limit is taken from the left and $p^* \simeq 0{\cdot}64087$ is the level of p that solves:

$$\lim_{q\to 0} \overline{F}(\overline{V}) = \frac{1-p}{p}.$$

Our objective is to show that there exists always a couple $(q, b_{\min})$ with $q \in \left[0, \left(\frac{\sqrt{2p-1}(1-\sqrt{2p-1})}{1-p}\right)\right[$ and $b_{\min} \in [(\overline{V}+\underline{V})/2, \upsilon]$ thus ensuring that condition (A.18) is met. Note that $\frac{(1-p)}{p} = \left.\frac{(1-p)(b_{\min}-\underline{V})}{p(\overline{V}-b_{\min})}\right|_{b_{\min}=(\overline{V}+\underline{V})/2}$.

Suppose $p \in]1/2, p^*]$, then it results

$$\lim_{q\to 0} \overline{F}(\overline{V}) \leq \frac{1-p}{p} < 1 = \lim_{q \to \left(\frac{\sqrt{2p-1}(1-\sqrt{2p-1})}{1-p}\right)^-} \overline{F}(\overline{V}).$$

By continuity of $\overline{F}(\overline{V})$ in q, these two inequalities imply that condition (A.18) is met for $b_{\min} = (\overline{V}+\underline{V})/2$ and some $q^* \in \left[0, \frac{\sqrt{2p-1}(1-\sqrt{2p-1})}{1-p}\right]$.

Now consider the case $p \geq p^*$, then we have

$$\left.\frac{(1-p)(b_{\min}-\underline{V})}{p(\overline{V}-b_{\min})}\right|_{b_{\min}=(\overline{V}+\underline{V})/2} = \frac{(1-p)}{p} < \lim_{q\to 0} \overline{F}(\overline{V})$$
$$\leq 1 = \left.\frac{(1-p)(b_{\min}-\underline{V})}{p(\overline{V}-b_{\min})}\right|_{b_{\min}=\upsilon}$$

that implies that by continuity of $\frac{(1-p)(\underline{V}-b_{\min})}{p(\overline{V}-b_{\min})}$ in $b_{\min}$, there is a $b_{\min} \in](\overline{V}+\underline{V})/2, \upsilon[$ so that condition (A.18) is met for $q = 0$. In other words for each level of $p > 1/2$, there exists an appropriate q^* and $b_{\min}$ that satisfies equation (A.18) and the conditions in Lemmas 12 and 10. ||

Steps 1 and 2 complete the proof of Lemma 6. ||

In order to conclude the proof of Proposition 3 in the case of two trading rounds, we must now consider the cases in which $p < 1/2$ and $p = 1/2$.

Case 2. $p < 1/2$.

The equilibrium in the last round of the game is known, so we analyse the first round of trade. By using the SP it is easy to characterize the equilibrium strategy in the first round of trade when $p < 1/2$. In this case, MM1$(\overline{V})$ always tries to buy the asset, while MM1$(\underline{V})$ randomizes between trying to sell it and mimicking MM1$(\overline{V})$. The equilibrium pay-offs are equal to $(2-3p)(\overline{V}-\underline{V})$ for MM1$(\overline{V})$ and $p(\overline{V}-\underline{V})$ for MM1$(\underline{V})$, while MM2 has an expected pay-off equal to 0.

Case 3. $p = 1/2$.

Finally, if $p = 1/2$, then at the first stage of the game all market-makers set bid and ask quotes equal to $\upsilon = (\overline{V} + \underline{V})/2$ and the posterior belief does not change. Such a pure strategy, pooling equilibrium exists only for $p = 1/2$ and is sustained by the following out of equilibrium path belief:

$$\Pr(\widetilde{V} = \overline{V}|a_{1,1}, b_{1,1}) = \begin{cases} 1 & \text{for } b_{1,1} > 1/2 \\ 0 & \text{for } a_{1,1} < 1/2. \end{cases}$$

This ends the proof of Proposition 3 when $T = 2$.

Before considering the case of a general T, note that MM1's ex-interim total equilibrium pay-offs for the game $\Gamma(2, p)$ are continuous piecewise-linear monotone functions in p.[18]

$$\pi_1^*(\overline{V}, 2, p) = \begin{cases} (2 - 3p)(\overline{V} - \underline{V}) & \text{if } p \le 1/2 \\ (1 - p)(\overline{V} - \underline{V}) & \text{if } p > 1/2 \end{cases}$$

$$\pi^*(\underline{V}, 2, p) = \begin{cases} p(\overline{V} - \underline{V}) & \text{if } p \le 1/2 \\ (3p - 1)(\overline{V} - \underline{V}) & \text{if } p > 1/2. \end{cases}$$

This suggests that we can apply the same method used in this section recursively to obtain the equilibrium when the market-makers, interactions continue for an arbitrary number of periods T.

Construction of an equilibrium for $T > 2$

Sketch: The equilibrium can be characterized recursively applying the method used for the two-period case.[19]

By fixing a number of repetitions T for all natural numbers $j \le T$ and all $t \le T$, we generate the numbers $r_{j,T}$ recursively starting from $r_{0,T} = 0$ and $r_{1,T} = 1$ as follows:

$$r_{j,t} = \begin{cases} 0 & \text{if } j \le 0 \\ 1 & \text{if } j \ge t \\ \dfrac{r_{j-1,t-1} + r_{j,t-1}}{2} & \text{elsewhere.} \end{cases}$$

In this way, for a fixed τ we partition the interval $[0, 1]$ in successively τ number of sub-intervals: $[r_{0,\tau}, r_{1,\tau}]$, $[r_{1,\tau}, r_{2,\tau}]$, ..., $[r_{\tau-1,\tau}, r_{\tau,\tau}]$. Take the game at time t, and let $\tau + 1$ be the number of trading rounds that remain to be played. Suppose that MM1($\overline{V}$) and MM1($\underline{V}$)'s equilibrium continuation pay-off is continuous and linear in the level of posterior belief p_{t+1}within each sub-interval $[r_{0,\tau}, r_{1,\tau}]$, ..., $[r_{\tau-1,\tau}, r_{\tau,\tau}]$, as is the case, for example, in the one-shot game and in the twice-repeated game. This allows us to construct the equilibrium strategies in exactly the same way that we constructed the equilibrium for the twice-repeated game. It turns out that the resulting equilibrium pay-off is still continuous and piecewise linear in the level of beliefs, so that we can use the argument recursively for any T. The only difference with the twice-repeated game is that now the belief p_t follows a process that makes it jump into different sub-intervals at each stage. Namely, if $p \in [r_{i-1,T}, r_{i,T}]$ and MM1 tries to buy (respectively to sell) the asset, then the posterior belief will belong to the interval $[r_{i-1,T-1}, r_{i,T-1}]$ (respectively $[r_{i-2,T-1}, r_{i-1,T-1}]$). Therefore, one has to take the piecewise linearity of MM1's continuation pay-off into account when writing the differential equations that define the informed market-maker's quotes distribution. Apart from this, the characterization of MMs' equilibrium strategies is analogous to that in the case of the twice-repeated game.

End of the proof of Proposition 3. ||

Proof of Lemma 4. Let $p_2 = \Pr(V = \overline{V}|a_{1,1}, b_{1,1} = b_{\text{inf}})$ and let $\upsilon_2 = p_2\overline{V} + (1 - p_2)\underline{V}$

$$E[a_{2,2}] = \int_{\upsilon_2}^{\overline{V}} x dF^*(x) + \overline{V}(1 - F^*(\overline{V})) = \upsilon_2 - p_2 \ln(p_2)(\overline{V} - \underline{V})$$

$$E[b_{2,2}] = \int_{\underline{V}}^{\upsilon_2} x dG^*(x) + \underline{V} G^*(\underline{V}) = \upsilon_2 - (1 - p_2)\ln(1 - p_2)(\overline{V} - \underline{V})$$

18. The same can be seen in the equilibrium pay-off of the one-shot game.
19. The complete proof is available from the authors upon request.

where $F^*(\cdot)$ and $G^*(\cdot)$ are given in Proposition 1. Differentiating this expression with respect to p_2 we obtain

$$\frac{\partial E[a_{2,2}]}{\partial p_2} = -(\overline{V} - \underline{V})\ln(p_2) > 0$$

$$\frac{\partial E[b_{2,2}]}{\partial p_2} = -(\overline{V} - \underline{V})\ln(1 - p_2) > 0.$$

If $p > 1/2$,

$$p_2 = \mathrm{Post}(a_{1,1}, b_{\min}) = \frac{\pi_1^*(\underline{V}, 2, p) - (a_{1,1} - \underline{V})(1 - F_2(a_{1,1}))}{(\overline{V} - \underline{V})}.$$

Using the expression of $F_2(a)$ provided by (A.5), and differentiating with respect to $a_{1,1}$ we obtain

$$\frac{\partial p_2}{\partial a_{1,1}} = \frac{(2p-1)(\overline{V} - \underline{V})}{(2a_{1,1} - \overline{V} - \underline{V})^2},$$

which is positive because $p > 1/2$. The result follows from $\frac{\partial E[a_{2,2}]}{\partial a_{1,1}} = \frac{\partial E[a_{2,2}]}{\partial p_2}\frac{\partial p_2}{\partial a_{1,1}}$ and $\frac{\partial E[b_{2,2}]}{\partial a_{1,1}} = \frac{\partial E[b_{2,2}]}{\partial p_2}\frac{\partial p_2}{\partial a_{1,1}}$. The result for $p < 1/2$ follows from the symmetry of the model. Finally, in order to prove that MM1's quotes in the second period do not depend on MM2's quotes in the first period, it is sufficient to observe that the distribution of $(a_{1,2}, b_{1,2})$ is only affected by p_2, which does not change with MM2's quotes. ||

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