Problem Set 1: Stocks

Problem 1

a. Energy stocks currently provide an expected return of 10% per year. WindPower, a large energy company, will pay a year-end dividend of \$2 per share. If the stock is selling at \$50 per share, what must be the market's expectation of the growth rate of MBI dividends?

b. If dividend growth forecasts for WindPower are revised downward to 2% per year, but everything else remains unchanged, what happens to the price of WindPower stock?

Problem 2

Promises Ltd. never paid any dividend. Investors expect that it will start paying a dividend of €1 per share in three years from now ($t = 3$) and that dividends will then growth at 8% per year. The required return on Promises Ltd. stock is 12% per year. What is today's price of Promises stock? [Hint: find first the price in two years (P_2).]

Problem 3

The MLK company has current ($t = 0$) after-tax net earnings per share of €4. MLK pays out 50% of its earnings in dividends. If MLK expects to keep the same dividend policy, and expects to earn a return on equity of 20% on its future investments, what should its current price per share be? Assume that MLK's required rate of return is 15%.

Problem 4

CashBurner Ltd's stock is trading at \$25 per share. Its dividends are expected to grow at 4% forever. Next year's earnings and dividends are expected to be \$4 and \$2 per share, respectively.

a. What is the expected return on CashBurner Ltd's stock?

b. What is the ROE of projects available to the firm?

c. How much is the PVGO?

d. How can you explain the fact that the PVGO is negative? What would you recommand to the management of CashBurner Ltd?

Problem Set 1: Stocks

Elements of Answer

Problem 1

a. $P_0 = \dfrac{D_1}{k-g} \Rightarrow 50 = \dfrac{2}{0.1-g} \Rightarrow g = 6\%$

b. Substitute in $g' = 2\% \Rightarrow P_0 = \dfrac{2}{0.1-0.02} = 25$, i.e., an immediate loss of 50%

Problem 2

We can apply the Dividend Discount Model at $t=3$: $P_2 = \dfrac{D_3}{k-g} = \dfrac{1\times 1}{0.12-0.08} = 25$ €

Since there is no dividend until $t = 2$, the stock price today is today's present value of P_2: $P_0 = \dfrac{P_2}{(1+k)^2} = \dfrac{25}{1.12^2} = 19.93$ €

Problem 3

Growth rate of dividends: $g = \text{ROE} \times \text{plowback ratio} = 0.2 \times (1-0.5) = 10\%$

Future dividend (at $t = 1$): $D_1 = (1+g) \times D_0 = (1+g) \times \text{dividend payout ratio} \times E_0 = 1.1 \times 0.5 \times 4 = 2.2$€

Share price: $P_0 = \dfrac{D_1}{k-g} = \dfrac{2.2}{0.15-0.1} = 44$€

Problem 4

a. $P_0 = \dfrac{D_1}{k-g}$ implies $k = g + \dfrac{D_1}{P_0} = 0.04 + \dfrac{2}{25} = 12\%$

b. $g = b \times ROE^{new-projects}$ implies $ROE^{new-projects} = \dfrac{g}{b}$

The retention ratio is $b = \frac{E_1-D_1}{E_1} = 0.5$

Therefore $ROE^{new-projects} = \frac{0.04}{0.5} = 8\%$

c. $P_0^{no-growth} = \dfrac{E_1}{k} = \dfrac{4}{0.12} = 33.33$ €

Therefore $PVGO = P_0 - P_0^{no-growth} = 25 - 33.33 = -8.33$ € per share

d. The negative PVGO means that the projects in which the company reinvests part of its earnings reduce the value of the company. This is happening because the ROE of these projects (8%) is less than the discount rate ($k = 12\%$; remember that the discount rate is the same as the expected return, and it is also the same as the cost of capital for the company: all are equal to k as given by the CAPM). Therefore, the firm invests in projects that earn less than their opportunity cost, which destroys value for shareholders, hence the negative PVGO.

The firm should stop investing in these negative NPV projects, and the value of the company would go up from \$25 to \$33 per share.

Problem Set 2: Bonds

Problem 1

Please find below details of four default-free bonds:

	Annual coupon rate	Face Value	Maturity	Price at t=0
Bond 1	0%	50	1 year	48.08
Bond 2	0%	50	2 years	45.35
Bond 3	0%	50	3 years	41.98
Bond 4	10%	1000	3 years	1104.6

a. Find the term structure of interest rates for t=1 year, 2 years, and 3 years.

b. Find the composition of the portfolio formed by zero-coupon bonds 1, 2, and 3 that replicates coupon paying bond 4.

c. Is there an arbitrage opportunity? Show all of your calculations to justify your answer. If there is an arbitrage opportunity, create a detailed arbitrage table.

d. Assume you buy bond 4 today and sell it in one year (right after the coupon payment). What is the holding period return of your investment if one year from now, the term structure is flat at 5%?

e. What is the forward rate between years 1 and 3 equal to?

Problem 2

A bond has been issued with an annual coupon rate of 10%. This bond has a sinking-fund provision: the first half of the issue will be reimbursed in two years and the other half in three years. You hold for 100 million of nominal value (face value) of this bond.

a. Write the three future annual cash flows.

b. The term structure is currently flat at 9%. What is the value of the bond and its yield-to-maturity?

c. How much do you stand to lose if the term structure moves uniformly from 9% to 9.1% within one day?

Problem 3

Please find below the prices and characteristics of three bonds, which you will assume are default-free:

<u>Bond 1:</u> Coupon rate = 5% (one coupon a year); Maturity = 3 years; Face value = 1000 euros; Price at date 0 = 1001.8 euros

<u>Bond 2:</u> Coupon rate = 7% (one coupon a year); Maturity = 3 years; Face value = 1000 euros; Price at date 0 = 1056.986 euros

<u>Bond 3:</u> Zero-Coupon; Maturity = 2 years; Face value = 1000 euros; Price at date 0 = 924.556 euros

a. What should the prices of zero-coupon bonds of face value 1000 euros and of, respectively, maturity 1 and 3 years be equal to so that there are no arbitrage opportunities?

We now consider a new bond issued by JunkBond Inc. The bond has face value 1000 euros and a coupon rate equal to 10%. The maturity date is 2 years from now. The bond has a default risk for its payoffs in year 2. With probability 0.8, the bond will pay off 1100 euros (as expected) while with probability 0.2, the bond will pay off nothing (the issuer defaults). At date 1, there is no default risk and the coupon will be paid as expected with probability 1.

Consider also the following derivative asset (called a Credit Default Swap, or CDS): the asset pays off 100 euros at date t=2 if JunkBond defaults and 0 euro otherwise. The derivative asset pays no cash flow at date t=1. The current price of that derivative asset is 15 euros.

b. In the absence of arbitrage opportunities, what should the price of the bond issued by JunkBond Inc. be equal to?

Problem 4

Consider a coupon bond paying a coupon rate of 7% over 4 years and with a face value of 100. The yield-to-maturity of this bond is equal to 4%.

a. What is the duration of this bond?

b. Using the bond's duration, give an estimate of the capital loss of this bond following a sudden 30 basis point increase in the the bond's yield.

Problem Set 2: Bonds

Elements of Answer

Problem 1

a. Using the prices of bond 1, bond 2, and bond 3:

$$r_1 = \frac{50}{48.08} - 1 = 4\%$$

$$r_2 = \left(\frac{50}{45.35}\right)^{1/2} - 1 = 5\%$$

$$r_3 = \left(\frac{50}{41.98}\right)^{1/3} - 1 = 6\%$$

b. 2 × (Bond 1) + 2 × (Bond 2) + 22 × (Bond 3)

c. Replicating portfolio price $= 2 \times 48.08 + 2 \times 45.35 + 22 \times 41.98 = 1110.42 > 1104.6$, thus there is an arbitrage opportunity.

Arbitrage strategy: short 2 bonds 1 + short 2 bonds 2 + short 22 bonds 3 + long 1 bond 4

	t=0	t=1	t=2	t=3
short 2 bonds 1	96.16	-100	0	0
short 2 bonds 2	90.70	0	-100	0
short 22 bonds 3	923.56	0	0	-1,100
long 1 bond 4	-1104.6	100	100	1,100
Total	+5.82	0	0	0

d. The price of bond 4 in one year right after the coupon payment will be $\frac{100}{1.05} + \frac{1100}{1.05^2} = 1092.97$

Therefore, HPR $= \frac{100 + 1092.97 - 1104.6}{1104.6} = 8\%$

e. $(1 + r_1)(1 + f_{1\to3})^2 = (1 + r_3)^3$ implies $f_{1\to3} = \left(\frac{(1+r_3)^3}{1+r_1}\right)^{1/2} - 1 = 7\%$

Problem 2

a. CF_1 = 10% interest payment on 100 million = 10 million

CF_2 = 10% interest payment on 100 million + repayment of half the issue (i.e., 50 million) = 60 million

CF_3 = 10% interest payment on 50 million (what is left), i.e., 5 million + repayment of the second half of the issue (i.e., 50 million) = 55 million

b. $P = \frac{10}{1.09} + \frac{60}{1.09^2} + \frac{55}{1.09^3} = 102.1452$ million

Yield-to-maturity = 9%

c. $P = \frac{10}{1.091} + \frac{60}{1.091^2} + \frac{55}{1.091^3} = 101.9275$ million

Loss = 102.1452 million − 101.9275 million = 217,700

Problem 3

a. We solve for the discount factors d_1, d_2, and d_3, using the prices of the three bonds:

$$\begin{cases} 1001.8 & = & 50 \times d_1 + 50 \times d_2 + 1050 \times d_3 \\ 1056.986 & = & 70 \times d_1 + 70 \times d_2 + 1070 \times d_3 \\ 924.556 & = & 1000 \times d_2 \end{cases}$$

Solving the system, we find: $d_1 = 0.9709$; $d_2 = 0.9246$; $d_3 = 0.8638$. Thus, the price of the zero coupon with T=1 is 970.9 € and that of the zero coupon with T=3 is 863.8 €.

b. We price the bond by arbitrage, that is, we look for the replicating portfolio of the risky bond. We denote n_1, n_2, and n_D respectively the number of zero coupons of maturity 1, zero-coupons of maturity 2, and of credit derivatives in the portfolio. The portfolio should have the same cash flows as the risky bond at date t=1, at date t=2 if the bond does not default, and at date t=2 if the bond defaults. That is, we must have:

$$\begin{cases} t = 1: & 100 & = & n_1 \times 1000 + n_2 \times 0 + n_D \times 0 \\ t = 2 if no default: & 1100 & = & n_1 \times 0 + n_2 \times 1000 + n_D \times 0 \\ t = 2 if default: & 0 & = & n_1 \times 0 + n_2 \times 1000 + n_D \times 100 \end{cases}$$

where the LHS of the equations are the CF of the risky bond at each date and in each scenario and the RHS are the CF of the portfolio.

Solving the system of equations, we $n_1 = 0.1$; $n_2 = 1.1$; $n_D = -11$.

Thus, in absence of arbitrage Junk Bond is worth $0.1 \times 970.9 + 1.1 \times 924.6 - 11 \times 15 = 949.15$€

Note that the true probability of default plays NO ROLE in this formula. A similar property will hold in models of option prices.

<u>Problem 4</u>

a. Bond price: $P = \frac{7}{1.04} + \frac{017}{1.04^2} + \frac{7}{1.04^3} + \frac{7}{1.04^4} = 110.89$

Bond duration: $D = \frac{1 \times \frac{7}{1.04} + 2 \times \frac{7}{1.04^2} + 3 \times \frac{7}{1.04^3} + 4 \times \frac{017}{1.04^4}}{110.89} = 3.65$ years

b. $\Delta P = -D \times \frac{\Delta r}{1+r} \times P = 3.65 \times \frac{0.003/1.04}{\times} 110.89 = -1.17$ so the capital loss is 1.17

Problem Set 3: Forwards and Futures

Problem 1

One year ago you entered in a short position in a forward contract on one share of stock ABC with expiration date in one year from now and delivery price €55. Stock ABC sells at €50 today. It has a market beta of 0.8 and it is not paying any dividend. Finally, the risk-free interest rate is 3% and the expected market return is 8%.

a. Assuming that all CAPM assumptions are satisfied, what is your expected payoff in one year from now?

b. Find the no-arbitrage forward price of a forward contract created now on one share of stock ABC with expiration date in one year from now.

c. What is today's present value of your position, i.e., how much money can you lock in today?

Problem 2

Consider a one-year forward contract on one ounce of gold. Suppose that it costs €2 per ounce per year to store gold, with the payment being made at the end of the year. Assume that the spot price today is €350 and the term structure of interest rates is flat at 4.5%.

a. What is the forward price of that contract?

b. What would be the forward price if the storage cost of €2 was paid at the beginning of the year?

Problem 3

Jean Cadillac, a Bordeaux wine merchant, proposes to enter into a business relationship with Château Doux Albion, by buying their wine at a fixed price prior to delivery. The spot price for Doux Albion's variety of wine is €80 per hectoliter. Jean Cadillac faces a risk-free borrowing and lending rate of 4% per year.

a. What would be the forward price per hectoliter for delivery in 12 months?

b. In this question only we assume that there are storage costs of €10 per hectoliter paid at the beginning of the year, and that the wine is stored in oak barrels during the second six months and its volume shrinks by 5% during this period. What would be the forward price for delivery in 12 months in this case?

c. In this question only we assume that wine cannot be sold short. What would be the range for the forward price for delivery in 12 months?

d. (More difficult) In this question only we assume that buying or selling wine leads to transactions costs of €5 per hectoliter. What would be the range for the forward price for delivery in 12 months?

Problem 4

You will find below the characteristics and prices of a number of AAA-rated coupon bonds:

- Bond 0: zero-coupon, maturity: 2 years, face value: €100 price at date 0: €92.46
- Bond 1: zero-coupon, maturity: 3 years, face value: €100 price at date 0: €87.63
- Bond 2: coupon rate = 2% (one coupon a year), maturity: 3 years, face value: €100 price at date 0: €93.15

- Bond 3: coupon rate = 4% (one coupon a year), maturity: 3 years, face value: €100 price at date 0: €98.68

a. Assuming that Bonds 0, 1 and 2 are correctly priced, find the term structure of interest rates.

b. Find the no-arbitrage price of a forward contract whose underlying asset is Bond 3, and whose maturity is Year 1, just after the payment of the first coupon paid by Bond 3.

Problem Set 3: Forwards and Futures

Elements of Answer

Problem 1

a. The expected payoff is $E[F_1 - S_1]$. We compute $E[S_1]$ using the CAPM formula $E\left[\frac{S_1 - S_0}{S_0}\right] = k = r_F + \beta \times (E[r_M] - r_F) = 0.03 + 0.8 \times (0.08 - 0.03) = 7\%$. Therefore, $E[S_1] = (1+k) \times S_0 = 1.07 \times 50 = 53.5$ € and the expected payoff of the short forward position is $55 - 53.5 = 1.5$ €.

b. $F_0 = S_0 \times (1 + r_F) = 50 \times 1.03 = 51.5$ €

c. This is the value of closing out your position today. To close out your position you need to take a long position in the forward contract of part b) and borrow $\frac{55-51.5}{1.03}$ at the risk-free rate. In one year time you will sell ABC stock at €55 through the short forward position, buy ABC stock at €51.5 through the long forward position, and pay back 55-51.5: your payoff will indeed be equal to 0. Therefore, the present value of your position today is $\frac{55-51.5}{1.03} = 3.40$ €

Problem 2

a. $F_0 = (S_0 + PV(C)) \times (1 + r)$ where PV stands for present value.

$F_0 = 350 \times 1.045 + 2 = 367.75$€

b. $F_0 = (350 + 2) \times 1.045 = 367.84$€

Problem 3

a. From the no arbitrage relation: $F_0 = S_0 \times (1 + r) = 80 \times 1.04 = 83.20$ €

b. Because of shrinkage, for forward delivery of 1 hectoliter in a year, $\frac{1}{0.95} = 1.0526$ hectoliter needs to be stored today, and will trigger storage costs. The no arbitrage formula becomes: $F_0 = 1.0526 \times (S_0 + C_0) \times (1 + r) = 98.53$ €

c. Only if $F_0 > S_0(1+r)^T$, opens an arbitrage opportunity, whereas $F_0 < S_0(1+r)^T$ cannot be exploited because of the short sale constraint. Thus, every forward price $F_0 \leq S_0(1+r)^T = 83.20$ € does not admit arbitrage.

d. Let k be the transaction cost per hectoliter of wine. Not that for a long or short position in wine, we need to count transaction costs for a round-trip, i.e., purchase and sale.

Suppose $F_0 > S_0(1+r)^T$. Then the payoffs of the arbitrage strategy are:

Date	0	T
Short forward	0	$F_0 - S_T$
Long in wine	$-(S_0 + k)$	$S_T - k$
Borrow risk-free	$S_0 + k$	$-(S_0 + k)(1 + r)$
Total	0	$F_0 - S_0(1 + r) - k(2 + r)$

Suppose $F_0 < S_0(1+r)^T$. Then the payoffs of the arbitrage strategy are:

Date	0	T
Long forward	0	$-F_0 + S_T$
Short in wine	$S_0 - k$	$-S_T - k$
Invest risk-free	$-(S_0 - k)$	$(S_0 - k)(1 + r)$
Total	0	$-F_0 + S_0(1 + r) - k(2 + r)$

Hence, in the first case, arbitrage is only worthwhile if $F_0 > S_0(1+r) + k(2+r)$. In the second case, only if $F_0 < S_0(1+r) - k(2+r)$. Thus, the range of prices where no profitable arbitrage opportunity exists is $F_0 \in [S_0(1+r) - k(2+r); S_0(1+r) + k(2+r)]$.

Thus, $F_0 \in [83.20 - 5 \times 2.04; 83.20 + 5 \times 2.04]$, in other words $F_0 \in [€73.00; €93.40]$ is the price range compatible with absence of arbitrage in this case.

Problem 4

a. From bonds 0 and 1, we get: $d_2 = 0.9246 \Leftrightarrow r_2 = 4\%$

$$d_3 = 0.8763 \Leftrightarrow r_3 = 4.5\%$$

From bond 2, we get: $d_1 = 0.9591 \Leftrightarrow r_1 = 4.26\%$

b. If we try to construct the same cash-and-carry strategy as in class:

	t=0	t=1	t=2	t=3
Short forward	0	$F_0 - S_1$		
Long bond 3	-98.68	$+4 + S_1$		
Borrow €98.68 for 1 year	$+98.68$	-98.68×1.0426		

Thus, a no arbitrage price F_0 verifies $F_0 = 98.68 \times 1.0426 - 4 = 98.88$€.

Remark: Note that things would have been more difficult if we had looked at a contract expiring at t=2. Then, coupons received at date t=1 would have to be reinvested between date 1 and date 2, thus at the forward rate $f_{1\to 2}$. Thus, the complete formula for the pricing of forward contracts with non-flat term structures:

$$F_0 = S_0 \times (1 + r_T)^T - \sum_{t<T} D_t \times (1 + f_{t\to T})^{T-t}$$

where the t represent the dates of dividend payments before T, and the $f_{t\to T}$ are the forward interest rates between dates t and T.

Problem Set 4: Options

Problem 1

An American investor buys one call option on the euro at an exercise price of \$1.1 per euro. The option premium is \$0.01 per euro.

a. For what range of exchange rates will the investor exercise the call option at expiration?

b. For what range of exchange rates will the investor realize a net profit, taking the original cost into account (but ignoring time discounting)?

c. If the investor had purchased a put with the same exercise price and premium instead of a call, how would your answers to the two questions above change?

Problem 2

You think LVMH's stock price is going to appreciate substantially in the next six months. LVMH's current price is equal to €100, and a European call option on LVMH with a six-month maturity and an exercise price €100 has a price of €10. You have €10,000 to invest, and you can:

(i) Invest €10,000 in the stock.

(ii) Invest €10,000 in 1,000 call options (one option gives you the right to buy one share of Peugeot in six months).

(iii) Buy 100 options and invest the remaining €9,000 in a risk-free money market fund paying 4% over six months (4% is the effective semi-annual rate, that is, the effective annual rate is 8.16%).

What is your holding period return if LVMH's stock price six months from now is (a) €80, (b) €100, (c) €110, (d) €120?

Problem 3

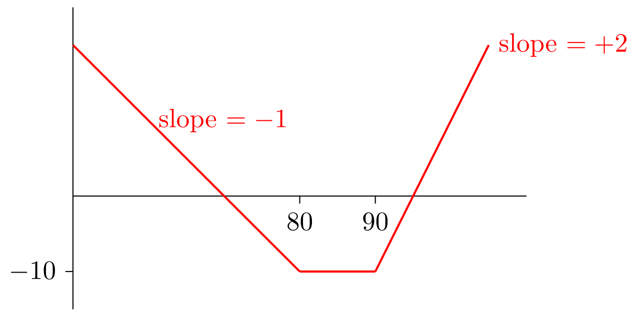
The following financial derivatives have the same underlying asset and the same maturity date:

- A European Call option with exercise price 80
- A European Call option with exercise price 90
- A forward contract with delivery price 70

a. Draw the payoff diagram of the forward contract (that is, plot the payoff at maturity of a long position in the forward as a function of the spot price of the underlying asset at maturity).

b. Draw the payoff diagram of a portfolio composed of one long position in the forward and one short position in the call option with exercise price 90.

c. Find a portfolio that contains some units of the three derivatives above and with a payoff at maturity as follows:



d. Suppose an investor decides to invest massively in the portfolio of question a). Which is most likely: (i) the investor thinks that the market under-estimates the volatility of the underlying asset; or (ii) the investor thinks that the market over-estimates the volatility of the underlying asset? Explain.

Problem 4

Suppose that you bought one forward contract (forward price $F_0 = 100$) and two European puts (strike $K = 100$) and sold one European call (strike $K = 120$) on the same underlying asset and the same maturity.

a. Draw carefully the payoff diagram of your portfolio at date T, i.e., your cash flows CF_T as a function of the underlying asset spot price S_T.

b. Which one(s) of the following four explanations is most likely to justify the purchase of this portfolio:

   (i) You think that the market underestimates the true value of the underlying asset.

   (ii) You think that the market overestimates the true value of the underlying asset.

   (iii) You think that the market underestimates the price volatility of the underlying asset.

   (iv) You think that the market overestimates the price volatility of the underlying asset.

c. Which one(s) of the following three reasons would best explain why you decided to sell the call with $K = 120$:

   (i) You wanted to reduce the overall initial cost of the portfolio.

   (ii) You wanted to benefit from a drop in the price of the underlying asset.

   (iii) You wanted to limit your downward exposure in case of an increase in the price volatility of the underlying asset.

Problem 5

You have the following information on two European options that have the same underlying asset:

Put option: maturity 2 years; strike price 100; put price 8

Call option: maturity 2 years; strike price 100; call price 5

Furthermore, you know that the underlying asset currently trades at a price 90, that the term structure of interest rates is flat, and that the annual risk-free rate is 5%.

a. Assume in this question only that the underlying asset does not pay any dividend between now and t=2 years. Is there an arbitrage opportunity? If yes, show the arbitrage strategy in a cash flow table.

Assume from now on that the underlying asset will pay a dividend of 15 at t=1 year.

b. Show that the put-call parity now writes $C_0 + \frac{K}{(1+r_f)^2} = P_0 + S_0 - \frac{D}{1+r_f}$. [Hint: find a portfolio containing a long position in the call and in the risk-free asset and another portfolio containing a long position in the put and in the stock and a short position in the risk-free asset, that have the same cash flows and apply the law of one price.]

c. Is there an arbitrage opportunity? If yes, show the arbitrage strategy in a cash flow table.

Problem 6

Stock XYZ is selling for €100 today (t=0). Two states of the economy are possible in one year (t=1). In the good state of the economy, the stock will sell for €120. In the bad state of the economy, it will sell for €90. XYZ will not pay any dividend in the years to come. The risk-free interest rate is 3%.

We consider the following European options:

	Underlying asset	Expiry	Strike price	Price at t=0
Call option 1	Stock XYZ	T=1	€100	
Call option 2	Stock XYZ	T=1	€110	€4.21
Put option 1	Stock XYZ	T=1		

Put 1 will pay €20 at date 1 in the bad state of the economy.

a. How much will Put 1 pay at date 1 in the good state of the economy?

b. What is the price of Put 1 today?

c. What is the price of Call 1 today?

d. What would be the price of Call 2 today if Call 2 was American instead of European?

Assume in questions e) and f) only that you can only trade stock XYZ and Put option 1 (you cannot trade the call options). We consider security ABC, which will pay €12 in the good state of the economy and €19 in the bad state of the economy at date 1.

e. Show how you can replicate the payoffs of this security (using only Put 1 and stock XYZ and assuming that you can buy or sell fractions of any security.) What is the no-arbitrage price of security ABC?

f. Assume that security ABC sells for €16 today. Find an arbitrage strategy and provide a table that contains the detail of your trades and transactions at dates 0 and 1.

Problem Set 4: Options

Elements of Answer

Problem 1

a. Exercise the call at maturity if USD/EUR > 1.1

b. Profit will be positive when USD/EUR $- 1.1 > 0.01$, i.e., when USD/EUR > 1.11

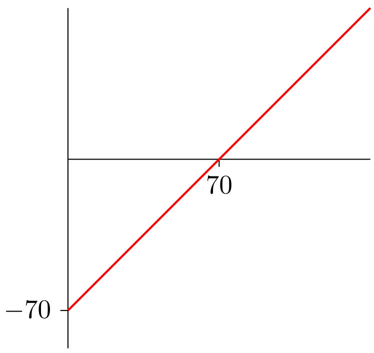
c. Exercise put if USD/EUR < 1.1. Profit will be positive when $1.10 -$ USD/EUR > 0.01, i.e., when USD/EUR < 1.09.

Problem 2

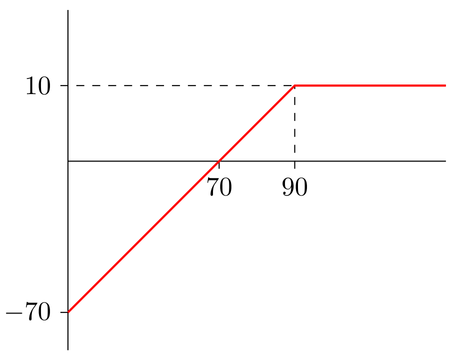
Stock price =	80	100	110	120
	Euro return			
(i) All stocks (100 shares)	8,000	10,000	11,000	12,000
(i) All options (1000 options)	0	0	10,000	20,000
(i) Bills + Options	9,360	9,360	10,360	11,360
	Rate of return			
(i) All stocks (100 shares)	−20%	0%	10%	20%
(i) All options (1000 options)	−100%	−100%	0%	100%
(i) Bills + Options	−6.4%	−6.4%	3.6%	13.6%

Problem 3

a.



b.

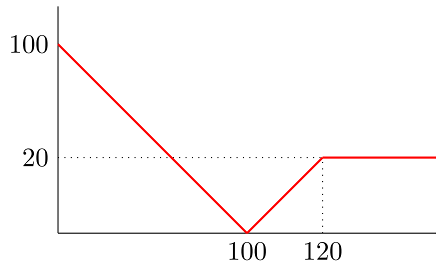


c. Sell 1 forward contract, buy 1 call option with $K = 80$, and buy 2 call options with $K = 90$.

d. That the market under-estimates volatility. Once we take into account the cash-flows at date 0, the strategy makes money if the price moves a lot and loses some if the price does not move too much.

Problem 4

a.



b. (ii) and/or (iii)

The strategy pays off if the price moves far away from 100, therefore (iii).

The payoff is the larger when the price approaches 0, therefore (ii).

c. (i)

Selling the call generates a positive payoff at t=0, therefore (i).

Problem 5

a. There is no arbitrage opportunity if the put-call parity holds:

$$C_0 + \frac{K}{(1+r_f)^T} = P_0 + S_0$$

Since instead $C_0 + \frac{K}{(1+r_f)^T} = 5 + \frac{100}{1.05^2} = 95.70$ and $P_0 + S_0 = 8 + 90 = 98$, the call is underpriced relative to put. The arbitrage strategy is the following:

	$t=0$	$t=T$ if $S_T < K$	$t=T$ if $S_T > K$
Buy the call	-5	0	$S_T - 100$
Invest $\frac{100}{1.05^2} = 90.70$ at the risk-free rate	-90.70	100	100
Sell the put	8	$S_T - 100$	0
Short-sell underlying asset	90	$-S_T$	$-S_T$
Total payoff	2.30	0	0

b. We follow the same logic leading to the put-call parity without dividends that we studied in class. The first portfolio is:

	t=1	$t=2$ if $S_T < K$	$t=2$ if $S_T > K$
Long position in the call	0	0	$S_T - K$
Invest $\frac{K}{(1+r_f)^2}$ at the risk-free rate at t=0 for 2 years	0	K	K
Total payoff	0	K	S_T

The second portfolio is:

	t=1	$t=2$ if $S_T < K$	$t=2$ if $S_T > K$
Long position in the put	0	$K - S_T$	0
Long position in the stock	D	S_T	S_T
Borrow $\frac{D}{1+r_f}$ at the risk-free rate at t=0 for 1 year	$-D$	0	0
Total payoff	0	K	S_T

where we borrow $\frac{D}{1+r_f}$ at t=0 for 1 year in order to offset the dividend payment at t=1.

The price of the first portfolio at t=0 is $C_0 + \frac{K}{(1+r_f)^T}$. The price of the second portfolio at t=0 is $P_0 + S_0 - \frac{D}{1+r_f}$. Since the two portfolios have the same cash flows at all futures dates, they should have the same price today:

$$C_0 + \frac{K}{(1+r_f)^T} = P_0 + S_0 - \frac{D}{1+r_f}$$

c. $C_0 + \frac{K}{(1+r_f)^T} = 95.70$ and $P_0 + S_0 - \frac{D}{1+r_f} = 83.71$, so the call is over-valued relative to the put. There is an arbitrage strategy:

	$t=0$	$t=1$	$t=T$ if $S_T < K$	$t=T$ if $S_T > K$
Sell the call	5		0	$100 - S_T$
Borrow $\frac{100}{1.05^2}$ at the risk-free rate at t=0 for 2 years	90.70		-100	-100
Buy the put	-8		$100 - S_T$	0
Buy the underlying asset	-90	15	S_T	S_T
Borrow $\frac{15}{1.05}$ at the risk-free rate at t=0 for 1 year	14.29	-15		
Total payoff	11.99	0	0	0

Problem 6

a. The strike price of Put 1 is €110. Therefore it pays nothing in the good state.

b. Put-call parity with Call 2 and Put 1: $4.21 + \frac{110}{1.03} = 100 + P$ implies $P = 11$ €.

c. Call 1 pays 0 in the bad state and 20 in the good state, i.e., its payoff is twice the payoff of Call 2. Its price is $2 \times 4.21 = 8.42$ €.

d. Same price.

e. Replicating portfolio with x_1 units of stock XYZ and x_2 units of Put 1:

$$\begin{cases} 12 = 120\ x_1 + 0 \\ 19 = 90\ x_1 + 20\ x_2 \end{cases}$$

which gives $x_1 = 0.1$ and $x_2 = 0.5$. The no-arbitrage price is $0.1 \times 100 + 0.5 \times 11 = 15.5$ €.

f. Security ABC is overpriced. Arbitrage strategy: sell short 1 security ABC + buy 0.1 share of XYZ + buy 0.5 Put

	Payoff at t=0	Payoff at t=1 Bad state	Payoff at t=1 Good state
Short 1 security ABC	+16	-19	-12
Long 0.1 share of stock	-10	+9	+12
Long 0.5 put option	-5.5	+10	0
Total payoff	+0.5	0	0