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This document is part of the course notes on Financial Economics and has been prepared by the professors of the Finance Department at HEC Paris.

- Capital budgeting

1 What is capital budgeting?

Firms carry on business by investing in a variety of real assets (machines, factories, trademarks...). Capital budgeting is the process by which management:

- proposes new investment projects;
- evaluates the projects;
- decides which projects to undertake and which projects to reject.

Here, we are mainly interested in the evaluation and the screening parts of the capital budgeting process. Note that the course does not address the issue of how to finance an investment project (issue debt or equity?). Investing and financing decisions can be considered separately because, under certain conditions, the value of investment projects is independent of the way the projects will be financed. The course on Corporate Finance will address firm's financing issues in detail.

An investment project is characterized by its cash flows $\{CF_0, CF_1, ..., CF_t, ..., CF_T\}$ where CF_t is the cash flow of the project at date t. The cash flows can be positive (cash inflows) or negative (cash outflows).

Example: Design of a new software to help medical doctors in their diagnoses.

Table 2: an example of cash-flows

Year	0	1	2	3	4	5
Cash Flow	-2000	750	750	775	900	50

How to evaluate this project? Can we simply sum the different cash-flows over time and accept the project if this sum is positive?

NO! Because this would neglect *the time value of money*.
Intuitively, $1 "today" is worth more than $1 "tomorrow" for three reasons:

- *Opportunity cost:* One dollar today can be invested and will earn interest.
- *Inflation:* the purchasing power of one dollar today is higher than the purchasing power of one dollar tomorrow.
- *Uncertainty*.

For this reason, comparing sums of money at different points in time is not straightforward. We first describe how to perform this comparison and we derive a simple rule (the Net Present Value rule) to accept or to reject an investment project. In a second step, we study some of the issues that arise in forecasting cash flows. Finally we present other rules that are used to select investment projects and show why they can be wrong.

2 Time Value of Money

2.1 Future Value and Compounding

Compounding is the process of computing the future value of a given sum of money today at some fixed date in the future. Central to this computation is the *interest rate*. An interest rate over a given period of time is a promised rate of return.

Example: What is the future value of \$100 in two years if the annual interest rate is 5%?

At the end of Year 1: $FV = 100(1+0.05) = 105$.

At the end of Year 2: $FV = 105(1.05) = 100(1.05)^2 = 110.25$.

Note that, for the second period, the interests received at the end of the first period ($\$5$) earn interests as well. This way of computing interests is called *compounding*.

Definition 1*: If r is the interest rate per year, the future value of a sum* S *in n years is:*

$$FV(r,n) = (1+r)^n S$$

Example: How many years are necessary to double a given initial stake, *S*? We are looking for *n* such that:

$$2S = (1+r)^n S$$

i.e..*n=ln(2)/ln(1+r)* For instance with $r = 5\%$, 14 years are necessary. With $r = 10\%$, only 7 years are necessary.

2.1.1 The frequency of compounding

Compounding can take place at a higher frequency than the year (e.g. every quarter, every month, every week or every day). However interest rates are usually stated "per year" (rather than the actual frequency of compounding). How to compute a future value in this case?

Example The interest rate per year is 10%, but compounding is per semester. The future value of \$100 in one year is computed as follows:

At the end of six months: $100(1+\tfrac{10\%}{2})=105$.
At the end of the year: $100(1+\tfrac{10\%}{2})(1+\tfrac{10\%}{2})=110.25$.

Note that the rate of return on the investment is not 10% but is in fact 10.25%. This rate of return is called the *Effective Annual Rate*. It depends on the frequency of compounding in the year and this is the one that must be used to compare investments with different compounding frequencies. More generally if r is the stated interest rate per year and if k is the compounding frequency, the future value of S in one year is:

$$FV(r,k)=\left(1+\frac{r}{k}\right)^k S$$

and the Effective Annual Rate, $\rho(k,r)$, is:

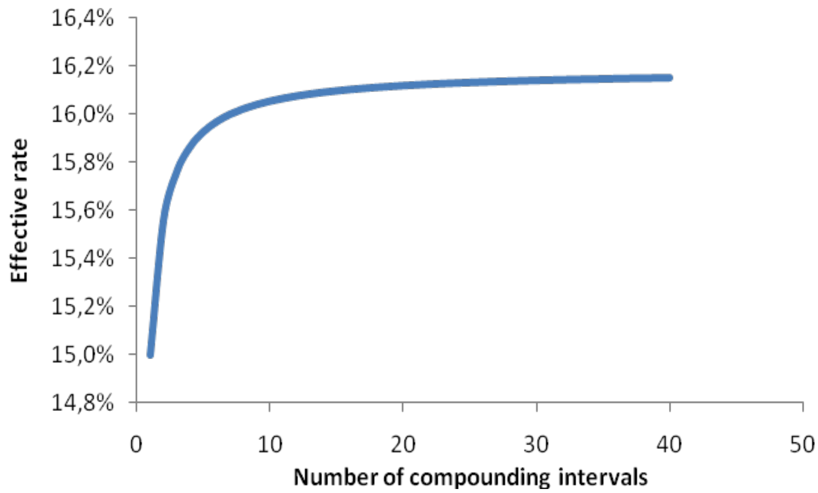
$$\rho(k,r)=\left(1+\frac{r}{k}\right)^k -1$$

For a given interest rate, as the compounding frequency increases, the Effective Annual Rate increases as well. This is shown in Graphic 2 for an interest rate equal to 15%. Note that as k becomes very large, the Effective Annual Rate reaches a limit. In fact, for k very large:

$$(1+\rho(k,r))=\left(1+\frac{r}{k}\right)^k \approx e^r$$

For instance with $r = 15\%$, $\rho(k,r)$ goes to 16,2% when interests are continuously compounded (i.e., when k goes to infinity).

Graphic 2: Effective Rate and Compounding Frequency



2.1.2 Interest Rates

In practice interest rates vary according to:

- **The unit of account**. Interest rate on dollar deposits are different from interest rates on Yen deposits.
- **The maturity**. In general, there is a relationship between the maturity of an investment or a loan and the interest rate. This relationship is depicted by the *yield curve* that we study in more details in the Financial Markets course.
- **Default risk**. Other things equal, interest rates are larger when default risk is higher as investors require a higher risk premium for a higher probability of default.

Table 3 : interest rates on treasury and corporate bonds as of july 2009

	US treasury bonds	US AA corporate bonds
2 year maturity	1.11	1.85
5 year maturity	2.52	3.51
10 year maturity	3.48	4.34

Table 4 : London Interbank Offered Rate (LIBOR) as of july 2009

	In US dollars	In euros
1 week maturity	0.29	0.46
1 month maturity	0.31	0.73
1 year maturity	1.60	1.5

Source: Yahoo finance & bbalibor

Inflation is an important determinant of the level of interest rate. Why? Suppose that the basket of goods which is used to compute the rate of inflation is worth \$100 today. The rate of inflation per year is $\pi = 2\%$. This means that the basket of goods will be worth \$102 in one year. Thus for investing \$100, you require at least an interest rate of 2% in order to maintain your purchasing power unchanged. Thus, the interest rate you will require should at least be equal to the inflation rate. In general, it will be larger. Suppose that this interest rate is $r = 5\%$, what is the increase in your purchasing power? Given an investment of \$100, you can purchase: $1.0294 = \frac{(1+r)*100}{(1+\pi)*100}$ baskets of goods in one year instead of one today. Thus your purchasing power has really increased by 2,94% (and not by 5%). This is called *the real rate of return* in contrast to r which is called *the nominal rate of return*. The real rate of return, r_e is given by:

$$1 + r_e = \frac{(1+r)}{(1+\pi)}$$

and a useful approximation is:

$$r \approx \pi + r_e$$

which shows that inflation is a main determinant of the nominal rate. The other determinant is the real interest rate which is determined by factors such as the productivity of capital goods, time preferences and risk aversion.

3 Present Value and Discounting

What is the amount of money that we must invest today in order to obtain a given future value S in n years? If the interest rate is r, then we search for the value PV that solves:

$$S = (1+r)^n PV$$

i.e. $PV(r,n) = \frac{S}{(1+r)^n}$

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Definition 2: *If r is the interest rate per year, the present value of a sum S received in n years is:*

$$PV(r,n) = \frac{S}{(1+r)^n}$$

Computing a present value is called *discounting* (the reverse of compounding) and r is often referred to as the *discount rate*. Note that when $r > 0$, the present value of one dollar in one year is less than one dollar, which reflects the time value of money.

3.1 Multiple Cash Flows

Result 1: If the interest rate is r, the present value of a stream of cash-flows $\{CF_0,...,CF_T\}$ is:

$$PV = CF_0 + \frac{CF_1}{(1+r)} + ... + \frac{CF_t}{(1+r)^t} + ... + \frac{CF_T}{(1+r)^T} = \sum_{t=0}^{t=T} \frac{CF_t}{(1+r)^t}$$

This formula is often called *the Discounted Cash Flow* (DCF) formula. Note that the value of the stream of cash flows is just the sum of the present value of each cash flow. This is in fact the amount one would have to save at date 0 to secure the cash flow CF_0 at date 0, CF_1 at date 1,..., CF_T at date T.

Example: Consider the investment project which is described in Table 2 and assume that the interest rate is $r = 11.5\%$. The present value is of this project is:

$$-2000 + \frac{750}{1+r} + \frac{750}{(1+r)^2} + \frac{775}{(1+r)^3} + \frac{900}{(1+r)^4} + \frac{50}{(1+r)^5} = 446.31$$

The interest rate can depend on the duration of the investment period. Let *r(t)* be the annual interest rate for an investment that matures in *t* periods. In this case the present value of the stream of cash flows $\{CF_0, CF_1, ..., CF_t, ..., CF_T\}$ is:

$$PV = \sum_{t=0}^{t=T} \frac{CF_t}{(1+r(t))^t}$$

Note that in this case each cash flow CF_t is discounted using the corresponding interest rate *r(t)*.

Important remark: The DCF formula can be used to compute the present value of any stream of future cash flows. This means that the formula can be applied to compute the present value of the cash flows associated with an investment project, as we do here but also to compute the present value (price) of **a financial asset** once the cash flows of the security have been forecasted. For instance, the price of a bond must be the present value of the stream of coupons and the reimbursement value that the bondholder can expect. This will be explained in more details in the Financial Markets course.

3.1.1 Annuities

Definition 3: *An ordinary annuity of length n is a sequence of **constant** cash-flows during n periods, where the cash flows are paid (received) at the end of each period.*

*An **immediate** annuity is an annuity in which the cash flows are paid at the beginning of each period. We only consider ordinary annuities below.*

Definition 4: *A perpetuity is an ordinary annuity with an infinite length.*

Result 2: The present value $P(C,r)$ of a perpetuity with a constant cash flow C when the interest rate is *r* is:

$$P(C,r) = \frac{C}{r}$$

Using this result we can now deduce the present value $A(C,r,n)$ of an ordinary annuity of length *n* with a cash flow C. Actually, a perpetuity with cash flow C has the same cash flows as a portfolio that is made of one ordinary annuity of length *n* with cash flow C and one perpetuity whose first cash flow is paid at the end of period *n+1*. It follows that:

$$P(C,r) = A(C,r,n) + \frac{P(C,r)}{(1+r)^n}$$

Result 3: The present value $A(C,r,n)$ of an ordinary annuity with a cash flow C when the interest is r is:

$$A(C,r,n) = \frac{C}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

Some applications

Example 1: You win a prize. You are offered two ways to cash-in the prize. In the first case, you receive \$1,000,000 immediately. In the second case, you receive a perpetuity of \$100,000 per year with a first payment today. The interest rate is 9.5%. Which one would you choose?

Example 2: You plan to borrow \$1,000,000, that you will reimburse in 25 years, with monthly payments (300 payments). If the interest rate is 16% per year, what is the monthly payment C?

Result 4: Consider a stream of cash-flows which grow at rate g in perpetuity, that is $CF_{t+1} = (1+g)CF_t$. The first cash flow is received in one year and is equal to C. The present value of this stream of cash flows is:

$$PV = \frac{C}{r-g} \quad if \quad g < r$$

4 Net Present Value

4.1 The Net Present Value Rule

Consider an investment project for which the initial investment outlay is *I* and which yields a stream of cash-flows $\{CF_1, CF_2, ..., CF_T\}$, up to year *T*.

The Net Present Value of this project (NPV) is the difference between the present value of the stream of cash flows associated with the investment and the initial investment outlay:

$$NPV = \sum_{t=1}^{t=T} \frac{CF_t}{(1+r)^t} - I$$

The NPV is the difference between the "income" generated by the investment (in present value) and its cost. The *NPV rule* consists of accepting an investment project if and only if its NPV is positive. For instance, as shown previously, the NPV of the investment project described in Table 2 is positive, therefore the project must be accepted.

If we evaluate *several investment projects*, two cases must be distinguished:

1) **If the projects are not mutually exclusive**: all the projects with a positive NPV must be undertaken.

2) **If the projects are mutually exclusive**: choose the project with the greatest positive NPV.

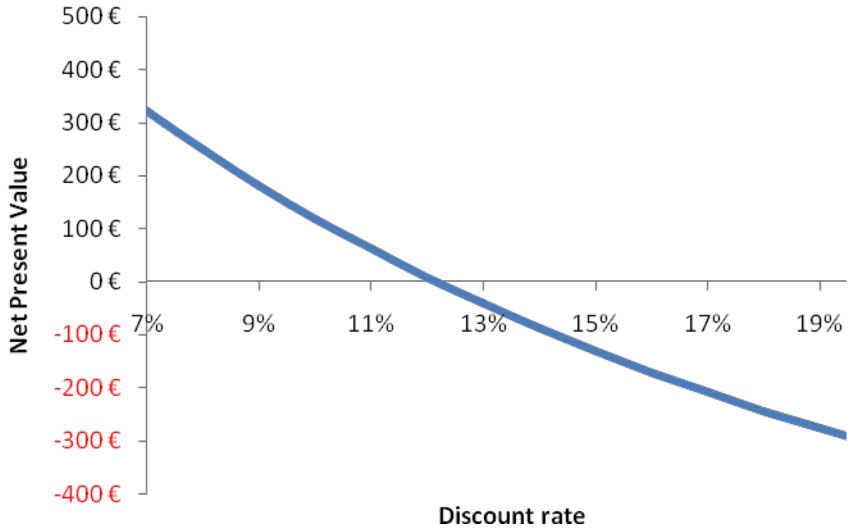
4.2 Opportunity Cost of Capital.

The NPV depends on the discount rate. For instance, consider the following investment project:

Year	0	1	2	3	4	5	6	7	8	9	10
Cash Flow	-1000	100	115	132	152	175	201	231	266	306	352

Graphic 3 illustrates the evolution of the NPV for this project as the interest rate increases between 7% and 20%. As can be seen the NPV decreases and is positive if r < 12.2% and is negative otherwise. Thus the discount rate is crucial for the investment decision. In general, the NPV will decrease with the interest rate but not always (more on this below).

Graphic 3: NPV and discount rate



How should we choose the discount rate? We must choose a discount rate equal to the interest rate that we could earn elsewhere **in the same risk profile** if we did not invest in the project under evaluation. This rate is called **the opportunity cost of capital**. Two problems arise. What does *elsewhere* mean? Investments in financial assets always provide an alternative investment. Thus we can use rates of return offered on financial assets to determine the discount rate. This is one function of financial markets. A more difficult problem is to evaluate the risk of the project and to find financial assets with a similar risk. The only case in which there is no difficulty is when the project is *riskless* (no uncertainty on the cash flows) in which case, the opportunity cost of capital is the rate of return on riskless financial asset (for instance, a treasury bill). For the other cases, we first need to define what is the risk of an asset. This will be considered in a future chapter.

Example: You can invest in a riskless project which will yield a cash flow equal to \$10,500 in one year and which requires an investment outlay equal to \$10,000. The rate of return on a one year treasury bill is 6%. Should you invest in the project? Investing \$10,000 in the treasury bill will yield a future value of *FV*=\$10,600 whereas the project has a future value of \$10,500. Thus the project should be rejected. In fact its NPV is:

$$NPV = \frac{10,500}{1.06} - 10,000 = -94 < 0$$

4.3 What is the NPV?

The NPV of an investment project is the increase in the wealth of the existing shareholders when the project is undertaken. Note that this vindicates the NPV rule: the shareholders do not want an investment project with a negative NPV to be chosen since this would decrease the value of their shares.

Example 1. Consider again the project described in Table 2. The firm with this project has one shareholder-manager who does not have the cash (\$2,000,000) which is necessary to undertake the project. The shareholder contacts one financier who will finance the investment project. The financier pays \$2,000,000 and receives interests at 11.5%. The initial shareholder-manager invests the amount provided by the financier, pays back the financier, and retains the NPV. He is richer by \$446,310.

Example 2. Now consider an entrepreneur with the following one-period project: a cash flow of \$240,000 in one year for an initial investment equal to \$200,000. The opportunity cost of capital is 10% Thus the *NPV*=\$18,181. Without undertaking the project, the entrepreneur has an income equal to \$100,000 in Year 0 and in Year 1. The consumption needs of the entrepreneur are at least equal to this amount in each year. Does it mean that the entrepreneur should not undertake the project?

NO! If there is a financial market, he can borrow money on the financial market and undertake the project without sacrifices in term of consumption, as long as the NPV is positive. He can even increase his consumption in Year 0 and Year 1 of an amount (in present value) equal to the NPV of the project.

Fisher's Separation Theorem. The previous examples show that all shareholders should agree on undertaking positive NPV projects, independently of their consumption plans. This result is the cornerstone of the *separation between ownership and management* since this implies that all managers have to do is to select investments with positive NPV (they do not need to know shareholders' desired consumption plans for instance). This result is known as *Fisher's separation theorem*. The second example also illustrates another function of financial markets: they allow consumption plans to be different from the cash flows associated with an investment project.

Problem: Moral-hazard: do managers have sufficient incentives to act in shareholders' best interests? Not necessarily. Thus there is a need to align shareholders and managers' incentives (incentive contracts, stock options, etc...).

4.4 Forecasting Cash-Flows

We can identify three different types of cash flows associated with a project:

- The initial investment outlay.
- Cash-flows from operations: The cash flows associated with the operations of the project.
- Cash-flows from liquidation: The cash flows at the end of the life of the project (assets sale etc...)

4.4.1 Some basic rules

Rule 1: Incremental Approach.

In general, the cash flows of a firm derive from several investment projects, undertaken at different points in time. A first important rule is to evaluate each new project in isolation of the other investments of the firms i.e., one must estimate the *incremental* revenues and costs associated with the project. However, the opportunity costs of the project must be taken into account. For instance, if the project partly uses existing assets of the firm, it might reduce the cash-flows generated by these assets. This reduction is a cost of the project. In order not to forget these opportunity costs in the calculation of the cash flows, it is often useful to compare the firm cash flows *with* and *without* the project.

Example-opportunity cost. Your firm owns a factory that can be sold at $2,000,000. Alternatively, the firm can use this factory to undertake a new investment project that will last 10 years. The cash flow of this project is expected to be $400,000 per year. In 10 years, the market value of the factory is expected to remain at $2,000,000. The opportunity cost of capital for this project is 12%.

Year	0	1	...	10
Cash Flow without the project	+2,000,000	0	...	0
Cash Flow with the project	0	+400,000	...	+2,400,000
Incremental Cash Flow	-2,000,000	+400,000	...	+2,400,000

Thus the NPV of the investment project is:

$$NPV = \frac{400,000}{0.12} * (1 - \frac{1}{(1.12)^{10}}) + \frac{2,000,000}{(1.12)^{10}} - 2,000,000 = 904,036.$$

Rule 2: Be cautious with the treatment of inflation. Cash-flows forecasts can be in nominal terms or in real terms (i.e. without taking into account the effect of inflation). The way forecasts are formulated influence the choice of the discount rate:

- Nominal cash flows must be discounted with a nominal interest rate.
- Real cash flows must be discounted with a real interest rate.

Of course, these two ways of computing the present value of a stream of cash flows yield the same present value.

Rule 3: Do not forget taxes. Cash flows must be calculated after taxes. Some expenses/losses are tax deductible. This should not be forgotten in computing the cash-flows.

Example: Consider an investment project which costs $100,000 and which yields a perpetual operating income per year equal to$20,000. The tax rate is 33%. The project can be financed either by issuing equity or by issuing debt bearing an interest rate of 6%. In all the cases the project is without risk and the opportunity cost of capital is 6%. Interest expenses are deductible from the taxable income. How should the project be financed?

Rule 4: Sunk costs are irrelevant. Sunk costs are costs that are related to the investment project but that have been incurred *before* the decision to undertake the investment project (e.g. past R&D expenses). Sunk costs incurred **prior** to $t = 0$ are irrelevant for NPV calculation at $t = 0$.