
HERDING AND CONTRARIAN BEHAVIOR IN FINANCIAL MARKETS

Author(s): Andreas Park and Hamid Sabourian

Source: *Econometrica*, Vol. 79, No. 4 (July, 2011), pp. 973-1026

Published by: [Econometric Society](#)

Stable URL: <http://www.jstor.org/stable/41237854>

Accessed: 26-11-2015 08:45 UTC

REFERENCES

Linked references are available on JSTOR for this article:

http://www.jstor.org/stable/41237854?seq=1&cid=pdf-reference#references_tab_contents

You may need to log in to JSTOR to access the linked references.

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



Wiley and Econometric Society are collaborating with JSTOR to digitize, preserve and extend access to *Econometrica*.

<http://www.jstor.org>

HERDING AND CONTRARIAN BEHAVIOR IN FINANCIAL MARKETS

BY ANDREAS PARK AND HAMID SABOURIAN¹

Rational herd behavior and informationally efficient security prices have long been considered to be mutually exclusive but for exceptional cases. In this paper we describe the conditions on the underlying information structure that are necessary and sufficient for informational herding and contrarianism. In a standard sequential security trading model, subject to sufficient noise trading, people herd if and only if, loosely, their information is sufficiently dispersed so that they consider extreme outcomes more likely than moderate ones. Likewise, people act as contrarians if and only if their information leads them to concentrate on middle values. Both herding and contrarianism generate more volatile prices, and they lower liquidity. They are also resilient phenomena, although by themselves herding trades are self-enforcing whereas contrarian trades are self-defeating. We complete the characterization by providing conditions for the absence of herding and contrarianism.

KEYWORDS: Social learning, herding, contrarians, asset price volatility, market transparency.

1. INTRODUCTION

IN TIMES OF GREAT ECONOMIC UNCERTAINTY, financial markets often appear to behave frantically, displaying substantial price spikes as well as drops. Such extreme price fluctuations are possible only if there are dramatic changes in behavior with investors switching from buying to selling or the reverse. This pattern of behavior and the resulting price volatility is often claimed to be inconsistent with rational traders and informationally efficient asset prices, and is attributed to investors' animal instincts. We argue in this paper, however, that such behavior can be the result of fully rational social learning where agents change their beliefs and behavior as a result of observing the actions of others.

¹This manuscript is a substantially revised and generalized version of our earlier paper "Herd Behavior in Efficient Financial Markets." Financial support from the ESRC (Grants RES-156-25-0023 and R00429934339) and the TMR Marie Curie Fellowship Program is gratefully acknowledged. Andreas thanks the University of Copenhagen for its hospitality while some of this research was developed. We thank three anonymous referees for detailed comments, and the co-editor for very helpful suggestions and advice. We are also grateful to Markus Brunnermeier, Christophe Chamley, Tony Doblas-Madrid, Scott Joslin, Rob McMillan, Peter Sorensen, and Wei Xiong for helpful discussions. Finally, we thank seminar participants at the following conferences, workshops, and departments for useful comments: St. Andrews ESRC Macro, Gerzensee Economic Theory, SAET Vigo Spain, ESRC World Economy Birkbeck, ESRC British Academy Joint Public Policy Seminar, Cambridge–Princeton Workshop, Cambridge Social Learning Workshop, Copenhagen Informational Herding Workshop, CEA, NFA, Said Oxford, St. Andrews, Leicester, Nottingham, Queen Mary London, Cambridge, LSE Finance, Naples, EUI Florence, Paris School of Economics, Yonsei Korea, Singapore National University, Singapore Management University, Queens, Rotman Toronto, Penn State, and Wharton.

One example of social learning is herd behavior in which agents switch behavior (from buying to selling or the reverse) *following the crowd*. So-called rational herding can occur in situations with information externalities, when agents' private information is swamped by the information derived from observing others' actions. Such "herders" rationally act against their private information and follow the crowd.²

It is not clear, however, that such herd behavior can occur in informationally efficient markets, where prices reflect all public information. For example, consider an investor with unfavorable private information about a stock. Suppose that a crowd of people buys the stock frantically. Such an investor will update his information and upon observing many buys, his expectation of the value of the stock will rise. At the same time, prices also adjust upward. Then it is not clear that the investor buys—to him the security may still be overvalued. So for herding, private expectations and prices must diverge.

In models with only two states of the world, such divergence is impossible, as prices always adjust so that there is no herding.³ Yet two-state models are rather special and herding can emerge once there are at least three states. In this paper, we characterize the possibility of herding in the context of a simple, informationally efficient financial market. Moreover, we show that (i) during herding, prices can move substantially and (ii) herding can induce lower liquidity and higher price volatility than if there were no herding.

Herd behavior in our setup is defined as any history-switching behavior in the direction of the crowd (a kind of momentum trading).⁴ Social learning can also arise as a result of traders switching behavior by acting *against the crowd*. Such contrarian behavior is the natural counterpart of herding, and we also characterize conditions for such behavior. Contrary to received wisdom that contrarian behavior is stabilizing, we also show that contrarian behavior leads to higher volatility and lower liquidity, just as herd behavior does.

The key insight of our characterization result is that social learning in financial markets occurs if and only if investors receive information that satisfies a compelling and intuitive property. Loosely, herding arises if and only if private information satisfies a property that we call U-shaped. An investor who receives such information believes that extreme states are more likely to have generated the information than more moderate ones. Therefore, when forming his posterior belief, the recipient of such a signal will shift weight away from the center to the extremes so that the posterior distribution of the trader

²See Banerjee (1992) or Bikhchandani, Hirshleifer, and Welch (1992) for early work on herding.

³With two states, the price will adjust so that it is always below the expectation of traders with favorable information and above the expectation of those with unfavorable information *irrespective* of what is observed. As we show, with more than two states, this strict separation no longer applies.

⁴We are concerned with short-term behavior. In the literature, there are also other definitions of herding; see Section 4 for a discussion.

is “fat-tailed.” The recipient of a U-shaped signal thus discounts the possibility of the intermediate value and as a consequence, will update the probabilities of extreme values faster than an agent who receives only the public information.

Contrarianism occurs if and only if the investor’s signal indicates that moderate states are more likely to have generated the signal than extreme states. We describe such signals as being “hill-shaped.” The recipient of a hill-shaped signal always puts more weight on middle outcomes relative to the market so that this trader’s posterior distribution becomes “thin-tailed.” He thus discounts the possibility of extreme states and, therefore, updates extreme outcomes slower than the market maker.

We follow the microstructure literature and establish our results in the context of a stylized trading model in the tradition of Glosten and Milgrom (1985). In such models, the bid and ask prices are set by a competitive market maker. Investors trade with the market maker either because they receive private information about the asset’s fundamental value or because they are “noise traders” and trade for reasons outside of the model.

The simplest possible Glosten–Milgrom trading model that allows herding or contrarianism is one with at least three states. For this case, we show that (i) a U-shaped (hill-shaped) signal is necessary for herding (contrarianism) and (ii) herding (contrarianism) occurs with positive probability if there exists at least one U-shaped (hill-shaped) signal and there is a sufficient amount of noise trading. The latter assumption on the minimum level of noise trading is not required in all cases and is made as otherwise the bid and ask spread may be too large to induce appropriate trading. In Section 9, we show that the intuition for our three-state characterization carries over to a setup with an arbitrary number of states.

We obtain our characterization results without restrictions on the signal structure. In the literature on asymmetric information, it is often assumed that information structures satisfy the monotone likelihood ratio property (MLRP). Such information structures are “well behaved” because, for example, investors’ expectations are ordered. It may appear that such a strong monotonicity requirement would prohibit herding or contrarianism. Yet MLRP not only admits the possibility of U-shaped signals (and thus herding) or hill-shaped signals (and thus contrarianism), but also the trading histories that generate herding and contrarianism are significantly simpler to describe with than without MLRP signals.

Our second set of results concerns the impact of social learning on prices. We first show that the range of price movements can be very large during both contrarianism and herding. We then compare price movements in our setup where agents observe one another with those in a hypothetical economy that is otherwise identical to our setup except that the informed traders do not switch behavior. We refer to the former as the *transparent* economy and to the latter as the *opaque* economy. In contrast to the transparent economy, in the opaque economy there is no social learning by assumption. We show, for the

case of MLRP, that once herding or contrarianism begins, prices respond more to individual trades relative to the situation without social learning so that price rises *and* price drops are greater in the transparent setup than in the opaque one.⁵ As a corollary, liquidity, measured by the inverse of the bid-ask spread, is lower with social learning than without.

The price volatility and liquidity results may have important implications for the discussion on the merits of “market transparency.” The price path in the opaque economy can be interpreted as the outcome of a trading mechanism in which people submit orders without knowing the behavior of others or the market price. Our results indicate that in the less transparent setup, price movements are less pronounced and liquidity is higher.

While the results on price ranges, volatility, and liquidity indicate similarities between herding and contrarianism, there is also a stark difference. Contrarian trades are self-defeating because a large number of such trades will cause prices to move against the crowd, thus ending contrarianism. During herding, on the other hand, investors continue to herd when trades are in the direction of the crowd, so herding is self-enforcing.

Examples of Situations That Generate U- and Hill-Shaped Signals

First, a U-shaped signal may be interpreted as a “volatility signal.”⁶ Very informally, an example is a signal that generates a mean preserving spread of a symmetric prior distribution. Conversely, a mean preserving signal that decreases the variance is hill-shaped.

Second, U-shaped and hill-shaped signals may also be good descriptions of situations with a potential upcoming event that has an uncertain impact. For example, consider the case of a company or institution that contemplates appointing a new leader who is an uncompromising “reformer.” If this person takes power, then either the necessary reforms take place or there will be strife with calamitous outcomes. Thus the institution will not be the same, as the new leader will be either very good or disastrous. Any private information signifying that the person is likely to be appointed exemplifies a U-shaped signal and any information revealing that this person is unlikely to be appointed (and thus the institution will carry on as before) represents a hill-shaped signal.⁷

⁵The increase in price volatility associated with herding is only relative to a hypothetical scenario. Even when herding is possible, in the long-run, volatility settles down and prices react less to individual trades. It is well known that the variance of martingale price processes such as ours is bounded by model primitives.

⁶We thank both Markus Brunnermeier and an anonymous referee for this interpretation.

⁷Other examples include an upcoming merger or takeover with uncertain merits, the possibility of a government stepping down, announcements of Food and Drug Administration (FDA) drug approvals, and outcomes of lawsuits. Degenerate examples for such signals were first discussed by Easley and O’Hara (1992) and referred to as “event uncertainty.”

Third, consider a financial institution (FI) that is a competitor to a bank that has recently failed. Suppose there are three possible scenarios: (i) the FI will also fail because it has deals with the failed bank that will not be honored and/or that the business model of the FI is as bad as that of the failed bank; (ii) the FI's situation is entirely unrelated to the bank and the latter's collapse will not affect the FI; (iii) the FI may benefit greatly from the bank's collapse, as it is able to attract the failed bank's customers and most capable employees. Cases (i) and (iii) resemble extreme outcomes and case (ii) resembles a middle outcome.

In this environment, some investor's information might have implied that the most likely outcome is either that the FI will also go down as well or that it will benefit greatly from the failed banks' demise. Such information is an example of a U-shaped signal. Alternatively, some investors' assessments might have implied that the most likely outcome is that the FI is unaffected. Such information is an example of a hill-shaped signal.

It is conceivable that in the Fall of 2008 (after the collapse of Lehman) and early 2009, many investors believed that for individual financial institutions the two extreme states (collapse or thrive) were the most likely outcomes. Then our theory predicts the potential for herd behavior, with investors changing behavior in the direction of the crowd, causing strong short-term price fluctuations. Hill-shaped private signals, signifying that the institutions were likely to be unaffected, may also have occurred, inducing contrarianism and changes of behavior against the crowd.

The Mechanism That Induces Herding and Contrarianism

Consider the above banking example and assume that all scenarios are equally likely. Let the value of the stock of the FI in each of the three scenarios (i), (ii), and (iii) be $V_1 < V_2 < V_3$, respectively. We are interested in the behavior of an investor, who has a private signal S , after different public announcements. Specifically, consider a good public announcement G that rules out the worst state, $\Pr(V_1|G) = 0$, and a bad public announcement B that rules out the best state, $\Pr(V_3|B) = 0$. Assume that the price of the stock is equal to the expected value of the asset conditional on the public information and that the investor buys (sells) if his expectation exceeds (is less than) the price. Note that the price will be higher after G and lower after B , compared to the ex ante situation when all outcomes are equally likely.

Both G and B eliminate one state, so that after each such announcement, there are only two states left. In two-state models, an investor has a higher (lower) expectation than the market if and only if his private information is more (less) favorable toward the better state than toward the worse state. Thus, in the cases of G and B , $E[V|G] \leq E[V|S, G]$ is equivalent to $\Pr(S|V_2) \leq \Pr(S|V_3)$ and $E[V|B] \leq E[V|S, B]$ is equivalent to $\Pr(S|V_2) \leq \Pr(S|V_1)$. Hence, for example, after good news G , an investors buys (sells) if he thinks, relative

to the market, that it is more (less) likely that the FI will thrive than being unaffected.

It follows from the above discussion that the investor buys after G and sells after B if and only if $\Pr(S|V_3) > \Pr(S|V_2)$ and $\Pr(S|V_1) > \Pr(S|V_2)$. Such an investor, loosely, herds in the sense that he acts like a momentum trader, buying with rising and selling with falling prices. The private information (conditional probabilities) that is both necessary and sufficient for such behavior thus has a *U shape*. Conversely, the investor sells after G and buys after B if and only if $\Pr(S|V_3) < \Pr(S|V_2)$ and $\Pr(S|V_1) < \Pr(S|V_2)$. Such an investor, loosely, trades contrary to the general movement of prices. The private information that is both necessary and sufficient to generate such a behavior thus has a *hill shape*.⁸

There are several points to note about this example. First, the public announcements G and B are degenerate as they each exclude one of the extreme states. Yet the same kind of reasoning holds if we replace G by an announcement that attaches arbitrarily small probability to the worst outcome, V_1 , and if we replace B by an announcement that attaches arbitrarily small probability to the best outcome, V_3 . Second, in the above illustration, G and B are *exogenous* public signals. In the security model described in this paper, on the other hand, public announcements or, more generally, public information, are created *endogenously* by the history of publicly observable transactions. Yet the intuition behind our characterization results is similar to the above illustration. The analysis in the paper involves describing public histories of trades that allow investors to almost rule out some extreme outcome, either V_1 or V_3 .⁹ Such histories are equivalent to public announcements G and B (or, to be more precise, to perturbations of G and B), and demonstrating their existence is crucial for demonstrating the existence of herding and contrarianism.

Overview

The next section discusses some of the related literature. Section 3 outlines the setup. Section 4 defines herding and contrarian behavior. Section 5 discusses the necessary and sufficient conditions that ensure herding and contrarianism. Section 6 discusses the special case of MLRP signals. Section 7 considers the resiliency, and fragility of herding and contrarianism and describes the range of prices for which there may be herding and contrarianism. Section 8 discusses the impact of social learning on prices with respect to volatility and liquidity. Section 9 extends the result to a setting with an arbitrary number of

⁸In our formal definition of herding and contrarianism, we benchmark behavior against the decision that the trader would take at the *initial* history, but the switching mechanism is akin to what we describe here.

⁹In the asset market described in the paper, every state has a positive probability at all finite trading histories because of the existence of noise traders. Therefore, we describe public histories at which the probabilities of extreme states are arbitrarily small, but not zero.

states. Section 10 discusses the relation of our findings to an earlier important paper on financial market herding. Section 11 concludes. Proofs that are not given in the text are either in the Appendix or in the Supplemental Material (Park and Sabourian (2011)).

2. RELATED LITERATURE

Extensive literature surveys on herding in financial markets are given in Brunnermeier (2001), Chamley (2004), and Vives (2008). Our work relates to the part of the literature that focuses on short-run herding. The work closest to ours is Avery and Zemsky (1998) (henceforth AZ), who were the first to present an intuitively appealing example of informational herding in financial markets. They argued that herd behavior with informationally efficient asset prices is not possible unless signals are “nonmonotonic” and they attributed the herding result in their example to “multidimensional uncertainty” (investors have a finer information structure than the market). In their main example, however, prices hardly move under herding. To generate extreme price movements (bubbles) with herding, AZ expanded their example to a second level of information asymmetry that leads to an even finer information partition. Yet even with these further informational asymmetries, the likelihood of large price movements during herding is extremely small (see Chamley (2004)).

The profession, for instance, Brunnermeier (2001), Bikhchandani and Sunil (2000), and Chamley (2004), has derived three messages from AZ’s paper. First, with “unidimensional” or “monotonic” signal structures, herding is impossible. Second, the information structure needed to induce herding is very special. Third, herding does not involve violent price movements except in the most unlikely environments.

AZ’s examples are special cases of our framework. Our paper demonstrates that the conclusions derived from AZ’s examples should be reconsidered. First, we show that it is U-shaped signals, and not multidimensionality or nonmonotonicity of the information structure, that is both necessary and sufficient for herding. Second, while AZ’s examples are intuitively appealing, due to their extreme nature (with several degenerate features), it may be argued that they are very special and, therefore, have limited economic relevance. Our results show instead that herding may apply in a much more general fashion and, therefore, there may be a great deal more rational informational herding than is currently expected in the literature. Third, we show that extreme price movements with herding are possible under not so unlikely situations, even with MLRP signals and without “further dimensions of uncertainty.” In Section 10, we discuss these concepts in detail by comparing and contrasting the work and conclusions of AZ with ours.

A related literature on informational learning explains how certain facets of market organization or incentives can lead to conformism and informational cascades. In Lee (1998), fixed transaction costs temporarily keep traders out

of the market. When they enter suddenly and en masse, the market maker absorbs their trades at a fixed price, leading to large price jumps after this “avalanche.” In Cipriani and Guarino (2008), traders have private benefits from trading in addition to the fundamental value payoff. As the private and public expectations converge, private benefits gain importance to the point when they overwhelm the informational rents. Then learning breaks down and an informational cascade arises. In Dasgupta and Prat (2008), an informational cascade is triggered by traders’ reputation concerns, which eventually outweigh the possible benefit from trading on information. Chari and Kehoe (2004) also studied a financial market with efficient prices; herding in their model arises with respect to a capital investment that is made outside of the financial market.

Our work also relates to the literature that shows how public signals can have a larger influence on stock price fluctuations than warranted by their information content. Beginning with He and Wang (1995), who described the relation between public information and nonrandom trading volume patterns caused by the dynamic trading activities of long-lived traders, this literature has identified how traders who care for future prices (as opposed to fundamentals) rely excessively on public expectations (see also Allen, Morris, and Shin (2006), Bacchetta and Wincoop (2006, 2008), Ozdenoren and Yuan (2007), and Goldstein, Ozdenoren, and Yuan (2010)).

All of the above contributions highlight important aspects, facets, and mechanisms that can trigger conformism in financial markets. Our findings complement the literature in that the effects that we identify may be combined with many of the above studies and they may amplify the effects described therein.

3. THE MODEL

We model financial market sequential trading in the tradition of Glosten and Milgrom (1985).

Security: There is a single risky asset with a value V from a set of three potential values $\mathbb{V} = \{V_1, V_2, V_3\}$ with $V_1 < V_2 < V_3$. Value V is the liquidation or true value when the game has ended and all uncertainty has been resolved. States V_1 and V_3 are the extreme states; state V_2 is the moderate state. The prior distribution over \mathbb{V} is denoted by $\Pr(\cdot)$. To simplify the computations, we assume that $\{V_1, V_2, V_3\} = \{0, \mathcal{V}, 2\mathcal{V}\}$, $\mathcal{V} > 0$, and that the prior distribution is symmetric around V_2 ; thus $\Pr(V_1) = \Pr(V_3)$.¹⁰

Traders: There is a pool of traders that consists of two kinds of agents: *noise traders* and *informed traders*. At each discrete date t , one trader arrives at the market in an exogenous and random sequence. Each trader can only trade once at the point in time at which he arrives. We assume that at each date, the

¹⁰The ideas of this paper remain valid without these symmetry assumptions.

entering trader is an informed agent with probability $\mu > 0$ and a noise trader with probability $1 - \mu > 0$.

The informed agents are risk neutral and rational. Each receives a private, conditionally independent and identically distributed (i.i.d.) signal about the true value of the asset V . The set of possible signals or *types* of informed agents is denoted by \mathbb{S} and consists of three elements S_1 , S_2 , and S_3 . The signal structure of the informed can, therefore, be described by a 3-by-3 matrix $\mathcal{I} = \{\Pr(S_i|V_j)\}_{i,j=1,2,3}$, where $\Pr(S_i|V_j)$ is the probability of signal S_i if the true value of the asset is V_j .

Noise traders have no information and trade randomly. These traders are not necessarily irrational, but they trade for reasons not included in this model, such as liquidity.¹¹

Market Maker: Trade in the market is organized by a market maker who has no private information. He is subject to competition and thus makes zero expected profits. In every period t , prior to the arrival of a trader, he posts a bid price bid^t at which he is willing to buy the security and an ask price ask^t at which he is willing to sell the security. Consequently, he sets prices in the interval $[\text{bid}^t, \text{ask}^t]$.

Traders' Actions: Each trader can buy or sell *one* unit of the security at prices posted by the market maker, or he can be inactive. So the set of possible actions for any trader is {buy, hold, sell}. We denote the action taken in period t by the trader who arrives at that date by a^t . We assume that noise traders trade with equal probability. Therefore, in any period, a noise-trader buy, hold, or sale occurs with probability $\gamma = (1 - \mu)/3$ each.

Public History: The structure of the model is common knowledge among all market participants. The identity of a trader and his signal are private information, but everyone can observe past trades and transaction prices. The history (public information) at any date $t > 1$, the sequence of the traders' past actions together with the realized past transaction prices, is denoted by $H^t = ((a^1, p^1), \dots, (a^{t-1}, p^{t-1}))$ for $t > 1$, where a^τ and p^τ are traders' actions and realized transaction prices at any date $\tau < t$, respectively. Also, H^1 refers to the initial history before any trade takes place.

Public Belief and Public Expectation: For any date t and any history H^t , denote the public belief/probability that the true liquidation value of the asset is V_i by $q_i^t = \Pr(V_i|H^t)$ for each $i = 1, 2, 3$. The public expectation, which we sometimes also refer to as the market expectation, of the liquidation value at H^t is given by $E[V|H^t] = \sum q_i^t V_i$. Also, we shall, respectively, denote the probability of a buy and the probability of a sale at any history H^t , when the true value of the asset is V_i , by $\beta_i^t = \Pr(\text{buy}|H^t, V_i)$ and $\sigma_i^t = \Pr(\text{sell}|H^t, V_i)$. For instance, suppose that at history H^t , the only informed type that buys is S_j . Then

¹¹As is common in the microstructure literature with asymmetric information, we assume that noise traders have positive weight ($\mu < 1$) to prevent "no-trade" outcomes à la Milgrom and Stokey (1982).

when the true value is V_i , the probability of a buy is given by the probability that there is a noise trader who buys plus the probability that there is an informed trader with signal S_j : $\beta_i^j = (1 - \mu)/3 + \mu \Pr(S_j|V_i)$.

The Informed Trader's Optimal Choice: The game played by the informed agents is one of incomplete information; therefore, the optimal strategies correspond to a perfect Bayesian equilibrium. Here, the equilibrium strategy for each trader simply involves comparing the quoted prices with his expected value, taking into account both the public history and his own private information. For simplicity, we restrict ourselves to equilibria in which each agent trades *only if* he is strictly better off (in the case of indifference, the agents do not trade). Therefore, the equilibrium strategy of an informed trader who enters the market in period t , receives signal S^t , and observes history H^t is (i) to buy if $E[V|H^t, S^t] > \text{ask}^t$, (ii) to sell if $\text{bid}^t > E[V|H^t, S^t]$, and (iii) to hold in all other cases.

The Market Maker's Price-Setting: To ensure that the market maker receives zero expected profits, the bid and ask prices must satisfy the following criteria at any date t and any history H^t : $\text{ask}^t = E[V|a^t = \text{buy at } \text{ask}^t, H^t]$ and $\text{bid}^t = E[V|a^t = \text{sell at } \text{bid}^t, H^t]$. Thus if there is a trade at H^t , the public expectation $E[V|H^{t+1}]$ coincides with the transaction price at time t (ask^t for a buy, bid^t for a sale). If the market maker always sets prices equal to the public expectation, $E[V|H^t]$, he makes an expected loss on trades with an informed agent. However, if the market maker sets an ask price and a bid price, respectively, above and below the public expectation, he gains on noise traders, as their trades have no information value. Thus, in equilibrium, the market maker must make a profit on trades with noise traders to compensate for any losses against informed types. This implies that if, at any history H^t , there is a possibility that the market maker trades with an informed trader, then there is a spread between the bid–ask prices at H^t and the public expectation $E[V|H^t]$ that satisfies $\text{ask}^t > E[V|H^t] > \text{bid}^t$.

Trading by the Informed Types and No Cascade Condition: At any history H^t , either informed types do not trade and every trade is by a noise trader or there is an informed type that would trade at the quoted prices. The game played by the informed types in the former case is trivial, as there will be no trade by the informed from H^t onward and an informational cascade occurs. The reason is that if there were no trades by the informed at H^t , no information will be revealed, and the expectations and prices remain unchanged; hence, by induction, we would have no trading by the informed and no information revelation at any date after H^t . In this paper, we thus consider only the case in which at every history, there is an informed type that would trade at the quoted prices.

Informative Private Signals: The private signals of the informed traders are informative at history H^t if

- (1) there exists $S \in \mathcal{S}$ such that $E[V|H^t, S] \neq E[V|H^t]$.

First note that (1) implies that at H^t there is an informed type that buys and an informed type that sells. To see this, observe that by (1) there must exist two signals S' and S'' such that $E[V|H^t, S'] < E[V|H^t] < E[V|H^t, S'']$. If no informed type buys at H^t , then there is no informational content in a buy and $\text{ask}^t = E[V|H^t]$. Then, by $E[V|H^t] < E[V|H^t, S'']$, type S'' must be buying at H^t ; a contradiction. Similarly, if no informed type sells at H^t , then $\text{bid}^t = E[V|H^t]$. Then, by $E[V|H^t] > E[V|H^t, S']$, type S' must be selling at H^t ; a contradiction. Second, it is also the case that if there is an informed type that trades at H^t , then (1) must hold. Otherwise, for every signal $S \in \mathbb{S}$, $E[V|H^t, S] = E[V|H^t] = \text{ask}^t = \text{bid}^t$ and the informed types that not trade at H^t .

It follows from the above discussion that (1) is both necessary and sufficient for trading by an informed type at H^t . Since we are interested in the case when the informed types trade, we therefore assume throughout this paper that (1) holds at every history H^t .¹²

One important consequence of condition (1) is that past behavior can be inferred from past transaction prices alone: since the bid and ask prices always differ by condition (1), one can infer behavior iteratively, starting from date 1. Therefore, all the results of this paper are valid if traders observe only past transaction prices and no-trades.

Long-Run Behavior of the Model: Since price formation in our model is standard, (1) also ensures that standard asymptotic results on efficient prices hold. More specifically, by standard arguments as in Glosten and Milgrom (1985), we have that transaction prices form a martingale process. Since by (1), buys and sales have some information content (at every date there is an informed type that buys and one that sells), it also follows that beliefs and prices converge to the truth in the *long run* (see, for instance, Proposition 4 in AZ). However, here we are solely interested in *short-run* behavior and fluctuations.

Conditional Signal Distributions: As we outlined in the Introduction, the possibility of herding or contrarian behavior for any informed agent with signal $S \in \mathbb{S}$ depends critically on the shape of the conditional signal distribution of S . Henceforth, we refer to the conditional signal distribution of the signal as the c.s.d. Furthermore, we will also employ the following terminology to describe four different types of c.s.d.s:

- increasing: $\Pr(S|V_1) \leq \Pr(S|V_2) \leq \Pr(S|V_3)$,
- decreasing: $\Pr(S|V_1) \geq \Pr(S|V_2) \geq \Pr(S|V_3)$,
- U-shaped: $\Pr(S|V_i) > \Pr(S|V_2)$ for $i = 1, 3$,
- hill-shaped: $\Pr(S|V_i) < \Pr(S|V_2)$ for $i = 1, 3$.

¹²A sufficient condition for (1) to hold at every H^t is that all minors of order 2 of matrix \mathcal{I} are nonzero.

An increasing c.s.d. is strictly increasing if all three conditional probabilities for the signal are distinct; a strictly decreasing c.s.d. is similarly defined.

For the results in our paper, it is also important whether the likelihood of a signal is higher in one of the extreme states V_1 or V_3 relative to the other extreme state. We thus define the *bias* of a signal S as $\Pr(S|V_3) - \Pr(S|V_1)$. A U-shaped c.s.d. with a negative bias, $\Pr(S|V_3) - \Pr(S|V_1) < 0$, will be labeled as an nU-shaped c.s.d. and a U-shaped c.s.d. with a positive bias, $\Pr(S|V_3) - \Pr(S|V_1) > 0$, will be labeled as a pU-shaped c.s.d. Similarly, we use nhill (phill) to describe a hill-shaped c.s.d. with a negative (positive) bias.

In describing the above properties of a type of c.s.d. for a signal, we shall henceforth drop the reference to the c.s.d. and attribute the property to the signal itself, when the meaning is clear. Similarly, when describing the behavior of a signal recipient, we attribute the behavior to the signal itself.

4. DEFINITIONS OF HERDING AND CONTRARIAN BEHAVIOR

In the literature, there are several definitions of herding. Some require “action convergence” or even complete informational cascades where all types take the same action in each state, irrespective of their private information; see Brunnermeier (2001), Chamley (2004), and Vives (2008). The key feature of this early literature was that herding can induce, after some date, the loss of all private information and wrong or inefficient decisions thereafter.

A situation like an informational cascade in which all informed types act alike may not, however, be very interesting in an informationally efficient financial market setting. In such a framework, prices account for the information contained in the traders’ actions. If all informed types act alike, then their actions would be uninformative and, as result, price would not move. Therefore, such uniformity of behavior cannot explain price movements, which are a key feature of financial markets. Moreover, if the uniform action involves trading, then a large imbalance of trades would accumulate without affecting prices—contrary to common empirical findings.¹³

Furthermore, as we have explained in the previous section, in our “standard” microstructure trading model, at any history, uniform behavior by the informed types is possible if and only if *all* private signals are uninformative, in the sense that the private expectations of all informed types are equal that of the market expectation (condition (1) is violated). The case of such uninformative private signals is trivial and uninteresting, as it implies that no further information is revealed, that all informed types have the same expectation, and that, because we assume that informed agents trade only if trading makes them strictly better off, a no-trade cascade results.

In this paper we thus focus on the social learning (learning from others) aspect of behavior for individual traders that is implied by the notion of herding from the earlier literature. Specifically, we follow Brunnermeier’s (2001,

¹³See, for instance, Chordia, Roll, and Subrahmanyam (2002).

Chap. 5) description of herding as a situation in which “an agent imitates the decision of his predecessor even though his own signal might advise him to take a different action,” and we consider the behavior of a *particular* signal type by looking at how the history of past trading can induce a trader to change behavior and trade against his private signal.¹⁴

Imitative behavior is not the only type of behavior that learning from others' past trading activities may generate: a trader may also switch behavior and go against what most have done in the past. We call such social learning contrarianism and differentiate it from herding by describing the latter as a history-induced switch of opinion *in the direction* of the crowd and the former as a history-induced switch *against the direction* of the crowd. The direction of the crowd here is defined by the “recent” price movement. Thus, there is a symmetry in our definitions, making herding the intuitive counterpart to contrarianism.

DEFINITION: Herding. A trader with signal S buy herds in period t at history H^t if and only if (i) $E[V|S] < bid^t$, (ii) $E[V|S, H^t] > ask^t$, or (iii-h) $E[V|H^t] > E[V]$. Sell herding at history H^t is defined analogously with the required conditions $E[V|S] > ask^t$, $E[V|S, H^t] < bid^t$, and $E[V|H^t] < E[V]$. Type S herds if he either buy herds or sell herds at some history.

Contrarianism. A trader with signal S engages in buy contrarianism in period t at history H^t if and only if (i) $E[V|S] < bid^t$, (ii) $E[V|S, H^t] > ask^t$, or (iii-c) $E[V|H^t] < E[V]$. Sell contrarianism at history H^t is defined analogously with the required conditions $E[V|S] > ask^t$, $E[V|S, H^t] < bid^t$, and $E[V|H^t] > E[V]$. Type S engages in contrarianism if he engages in either buy contrarianism or sell contrarianism at some history.

Both with buy herding and buy contrarianism, type S prefers to sell at the initial history before observing other traders' actions (condition (i)), but prefers to buy after observing the history H^t (condition (ii)). The key differences between buy herding and buy contrarianism are conditions (iii-h) and (iii-c). The former requires the public expectation, which is the last transaction price and an *average* of the bid and ask prices, to rise at history H^t so that a change of action from selling to buying at H^t is *with* the general movement of the prices (crowd), whereas the latter condition requires the public expectation to have dropped so that a trader who buys at H^t acts *against* the movement of prices.

Henceforth, we refer to a *buy herding history* as one at which some type, were they to trade, would buy herd at that history; similarly for a *buy contrarianism history*.

¹⁴Vives (2008, Chap. 6) adopted a similar view of herding, defining it as a situation in which agents put “too little” weight on their private signals with respect to a well defined welfare benchmark.

Our definition of herding is identical to that in Avery and Zemsky (1998),¹⁵ and it has also been used in other work on social learning in financial markets (see, for instance, Cipriani and Guarino (2005) or Drehmann, Oechssler, and Roider (2005)). It describes histories at which a trader acts against his signal (judgement) and follows the trend, where “against his signal” is defined by comparing the herding action to the benchmark without public information. “The trend” is identified by price movements based on the idea that prices rise (fall) when there are more (less) buys than sales. The contrarianism definition, on the other hand, captures the contra-trend action that is also against one’s signal.¹⁶

Our definitions also capture well documented financial market trading behavior. In particular, our herding definition is a formalization of the idea of rational *momentum* trading. It also captures the situation in which traders behave as if their demand functions are increasing. Contrarianism has a *mean-reversion* flavor. Both momentum and contrarian trading have been analyzed extensively in the empirical literature and have been found to generate abnormal returns over some time horizon.¹⁷ Our analysis thus provides a characterization for momentum and mean-reversion behavior, and shows that it can be rational.

As we have argued above, in our setup we assume that trades are informative to avoid the uninteresting case of a no-trade cascade. Thus, we cannot have a situation in which *all* informed types act alike.¹⁸ Nevertheless, our setup allows for the possibility that a very large portion of informed traders are involved in herding or contrarianism. The precise proportion of such informed traders is, in fact, determined exogenously by the information structure through the like-

¹⁵Avery and Zemsky’s (1998) definition of contrarianism is stronger than ours (they also imposed an additional bound on price movements that reflects the expectations that would obtain if the traders received an infinite number of draws of the same signal). We adopt the definition of contrarianism above because, as we explained before, it is a natural and simple counterpart to the definition of herd behavior.

¹⁶Herding and contrarianism here refer to extreme switches of behavior from selling to buying or the reverse. One could expand the definition to switches from holding to buying or to selling (or the reverse). For consistency with the earlier literature, we use the extreme cases where switches do not include holding.

¹⁷In the empirical literature, contrarian behavior is found to be profitable in the very short run (1 week and 1 month, Jegadeesh (1990) and Lehmann (1990)) and in the very long run (3–5 years, de Bondt and Thaler (1985)). Momentum trading is found to be profitable over the medium term (3–12 months, Jegadeesh and Titman (1993)) and exceptionally unprofitable beyond that (the 24 months following the first 12, Jegadeesh and Titman (2001)). Sadka (2006) studied systematic liquidity risk and linked a component of it, which is caused by informed trading, to returns on momentum trading.

¹⁸Note that even though with no cascades (the market expects that), the different types do not take the same actions, in degenerate settings when a particular type is not present in certain states, it is possible that in these particular states all informed types that occur with positive probability take the same action. An example of such a degenerate situation is the information structure in Avery and Zemsky (1998) that we discuss in Section 10.

likelihood that a trader receives the relevant signal. We discuss this point further in Section 11 and show that this proportion can be arbitrarily large.

Although the informative trade assumption ensures that asymptotically the true value of the security is learned, our definition of herding and contrarianism admits the possibility of switches of trades “in the wrong direction” in the *short* run. For instance, traders may buy herd even though the true value of the asset is V_1 (the lowest).

Finally, social learning can have efficiency consequences. In any setting where agents learn from the actions of others, an informational externality is inevitable as future agents are affected when earlier agents take actions that reveal private information. For example, in the early herding models of Bikhchandani, Hirshleifer, and Welch (1992) and Banerjee (1992), once a cascade begins, no further information is revealed. However, in this paper, we do not address the efficiency element, because it requires a welfare benchmark, which “is generally lacking in asymmetric information models of asset markets” (see Avery and Zemsky (1998, p. 728)). Instead, our definition is concerned with observable behavior and outcomes that are sensitive to the details of the trading history. Such sensitivities may dramatically affect prices in financial markets.

5. CHARACTERIZATION RESULTS: THE GENERAL CASE

The main characterization result for herding and contrarianism is as follows:

THEOREM 1: (a) **HERDING.** (i) *Necessity:* If type S herds, then S is U-shaped with a nonzero bias. (ii) *Sufficiency:* If there is a U-shaped type with a nonzero bias, there exists $\mu_h \in (0, 1]$ such that some informed type herds when $\mu < \mu_h$.

(b) **CONTRARIANISM.** (i) *Necessity:* If type S acts as a contrarian, then S is hill-shaped with a nonzero bias. (ii) *Sufficiency:* If there is a hill-shaped type with a nonzero bias, there exists $\mu_c \in (0, 1]$ such that some informed type acts as a contrarian when $\mu < \mu_c$.

Theorem 1 does not specify when we have buy or sell herding or when we have buy or sell contrarianism. In what follows, we first consider the necessary and then the sufficient conditions for each of these cases. The proof of Theorem 1 then follows from these characterization results at the end of this section.

We begin by stating three useful lemmas. The first lemma provides a characterization for the difference between private and public expectations.

LEMMA 1: For any S , time t , and history H^t , $E[V|S, H^t] - E[V|H^t]$ has the same sign as

$$(2) \quad q_1' q_2' [\Pr(S|V_2) - \Pr(S|V_1)] + q_2' q_3' [\Pr(S|V_3) - \Pr(S|V_2)] \\ + 2q_1' q_3' [\Pr(S|V_3) - \Pr(S|V_1)].$$

Second, as the prior on the liquidation values is symmetric, it follows that the expectation of the informed is less (greater) than the public expectation at the initial history if and only if his signal is negatively (positively) biased.

LEMMA 2: *For any signal S , $E[V|S]$ is less than $E[V]$ if and only if S has a negative bias, and $E[V|S]$ is greater than $E[V]$ if and only if S has a positive bias.*

An immediate implication of this lemma is that someone sells at the initial history only if this type's signal is negatively biased and buys only if the signal is positively biased.

Third, note that herding and contrarianism involve switches in behavior after changes in public expectations relative to the initial period. A useful way to characterize these changes is the following lemma.

LEMMA 3: *If $E[V|H^t] > E[V]$, then $q_3^t > q_1^t$, and if $E[V] > E[V|H^t]$, then $q_1^t > q_3^t$.*

Thus the public expectation rises (falls) if and only if the public belief attaches a lower (higher) probability to the lowest value, V_1 , than to the highest value, V_3 .

5.1. Necessary Conditions

Herding and contrarianism by a given signal type involve buying at some history and selling at another. Our first result establishes that this cannot happen if the signal is decreasing or increasing. Consider a decreasing type S . Since the ask price always exceeds the public expectation, it follows that, at any H^t , type S does not buy if the expectation of S , $E[V|S, H^t]$, is no more than the public expectation, $E[V|H^t]$. The latter must indeed hold because, for any two valuations V_ℓ and V_h such that $V_\ell < V_h$, the likelihood that a decreasing type S attaches to V_ℓ relative V_h at any history H^t cannot exceed that of the market:

$$\frac{\Pr(V_\ell|S, H^t)}{\Pr(V_h|S, H^t)} = \frac{\Pr(V_\ell|H^t) \Pr(S|V_\ell)}{\Pr(V_h|H^t) \Pr(S|V_h)} \leq \frac{\Pr(V_\ell|H^t)}{\Pr(V_h|H^t)}.$$

Formally, every term in (2) is nonpositive when S is decreasing; therefore, it follows immediately from Lemma 1 that $E[V|S, H^t] \leq E[V|H^t]$.

An analogous set of arguments demonstrates that an increasing type does not sell at any history. Therefore, we can state the following proposition.

PROPOSITION 1: *If S is decreasing, then type S does not buy at any history. If S is increasing, then type S does not sell at any history. Thus recipients of such signals cannot herd or behave as contrarians.*

Proposition 1 demonstrates that any herding and contrarian type must be either U- or hill-shaped. We next refine these necessary conditions further and state the main result of this subsection as follows.

PROPOSITION 2: (a) *Type S buy herds only if S is nU-shaped and sell herds only if S is pU-shaped.* (b) *Type S acts as a buy contrarian only if S is nhill-shaped and acts as a sell contrarian only if S is phill-shaped.*

A sketch of the proof of Proposition 2 for buy herding and buy contrarianism is as follows. Suppose that S buy herds or acts as a buy contrarian. Then it must be that at H^1 , type S sells and thus his expectation is below the public expectation. By Lemma 2, this implies that S is negatively biased.¹⁹ Thus, by Proposition 1, S is either nU- or nhill-shaped.

The proof is completed by showing that buy herding is inconsistent with an nhill-shaped c.s.d. and that buy contrarianism is inconsistent with an nU-shaped c.s.d. To see the intuition, for example, for the case of buy herding, note that in forming his belief, a trader with an nhill-shaped c.s.d. puts less weight on the tails of his belief (and thus more on the center) relative to the public belief; furthermore, the shift from the tail towards the center is more for value V_3 than for V_1 because of the negative bias. When buy herding occurs, the public belief must have risen and thus, by Lemma 3, the public belief attaches more weight to V_3 relative to V_1 (i.e., $q_1^t < q_3^t$). Such a redistribution of probability mass ensures that S 's expectation is less than that of the public. Hence an nhill-shaped S cannot be buying.

The arguments for sell herding and sell contrarianism are analogous except that here the bias has to be positive to ensure that the informed type buys at the initial history.

5.2. Sufficient Conditions

The above necessary conditions—U shape for herding and hill shape for contrarianism—turn out to be almost sufficient as well. Before stating the sufficiency results, it will be useful to discuss two of the ideas that provide insight into the analysis.

(I) *With a U-shaped signal S , the difference between the private and the public expectation, $E[V|S, H^1] - E[V|H^1]$, is positive at histories at which the public probability of state V_1 is sufficiently small relative to the probability of the other states (both q_1^t/q_2^t and q_1^t/q_3^t are close to zero) and is negative at histories at which the public probability of state V_3 is sufficiently small relative to the probability of the other states (both q_3^t/q_1^t and q_3^t/q_2^t are close to zero). For a hill-shaped signal S the sign of this difference is the reverse.*

¹⁹The bias is only required because we assume that priors are symmetric.

The intuition for this statement relates to the example from the Introduction. Consider first any history H^t at which both q_1^t/q_2^t and q_1^t/q_3^t are close to zero. Then there are effectively two states V_2 and V_3 at H^t . This means that at such a history, the difference between the expectations of an informed type S and of the public (which has no private information), $E[V|S, H^t] - E[V|H^t]$, has the same sign as $\Pr(S|V_3) - \Pr(S|V_2)$.²⁰ The latter is positive for a U-shaped S and negative for a hill-shaped S . Thus, at such history, $E[V|S, H^t] - E[V|H^t]$ is positive if S is U-shaped and is negative if S is hill-shaped.

By a similar reasoning, the opposite happens at any history H^t at which both q_3^t/q_1^t and q_3^t/q_2^t are close to zero. At such a history, there are effectively two states, V_1 and V_2 ; therefore, $E[V|S, H^t] - E[V|H^t]$ has the same sign as $\Pr(S|V_2) - \Pr(S|V_1)$.²¹ Since the latter is negative for a U-shaped S and positive for a hill-shaped S , it follows that at such a history, $E[V|S, H^t] - E[V|H^t]$ is negative if S is U-shaped and is positive if S is hill-shaped.

(II) *The probability of noise trading may have to be sufficiently large to ensure that the bid–ask spread is not too wide both at the initial history and later at the history at which the herding or contrarian candidate changes behavior.*

In (I), we have compared the private expectation of the informed trader with that of the public. To establish the existence of herding or contrarian behavior, however, we must compare the private expectations with the bid and ask prices. The difference is that bid and ask prices form a spread around the public expectation. To ensure the possibility of herding or contrarianism, this spread must be sufficiently “tight.” Tightness of the spread, in turn, depends on the extent of noise trading: the more noise there is (the smaller the likelihood of the informed types, μ), the tighter is the spread.

More specifically, to ensure buy herding or buy contrarianism (the other cases are similar) by an informed type, a minimal amount of noise trading may be necessary, so that the informed type (i) sells initially and (ii) switches to buying after some history H^t . Below, we formalize these minimal noise trading restrictions by introducing two bounds, one to ensure a tight spread for the *initial* sell and the second to ensure a tight spread for the subsequent *switch* to buying, and we require the likelihood of informed trading μ to be less than both these bounds.

Appealing to the above ideas, we can now state a critical lemma. To save space, we state the result only for the case of buy herding and buy contrarianism.

²⁰Formally, at any H^t at which both q_1^t/q_2^t and q_1^t/q_3^t are close to zero, the first and the third terms in (2) are arbitrarily small; moreover, the second term in (2) has the same sign as $\Pr(S|V_3) - \Pr(S|V_2)$; therefore, it follows from Lemma 1 that $E[V|S, H^t] - E[V|H^t]$ has the same sign as $\Pr(S|V_3) - \Pr(S|V_2)$.

²¹This claim also follows from Lemma 1: at such history, the second and the third terms in (2) are arbitrarily small and the second term in (2) has the same sign as $\Pr(S|V_3) - \Pr(S|V_2)$.

LEMMA 4: (i) *Suppose that signal S is nU -shaped. Then there exist μ^i and $\mu_{bh}^s \in (0, 1]$ such that S buy herds if $\mu < \mu_{bh} \equiv \min\{\mu^i, \mu_{bh}^s\}$ and if the following statement holds:*

(3) *For any $\varepsilon > 0$, there exists a history H^l such that $q_1^l/q_l^i < \varepsilon$ for all $l = 2, 3$.*

(ii) *Suppose signal S is n hill-shaped. Then there exist μ^i and $\mu_{bc}^s \in (0, 1]$ such that S acts as a buy contrarian if $\mu < \mu_{bc} \equiv \min\{\mu^i, \mu_{bc}^s\}$ and if the following statement holds:*

(4) *For any $\varepsilon > 0$, there exists a history H^l such that $q_3^l/q_1^i < \varepsilon$ for all $l = 1, 2$.*

Conditions (3) and (4) above ensure that there are histories at which the probability of an extreme state, V_1 or V_3 , can be made arbitrarily small relative to the other states. Value μ_{bh} is the minimum of the two bounds μ^i and μ_{bh}^s , mentioned in (II), that respectively ensure that spreads are small enough at the initial history and at the time of the switch of behavior by a buy herding type. Similarly, μ_{bc} is the minimum of the two bounds μ^i and μ_{bc}^s that respectively ensure that spreads are small enough at the initial history and at the time of the switch of behavior by a buy contrarian type S . Below we will discuss how restrictive these bounds are and whether they are necessary for the results.

A sketch of the proof for part (i) of Lemma 4 is as follows. First, by Lemma 2, S having a negative bias implies that $E[V|S] < E[V]$. Then it follows from the arguments outlined in (II) above that one can find an upper bound $\mu^i > 0$ on the size of the informed trading such that if $\mu < \mu^i$, then at H^1 the bid price bid^1 is close enough to the public expectation $E[V]$ so that $E[V|S]$ is also below bid^1 , that is, S sells at H^1 .

Second, since S is U -shaped, it follows from the arguments outlined in (I) that at any history H^l at which q_1^l/q_2^i and q_1^l/q_3^i are sufficiently small, $E[V|S, H^l] > E[V|H^l]$ (by condition (3) such history H^l exists). Then, by the arguments outlined in (II) above, one can find an upper bound $\mu_{bh}^s > 0$ such that if $\mu < \mu_{bh}^s$, then at H^l the ask price ask^l is sufficiently close to $E[V|H^l]$ so that $E[V|S, H^l]$ also exceeds ask^l , that is, S switches to buying.

Finally, since q_1^l/q_3^i is small at H^l , by Lemma 3, $E[V|H^l] > E[V]$. Thus, the switch to buying at H^l by S is in the direction of the crowd.

The argument for contrarianism in part (ii) is analogous except that to ensure $E[V|S, H^l]$ exceeds $E[V|H^l]$ at some history H^l for a hill-shaped S , by (I), H^l must be such that q_3^l/q_1^i and q_3^l/q_2^i are sufficiently small (condition (4) ensures that such history H^l exists). Then by (II) there exists $\mu_{bc}^s > 0$ such that if $\mu < \mu_{bc}^s$, ask^l is sufficiently close to $E[V|H^l]$ so that $E[V|S, H^l] > \text{ask}^l$. Furthermore, at such a history H^l , S will be buying *against* the crowd because when q_3^l/q_1^i is small, which is the case at H^l , by Lemma 3, $E[V|H^l] < E[V]$.

A set of results similar to Lemma 4 can be obtained for the cases of sell herding, and sell contrarianism except that to ensure sell herding, the appropriate assumptions are that S is pU and (4) holds, and to ensure sell contrarian, we need that S is phill and (3) holds.²²

The sufficiency results in Lemma 4 are nonvacuous provided that conditions (3) and (4) are satisfied. As these conditions depict properties of endogenous variables, to complete the analysis, we need to show the existence of the histories assumed by conditions (3) and (4).

In some cases, this is a straightforward task. The easiest case arises when at any t there is a trade that reduces *both* q_1^t/q_2^t and q_1^t/q_3^t or *both* q_3^t/q_1^t and q_3^t/q_2^t uniformly (independent of time). For example, suppose that the probability of a buy is uniformly increasing in the liquidation value at any date and history:

$$(5) \quad \text{for some } \varepsilon > 0, \beta_j^t > \beta_i^t + \varepsilon \text{ for any } j > i \text{ and any } t.$$

Since $q_i^{t+1}/q_j^{t+1} = (q_i^t \beta_i^t)/(q_j^t \beta_j^t)$ when there is a buy at date t , it follows that in this case a buy reduces *both* q_1^t/q_2^t and q_1^t/q_3^t uniformly. Thus if (5) holds, a sufficiently large number of buys induces the histories described in (3). Similarly, with a sale, $q_i^{t+1}/q_j^{t+1} = (q_i^t \sigma_i^t)/(q_j^t \sigma_j^t)$. Thus, if the probability of a sale is uniformly decreasing in the liquidation value at any H^t ,

$$(6) \quad \text{for some } \varepsilon > 0, \sigma_i^t > \sigma_j^t + \varepsilon \text{ for any } j > i \text{ and any } t,$$

then a sale reduces *both* q_3^t/q_1^t and q_3^t/q_2^t uniformly. Thus, if (6) holds, a sufficiently large number of sales induces the histories described in (4).²³

Demonstrating conditions (3) and (4) generally, however, requires a substantially more complex construction. In particular, in some cases there are no paths that result in both q_1^t/q_2^t and q_1^t/q_3^t decreasing at *every* t or in both q_3^t/q_1^t and q_3^t/q_2^t decreasing at *every* t .²⁴ For these cases, we construct outcome paths consisting of two different stages. For example, to ensure (3), the path is constructed so that in the first stage, q_1^t/q_2^t becomes small while ensuring that q_1^t/q_3^t does not increase by too much. Then in the second stage, once q_1^t/q_2^t is

²²These differences arise because with sell herding or sell contrarianism the informed needs to buy initially and switch to selling later. To ensure the former, by Lemma 2, we need to assume a positive bias and, to ensure the latter, the appropriate “extreme” histories at which the switches happen are the opposite of the buy herding and buy contrarian case. Also, the values for the bounds on the size of informed trading might be different for sell herding and sell contrarianism from those for buy herding and buy contrarianism.

²³These monotonicity properties of the probability of a buy and the probability of a sale as defined in (5) and (6) are satisfied, for instance, by MLRP information structures; see the next section.

²⁴For instance, if for every action a^t (= buy, sell, or hold), the probability of a^t in state V_1 is no less than the probability of a^t in the other two states, there will not be a situation that reduces both q_1^t/q_2^t and q_1^t/q_3^t .

sufficiently small, the continuation path makes q_1^t/q_3^t small while ensuring that q_1^t/q_2^t does not increase by too much. A similar construction is used for (4).

Such constructions work for most signal distributions. The exceptions are cases with two U-shaped signals with opposite biases or two hill-shaped signals with opposite biases. In these cases, we can show, depending on the bias of the third signal, that either (3) or (4) holds, but we cannot show both. For example, if one of the three signals is nU and another is pU, then we can show that (3) holds if the third signal has a nonnegative bias and (4) holds if the third signal has a nonpositive bias. This implies, by Lemma 4(i), that in the former case, the nU type buy herds, and in the latter case, by an analogous argument, the pU type sell herds (similarly for the contrarian situation).

The next proposition is our main sufficiency result. It follows from the discussion above, concerning (3) and (4), and Lemma 4 (for completeness, we state the result for buy and sell herding and for buy and sell contrarianism).

PROPOSITION 3: (a) *Let S be nU-shaped. If another signal is pU-shaped, assume the third signal has a non negative bias. Then there exists $\mu_{bh} \in (0, 1]$ such that S buy herds if $\mu < \mu_{bh}$.*

(b) *Let S be pU-shaped. If another signal is nU-shaped, assume the third signal has a nonpositive bias. Then there exists $\mu_{sh} \in (0, 1]$ such that S sell herds if $\mu < \mu_{sh}$.*

(c) *Let S be nhill-shaped. If another signal is phill-shaped assume the third signal has a nonpositive bias. Then there exists $\mu_{bc} \in (0, 1]$ such that S is a buy contrarian if $\mu < \mu_{bc}$.*

(d) *Let S be phill-shaped. If another signal is nhill-shaped, assume the third signal has a nonnegative bias. Then there exists $\mu_{sc} \in (0, 1]$ such that S is a sell contrarian if $\mu < \mu_{sc}$.*

Discussion of the Noise Restriction

For each of our sufficiency results above (Lemma 4 and Proposition 3), we assume that μ is less than some upper bound. We will now discuss whether these bounds are necessary for our results, and whether they are restrictive.

Consider the case of buy herding by an nU-shaped type S as in Lemma 4(i). For this sufficiency result, we assume two restrictions: $\mu < \mu^i$ and $\mu < \mu_{bh}^s$. These conditions respectively ensure that the spread is small enough at the initial history and at history H^t at which there is a switch in behavior. In the Appendix, we show that there exists a unique value for the first bound, $\mu^i \in (0, 1]$, such that $\mu < \mu^i$ is also necessary for buy herding.

In general, we cannot find a unique upper bound for the second value, μ_{bh}^s , such that $\mu < \mu_{bh}^s$ is also necessary for buy herding. The reason is that the upper bound that ensures that the spread is not too large at the history H^t at which S switches to buying may depend on what the types other than S do at H^t . Since there may be more than one history at which S switches to buying, this upper bound may not be unique.

The above difficulty with respect to the necessity of the second noise condition $\mu < \mu_{bh}^s$ does not arise, for instance, when types other than S always take the same action.²⁵ More generally, we show in the Supplemental Material (Proposition 3a), that $\mu < \mu_{bh}^s$ is also *necessary* for buy herding provided there is at most one U-shaped signal.

A similar argument applies to contrarian behavior. The noise restriction with respect to the initial trade is necessary, whereas the noise restriction with respect to the switch of behavior is necessary as long as there is at most one hill-shaped type.

Finally, note that the bounds on μ in our sufficiency results do not always constitute a restriction. Consider again the case of buy herding for an nU type S . If at H^1 , type S has the lowest expectation among all informed types, then his expectation must be less than the bid price at H^1 and, thus, there is no need for any restriction on the size of the informed at H^1 (μ^i can be set to 1). Likewise, if at H^t at which buy herding occurs, type S has the highest expectation (for example, this happens if no type other than S buys at H^t), then his expectation must be greater than the ask price at H^t and, therefore, no restriction on the size of the informed is needed (μ_{bh}^s can be set to 1). Consequently, to obtain our sufficiency results, there are restrictions on the value of μ only if at H^1 or at the switch history H^t the expectation of the herding candidate is in between those of the other signal types.²⁶

5.3. Proof of Theorem 1

The necessity part in Theorem 1 follows immediately from Proposition 2. The proof of the sufficiency part for case (a) of herding is as follows. Let $\mu_h = \min\{\mu_{sh}, \mu_{bh}\}$, where μ_{bh} and μ_{sh} are the bounds for herding given in Proposition 3. Also, assume that $\mu < \mu_h$ and that there exists a U-shaped signal as in part (a) of Theorem 1. Then there are two possibilities: either there is another U-shaped signal with the opposite bias or there is not. If there is no other U-shaped signal with the opposite bias, then by part (a) of Proposition 3, the U-shaped type buy herds if it has a negative bias, and by part (b) of Proposition 3, the U-shaped type sell herds if it has a positive bias. If there is another U-shaped signal with the opposite bias, then by parts (a) and (b) of Proposition 3, one of the U-shaped signals must herd. This is because if the third signal is weakly positive, then the U-shaped signal with a negative bias buy herds, and if the third signal is weakly negative, then the U-shaped signal

²⁵For example, this happens when the information structure satisfies MLRP; see the next section.

²⁶Romano and Sabourian (2008) extended the present model to the case where traders can trade a continuum of quantities. In this setting, they showed that no restrictions on μ are needed because at each history, each quantity is traded by a specific signal type.

with a positive bias sell herds. The reasoning for sufficiency in part (b) of Theorem 1 is analogous and is obtained by setting $\mu_c = \min\{\mu_{sc}, \mu_{bc}\}$, where μ_{sc} and μ_{bc} are the bounds for contrarianism from Proposition 3.²⁷ *Q.E.D.*

6. SOCIAL LEARNING WITH MLRP INFORMATION STRUCTURE

The literature on asymmetric information often assumes that the information structure is monotonic in the sense that it satisfies the monotone likelihood ratio property (MLRP). Here this means that for any signals $S_l, S_h \in \mathbb{S}$ and values $V_l, V_h \in \mathbb{V}$ such that $S_l < S_h$ and $V_l < V_h$, $\Pr(S_h|V_l)\Pr(S_l|V_h) < \Pr(S_h|V_h)\Pr(S_l|V_l)$.

MLRP is very restrictive (it is stronger than first order stochastic dominance) and at first might seem to be too strong to allow herding or contrarianism. This turns out to be false. Not only does MLRP not exclude such possibilities, it actually enables one to derive a sharper set of results for the existence of herding and contrarianism. Moreover, with MLRP the histories that can generate herding or contrarianism can be easily identified.

MLRP is a set of restrictions on the conditional probabilities for the *entire* signal structure and is equivalent to assuming that all minors of order 2 of the information matrix \mathcal{I} are positive. Herding or contrarianism, on the other hand, relates to the c.s.d. of a signal being U- or hill-shaped, that is, to the individual row in matrix \mathcal{I} that corresponds to the signal. Therefore, to analyze the possibility of herding or contrarianism with MLRP, we need to consider the c.s.d. of the different signals under MLRP. In the next lemma, we describe some useful implications of MLRP.

LEMMA 5: *Assume $S_1 < S_2 < S_3$ and that the information structure satisfies MLRP. Then the following statements hold:*

- (i) $E[V|S_1, H^t] < E[V|S_2, H^t] < E[V|S_3, H^t]$ at any t and any H^t .
- (ii) In any equilibrium, S_1 types always sell and S_3 types always buy.
- (iii) S_1 is strictly decreasing and S_3 is strictly increasing.
- (iv) The probability of a buy is uniformly increasing in the liquidation value as specified in (5) and the probability of a sale is uniformly decreasing as specified in (6).

Part (i) states that MLRP imposes a natural order on the signals in terms of their conditional expectations after any history. Part (iv) implies that with MLRP, the probability of a buy is uniformly increasing and the probability of a sell is uniformly decreasing in the liquidation values. Parts (ii) and (iii) state that MLRP restricts the behavior and the shape of the lowest and the highest signals S_1 and S_3 . In particular, these two types do not change behavior and they are decreasing and increasing, respectively.

²⁷The bounds μ_{bh} , μ_{sh} , μ_{sc} , and μ_{bc} in the proof of Proposition 3 are such that $\mu_h = \mu_c$.

Lemma 5, however, does not impose any restrictions on the behavior or the shape of the middle signal S_2 . In fact, MLRP is consistent with a middle signal S_2 that is decreasing, increasing, hill-shaped, or U-shaped with a negative or a positive bias: Table I describes a robust example of all these possibilities. Thus, with MLRP, S_2 is the only type that can display herding or contrarian behavior. We can then state the following characterization result for MLRP information structures (again we omit the analogous sell herding and sell contrarian cases).

THEOREM 2: *Assume $S_1 < S_2 < S_3$ and the signal structure satisfies MLRP. (a) If S_2 is nU, there exists $\mu_{bh} \in (0, 1]$ such that S_2 buy herds if and only if $\mu < \mu_{bh}$. (b) If S_2 is nhill, there exists $\mu_{bc} \in (0, 1]$ such that S_2 acts as a buy contrarian if and only if $\mu < \mu_{bc}$.*

PROOF: Fix the critical levels $\mu_{bh} \equiv \min\{\mu^i, \mu_{bh}^s\}$ for buy herding and $\mu_{bc} \equiv \min\{\mu^i, \mu_{bc}^s\}$ for buy contrarianism, where μ^i is defined in Lemma 8, and μ_{bh}^s and μ_{bc}^s are, respectively, defined in (12) and (13) in the Appendix. The “if” part follows from Lemma 4: By Lemma 5(iv), conditions (5) and (6) hold. As outlined in the last section, both q_1^i/q_2^i and q_1^i/q_3^i can be made arbitrarily close to zero by considering histories that involve a sufficiently large number of buys, and both q_3^i/q_1^i and q_3^i/q_2^i can be made arbitrarily close to zero by considering histories with a sufficiently large number of sales. Then conditions (3) and (4) hold, and by Lemma 4, an nU-shaped S_2 buy herds and an nhill-shaped S_2 acts as a buy contrarian.

The “only if” part follows from Proposition 3a in the Supplemental Material.²⁸ *Q.E.D.*

The “if” part of the above proof demonstrates that with MLRP, it is strikingly easy to describe histories that induce herding by a U-shaped type or contrarianism by a hill-shaped type: an nU-shaped S_2 buy herds after a sufficient number of buys and an nhill-shaped S_2 acts as a buy contrarian after a sufficient number of sales.²⁹

²⁸The “only if” part of the theorem also follows from the discussion in the previous section on noise trading: if types other than S_2 always take the same action, then there is a unique upper bound on the size of the informed trading that ensures that the spreads are sufficiently tight for S_2 to buy herd (or to act as a buy contrarian). By Lemma 5, S_3 always buys and S_1 always sells. Therefore, with MLRP, the upper bound μ_{bh} is unique.

²⁹Note that the bounds on the size of the informed μ_{bh} and μ_{bc} in Theorem 2 must be strictly below 1. To see this, recall that by part (i) of Lemma 5, the expectation of the herding or contrarian candidate type S_2 is always between those of the other two types at every history. Also, by part (ii) of Lemma 5, S_1 always sells and S_3 always buys. Therefore, if μ is arbitrarily close to 1, then, to ensure zero profits for the market maker, $E[V|S_1, H'] < \text{bid}' < E[V|S_2, H']$ and $E[V|S_2, H'] < \text{ask}' < E[V|S_3, H']$ for every H' . This implies that S_2 does not trade; a contradiction.

TABLE I
AN EXAMPLE OF AN MLRP SIGNAL DISTRIBUTION^a

Pr(S V)	V ₁	V ₂	V ₃
S ₁	$\delta(1 - \beta)/(\beta + \delta(1 - \beta))$	$\delta(1 - \alpha)$	0
S ₂	$\beta/(\beta + \delta(1 - \beta))$	α	$\beta/(\beta + (1 - \delta)(1 - \beta))$
S ₃	0	$(1 - \delta)(1 - \alpha)$	$(1 - \beta)(1 - \delta)/(\beta + (1 - \delta)(1 - \beta))$

^aFor $\alpha, \delta \in (0, 1)$ and $\beta \in (0, \alpha)$, the above distribution satisfies MLRP. Moreover, S₂ is nU-shaped if $\beta \in (\alpha(1 - \delta)/(1 - \alpha\delta), \alpha)$ and $\delta < 1/2$, is pU-shaped if $\beta \in (\alpha\delta/(1 - \alpha(1 - \delta)), \alpha)$ and $\delta > 1/2$, is nhill-shaped if $\beta \in (0, \alpha\delta/(1 - \alpha(1 - \delta)))$ and $\delta < 1/2$, is phill-shaped if $\beta \in (0, \alpha(1 - \delta)/(1 - \alpha\delta))$ and $\delta > 1/2$, is decreasing if $\beta \in (\alpha\delta/(1 - \alpha(1 - \delta)), \alpha(1 - \delta)/(1 - \alpha\delta))$ and $\delta < 1/2$, and is increasing if $\beta \in (\alpha(1 - \delta)/(1 - \alpha\delta), \alpha\delta/(1 - \alpha(1 - \delta)))$ and $\delta > 1/2$.

7. RESILIENCE, FRAGILITY, AND LARGE PRICE MOVEMENTS

We now consider the robustness of herding and contrarianism, and describe the range of prices for which herding and contrarianism can occur. Throughout this section, we assume that signals satisfy the well behaved case of MLRP (we will return to this later), and perform the analysis for buy herding and buy contrarianism; the other cases are analogous.

We first show that buy herding persists if and only if the number of sales during an episode of buy herding is not too large. This implies, in particular, that buy herding behavior persists if the buy herding episode consists only of buys. We also show that during a buy herding episode, as the number of buys increases, it takes more sales to break the herd. For buy contrarianism, the impact of buys and sales work in reverse: in particular, buy contrarianism persists if and only if the number of buys during an episode of buy contrarianism is not too large. This means that buy contrarianism does not end if the buy contrarianism episode consists only of sales. We also show that during a buy contrarianism episode, as the number of sales increases, it takes more buys to end buy contrarianism.

PROPOSITION 4: Assume MLRP. Consider any history $H^r = (a^1, \dots, a^{r-1})$ and suppose that H^r is followed by $b \geq 0$ buys and $s \geq 0$ sales in some order; denote this history by $H^i = (a^1, \dots, a^{r+b+s-1})$.³⁰

- (a) If there is buy herding by S at H^r , then there exists an increasing function $\bar{s}(\cdot) > 1$ such that S continues to buy herd at H^i if and only if $s < \bar{s}(b)$.
- (b) If there is buy contrarianism by S at H^r , then there exists an increasing function $\bar{b}(\cdot) > 1$ such that S continues to act as a buy contrarian at H^i if and only if $b < \bar{b}(s)$.

One implication of the above result is that herding is resilient and contrarianism is self-defeating. The reason is that when buy herding or buy contrarian-

³⁰We will henceforth omit past prices from the history H^i to simplify the exposition.

ism begins, buys become more likely relative to a situation where the herding or contrarian type does not switch. Thus, in both buy herding and buy contrarianism there is a general bias toward buying (relative to the case of no social learning). By Proposition 4, buy herding behavior persists if there are not too many sales and buy contrarian ends if there is a sufficiently large number of buys. Thus herding is more likely to persist, whereas contrarianism is more likely to end.

To see the intuition for Proposition 4, consider the case of buy herding in part (a). At any history, the difference between the expectation of the herding type S and that of the market is determined by the relative likelihood that they attach to each of the three states. Since the herding type S must have an nU-shaped c.s.d., it follows that in comparing the expectation of the herding type S with that of the market there are two effects: first, S attaches more weight to V_3 relative to V_2 than the market and, second, S attaches more weight to V_1 relative to both V_2 and V_3 than the market. Since $V_1 < V_2 < V_3$ and at any history H' with buy herding, the expectation of the herding type S exceeds that of the market, it then follows that at H' the first effect must dominate the second one, that is, q'_1/q'_2 and q'_1/q'_3 are sufficiently small so that the first effect dominates. Also, by Lemma 5(iv), when MLRP holds, buys reduce the probability of V_1 relative to the other states. Therefore, further buys after H' make the second effect more insignificant and thereby ensure that the expectation of the herding type S remains above the ask price.

On the other hand, by Lemma 5(iv), when MLRP holds, sales reduce the probability of V_3 relative to the other states; thus sales after H' make the first effect less significant. Therefore, with sufficiently many sales, the expectation of the herding type S will move below the ask price so that type S will no longer buy. This ends herding.

The intuition for the buy contrarian case is analogous except that the effect of further buys and further sales work in the opposite direction.

Next, we show that with MLRP, large price movements are consistent with both herding and contrarianism. In fact, the range of price movements in both cases can include (almost) the entire set of feasible prices. Specifically, for buy herding, the range of feasible prices is $[V_2, V_3]$, and for buy contrarianism, the range is $[V_1, V_2]$.³¹ As argued above, with MLRP, buys increase prices and sales decrease prices. Furthermore, by Proposition 4, buy herding persists when there are only buys and buy contrarianism persists when there are only sales. Thus, once buy herding starts, a large number of buys can induce prices to rise to levels arbitrarily close to V_3 without ending buy herding, and once buy contrarianism starts, large numbers of sales can induce prices to fall to levels arbitrarily close to V_1 without ending buy contrarianism.

³¹The reason is, by Lemma 3, at any date t , buy herding implies $q'_3 > q'_1$, and buy contrarian implies $q'_1 > q'_3$.

We complete the analysis by showing that there exists a set of priors on V such that herding and contrarianism can start when prices are close to the middle value, V_2 . Together with the arguments in the last paragraph, we have that herding and contrarian prices can span almost the entire range of feasible prices. Formally, we have the following proposition.

PROPOSITION 5: *Let signals obey MLRP.*

(a) *Consider any history $H^r = (a^1, \dots, a^{r-1})$ at which there is buy herding (contrarianism). Then for any $\varepsilon > 0$, there exists history $H^t = (a^1, \dots, a^{t-1})$ following H^r such that there is buy herding (contrarianism) at every $H^\tau = (a^1, \dots, a^{\tau-1})$, $r \leq \tau \leq t$, and $E[V|H^{r+\tau}]$ exceeds $V_3 - \varepsilon$ (is less than $V_1 + \varepsilon$).*

(b) *Suppose the assumptions in Theorem 2 that ensure buy herding (contrarianism) hold. Then for every $\varepsilon > 0$, there exists a $\delta > 0$ such that if $\Pr(V_2) > 1 - \delta$, there is a history $H^t = (a^1, \dots, a^{t-1})$ and a date $r < t$ such that (i) there is buy herding (contrarianism) at every $H^\tau = (a^1, \dots, a^{\tau-1})$, $r \leq \tau \leq t$, (ii) $E[V|H^r] < V_2 + \varepsilon$ ($E[V|H^r] > V_2 - \varepsilon$), and (iii) $E[V|H^t] > V_3 - \varepsilon$ ($E[V|H^t] > V_1 + \varepsilon$).*

The results of this section (and those in the next section on volatility) assume that the information structure satisfies the well behaved case of MLRP. This ensures that the probabilities of buys and sales are uniformly increasing and decreasing in V (see Lemma 5(iv)). As a result, we have that the relative probability q_1^t/q_ℓ^t falls with buys and rises with sales for all $\ell = 2, 3$, and the opposite holds for q_3^t/q_ℓ^t for all $\ell = 1, 2$. This monotonicity in the relative probabilities of the extreme states is the feature that allows us to establish our persistence and fragility results. If MLRP were not to hold, then the probability of buys and sales may not be monotonic in V , and the results of this section may not hold.³² An example of this is the herding example in Avery and Zemsky (1998); also see Section 10.

8. THE IMPACT OF SOCIAL LEARNING ON VOLATILITY AND LIQUIDITY

In this section, we are concerned with the impact of social learning on price movements. In particular, we ask the following questions: Do buys move prices more with than without social learning? Will sales move prices more with than without social learning?

To address these questions, we compare price movements in our setup with those in a hypothetical *benchmark* economy in which informed traders do not switch behavior. This economy is identical to our setup except that each informed type always takes the same action as the one he chooses at the initial

³²The monotonicity of the probability of buys and sales in V can hold under weaker conditions than MLRP. For example, we could assume the existence of a strictly increasing and a strictly decreasing signal. All the results of this paper with MLRP also hold with this weaker assumption.

history (before receiving any public information). Consequently, in the hypothetical benchmark economy, informed traders act as if they do not observe prices and past actions of others. We thus refer to this world as the *opaque* market and discuss examples for such situations at the end of the section. In contrast, in the standard setting, traders observe and learn from the actions of their predecessors. To highlight the difference, in this section we refer to the standard case as the *transparent* market. In both the transparent and the opaque economies, the market maker correctly accounts for traders' behavior when setting prices.

Volatility

We show that with MLRP signals, at any histories at which either herding or contrarianism occurs, trades move prices more in the transparent market than in the opaque one. We found it interesting that larger price movements in the transparent market occur both after buys *and* after sales. Moreover, the result holds for MLRP information structures which, taken at face value, are "well behaved."

We present the result for the case of buy herding and buy contrarianism; the results for sell herding and sell contrarianism are identical and will thus be omitted. Specifically, fix any history H^r at which buy herding starts and consider the difference between the most recent transaction price in the transparent market with that in the opaque market at any buy herding history that follows H^r . Assuming MLRP signals, we show (a) that the difference between the two prices is positive if the history since H^r consists of only buys, (b) that the difference is negative if the history since H^r consists of only sales and the number of sales is not too large,³³ and (c) that the difference is positive if the history following H^r is such that the number of buys is arbitrarily large relative to the number of sales. We also show an analogous result for buy contrarianism.

Formally, for any history H^r , let $E_o[V|H^r]$, $q_{i,o}^r$, $\beta_{i,o}^r$, and $\sigma_{i,o}^r$ be, respectively, the market expectation, the probability of V_i , the probability of a buy in state V_i , and the probability of a sale in state V_i in the opaque market at H^r . Then we can show the following situations.

PROPOSITION 6: *Assume MLRP. Consider any finite history $H^r = (a^1, \dots, a^{r-1})$ at which the priors in the two markets coincide: $q_i^r = q_{i,o}^r$ for $i = 1, 2, 3$. Suppose that H^r is followed by $b \geq 0$ buys and $s \geq 0$ sales in some order; denote this history by $H^r = (a^1, \dots, a^{r+b+s-1})$.*

I. Assume that there is buy herding at H^r for every $\tau = r, \dots, r + b + s$.

(a) Suppose $s = 0$. Then $E[V|H^r] > E_o[V|H^r]$ for any $b > 0$.

³³Note that, by Proposition 5, buy herding cannot persist with an arbitrarily large number of sales.

(b) Suppose $b = 0$. Then there exists $\bar{s} \geq 1$ such that $E[V|H^r] < E_o[V|H^r]$ for any $s \leq \bar{s}$.

(c) For any s , there exists \bar{b} such that $E[V|H^r] > E_o[V|H^r]$ for any $b > \bar{b}$.

II. Assume that there is buy contrarianism at H^r for every $\tau = r, \dots, r + b + s$.

(a) Suppose $b = 0$. Then $E[V|H^r] < E_o[V|H^r]$ for any $s > 0$.

(b) Suppose $s = 0$. Then there exists $\bar{b} \geq 1$ such that $E[V|H^r] > E_o[V|H^r]$ for any $b \leq \bar{b}$.

(c) For any b , there exists \bar{s} such that $E[V|H^r] < E_o[V|H^r]$ for any $s > \bar{s}$.

The critical element in demonstrating the result is the U-shaped nature of the herding candidate's signal and the hill-shaped nature of the contrarian candidate's signal in combination with the public belief once herding/contrarianism starts. To see this, consider any buy herding history $H^r = (a^1, \dots, a^{r+b+s-1})$ that satisfies the above proposition for the case described in part (I); the arguments for a buy contrarian history described in part (II) are analogous. Then the prices in the transparent and opaque markets differ because at any buy herding history in the transparent market, the market maker assumes that the buy herding candidate S buys, whereas in the opaque market, the market maker assumes that S sells.³⁴ Since the buy herding type must have a U-shaped signal, we also have $\Pr(S|V_3) > \Pr(S|V_2)$. Then the following must hold: (i) the market maker upon observing a buy increases his belief about the likelihood of V_3 relative to that of V_2 faster in the transparent market (where S is a buyer) than in the opaque market (where S is a seller) and (ii) the market maker upon observing a sale decreases his belief about the likelihood of V_3 relative to V_2 faster in the transparent market than in the opaque market. Now if it is also the case that the likelihood of V_1 is small relative to that of V_3 in both worlds, then it follows from (i) and (ii), respectively, that the market expectation (which is the most recent transaction price) in the transparent market exceeds that in the opaque market after a buy and it is less after a sale.

At H^r in both markets the likelihoods of each state coincide ($q_i^t = q_{i,o}^t$); moreover, the likelihood of V_1 in both markets is small relative to that of V_3 (to ensure buy herding). Then the following two conclusions follow from the discussion in the previous paragraph: First, if H^r involves only a single buy after H^r (i.e., if $s = 0$ and $b = 1$), then $E[V|H^r] > E_o[V|H^r]$. Second, if H^r involves only a single sale after H^r (i.e., if $b = 0$ and $s = 1$), then $E[V|H^r] < E_o[V|H^r]$. Part I(b) follows from the latter. To complete the intuition for I(a) and I(c), note that further buys after H^r reduce the probabilities of V_1 relative to V_3 in both markets (see Lemma 5(iv)). Thus if either the history after H^r involves no sales (as in part I(a)) or the number of buys is large relative to the number of sales (as in part I(c)), then the first conclusion is reinforced and $E[V|H^r]$ remains above $E_o[V|H^r]$ after any such histories.

³⁴If we assume $S_1 < S_2 < S_3$, then with MLRP, the buy herding (buy contrarian) candidate must be S_2 .

Notice that with MLRP, any sale beyond H' increases the probability of V_1 relative to V_3 (and relative to V_2) both in the transparent and in the opaque market. Furthermore, the increase may be larger in the latter than in the former. As a result, for the buy herding case, we cannot show that, in general, prices in the transparent market fall more than in the opaque market after *any arbitrary* number of sales. However, if the relative likelihood of a sale in state V_1 to V_3 in the transparent market is no less than that in the opaque market (i.e., $(\sigma_1/\sigma_3) \geq (\sigma_{1,o}/\sigma_{3,o})$), then we can extend the conclusion in part I(b) to show that the price in the transparent market falls more than in the opaque market after *any arbitrary* number of sales (the proof is given in the Supplemental Material).³⁵

Proposition 6 of course does not address the likelihood of a buy or a sale after herding or contrarianism begins. It is important to note, however, that once buy herding or buy contrarianism starts, there will also be more buys in the transparent market compared to the opaque market because the herding type buys at such histories. Thus, given the conclusions of Proposition 6, price paths must have a stronger upward bias in the transparent market than in the opaque market.

Finally, it is often claimed that herding generates excess volatility, whereas contrarianism tends to stabilize markets because the contrarian types act against the crowd. The conclusions of this section are consistent with the former claim but contradict the latter. Both herding and contrarianism increase price movements compared to the opaque market and they do so for similar reasons—namely because of the U-shaped nature of the herding type's c.s.d. and the hill-shaped nature of the contrarian type's c.s.d.

Liquidity

In sequential trading models in the tradition of Glosten and Milgrom (1985), liquidity is measured by the bid–ask spread, as a larger spread implies higher adverse selection costs and thus lower liquidity. Since, at any date, market expectations after a buy and market expectations after a sale respectively coincide with the ask and the bid price at the previous date, the next corollary to Proposition 6 follows:

COROLLARY: *At any history H' at which type S engages in buy herding or buy contrarianism, two situations occur:*

- (a) *The ask price when the buy herding or buy contrarian candidate S rationally buys exceeds the ask price when he chooses not to buy.*
- (b) *The bid price when the buy herding or buy contrarian candidate S rationally buys is lower than the bid price when he chooses to sell.*

³⁵The condition $(\sigma_1/\sigma_3) \geq (\sigma_{1,o}/\sigma_{3,o})$ is satisfied if, for example, the bias of the herding candidate is close to zero.

Part (a) of the result follows from Proposition 6.I(a) and II(b) when $b = 1$ and part (b) follows from Proposition 6.I(b) and II(a) when $s = 1$.³⁶ The above corollary implies that in equilibrium, liquidity (as measured by the bid–ask spread) is lower when some informed types herd or act as a contrarian than when they do not.

Interpretation of the Opaque Market and the Volatility Result

One can think of the traders in the opaque market as automata that always buy or sell depending on their signals. One justification for such naive behavior is that traders do not observe or remember the public history of actions and prices (including current prices).

Alternatively, the nonchanging behavior may represent actions of rational traders in a trading mechanism where traders submit their orders through a market maker some time *before* the orders get executed. The market maker would receive these orders in some sequence and he would execute them sequentially at prices which reflect all the information contained in the orders received so far. The actions of other traders and the prices are unknown at the time of the order submission and, thus, as in the opaque market, the order of each trader is independent of these variables.³⁷ As traders effectively commit to a particular trade before any information is revealed, the price sequence in this alternative model would coincide with the price sequence in the opaque market that we depict above. Therefore, Proposition 6 can also be used to claim that volatility is greater in the transparent market than in this alternative setup in which all orders are submitted before any execution.

A slightly more transparent market than the opaque one is one where each trader with herding/contrarian signal S compares his prior expectations, $E[V|S]$, with the current price and buys if $E[V|S]$ exceeds the ask price, sells if $E[V|S]$ is less than the bid price, and does not trade otherwise. In this “almost opaque” market, there is a different kind of nontransparency in that at each period, the traders do not observe or recall past actions and prices, but they know the bid and ask prices at that period; furthermore, they act semi-rationally by comparing their private expectation with current prices without learning about the liquidation value from the current price (e.g., for cognitive reasons).

For the case of herding, the same excess volatility result as in part (I) of Proposition 6 holds if we compare the transparent market with the above almost opaque market. To see this, note that at the initial history H^1 , every buy

³⁶We show in the Appendix that the proof of Proposition 6 in these cases does not require MLRP. Thus, the corollary is stated without assuming MLRP information structure; see footnotes 46 and 47.

³⁷A possible example of such mechanism is a market after a “circuit breaker” is introduced. The latter triggers a trading halt after “large” movements in stock prices. Before trading recommences, traders submit their orders without knowing others’ actions. We thank Markus Brunnermeier for this interpretation.

herding type sells. Also, at every buy herding history, the prices are higher than at H^1 ; therefore, in an almost opaque market, the herding type must also sell at every buy herding history.³⁸ Since Proposition 6 compares price volatility *only* at histories at which buy herding occurs, it follows that the same excess volatility result holds if we compare the transparent with the almost opaque market.

9. HERDING AND CONTRARIANISM WITH MANY STATES

Our results intuitively extend to cases with more signals and more values. In fact, with three states and an arbitrary number of signals, our characterization results, in terms of U-shaped signals for herding and hill-shaped signals for contrarianism, and all our conclusions in the previous two sections with respect to fragility, persistence, large price movements, liquidity, and price volatility remain unchanged.³⁹

With more than three states, U shape and Hill shape are no longer the only possible signal structures that can lead to herding and contrarianism. The intuition for our results with many states does, however, remain the same: the herding type must distribute probability weight to the tails and the contrarian types must distribute weight to the middle.

Assume there are $N > 2$ states and N signals. Denote the value of the asset in state j by V_j and assume that $V_1 \leq V_2 \leq \dots \leq V_N$. Signal S is said to have an increasing c.s.d. if $\Pr(S|V_i) \leq \Pr(S|V_{i+1})$ for all $i = 1, \dots, N-1$ and a decreasing c.s.d. if $\Pr(S|V_i) \geq \Pr(S|V_{i+1})$ for all $i = 1, \dots, N-1$.

By the same reasoning as in Lemma 1, one can show that $E[V|S, H'] - E[V|H']$ has the same sign as

$$(7) \quad \sum_{j=1}^{N-1} \sum_{i=1}^{N-j} (V_{i+j} - V_i) \cdot q_i q_{i+j} [\Pr(S|V_{i+j}) - \Pr(S|V_i)].$$

For an increasing c.s.d., (7) is always nonnegative since $\Pr(S|V_{i+j}) - \Pr(S|V_i)$ is nonnegative for all i, j ; similarly, for decreasing c.s.d.s, (7) is always nonpositive since $\Pr(S|V_{i+j}) - \Pr(S|V_i)$ is nonpositive for all i, j . Therefore, an increasing or decreasing type cannot switch behavior and we have the following necessity result which is analogous to Proposition 1.

LEMMA 6: *An increasing or decreasing S never switches from buying to selling or vice versa.*

³⁸Since at a buy contrarian history, prices are lower than at the initial history, the same claim cannot be made for the contrarian case.

³⁹With three states, hill and U shape are still well defined, irrespective of the number of signals; even with a continuum of signals, these concepts can be defined in terms of conditional densities.

Next, we describe two sufficient conditions that yield herding and contrarianism that have a similar flavor as our sufficiency results in Section 5. We will focus only on buy herding and buy contrarianism; sell herding and sell contrarianism are analogous.

In line with the previous analysis, we assume for the remainder of this section that the values of the asset are distinct in each state and that they are on an equal grid. Moreover, we assume that the prior probability distribution is symmetric. Thus we set $\{V_1, V_2, \dots, V_N\} = \{0, \mathcal{V}, 2\mathcal{V}, \dots, (N-1)\mathcal{V}\}$ and $\Pr(V_i) = \Pr(V_{N+1-i})$ for all i .

We begin with the analysis of the decision problem of selling at H^1 and generalize the concept of a negative bias as follows. Signal S is said to have a *negative bias* if for any pair of values that are equally far from the middle value, the signal happens more frequently when the true value is the smaller one than when it is the larger one: $\Pr(S|V_i) > \Pr(S|V_{N+1-i})$ for all $i < (N+1)/2$. In the Supplemental Material, we show that this property ensures that $E[V|S] < E[V]$. This means, by a similar argument as with the three value case, that a negatively biased S must be selling at H^1 if μ is sufficiently small.

Next we generalize the sufficient conditions for switching to buying at some history. Recall that in the three value case, we considered histories at which the probability of one extreme value was small to the point where it can be effectively ignored. Then the expectation of the informed exceeds that of the market if the informed puts more weight on the larger remaining value than on the smaller remaining one.

The sufficient conditions that we describe for the switches in the general case have a similar intuition and are very simple, as we impose restrictions only on the most extreme values. Specifically, to ensure buy herding, we assume $\Pr(S|V_{N-1}) < \Pr(S|V_N)$ and consider histories at which the probabilities of all values are small relative to the two largest values V_{N-1} and V_N . Since at such histories all but the two largest values can be ignored, it must be that the price must have risen and the expectation of type S must exceed that of the public expectation if $\Pr(S|V_{N-1}) < \Pr(S|V_N)$. If, in addition, the bid-ask spread is not too large (enough noise trading), the expectation of S will also exceed the ask price at H^1 and type S switches from selling to buying after a price rise. Similarly, to ensure buy contrarianism, we assume $\Pr(S|V_1) < \Pr(S|V_2)$ and consider histories at which the probabilities of all values are small relative to the two smallest ones V_1 and V_2 . Since at such histories all but the two smallest values can be ignored and the price must have fallen, the expectation of type S must exceed that of the public expectation if $\Pr(S|V_1) < \Pr(S|V_2)$. If, in addition, the bid-ask spread is not too large at such histories, then S switches from selling to buying after a price fall. Formally, we can show the following analogous result to Lemma 4.

LEMMA 7: (i) Suppose S is negatively biased and satisfies $\Pr(S|V_{N-1}) < \Pr(S|V_N)$. Then there exists a $\mu_{bh} \in (0, 1]$ such that S buy herds if $\mu < \mu_{bh}$ and if

$$(8) \quad \text{for all } \varepsilon > 0 \text{ there exists } H^l \text{ such that } q_i^l/q_i^l < \varepsilon \text{ for all } l = N - 1, N \text{ and } i < N - 1.$$

(ii) Suppose S is negatively biased and satisfies $\Pr(S|V_1) < \Pr(S|V_2)$. Then there exists a $\mu_{bc} \in (0, 1]$ such that S is a buy contrarian if $\mu < \mu_{bc}$ and if

$$(9) \quad \text{for all } \varepsilon > 0 \text{ there exists } H^i \text{ such that } q_i^i/q_i^i < \varepsilon \text{ for all } l = 1, 2 \text{ and } i > 2.$$

The simplest way to ensure the existence of histories that satisfy (8) and (9) is to assume MLRP. Then, as in Lemma 5(iv) for the three state case, the probability of a buy is increasing and the probability of a sale is decreasing in V . As a result, with MLRP we can always ensure (8) by considering histories that contain a sufficiently large number of buys and ensure (9) by considering histories that contain a sufficiently large number of sales. Hence, Lemma 7 yields the following situations for buy herding and buy contrarianism.⁴⁰

THEOREM 3: Assume MLRP and suppose signal S is negatively biased.

(a) If $\Pr(S|V_{N-1}) < \Pr(S|V_N)$, then there exists $\mu_{bh} \in (0, 1]$ such that S buy herds if $\mu < \mu_{bh}$.

(b) If $\Pr(S|V_1) < \Pr(S|V_2)$, then there exists $\mu_{bc} \in (0, 1]$ such that S is a buy contrarian if $\mu < \mu_{bc}$.

The description in this section has assumed that each state is associated with a unique liquidation value of the underlying security. There can be, however, other uncertainties that do not affect the liquidation value but that do have an impact on the price. One example is a situation in which some agents may have superior information about the distribution of information in the economy (e.g., as in Avery and Zemsky's (1998) "composition uncertainty"; see the next section). In the Supplemental Material, we prove all the sufficiency results from this section for such a generalized setup.

10. AVERY AND ZEMSKY (1998)

As mentioned in the literature review, Avery and Zemsky (1998) (AZ) argue that herd behavior with informationally efficient asset prices is not possible

⁴⁰Conditions that ensure sell herding and sell contrarian are defined analogously. In particular, to ensure the initial buy, we need to assume a positive bias $\Pr(S|V_i) < \Pr(S|V_{N+1-i})$ for all $i < (N + 1)/2$. For the switches, we reverse the two conditions that ensure switching for buy herding and buy contrarian: for sell herding, we need $\Pr(S|V_1) > \Pr(S|V_2)$; for sell contrarian, we need $\Pr(S|V_{N-1}) > \Pr(S|V_N)$.

unless signals are “nonmonotonic” and uncertainty is “multidimensional.” AZ reached their conclusions by (i) showing that herding is not possible when the information structure satisfies their definition of monotonicity and (ii) providing an example of herding that has “multidimensional uncertainty.” In this section, we explain why our conclusions differ from theirs. We will also discuss the issue of price movements (or lack thereof) in their examples.

AZ’s conclusion with respect to monotonicity arises because their adopted definition is nonstandard and excludes herding almost by assumption. Specifically, they define a monotonic information structure as one that satisfies

$$(10) \quad \forall S, \exists w \text{ s.t. } \forall H', E(V|H', S) \text{ is weakly between } w \text{ and } E(V|H').$$

This definition neither implies nor is implied by the standard MLRP definition of monotonicity. Also, in contrast to MLRP, it is not a condition on the primitives of the signal distribution. Instead, it is a requirement on endogenous variables that must hold for *all trading histories*.⁴¹ Furthermore, it precludes herding almost by definition.⁴²

AZ’s example of herding uses *event uncertainty*, a concept first introduced by Easley and O’Hara (1992). Specifically, in their example, the value of the asset and the signals can take three values $\{0, \frac{1}{2}, 1\}$ and the information structure can be described by

$\Pr(S V)$	$V_1 = 0$	$V_2 = \frac{1}{2}$	$V_3 = 1$
$S_1 = 0$	p	0	$1 - p$
$S_2 = \frac{1}{2}$	0	1	0
$S_3 = 1$	$1 - p$	0	p

for some $p > 1/2$. The idea behind the notion of event uncertainty as used by AZ is that, first, informed agents know *if* something (an event) has happened (they know whether $V = V_2$ or $V \in \{V_1, V_3\}$). Second, they receive noisy information with precision p about how this event has influenced the asset’s liquidation value. This two-stage information structure makes the uncertainty multidimensional. Thus multidimensionality is equivalent to informed traders having a *finer information partition* than the market maker. AZ attributed herding to this feature of their example.

Multidimensionality is, however, neither necessary nor sufficient for herding and it is relevant to herding only to the extent that it may generate U-

⁴¹Condition (10) does not imply that each signal has an increasing or decreasing c.s.d.; however, one can show that if every signal has either a strictly increasing or a strictly decreasing c.s.d., then the information structure satisfies (10).

⁴²For example, for buy herding by type S to occur at some history H' , we must have $E[V|S] < E[V]$ and a subsequent price rise $E[V] < E[V|H']$; but then (10) implies immediately that $w < E[V|H', S] < E[V|H']$ and hence buy herding by S at H' is not possible.

shaped signals. First, since AZ's example has three states and three signals, it is a special case of our main setup and our characterization results apply. Specifically, the two herding types in AZ's example are S_1 and S_3 . In addition to having finer partitions of the state space than the market maker, both types are also U-shaped and so our Proposition 3 explains the possibility of buy herding by S_1 and sell herding by S_3 .⁴³ Second, our Proposition 3 demonstrates that there would also be herding if the AZ example is changed in such a way that all signals occur with positive probability in all states, while maintaining the U-shaped nature of signals S_1 and S_3 .⁴⁴ Such an information structure is no longer multidimensional (the informed trader's partition would be the same as the market maker's). Third, consider an information structure for which signals S_1 and S_3 are such that informed traders know whether or not $V = 0$ has occurred (and the market did not). Such signals are multidimensional (they generate a finer partition), but they are not U-shaped and thus do not admit herding.

Our general analysis with three states also provides us with an appropriate framework to understand the nature of histories that generate herding and contrarianism. For example, as we explained in Section 7, to induce buy herding, the trading history must be such that the probabilities of the lowest state V_1 is sufficiently small relative to the two other states. With MLRP, such beliefs arise after very simple histories consisting of a sufficiently large number of buys. In AZ's example, one also needs to generate such beliefs, but the trading histories that generate them are more complicated and involve a large number of holds followed by a large number of buys.⁴⁵

Turning to price movements, in AZ's event uncertainty example, herding has limited capacity to explain price volatility, as herding is fragile and price movements during herding are strictly limited. To allow for price movements during herding, AZ introduced a further level of informational uncertainty to their event uncertainty example. Specifically, they assumed additionally that

⁴³In AZ's example, not all signals arise in all states (in states V_1 and V_3 only signals S_1 and S_3 arise; in state V_2 only signal S_2 arises). Thus when there is herding (by types S_1 or S_3), all informed types that occur with positive probability act alike. Nevertheless, it is important to note that in AZ's example, herding does not constitute an informational cascade since at any history, not all types take the same action. The reason is that it is never common knowledge that there is herding. Instead, at any finite history, the market maker attaches nonzero probability to *all* three states and thus he always attaches nonzero probability to the state in which some trades are made by type S_2 . Moreover, the market expects that when type S_1 buy herds, type S_2 sells, and when type S_3 sell herds, type S_2 buys. See also our discussion in footnote 18.

⁴⁴For example, take $\Pr(S_1|V_i) = p(1 - \varepsilon)$ and $\Pr(S_2|V_i) = \Pr(S_3|V_2) = \varepsilon$ for all $i = 1, 3$ for $0 < \varepsilon < (1 - p)/2$.

⁴⁵The holds bring down the probability of V_1 relative to V_2 and the number of buys is chosen to bring down the probability of V_1 relative to V_3 sufficiently while not increasing the probability of V_1 relative to V_2 by too much. Formally, their construction is similar to Subcase D2 in the proof of our Proposition 3.

for each signal there are high and low quality informed traders. They also assumed that there is uncertainty about the proportion of each type of informed trader. AZ claimed that this additional level of uncertainty, which they label *composition uncertainty*, complicates learning and allows for large price movements during a buy herding phase (they do this by simulation).

A state of the world in AZ's example with composition uncertainty refers to both the liquidation value of the asset and the proportion of different types of informed traders in the market (the latter influences the prices). Thus, there is more than one state associated with a given value V of the asset. This example is, therefore, formally a special case of the multistate version of our model, and the possibility of herding follows from Lemma IV in the Supplemental Material. More specifically, our result establishes that to ensure herding in AZ's example with composition uncertainty, we need the analogue of U-shaped signals with the property that the probability of a signal in each state with $V = 1/2$ is less than the probability of the signal in each of the states with $V = 0$ or with $V = 1$. This is indeed the case in the example with composition uncertainty. Therefore, herding there is also due to U-shaped signals. (See the discussion in Section F of the Supplemental Material.)

To understand the differences in price movements and persistence, recall our discussion of fragility in Section 7 with regard to buy herding by type S at some history H^t . The probability of the lowest state V_1 relative to those of the other two states, q'_1/q'_2 and q'_1/q'_3 , must be sufficiently small to start herding, and these relative probabilities need to remain small for herding to persist beyond H^t . With MLRP, since $\Pr(\text{buy}|V)$ is increasing in V , further buys reduce both q'_1/q'_2 and q'_1/q'_3 ; at the same time, further buys increase prices. Thus herding can persist and prices can move significantly.

In AZ's example *without* composition uncertainty, while buys result in price increases, during any buy herding phase we have that $\Pr(\text{buy}|V_2) < \Pr(\text{buy}|V_1) = \Pr(\text{buy}|V_3)$. Thus, once buy herding begins, further buys cannot ensure that the relative probabilities of V_1 remain low, as buys increase q'_1/q'_2 , while leaving q'_1/q'_3 unaffected. Hence, buys during buy herding in AZ's example *without* composition uncertainty are self-defeating. In AZ's example *with* composition uncertainty, we have that $\Pr(\text{buy}|V_2) < \Pr(\text{buy}|V_1) < \Pr(\text{buy}|V_3)$ once herding starts. Further buys during herding thus reduce q'_1/q'_3 while increasing q'_1/q'_2 . As the former offsets somewhat the effect of the latter, buy herding may persist (and allow price increases) for longer than without composition uncertainty.

In conclusion, what makes herding less fragile and more consistent with significant price movements are the relative probabilities of a specific trade in the different states of the world and not so much the addition of extra dimensions of uncertainty. In fact, when the probability of a buy is increasing in the value of the asset, buy herding is least fragile and most consistent with large price movements. As we have shown, this can happen with only three states, without different dimensions of uncertainty and with "well behaved" information structures satisfying MLRP.

11. EXTENSIONS, DISCUSSION AND CONCLUSION

Herding and contrarian behavior are examples of history-dependent behavior that may manifest itself in real market data as momentum or mean reversion. Understanding the causes for the behavior that underlies the data can thus help interpret these important nonstationarities. In the first part of this paper, we characterized specific circumstances under which herding and contrarian behavior can and cannot occur in markets with efficient prices. In the second part, we showed that both herding and contrarianism can be consistent with large movement in prices, and that they both can reduce liquidity and increase volatility relative to situations where these kinds of social learning are absent.

In the early literature on informational social learning (e.g., Banerjee (1992)), herding was almost a generic outcome and would arise, loosely, with any kinds of signals. Herding as defined in our setup does not arise under all circumstances, but only under those that we specify here. Namely, the underlying information generating process must be such that there are some signals that people receive under extreme outcomes more frequently than under moderate outcomes. Therefore, to deter herding, mixed messages that predict extreme outcomes (U-shaped signal) should be avoided.

It is important to note that, depending on the information structure, the prevalence of types who herd (or act as contrarians) can vary and in some cases they can be very substantial. For example, consider the MLRP information structure in Table I when S_2 is U-shaped. Then, in *any* state, the likelihood that an informed trader is a herding type S_2 has lower bound α . This bound can range between 0 and 1. Thus, when α is sufficiently close to 1, the likelihood that an informed trader is a herding type is arbitrarily close to 1 and the impact of herding switches can then be very significant.

In this paper, we have presented the results for which we were able to obtain clear-cut analytical results. In the Supplemental Material, we also explore other implications with numerical simulations. First, an important implication of our analysis for applied research is that when social learning arises according to our definition, simple summary statistics such as the number of buys and sales are not sufficient statistics for trading behavior. Instead, as some types of traders change their trading modes during herding or contrarianism, prices become history-dependent. Thus as the entry order of traders is permuted, prices with the same population of traders can be strikingly different, as we illustrate with numerical examples. Second, herding results in price paths that are very sensitive to changes in some key parameters. Specifically, in the case with MLRP, comparing the situation where the proportion of informed agents is just below the critical levels described in Theorem 2 with that where the proportion is just above that threshold (so there is no herding), prices deviate substantially in the two cases. Third, herding slows down the convergence to the true value if the herd moves away from that true value, but it accelerates convergence if the herd moves in the direction of the true value.

APPENDIX A: OMITTED PROOFS

A.1. Proof of Proposition 2

To save space, we shall prove the result for the case of buy herding and buy contrarian; the proof for the sell cases is analogous. Thus suppose that S buy herds or acts as a buy contrarian at some H^t . The proof proceeds in several steps.

Step 1— S has a negative bias. Buy herding and buy contrarianism imply $E[V|S] < \text{bid}_1$. Since $\text{bid}_1 < E[V]$, we must have $E[V|S] < E[V]$. Then by Lemma 2, S has a negative bias.

Step 2— $(\Pr(S|V_1) - \Pr(S|V_2))(q_3^t - q_1^t) > 0$. It follows from the definition of buy herding and buy contrarianism that $E[V|S, H^t] > \text{ask}^t$. Since $E[V|H^t] < \text{ask}^t$, we must have $E[V|S, H^t] > E[V|H^t]$. By Lemma 1, this implies that (2) is positive at H^t . Also, by the negative bias (Step 1), the third term in (2) is negative. Therefore, the sum of the first two terms in (2) is positive: $q_3^t(\Pr(S|V_3) - \Pr(S|V_2)) + q_1^t(\Pr(S|V_2) - \Pr(S|V_1)) > 0$. But this means, by negative bias, that $(\Pr(S|V_1) - \Pr(S|V_2))(q_3^t - q_1^t) > 0$.

Step 3a—If S buy herds at H^t , then S is nU -shaped. It follows from the definition of buy herding that $E[V|H^t] > E[V]$. By Lemma 3, this implies that $q_3^t > q_1^t$. Then it follows from Step 2 that $\Pr(S|V_1) > \Pr(S|V_2)$. Also, since S buy herds, by Lemma 1, S cannot have a decreasing c.s.d. and we must have $\Pr(S|V_2) < \Pr(S|V_3)$. Thus, S is nU -shaped.

Step 3b—If S acts as a buy contrarian at H^t , then S is $nHill$ -shaped. It follows from the definition of buy contrarianism that $E[V|H^t] < E[V]$. By Lemma 3, this implies that $q_3^t < q_1^t$. But then it follows from Step 2 that $\Pr(S|V_1) < \Pr(S|V_2)$. Since, by Step 1, S has a negative bias, we have $\Pr(S|V_2) > \Pr(S|V_1) > \Pr(S|V_3)$. Thus S is $nHill$ -shaped.

A.2. Proof of Lemma 4

First consider the decision problem of type S at H^1 . If S has a negative bias, by Lemma 2, $E[V|S] < E[V]$. Also, $E[V] - \text{bid}^1 > 0$ and $\lim_{\mu \rightarrow 0} E[V] - \text{bid}^1 = 0$. We can thus establish the following lemma.

LEMMA 8: *If S has a negative bias, then there exists $\mu^i \in (0, 1]$ such that $E[V|S] - \text{bid}^1 < 0$ if and only if $\mu < \mu^i$.*

Simple calculations (see the Supplemental Material for details) establish the following useful characterization of the buying decision of type S at any H^t .

LEMMA 9: $E[V|S, H^t] - \text{ask}^t$ has the same sign as

$$(11) \quad q_1^t q_2^t [\beta_1^t \Pr(S|V_2) - \beta_2^t \Pr(S|V_1)] + q_2^t q_3^t [\beta_2^t \Pr(S|V_3) - \beta_3^t \Pr(S|V_2)] \\ + 2q_1^t q_3^t [\beta_1^t \Pr(S|V_3) - \beta_3^t \Pr(S|V_1)].$$

To establish buy herding or buy contrarianism, we need to show that (11) is positive at some history H^t . To analyze the sign of the expression in (11), we first show that the signs of the first and the second terms in (11) are, respectively, determined by the signs of the expressions $\Pr(S|V_2) - \Pr(S|V_1)$ and $\Pr(S|V_3) - \Pr(S|V_2)$ if and only if μ is sufficiently small. To establish this, let, for any $i = 1, 2$ and any signal type S' , $m^i \equiv \Pr(S|V_{i+1}) - \Pr(S|V_i)$, $M^i(S') \equiv \Pr(S'|V_i) \Pr(S|V_{i+1}) - \Pr(S'|V_{i+1}) \Pr(S|V_i)$, and

$$\mu_i(S') \equiv \begin{cases} \frac{m^i}{m^i - 3M^i(S')}, & \text{if } m^i \text{ and } M^i(S') \text{ are nonzero} \\ & \text{and have opposite signs,} \\ 1, & \text{otherwise.} \end{cases}$$

Clearly, $\mu_i(S') \in (0, 1]$. The next lemma shows that for some S' , $\mu_1(S')$ and $\mu_2(S')$ are, respectively, the critical bounds on the value of μ that characterize the signs of the first and the second terms in (11).

LEMMA 10: *In any equilibrium the following cases hold:*

(i) *Suppose that $\Pr(S|V_3) > \Pr(S|V_2)$. Then at any H^t at which S' buys and $S'' \neq S$, S' does not, the second term in (11) is positive if and only if $\mu < \mu_2(S')$.*

(ii) *Suppose that $\Pr(S|V_2) > \Pr(S|V_1)$. Then at any H^t at which S' buys and $S'' \neq S$, S' does not, the first term in (11) is positive if and only if $\mu < \mu_1(S')$.*

PROOF: First we establish (i). By simple computation, it follows that the second term in (11) equals $\gamma m^2 + \mu M^2(S')$. Also, since in this case $\Pr(S|V_3) > \Pr(S|V_2)$, we have from the definition of m^2 and $M^2(S')$ that $\gamma m^2 + \mu M^2(S') > 0$ if and only if $\mu < \mu_2(S')$. This completes the proof of (i).

The proofs of (ii) is analogous: By simple computation, it follows that the first term in (11) equals $\gamma m^1 + \mu M^1(S')$. Also, since in this case $\Pr(S|V_1) < \Pr(S|V_2)$, we have from the definition of m^1 and $M^1(S')$ that $\gamma m^1 + \mu M^1(S') > 0$ if and only if $\mu < \mu_1(S')$. *Q.E.D.*

Each of the two cases in Lemma 10 provides a set of conditions that determine the sign of one of the terms in (11). If the other terms in (11) are sufficiently small, then these conditions also determine whether S is a buyer. Specifically, if q_1^t is arbitrarily small relative to q_2^t and q_3^t , then the first and the last terms are close to zero (as they are multiplied by q_1^t and the second term is not) and can be ignored; thus, at such a history, type S buys if the second term in (11) is positive. Also, if q_3^t is arbitrarily small relative to q_1^t and q_2^t , then the last two terms are close to zero (as they are multiplied by q_3^t and the first term is not) and can be ignored; thus, at such history, type S buys if the first term in (11) is positive.

We can now prove Lemma 4 by appealing to Lemmas 3, 8, and 10 as follows:

PROOF OF LEMMA 4: Consider case (i). Let μ^i be as defined in Lemma 8 and let

$$(12) \quad \mu_{bh}^s \equiv \min_{S'} \mu_2(S').$$

Assume also that $\mu < \mu_{bh} \equiv \min\{\mu^i, \mu_{bh}^s\}$. Since by assumption S has a negative bias and $\mu < \mu^i$, it follows from Lemma 8 that S sells at the initial history. Also, since S is U-shaped, we have $\Pr(S|V_3) > \Pr(S|V_2)$. Therefore, by $\mu < \mu_{bh}^s$ and Lemma 10(i), there exists some $\eta > 0$ such that the second term in (11) always exceeds η .

By condition (3), there exists a history H^t such that $q_1^t/q_3^t < 1$ and $\frac{q_1^t}{q_3^t} + \frac{2q_1^t}{q_2^t} < \eta$. Then by the former inequality and Lemma 3, we have $E[V|H^t] > E[V]$. Also, since the sum of the first and the third terms in (11) is greater than $-q_2q_3(\frac{q_1^t}{q_3^t} + \frac{2q_1^t}{q_2^t})$, it follows from $\frac{q_1^t}{q_3^t} + \frac{2q_1^t}{q_2^t} < \eta$ that the sum must also be greater than $-\eta$. This, together with the second term in (11) exceeding η , implies that (11) is greater than zero and hence S must be buying at H^t .

The proof of (ii) is analogous. Let

$$(13) \quad \mu_{bc}^s \equiv \min_{S'} \mu_1(S')$$

and assume that $\mu < \mu_{bc} \equiv \min\{\mu^i, \mu_{bc}^s\}$. Then by the same reasoning as above, S sells at H^1 . Also, since S has a hill shape, we have $\Pr(S|V_1) > \Pr(S|V_2)$. Therefore, by $\mu < \mu_{bc}^s$ and Lemma 10(ii), there exists some $\eta > 0$ such that the first term in (11) always exceeds η .

By condition (4), there exists a history H^t such that $q_3^t/q_1^t < 1$ and $\frac{q_3^t}{q_1^t} + \frac{2q_3^t}{q_2^t} < \eta$. Then by the former inequality and Lemma 3, we have $E[V|H^t] < E[V]$. Also, since the sum of the second and the third terms in (11) is greater than $-q_1q_2(\frac{q_3^t}{q_1^t} + \frac{2q_3^t}{q_2^t})$, it follows from $\frac{q_3^t}{q_1^t} + \frac{2q_3^t}{q_2^t} < \eta$ that the sum must also be greater than $-\eta$. Since the first term in (11) exceeds η , this implies that (11) is greater than zero and hence S must be buying at H^t . Q.E.D.

A.3. Proof of Proposition 3

Below we provide a proof for part (a) of the proposition; the arguments for the other parts are analogous and, therefore, are omitted.

The proof of part (a) is by contradiction. Suppose that S is nU-shaped and that all the other assumptions in part (a) of the proposition hold. Also assume, contrary to the claim in part (a), that S does not buy herd. Then by Lemma 4(i), we have a contradiction if it can be shown that (3) holds. This is indeed what we establish in the rest of the proof.

First note that the no buy herding supposition implies that S does not buy at any history H^t . Otherwise, since S has a negative bias, by Step 2 in the proof

of Proposition 2, $(\Pr(S|V_1) - \Pr(S|V_2))(q_3^t - q_1^t) > 0$. Because S is U-shaped this implies that $q_3^t > q_1^t$; but then since by assumption $\mu < \mu^i$, it follows from Lemma 8 that S buy herds—a contradiction.

Next, we describe conditions that ensure that q_1^t/q_i^t is decreasing in t for any $l = 2, 3$. Denote an infinite path of actions by $H^\infty = \{a^1, a^2, \dots\}$. For any date t and any finite history $H^t = \{a^1, \dots, a^{t-1}\}$, let a_k^t be the action that would be taken by type $S_k \in \mathbb{S} \setminus S$ at H^t ; thus if the informed trader at date t receives a signal $S_k \in \mathbb{S} \setminus S$, then a^t , the actual action taken at H^t , equals a_k^t . Also denote the action taken by S at H^t by $a^t(S)$. Then we have the following lemma.

LEMMA 11: *Fix any infinite path $H^\infty = \{a^1, a^2, \dots\}$ and any signal $S_k \in \mathbb{S} \setminus S$. Let $S_{k'} \in \mathbb{S} \setminus S$ be such that $S_{k'} \neq S_k$. Suppose that $a^t = a_k^t$. Then for any date t and $l = 2, 3$, we have the following cases:*

- (A) *If $a_k^t = a_{k'}^t$, then q_1^t/q_i^t is decreasing.*
- (B) *If $a_k^t = a^t(S)$ and the inequality $\Pr(S_{k'}|V_l) \leq \Pr(S_k|V_l)$ holds, then q_1^t/q_i^t is nonincreasing. Furthermore, if the inequality is strict, then q_1^t/q_i^t is decreasing.*
- (C) *If $a_k^t \neq a_{k'}^t$, and $a_k^t \neq a^t(S)$ and the inequality $\Pr(S_k|V_l) \geq \Pr(S_{k'}|V_l)$ holds, then q_1^t/q_i^t is nonincreasing. Furthermore, if the inequality is strict, then q_1^t/q_i^t is decreasing.*

PROOF: Fix any $l = 2, 3$. Since $\frac{q_1^{t+1}}{q_i^{t+1}} = \frac{q_1^t \Pr(a^t|H^t, V_l)}{q_i^t \Pr(a^t|H^t, V_l)}$, to establish that q_1^t/q_i^t is decreasing, it suffices to show that $\Pr(a^t|H^t, V_l)$ is (greater) no less than $\Pr(a^t|H^t, V_l)$. Now consider each of the following three cases:

(A) Since signal S is nU-shaped, the combination of S_k and $S_{k'}$ is phill-shaped. This together with $a^t = a_k^t = a_{k'}^t$ implies that $\Pr(a^t|H^t, V_l)$ exceeds $\Pr(a^t|H^t, V_l)$.

(B) If $\Pr(S_{k'}|V_l) \leq \Pr(S_k|V_l)$, we have $\Pr(S_k|V_l) + \Pr(S|V_l) \geq \Pr(S_{k'}|V_l) + \Pr(S|V_l)$. This, together with $a^t = a_k^t = a^t(S)$ implies that $\Pr(a^t|H^t, V_l) \geq \Pr(a^t|H^t, V_l)$. Furthermore, the latter inequality must be strict if $\Pr(S_{k'}|V_l)$ were less than $\Pr(S_k|V_l)$.

(C) If $\Pr(S_k|V_l) \geq \Pr(S_{k'}|V_l)$, $a_k^t \neq a_{k'}^t$, and $a_k^t \neq a^t(S)$, we have immediately that $\Pr(a^t|H^t, V_l) \geq \Pr(a^t|H^t, V_l)$. Furthermore, the latter inequality is strict if $\Pr(S_{k'}|V_l)$ were less than $\Pr(S_k|V_l)$. This concludes the proof of Lemma 11.

Q.E.D.

Now we show that (3) holds and thereby obtain the required contradiction. This will be done for each feasible c.s.d. combination of signals.

Case A—Either There Exists a Signal That Is Decreasing or There Are Two Hill-Shaped Signals Each With a Nonnegative Bias: Consider an infinite path of actions consisting of an infinite number of buys. We demonstrate (3) by showing that along this infinite history, at any date t , both q_1^t/q_2^t and q_1^t/q_3^t are decreasing and hence converge to zero (note that there are a finite number of states and signals). We show this in several steps.

Step 1—If more than one informed type buys at t , then q_1^t/q_2^t and q_1^t/q_3^t are both decreasing at any t . Since S does not buy at any t , this follows immediately from Lemma 11(A).

Step 2—If exactly one informed type buys at period t , then (i) q_1^t/q_2^t is decreasing and (ii) q_1^t/q_3^t is decreasing if the informed type that buys has a nonzero bias and is nonincreasing otherwise. Let S_i be the only type that buys at t . This implies that S_i cannot be decreasing; therefore, by assumption, S_i must be phill-shaped and the step follows from Lemma 11(C).

Step 3—If a type has a zero bias, he cannot be a buyer at any date t . Suppose not. Then there exist a type S_i with a zero bias such that $E[V|H^t, S_i] - E[V|H^t] > 0$. By Lemma 1, we then have

$$(14) \quad [\Pr(S_i|V_3) - \Pr(S_i|V_2)](q_3^t - q_1^t) > 0.$$

Also, by Steps 1 and 2, q_1^t/q_3^t is nonincreasing at every t . Moreover, by assumption, $q_1^t/q_3^t = 1$. Therefore, $q_1^t/q_3^t \leq 1$. Since S_i buys at t , S_i must be hill-shaped, contradicting (14).

Step 4— q_1^t/q_2^t and q_1^t/q_3^t are both decreasing at any t . This follows Steps 1–3.

Case B—There Exists an Increasing S_i Such That $\Pr(S_i|V_k) \neq \Pr(S_i|V_{k'})$ for Some k and k' : Let S_j be the third signal other than S and S_i . Now we obtain (3) in two steps.

Step 1—If $\Pr(S_i|V_1) = \Pr(S_i|V_2)$, then for any $\varepsilon > 0$, there exists a finite history $H^t = \{a^1, \dots, a^{t-1}\}$ such that $q_1^t/q_2^t < \varepsilon$. Consider an infinite path $H^\infty = \{a^1, a^2, \dots\}$ such that $a^t = a_j^t$ (recall that a_j^t is the action taken by S_j at history $H^t = (a^1, \dots, a^{t-1})$). Note that S is nU-shaped and $\Pr(S_i|V_1) = \Pr(S_i|V_2) < \Pr(S_i|V_3)$. Therefore, $\Pr(S_j|V_2) > \max\{\Pr(S_j|V_1), \Pr(S_j|V_3)\}$.

Then it follows from Lemma 11 that q_1^t/q_2^t is decreasing if $a^t \neq a^t(S)$ and it is constant if $a^t = a^t(S)$. To establish the claim, it suffices to show that $a^t \neq a^t(S)$ infinitely often. Suppose not. Then there exists T such that for all $t > T$, $a_j^t = a^t(S)$. Since type S does not buy at any date and there cannot be more than one informed type holding at any date (there is always a buyer or a seller), we must have S_j (and S) selling at every $t > T$. Then, by Lemma 1, we have

$$(15) \quad \frac{q_3^t}{q_1^t} [\Pr(S_j|V_3) - \Pr(S_j|V_2)] + [\Pr(S_j|V_2) - \Pr(S_j|V_1)] \\ + 2 \frac{q_3^t}{q_2^t} [\Pr(S_j|V_3) - \Pr(S_j|V_1)] < 0$$

for all $t > T$. Also, by $\Pr(S_i|V_1) = \Pr(S_i|V_2) < \Pr(S_i|V_3)$, we have $\Pr(S_j|V_l) + \Pr(S|V_l) > \Pr(S_j|V_3) + \Pr(S|V_3)$ for $l = 1, 2$. Therefore, $\frac{q_3^t}{q_1^t} \rightarrow 0$ as $t \rightarrow \infty$ for any $l = 1, 2$. This, together with $\Pr(S_j|V_2) > \Pr(S_j|V_1)$, contradicts (15).

Step 2—For any $\varepsilon > 0$, there exists a history H^t such that $q_1^t/q_2^t < \varepsilon$ for any $l = 2, 3$. Fix any $\varepsilon > 0$. Let H^t be such that $q_1^t/q_2^t < \varepsilon$ if $\Pr(S_i|V_1) = \Pr(S_i|V_2)$ (by

the previous step such a history exists) and be the empty history H^1 otherwise. Consider any infinite path $H^\infty = \{H^\tau, a^\tau, a^{\tau+1}, \dots\}$, where for any $t \geq \tau$, a^t is the action that type S_i takes at history $H^t = \{H^\tau, a^\tau, \dots, a^{t-1}\}$; that is, we first have the history H^τ and then we look at a subsequent history that consists only of the actions that type S_i takes.

Since S_i is increasing, it follows from Proposition 1 that at any history, S_i does not sell. Also by the supposition, S does not buy at any history. Therefore, S_i and S always differ at every history H^t with $t \geq \tau$ (there cannot be more than one type holding). But since a^t is the action that type S_i takes at history H^t , S_i is increasing and $\Pr(S_i|V_k) \neq \Pr(S_i|V_{k'})$ for some k and k' , it then follows from parts (A) and (C) of Lemma 11 that for every $t \geq \tau$, (i) $\frac{q_1^t}{q_3^t}$ is decreasing and (ii) $\frac{q_1^t}{q_2^t}$ is nonincreasing. This, together with $q_1^\tau/q_2^\tau < \varepsilon$ when $\Pr(S_i|V_1) = \Pr(S_i|V_2)$, establishes that there exists t such that $q_1^t/q_l^t < \varepsilon$ for any $l = 2, 3$.

Case C: There Are Two Hill-Shaped Signals and One Has a Negative Bias: Let S_i be the hill-shaped signal with the negative bias. Also, let S_j be the other hill-shaped signal. Since both S and S_i have negative biases, S_j must have a positive bias.

Next fix any $\varepsilon > 0$, and define y and φ_{lm} for any $l, m = 1, 2, 3$, as

$$(16) \quad y := \frac{[\Pr(S_i|V_2) - \Pr(S_i|V_1)]}{2[\Pr(S_i|V_1) - \Pr(S_i|V_3)]} > 0,$$

$$\varphi_{lm} := \max \left\{ \frac{\gamma + \mu \Pr(S_i|V_l)}{\gamma + \mu \Pr(S_i|V_m)}, \frac{\gamma + \mu(1 - \Pr(S|V_l))}{\gamma + \mu(1 - \Pr(S|V_m))}, \frac{\gamma + \mu \Pr(S_j|V_l)}{\gamma + \mu \Pr(S_j|V_m)} \right\}.$$

Since both S_i and S_j are hill-shaped, we have $\varphi_{12} < 1$. This implies that there exists an integer $M > 0$ and $\delta \in (0, \varepsilon)$ such that $y(\varphi_{12})^M < \varepsilon$ and $\delta(\varphi_{13})^M < \varepsilon$.

Consider the infinite path $H^\infty = \{a^1, a^2, \dots\}$, where $a^t = a_j^t$ at every t . Then we make the following claims.

Claim 1— q_1^t/q_3^t is decreasing at every t . As S_i and S_j have a negative and a positive bias, respectively, by Lemma 11, q_1^t/q_3^t is decreasing at every t .

Claim 2— q_1^t/q_2^t converge to zero if there exists T such that $a_i^t \neq a_j^t$ for all $t > T$. Since S_j is hill-shaped, this follows immediately from parts (A) and (C) of Lemma 11.

Claim 3—There exists a history H^τ such that $q_1^\tau/q_3^\tau < \delta$ and $q_1^\tau/q_2^\tau < y$. Suppose not. Then by Claims 1 and 2, there exists a date τ such that $q_1^\tau/q_3^\tau < \delta$ and $a_i^\tau = a_j^\tau$. Since S does not buy at any history, it follows that S_i and S_j must be buying at τ (there is always at least one buyer and seller; thus S_i and S_j cannot both be holding at τ). Then $E[V|S_i, H^\tau] - E[V|H^\tau] > 0$. By Proposition 2, this

implies

$$\begin{aligned}
 & [\Pr(S_i|V_3) - \Pr(S_i|V_2)] + \frac{q_1^\tau}{q_3^\tau} [\Pr(S_i|V_2) - \Pr(S_i|V_1)] \\
 & + \frac{2q_1^\tau}{q_2^\tau} [\Pr(S_i|V_3) - \Pr(S_i|V_1)] > 0.
 \end{aligned}$$

Since S_i is nhill-shaped, it follows from the last inequality that

$$\begin{aligned}
 (17) \quad \frac{q_1^\tau}{q_2^\tau} & < \frac{[\Pr(S_i|V_3) - \Pr(S_i|V_2)] + \frac{q_1^\tau}{q_3^\tau} [\Pr(S_i|V_2) - \Pr(S_i|V_1)]}{2[\Pr(S_i|V_1) - \Pr(S_i|V_3)]} \\
 & < \frac{\frac{q_1^\tau}{q_3^\tau} [\Pr(S_i|V_2) - \Pr(S_i|V_1)]}{2[\Pr(S_i|V_1) - \Pr(S_i|V_3)]}.
 \end{aligned}$$

As $q_1^\tau/q_3^\tau < \delta$ and $\delta < 1$, we have $q_1^\tau/q_2^\tau < y$. This contradicts the supposition.

To complete the proof for this case, fix any τ and H^τ such that $q_1^\tau/q_3^\tau < \delta$ and $q_1^\tau/q_2^\tau < y$ (by Claim 3, such a history exists). Consider a history \bar{H}^t that consists of path $H^\tau = (a^1, \dots, a^{\tau-1})$ followed by M periods of buys. Thus $t = \tau + M$ and $H^t = \{H^\tau, \bar{a}^1, \dots, \bar{a}^M\}$, where for any $m \leq M$, $\bar{a}^m = \text{buy}$. Since a buy must be either from S_j or S_i or both, it then follows from the definitions of φ_{13} , M , and δ , and from $q_1^\tau/q_3^\tau < \delta$ that

$$(18) \quad q_1^t/q_3^t \leq (\varphi_{13})^M (q_1^\tau/q_3^\tau) < (\varphi_{13})^M \delta < \varepsilon.$$

Also, since $q_1^\tau/q_2^\tau < y$, we have

$$(19) \quad q_1^t/q_2^t < (\varphi_{12})^M (q_1^\tau/q_2^\tau) < (\varphi_{12})^M y < \varepsilon.$$

Since the initial choice of ε was arbitrary, (3) follows immediately from (18) and (19).

Case D—There Exists a U-Shaped Signal $S_i \in \mathbb{S} \setminus S$: Since both S and S_i are U-shaped it follows that the third signal S_j is hill-shaped. Moreover, by assumption, S_j must have a nonnegative bias. To establish (3), fix any $\varepsilon > 0$ and consider the two possible subcases that may arise.

Subcase D1— S_i Has a Zero Bias: We establish the result using two claims.

*Claim 1—*There exists a history H^τ such that $q_1^\tau/q_3^\tau < \varepsilon$. Consider the infinite path $H^\infty = \{a^1, a^2, \dots\}$ such that $a^t = \text{buy}$ for each t . Since a buy must be from either S_j or S_i or both, and S_i has a zero bias, it follows from parts (A) and (C) of Lemma 11 that q_1^t/q_3^t is nonincreasing at every t . Furthermore, q_1^t/q_3^t is decreasing if $a^t = a_j^t$. Therefore, the claim follows if S_j buys infinitely often along the path H^∞ . To show that the latter is true, suppose it is not. Then

there exists T such that for all $t \geq T$, $a_j^t \neq a_i^t = \text{buy}$. Then for all $t > T$, by Lemma 9,

$$(20) \quad \frac{q_2^t}{q_1^t} [\beta_2^t \Pr(S_j|V_3) - \beta_3^t \Pr(S_j|V_2)] + \frac{q_2^t}{q_3^t} [\beta_1^t \Pr(S_j|V_2) - \beta_2^t \Pr(S_j|V_1)] + [\beta_1^t \Pr(S_j|V_3) - \beta_3^t \Pr(S_j|V_1)] < 0.$$

Also, since S_i is U-shaped, both $\frac{q_2^t}{q_1^t}$ and $\frac{q_2^t}{q_3^t}$ must be decreasing at every $t > T$. But this is a contradiction because at every $t > T$, the last term in (20) is positive: $\beta_1^t \Pr(S_j|V_3) - \beta_3^t \Pr(S_j|V_1) = \gamma(\Pr(S_j|V_3) - \Pr(S_j|V_1)) > 0$ (the equality follows from S_i 's zero bias).

Claim 2—There exists a history H^t such that $q_1^t/q_l^t < \varepsilon$ for all $l = 2, 3$. By the previous claim, there exists a history H^τ such that $q_1^\tau/q_3^\tau < \varepsilon$. Next consider a history $H^\infty = \{H^\tau, a^\tau, a^{\tau+1}, \dots\}$ that consists of path H^τ followed by a sequence of actions $\{a^\tau, a^{\tau+1}, \dots\}$ such that $a^t = a_j^t$ at every history $H^t = \{H^\tau, a^\tau, \dots, a^{t-1}\}$. Since S_i has a zero bias, it follows from Lemma 11 that at every $t > \tau$, q_1^t/q_3^t is nonincreasing. Also, we have $q_1^\tau/q_3^\tau < \varepsilon$; therefore we have that at every $t > \tau$, $q_1^t/q_3^t < \varepsilon$. Furthermore, since S and S_i are U-shaped, and S_j is hill-shaped, by Lemma 11, q_1^t/q_2^t is decreasing at every $t > \tau$; hence there must exist $t > \tau$ such that $q_1^t/q_2^t < \varepsilon$.

Since the initial choice of ε was arbitrary, (3) follows from Claim 2.

Subcase D2—Both S_i And S_j Have Nonzero Bias: Consider first the infinite path $H^\infty = \{a^1, a^2, \dots\}$ such that $a^t = a_j^t$ at every history $H^t = \{a^1, \dots, a^{t-1}\}$. Then the following claims must hold.

Claim 1— q_1^t/q_2^t is decreasing at every t . Since S_j and S_i are, respectively, hill-shaped and U-shaped, it follows from Lemma 11 that q_1^t/q_2^t is decreasing.

Claim 2—If S_i has a negative bias, then q_1^t/q_3^t is decreasing at every t . Since S_j has a positive bias and S_i has a negative bias, by Lemma 11, q_1^t/q_3^t is decreasing at every t .

Claim 3—If S_j has a positive bias and there exists a period T such that for all $t > T$, $a_j^t = \text{buy}$, then q_1^t/q_3^t is decreasing at every $t > T$. Since S_j has a positive bias and S does not buy at any date, by Lemma 11, q_1^t/q_3^t must be decreasing at every $t > T$.

Before stating the next claim, consider φ_{ml} defined in (16). If both S_i and S_j have positive biases, $\varphi_{13} < 1$. Thus, if S_i has a positive bias, there exists an integer M such that

$$(21) \quad \begin{aligned} \text{(i)} \quad & (\varphi_{ml})^M < \varepsilon \text{ if } S_j \text{ has a positive bias and} \\ \text{(ii)} \quad & \left(\frac{\gamma + \mu \Pr(S_i|V_l)}{\gamma + \mu \Pr(S_i|V_m)} \right)^M < \varepsilon. \end{aligned}$$

Fix any such M . Then there also exists $\delta \in (0, \varepsilon)$ such that

$$(22) \quad \delta(\varphi_{12})^M < \varepsilon.$$

Claim 4—If S_j has a zero bias, then there exists a history H^τ such that $q_1^\tau/q_2^\tau < \delta$ and $q_1^\tau/q_3^\tau = 1$. Since $q_1^1/q_3^1 = 1$, it follows that at date 1, S_j holds. By recursion, it follows that at every history $H^t = \{a^1, \dots, a^{t-1}\}$, we have $q_1^t/q_3^t = 1$ and the claim follows from Claim 1.

Claim 5—If both S_i and S_j have positive biases, then there exists a history H^τ such that $q_1^\tau/q_2^\tau < \delta$ and $q_1^\tau/q_3^\tau < x$, where δ satisfies (21) and

$$x \equiv \frac{[\Pr(S_i|V_3) - \Pr(S_i|V_2)] + 2\varepsilon[\Pr(S_i|V_3) - \Pr(S_i|V_1)]}{[\Pr(S_i|V_1) - \Pr(S_i|V_2)]}.$$

Suppose not. Then by Claims 1 and 3, there exists date τ such that $q_1^\tau/q_2^\tau < \delta$ and $a_j^\tau \neq \text{buy}$. Since S also does not buy at H^τ , it follows that only S_i buys at τ . Then $E[V|S_i, H^\tau] > E[V|H^\tau]$. By Proposition 2, this implies

$$\begin{aligned} & [\Pr(S_i|V_3) - \Pr(S_i|V_2)] + \frac{q_1^\tau}{q_3^\tau} [\Pr(S_i|V_2) - \Pr(S_i|V_1)] \\ & + \frac{2q_1^\tau}{q_2^\tau} [\Pr(S_i|V_3) - \Pr(S_i|V_1)] > 0. \end{aligned}$$

Since $q_1^\tau/q_2^\tau < \delta < \varepsilon$ and S_i is pU-shaped, we can rearrange the above expression to show that

$$\begin{aligned} \frac{q_1^\tau}{q_3^\tau} & \leq \frac{[\Pr(S_i|V_3) - \Pr(S_i|V_2)] + \frac{2q_1^\tau}{q_2^\tau} [\Pr(S_i|V_3) - \Pr(S_i|V_1)]}{\Pr(S_i|V_1) - \Pr(S_i|V_2)} \\ & < \frac{[\Pr(S_i|V_3) - \Pr(S_i|V_2)] + 2\varepsilon[\Pr(S_i|V_3) - \Pr(S_i|V_1)]}{[\Pr(S_i|V_1) - \Pr(S_i|V_2)]} = x. \end{aligned}$$

Claim 6—If S_i has a positive bias, then there exists a history \bar{H}^t such that $q_1^l/q_l^l < \varepsilon$ for any $l = 2, 3$. Fix any history $H^\tau = (a^1, \dots, a^{\tau-1})$ such that $q_1^\tau/q_2^\tau < \delta$ and $q_1^\tau/q_3^\tau = 1$ if S_j has a zero bias, and $q_1^\tau/q_2^\tau < \delta$ and $q_1^\tau/q_3^\tau < x$ if S_j has a positive bias (by Claims 4 and 5 such histories exist). Next, consider a history \bar{H}^t that consists of path H^τ followed by M periods of buys. Thus $t = \tau + M$ and $H^t = \{h^\tau, \bar{a}^1, \dots, \bar{a}^M\}$, where for any $m \leq M$, $\bar{a}^m = \text{buy}$. Since a buy must be either from S_j or S_i or both, it then follows from the definitions of φ_{12} in (16), from (22), and from $q_1^\tau/q_2^\tau < \delta$ that $\frac{q_1^t}{q_2^t} \leq \frac{q_1^\tau}{q_2^\tau} (\varphi_{12})^M < \delta (\varphi_{12})^M < \varepsilon$. To show that $q_1^t/q_3^t < \varepsilon$, consider the two cases of S_j having a zero bias and S_j having a positive bias separately. In the latter case, we have $q_1^\tau/q_3^\tau < x$. Then, by (i) in (21), we have $\frac{q_1^t}{q_3^t} < \frac{q_1^\tau}{q_3^\tau} (\varphi_{13})^M < x (\varphi_{13})^M < \varepsilon$. In the former case, since $q_1^\tau/q_3^\tau = 1$, it must be that $\bar{a}^m = a_i^{\tau+m} \neq a_j^{\tau+m}$. Hence, $\frac{q_1^t}{q_3^t} < 1$. Re-

cursively, it then follows that $\bar{a}^{m'} = a_i^{t+1} \neq a_j^{\tau+m'}$ for all $m' \leq m$. Thus, by (ii) in (21), $\frac{q_1^t}{q_3^t} < \frac{q_1^t}{q_3^t} \left(\frac{\gamma + \mu \Pr(S_i|V_i)}{\gamma + \mu \Pr(S_i|V_m)} \right)^M < \varepsilon$.

Since the initial choice of ε was arbitrary, (3) follows from Claims 1, 2, and 5.

A.4. Proof of Proposition 4

(a) At H^t , buy herding occurs if and only if $E[V|S, H^t] - \text{ask}^t > 0$ and $E[V|H^t] - E[V] > 0$. Thus, to demonstrate the existence of the function \bar{s} , we need to characterize the expressions $E[V|S, H^t] - \text{ask}^t$ and $E[V|H^t] - E[V]$ for different values of b and s .

Let $\beta_i = \Pr(\text{buy}|V_i)$ and $\sigma_i = \Pr(\text{sale}|V_i)$ at every buy herding history (these probabilities are always the same at every history at which S buy herds). Note that, by Lemma 9, $E[V|S, H^t] - \text{ask}^t$ has the same sign as

$$(23) \quad \begin{aligned} & \left(\frac{\beta_1}{\beta_3} \right)^b \left(\frac{\sigma_1}{\sigma_3} \right)^s q_2^t q_1^t [\beta_1 \Pr(S|V_2) - \beta_2 \Pr(S|V_1)] \\ & + q_3^t q_2^t [\beta_2 \Pr(S|V_3) - \beta_3 \Pr(S|V_2)] \\ & + 2 \left(\frac{\beta_1}{\beta_2} \right)^b \left(\frac{\sigma_1}{\sigma_2} \right)^s q_3^t q_1^t [\beta_1 \Pr(S|V_3) - \beta_3 \Pr(S|V_1)]. \end{aligned}$$

Also, by MLRP and Lemma 5(iv), we have

$$(24) \quad \beta_1 < \beta_2 < \beta_3 \quad \text{and} \quad \sigma_3 < \sigma_2 < \sigma_1.$$

Since by Proposition 2, S must have an nU-shaped c.s.d., it then follows that

$$(25) \quad \begin{aligned} & \beta_1 \Pr(S|V_2) - \beta_2 \Pr(S|V_1) < 0, \quad \beta_1 \Pr(S|V_3) - \beta_3 \Pr(S|V_1) < 0, \\ & \beta_2 \Pr(S|V_3) - \beta_3 \Pr(S|V_2) > 0. \end{aligned}$$

(The last inequality in (25) follows from the first two and from (11) being positive at H^t .) Thus, the first and the third terms in (23) are negative, and the second is positive. Hence it follows from (24) that the expression in (23) satisfies the following three properties: (i) it increases in b , (ii) it decreases in s , and (iii) for any b , it is negative for sufficiently large s . By (24), the expression $E[V|H^t] - E[V]$ must also satisfy (i)–(iii) (note that q_3^t/q_1^t is increasing in b and decreasing in s). Since (23) and $E[V|H^t] - E[V]$ are both increasing in b and since, by assumption, there is buy herding at H^t , it must be that for any b , both (23) and $E[V|H^t] - E[V]$ are positive when $s = 0$. Thus, it follows from (ii) and (iii) that for any b , there exists an integer $\bar{s} > 1$ such that both (23) and $E[V|H^t] - E[V]$ are positive for any integer $s < \bar{s}$, and either (23) or $E[V|H^t] - E[V]$ is nonpositive for any integer $s \geq \bar{s}$.

To complete the proof of this part, we need to show that \bar{s} is increasing in b . To show this, suppose otherwise; then there exist b' and b'' such that $b' < b''$ and $\bar{s}' > \bar{s}''$, where \bar{s}' and \bar{s}'' are, respectively, the critical values of sales corresponding to b' and b'' described in the previous paragraph. Now since $\bar{s}' > \bar{s}''$, it follows that both (23) and $E(V|H') - E(V)$ are positive if $b = b'$ and $s = \bar{s}'$. But since both (23) and $E(V|H') - E(V)$ are increasing in b , we must then have that both (23) and $E(V|H') - E(V)$ are positive if $b = b''$ and $s = \bar{s}''$. By the definition of \bar{s}'' , this is a contradiction.

(b) At H' , buy contrarianism occurs if and only if $E[V|S, H'] - ask' > 0$ and $E[V] - E[V|H'] > 0$. By Lemma 9, $E[V|S, H'] - ask'$ has the same sign as

$$(26) \quad q_2^t q_1^t [\beta_1 \Pr(S|V_2) - \beta_2 \Pr(S|V_1)] \\ + \left(\frac{\beta_3}{\beta_1}\right)^b \left(\frac{\sigma_3}{\sigma_1}\right)^s q_3^t q_2^t [\beta_2 \Pr(S|V_3) - \beta_3 \Pr(S|V_2)] \\ + 2\left(\frac{\beta_3}{\beta_2}\right)^b \left(\frac{\sigma_3}{\sigma_2}\right)^s q_3^t q_1^t [\beta_1 \Pr(S|V_3) - \beta_3 \Pr(S|V_1)].$$

Also, with buy contrarianism, S must have an nhill-shaped c.s.d. and, therefore, $\beta_1 \Pr(S|V_2) - \beta_2 \Pr(S|V_1) > 0$, $\beta_1 \Pr(S|V_3) - \beta_3 \Pr(S|V_1) < 0$, and $\beta_2 \Pr(S|V_3) - \beta_3 \Pr(S|V_2) < 0$. Thus, the second and the third terms in (26) are negative, and the first is positive. Hence, by (24), the expression in (26) satisfies the following properties: (i) it increases in s , (ii) it decreases in b , and (iii) for each s , it is negative for sufficiently large b .

The expression $E(V) - E(V|H')$ also satisfies the same three properties. The existence of the function \bar{b} is now analogous to that for part (a), with reversed roles for buys and sales.

A.5. Proof of Proposition 5

We show the proof for buy herding; the proof for buy contrarianism is analogous.

(a) In the proof of Proposition 4, we have shown for the case of buy herding that if the history following H' consists only of buys, then type S herds at any point during that history. What remains to be shown is that for an arbitrary number of buys after herding has started, the price will approach V_3 . Observe that $E[V|H'] = \sum_i V_i q_i^t = q_3^t (\frac{q_2^t}{q_3^t} V_2 + V_3)$. Also, q_2^t/q_3^t is arbitrarily small at any history H' that includes a sufficiently large number of buys as outlined following conditions (5) and (6). Consequently, for every $\varepsilon > 0$, there exists a history H' consisting of H' followed by sufficiently many buys such that $E[V|H'] > V_3 - \varepsilon$.

(b) Since the assumptions of Theorem 2 that ensure buy herding hold, S_2 must be selling initially and also $\mu < \mu_{bh}^s \leq \mu_2(S_3)$. The latter implies that there exists $\eta > 0$ such that

$$(27) \quad [\beta_2^t \Pr(S_2|V_3) - \beta_3^t \Pr(S_2|V_2)] > \eta \quad \text{for every } t.$$

By MLRP, type S_1 does not buy at any history. Therefore, for any history H^r consisting only of $r - 1$ buys, it must be that $q_1^r/q_3^r \leq (\max\{\frac{\gamma+\mu \Pr(S_3|V_1)}{\gamma+\mu \Pr(S_3|V_3)}, \frac{\gamma+\mu(1-\Pr(S_1|V_1))}{\gamma+\mu(1-\Pr(S_1|V_3))}\})^{r-1}$. Also, by MLRP, S_3 is strictly increasing and S_1 is strictly decreasing. Thus, there must exist $r > 1$ such that $q_1^r/q_3^r < \eta/2$. Fix one such r . Then it follows from (27) that

$$(28) \quad [\beta_2^r \Pr(S_2|V_3) - \beta_3^r \Pr(S_2|V_2)] \\ + \frac{q_1^r}{q_3^r} [\beta_1^r \Pr(S_2|V_2) - \beta_2^r \Pr(S_2|V_1)] > \eta/2.$$

Next, fix any $\varepsilon > 0$. Note that there exists $\delta > 0$ such that if $q_2^1 > 1 - \delta$, then $\text{ask}^r = E[V|H^r, \text{buy}] = q_2^r V_2 + q_3^r V_3 \in (V_2, V_2 + \varepsilon)$ and

$$(29) \quad 2 \frac{q_1^r}{q_2^r} [\beta_3^r \Pr(S|V_1) - \beta_1^r \Pr(S|V_3)] < \eta/2.$$

Fix any such δ . Then it follows from (28) and (29) that

$$q_2^r q_3^r [\beta_2^r \Pr(S_2|V_3) - \beta_3^r \Pr(S_2|V_2)] + q_1^r q_2^r [\beta_1^r \Pr(S_2|V_2) - \beta_2^r \Pr(S_2|V_1)] \\ + 2q_1^r q_3^r [\beta_1^r \Pr(S_2|V_3) - \beta_3^r \Pr(S_2|V_1)] > 0.$$

Since S_2 sells initially and $q_1^r/q_3^r < \eta/2 < 1$, it follows from the last inequality that S_2 is buy herding at H^r at an ask price that belongs to the interval $(V_2, V_2 + \varepsilon)$.

Next, as shown in part (a), there must also exist a history H^t with $t = r + b$ following H^r such that there is buy herding at any history H^τ , $r \leq \tau \leq t$, and $E[V|H^t] > V_3 - \varepsilon$.

A.6. Proof of Proposition 6

We shall prove the two results for the case of buy herding; the proof for the buy contrarian case is analogous and will be omitted.

PROOF OF PART I(A) OF PROPOSITION 6: Let β_i and σ_i be, respectively, the probability of a buy and the probability of a sale in the transparent world at any date $\tau = r, \dots, r + b + s$. Also, let $\beta_{i,o}$ and $\sigma_{i,o}$ be the analogous probabilities

in the opaque world. Then

$$\begin{aligned} E[V|H^t] - E_o[V|H^t] &= \mathcal{V}\{(q_2^t - q_{2,o}^t) + 2(q_3^t - q_{3,o}^t)\} \\ &= \mathcal{V}\left\{q_2^t \left(\frac{\beta_2^b \sigma_2^s}{\sum_i q_i^t \beta_i^b \sigma_i^s} - \frac{\beta_{2,o}^b \sigma_{2,o}^s}{\sum_i q_i^t \beta_{i,o}^b \sigma_{i,o}^s} \right) \right. \\ &\quad \left. + 2q_3^t \left(\frac{\beta_3^b \sigma_3^s}{\sum_i q_i^t \beta_i^b \sigma_i^s} - \frac{\beta_{3,o}^b \sigma_{3,o}^s}{\sum_i q_i^t \beta_{i,o}^b \sigma_{i,o}^s} \right) \right\}. \end{aligned}$$

Therefore, $E[V|H^t] - E_o[V|H^t]$ has the same sign as

$$\begin{aligned} (30) \quad & q_2^t q_1^t [(\beta_2 \beta_{1,o})^b (\sigma_2 \sigma_{1,o})^s - (\beta_{2,o} \beta_1)^b (\sigma_{2,o} \sigma_1)^s] \\ & + q_3^t q_2^t [(\beta_3 \beta_{2,o})^b (\sigma_3 \sigma_{2,o})^s - (\beta_{3,o} \beta_2)^b (\sigma_{3,o} \sigma_2)^s] \\ & + 2q_3^t q_1^t [(\beta_3 \beta_{1,o})^b (\sigma_3 \sigma_{1,o})^s - (\beta_{3,o} \beta_1)^b (\sigma_{3,o} \sigma_1)^s]. \end{aligned}$$

Suppose that S buy herds at H^t . Then, by Lemma 9, we have

$$\begin{aligned} (31) \quad & q_2^t q_1^t [\beta_1 \Pr(S|V_2) - \beta_2 \Pr(S|V_1)] + q_3^t q_2^t [\beta_2 \Pr(S|V_3) - \beta_3 \Pr(S|V_2)] \\ & + 2 q_3^t q_1^t [\beta_1 \Pr(S|V_3) - \beta_3 \Pr(S|V_1)] > 0. \end{aligned}$$

By simple computation we also have

$$\begin{aligned} (32) \quad & \beta_2 \beta_{1,o} - \beta_{2,o} \beta_1 = \mu [\beta_1 \Pr(S|V_2) - \beta_2 \Pr(S|V_1)], \\ & \beta_3 \beta_{1,o} - \beta_{3,o} \beta_1 = \mu [\beta_1 \Pr(S|V_3) - \beta_3 \Pr(S|V_1)], \\ & \beta_3 \beta_{2,o} - \beta_{3,o} \beta_2 = \mu [\beta_2 \Pr(S|V_3) - \beta_3 \Pr(S|V_2)]. \end{aligned}$$

Therefore, it follows from (31) that

$$\begin{aligned} (33) \quad & q_2^t q_1^t [\beta_2 \beta_{1,o} - \beta_{2,o} \beta_1] + q_3^t q_2^t [\beta_3 \beta_{2,o} - \beta_{3,o} \beta_2] \\ & + 2q_3^t q_1^t [\beta_3 \beta_{1,o} - \beta_{3,o} \beta_1] > 0. \end{aligned}$$

To prove I(a) in Proposition 6, suppose that $s = 0$ (thus $t = b$). Then by expanding (30), it must be that $E[V|H^t] - E_o[V|H^t]$ has the same sign as

$$\begin{aligned} (34) \quad & q_2^t q_1^t \left\{ (\beta_2 \beta_{1,o} - \beta_{2,o} \beta_1) \sum_{\tau=0}^{b-1} (\beta_2 \beta_{1,o})^{b-1-\tau} (\beta_{2,o} \beta_1)^\tau \right\} \\ & + q_3^t q_2^t [(\beta_3 \beta_{2,o}) - (\beta_{3,o} \beta_2)] \sum_{\tau=0}^{b-1} (\beta_3 \beta_{2,o})^{b-1-\tau} (\beta_{3,o} \beta_2)^\tau \end{aligned}$$

$$+ 2q_3^r q_1^r \left\{ (\beta_3 \beta_{1,o} - \beta_{3,o} \beta_1) \sum_{\tau=0}^{b-1} (\beta_3 \beta_{1,o})^{b-1-\tau} (\beta_{3,o} \beta_1)^\tau \right\}.$$

Also, by MLRP, $\beta_3 > \beta_2 > \beta_1$ and $\beta_{3,o} > \beta_{2,o} > \beta_{1,o}$. Therefore,

$$(35) \quad \sum_{\tau=0}^{b-1} (\beta_3 \beta_{2,o})^{b-1-\tau} (\beta_{3,o} \beta_2)^\tau > \sum_{\tau=0}^{b-1} (\beta_2 \beta_{1,o})^{b-1-\tau} (\beta_{2,o} \beta_1)^\tau,$$

$$(36) \quad \sum_{\tau=0}^{b-1} (\beta_3 \beta_{2,o})^{b-1-\tau} (\beta_{3,o} \beta_2)^\tau > \sum_{\tau=0}^{b-1} (\beta_3 \beta_{1,o})^{b-1-\tau} (\beta_{3,o} \beta_1)^\tau.$$

Also, by (25) and (32), the first and the third terms in (34) are negative and the second is positive. Therefore, by (33), (35), and (36), $E[V|H^t] - E_o[V|H^t] > 0$ for $s = 0$. This completes the proof of part I(a) of Proposition 6.⁴⁶ *Q.E.D.*

PROOF OF PART I(B) OF PROPOSITION 6: Suppose that $b = 0$ and $s = 1$ ($t = r + 1$). Since S buys in the transparent world, $E[V|S, H^r] - \text{bid}^r > 0$. Simple computations analogous to the proof of Lemma 9 show that this is equivalent to

$$(37) \quad q_2^r q_1^r [\sigma_1 \Pr(S|V_2) - \sigma_2 \Pr(S|V_1)] + q_3^r q_2^r [\sigma_2 \Pr(S|V_3) - \sigma_3 \Pr(S|V_2)] + 2q_3^r q_1^r [\sigma_1 \Pr(S|V_3) - \sigma_3 \Pr(S|V_1)] > 0.$$

Also, by the definition of σ_i and σ_i , we have

$$(38) \quad \begin{aligned} \sigma_3 \sigma_{2,o} - \sigma_{3,o} \sigma_2 &= -\mu [\sigma_2 \Pr(S|V_3) - \sigma_3 \Pr(S|V_2)], \\ \sigma_3 \sigma_{1,o} - \sigma_{3,o} \sigma_1 &= -\mu [\sigma_1 \Pr(S|V_3) - \sigma_3 \Pr(S|V_1)], \\ \sigma_2 \sigma_{1,o} - \sigma_{2,o} \sigma_1 &= -\mu [\sigma_1 \Pr(S|V_2) - \sigma_2 \Pr(S|V_1)]. \end{aligned}$$

Therefore, (37) is equivalent to

$$(39) \quad q_2^r q_1^r [\sigma_2 \sigma_{1,o} - \sigma_{2,o} \sigma_1] + q_3^r q_2^r [\sigma_3 \sigma_{2,o} - \sigma_{3,o} \sigma_2] + 2q_3^r q_1^r [\sigma_3 \sigma_{1,o} - \sigma_{3,o} \sigma_1] < 0.$$

Since the left hand side of (39) is the same as the expression in (30) when $b = 0$ and $s = 1$, it follows that in this case, $E[V|H^t] - E_o[V|H^t] < 0$.⁴⁷ This completes the proof of this part. *Q.E.D.*

⁴⁶Note that MLRP is assumed in the above proof so as to establish conditions (35), and (36). When $b = 1$ and $s = 0$, these conditions, and hence MLRP, are not needed, as $E[V|H^t] - E_o[V|H^t] > 0$ follows immediately from (30) and (33).

⁴⁷This claim also does not require the assumption of MLRP.

PROOF OF I(C) IN PROPOSITION 6: First, note that (30) can be written as

$$(40) \quad q_2^r q_1^r \frac{(\beta_{2,o} \beta_1)^b}{(\beta_3 \beta_{2,o})^b} \left[\frac{(\beta_2 \beta_{1,o})^b}{(\beta_{2,o} \beta_1)^b} (\sigma_2 \sigma_{1,o})^s - (\sigma_{2,o} \sigma_1)^s \right] \\ + q_3^r q_2^r \left[(\sigma_3 \sigma_{2n})^s - \frac{(\beta_{3,o} \beta_2)^b}{(\beta_3 \beta_{2,o})^b} (\sigma_{3,o} \sigma_2)^s \right] \\ + 2q_3^r q_1^r \frac{(\beta_{3,o} \beta_1)^b}{(\beta_3 \beta_{2,o})^b} \left[\frac{(\beta_3 \beta_{1,o})^b}{(\beta_{3,o} \beta_1)^b} (\sigma_3 \sigma_{1,o})^s - (\sigma_{3,o} \sigma_1)^s \right].$$

Fix s and let $b \rightarrow \infty$. Then since by (25) and (32), $\beta_3 \beta_{2,o} > \beta_{3,o} \beta_2$, we have that the second term in (40) converges to $q_3^r q_2^r (\sigma_3 \sigma_{2n})^s$ as $b \rightarrow \infty$. Also, since $\beta_3 > \beta_2 > \beta_1$, it follows that $\beta_{2,o} \beta_1 < \beta_{2,o} \beta_3$ and $\beta_{3,o} \beta_2 > \beta_{3,o} \beta_1$. The former, together with (25) and (32), implies that the first term in (40) vanishes as $b \rightarrow \infty$. The latter, together with (25) and (32), implies that $\beta_3 \beta_{2,o} > \beta_{3,o} \beta_1$; therefore, using (25) and (32) again, the last term in (40) also vanishes. Consequently, as $b \rightarrow \infty$, the expression in (40) converges to $q_3^r q_2^r (\sigma_3 \sigma_{2n})^s$. Since $(\sigma_3 \sigma_{2n})^s > 0$ and $E[V|H^i] - E_o[V|H^i]$ has the same sign as the expression in (40), the claim in I(c) of the proposition is established. *Q.E.D.*

REFERENCES

- ALLEN, F., S. MORRIS, AND H. S. SHIN (2006): "Beauty Contests and Iterated Expectations in Asset Markets," *Review of Financial Studies*, 19, 719–752. [980]
- AVERY, C., AND P. ZEMSKY (1998): "Multi-Dimensional Uncertainty and Herd Behavior in Financial Markets," *American Economic Review*, 88, 724–748. [979,986,987,999,1006]
- BACCHETTA, P., AND E. V. WINCOOP (2006): "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?" *American Economic Review*, 96, 552–576. [980]
- (2008): "Higher Order Expectations in Asset Pricing," *Journal of Money, Credit and Banking*, 40, 837–866. [980]
- BANERJEE, A. (1992): "A Simple Model of Herd Behavior," *Quarterly Journal of Economics*, 107, 797–817. [974,987,1010]
- BIKHCHANDANI, S., AND S. SUNIL (2000): "Herd Behavior in Financial Markets: A Review," Staff Paper WP/00/48, IMF. [979]
- BIKHCHANDANI, S., D. HIRSHLEIFER, AND I. WELCH (1992): "Theory of Fads, Fashion, Custom, and Structural Change as Informational Cascades," *Journal of Political Economy*, 100, 992–1026. [974,987]
- BRUNNERMEIER, M. K. (2001): *Asset Pricing Under Asymmetric Information—Bubbles, Crashes, Technical Analysis and Herding*. Oxford, England: Oxford University Press. [979,984]
- CHAMLEY, C. (2004): *Rational Herds*. Cambridge, United Kingdom: Cambridge University Press. [979,984]
- CHARI, V., AND P. J. KEHOE (2004): "Financial Crises as Herds: Overturning the Critiques," *Journal of Economic Theory*, 119, 128–150. [980]
- CHORDIA, T., R. ROLL, AND A. SUBRAHMANYAM (2002): "Order Imbalance, Liquidity, and Market Returns," *Journal of Financial Economics*, 65, 111–130. [984]
- CIPRIANI, M., AND A. GUARINO (2005): "Herd Behavior in a Laboratory Financial Market," *American Economic Review*, 95, 1427–1443. [986]

- (2008): “Herd Behavior and Contagion in Financial Markets,” *The B.E. Journal of Theoretical Economics (Contributions)*, 8, Article 24. [980]
- DASGUPTA, A., AND A. PRAT (2008): “Information Aggregation in Financial Markets With Career Concerns,” *Journal of Economic Theory*, 143, 83–113. [980]
- DE BONDT, W. F. M., AND R. THALER (1985): “Does the Stock Market Overreact?” *The Journal of Finance*, 40, 793–805. [986]
- DREHMANN, M., J. OECHSSLER, AND A. ROIDER (2005): “Herding and Contrarian Behavior in Financial Markets: An Internet Experiment,” *American Economic Review*, 95, 1403–1426. [986]
- EASLEY, D., AND M. O’HARA (1992): “Time and the Process of Security Price Adjustment,” *Journal of Finance*, 47, 577–605. [976,1007]
- GLOSTEN, L., AND P. MILGROM (1985): “Bid, Ask and Transaction Prices in a Specialist Market With Heterogenously Informed Traders,” *Journal of Financial Economics*, 14, 71–100. [975,980, 983,1002]
- GOLDSTEIN, I., E. OZDENOREN, AND K. YUAN (2010): “Trading Frenzy and Its Impact on Real Investment,” Working Paper, The Wharton School, University of Pennsylvania. [980]
- HE, H., AND J. WANG (1995): “Differential Information and Dynamic Behavior of Stock Trading Volume,” *The Review of Financial Studies*, 8, 919–972. [980]
- JEGADEESH, N. (1990): “Evidence of Predictable Behavior of Security Returns,” *The Journal of Finance*, 45, 881–898. [986]
- JEGADEESH, N., AND S. TITMAN (1993): “Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency,” *The Journal of Finance*, 48, 65–91. [986]
- (2001): “Profitability of Momentum Strategies: An Evaluation of Alternative Explanations,” *The Journal of Finance*, 56, 699–720. [986]
- LEE, I. H. (1998): “Market Crashes and Informational Avalanches,” *Review of Economic Studies*, 65, 741–759. [979]
- LEHMANN, B. N. (1990): “Fads, Martingales, and Market Efficiency,” *The Quarterly Journal of Economics*, 105, 1–28. [986]
- MILGROM, P., AND N. STOKEY (1982): “Information, Trade and Common Knowledge,” *Journal of Economic Theory*, 26, 17–27. [981]
- OZDENOREN, E., AND K. YUAN (2007): “Feedback Effects and Asset Prices,” *Journal of Finance*, 6, 1939–1975. [980]
- PARK, A., AND H. SABOURIAN (2011): “Supplement to ‘Herding and Contrarian Behavior in Financial Markets’,” *Econometrica Supplemental Material*, 79, http://www.econometricsociety.org/ecta/Supmat/8602_proofs.pdf. [979]
- ROMANO, M. G., AND H. SABOURIAN (2008): “Herd Behaviour in Financial Markets With Continuous Investment Decisions,” Unpublished Manuscript, University of Cambridge. [994]
- SADKA, R. (2006): “Momentum and Post-Earnings-Announcement Drift Anomalies: The Role of Liquidity Risk,” *Journal of Financial Economics*, 80, 309–349. [986]
- VIVES, X. (2008): *Information and Learning in Markets*. Princeton, NJ: Princeton University Press. [979,984,985]

Dept. of Economics, University of Toronto, Toronto, Ontario M5S 3G7, Canada; andreas.park@utoronto.ca

and

Faculty of Economics, Cambridge University, Sidgwick Avenue, Cambridge CB3 9DD, United Kingdom; hamid.sabourian@econ.cam.ac.uk.

Manuscript received May, 2009; final revision received October, 2010.