Practice Final Exam

2 hours

Allowed: all documents and a calculator
FORBIDDEN: cell phones, tablets or laptops

Show all calculations when needed – no calculation, no points! All answers with two decimal places (for ex., 12.34 €, or 12.34%).

Problem 1

Company XYZ pays dividends annually. Its next dividends will be paid in one year from now and will be equal to €6 per share. These dividends are expected to grow forever at a constant rate of 5% per year. Next year's earnings per share of company XYZ are expected to be equal to €8. The required rate of return on stock XYZ is 10% per year.

a. What is the price of a share of XYZ stock?

b. How does the price of a share of XYZ stock change if the systematic risk of the company (as measured by its market beta) increases while holding everything else (risk-free rate, expected market return, dividends, earnings, etc.) constant?

c. What is the return on investment (also called return on equity) of the new projects available to company XYZ and in which XYZ reinvests part of its earnings?

d. What would be the price of a share of XYZ stock if the company decided to pay out all of its earnings as dividends forever (in which case its next year's earnings would be €8 per share and would remain constant at that level forever)?

e. How much is XYZ's Present Value of Growth Opportunities?

Problem 2

Please find below details of three default-free fixed rate bonds with annual coupon payments:

	Coupon rate	Face Value	Maturity	Price at t=0
Bond 1	0%	100	1 year	97.09
Bond 2	10%	100	2 years	111.41
Bond 3	15%	100	3 years	127.77

You will assume that it is possible to buy or sell any fraction of the three bonds (ignore integer constraints).

a. Find the term structure of interest rates at t=0.

b. What is the forward rate for investing/borrowing between years 1 and 3 (that might be denoted as $f_{1\to3}$ or $r_1(2Y)$) implied by this term structure?

c. Suppose for this question (and for this question only) that you cannot short-sell any of the three bonds. If you wish to build a portfolio with two of the three bonds and a duration of 2 years, which one of the three bonds will necessarily have to be in your portfolio? Why?

d. Suppose that you buy Bond 3 today and sell it back one year later. If the term structure has not shifted during this time (i.e., the 1-year, 2-year and 3-year interest rates are the same next year as they are today), what is your one-year holding period return equal to?

Problem 3

Consider a forward contract on 100 shares of stock ABC with maturity in exactly one year from now. Today the spot price for one share of ABC is €12 and the one-year risk-free interest rate is 3%. In 9 months from now, ABC will pay a dividend of €0.15 per share.

a. What is the no-arbitrage forward price for this contract?

b. Assume in this question only that the forward contract trades at a forward price of €1,218. Formulate an arbitrage strategy. Show all components and cash flows of this strategy in a cash flow table.

Suppose that one year ago you entered in a long position into this contract (which was then a two-year contract) at a forward price of €1,200 and that today you close your position at the no-arbitrage price you computed in question a.

c. What is the value today of your (initial) long position, that is, what is the profit you earn today by closing your position?

Problem 4

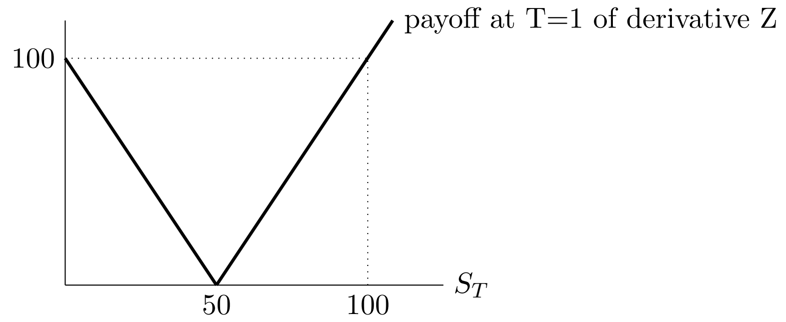
The risk-free rate is 0%. Consider the following derivative contracts written on the same underlying asset. The underlying asset will have a price of S_T at the maturity date of the derivative contracts.

The first derivative contract is a European call option with an exercise price of $K = 50$ and maturity $T = 1$ year with a current price of $C_0 = 5$.

The second derivative contract is a European put with the same exercise price of $K = 50$, a maturity of $T = 1$ year and a current price of $P_0 = 15$.

a. What is the current price of the underlying asset such that there is no arbitrage opportunity?

The third derivative contract Z has a maturity of $T = 1$ year and will pay at maturity a cash flow equal to the absolute value of $2 \times (S_T - K)$, that is, twice the absolute difference between S_T and $K = 50$ (the payoff diagram of derivative Z is plotted below). The current price of derivative Z is $Z_0 = 45$.



b. Build an arbitrage portfolio with the following three assets:

(i) the European call option described above;
(ii) the European put option described above;
(iii) the derivative contract Z described above.

Show the arbitrage strategy in a cash flow table.

The fourth derivatives is a forward contract with maturity $T = 1$ year on the same underlying asset as the other derivative contracts with a forward price of $F_0 = 50$ at $t = 0$.

c. Build an arbitrage portfolio with the following three assets:

   (i) the European call option described above;

   (ii) the European put option described above;

   (iii) the forward contract described above.

   Show the arbitrage strategy in a cash flow table.

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Elements of Answer

Problem 1

a. $P_0 = \frac{D_1}{k-g} = \frac{6}{0.10 - 0.05} = 120$ €

b. If the market beta increases, then the required rate of return k increases, and the price decreases.

c. Payout ratio $= 1 - b = \frac{D_1}{E_1} = \frac{6}{8} = 0.75$, therefore the retention ratio is $b = 0.25$

$ROE^{new-projects} = \frac{g}{b} = \frac{0.05}{0.25} = 20\%$

d. $P_0^{no-growth} = \frac{E_1}{k} = \frac{8}{0.10} = 80$ €

e. $PVGO = P_0 - P_0^{no-growth} = 120 - 80 = 40$ € per share

Problem 2

a. $97.09 = \frac{100}{1+r_1}$ implies $r_1 = 3\%$

$111.41 = \frac{10}{1.03} + \frac{110}{(1+r_2)^2}$ implies $r_2 = 4\%$

$127.77 = \frac{15}{1.03} + \frac{15}{1.04^2} + \frac{115}{(1+r_3)^3}$ implies $r_3 = 5\%$

b. $(1+r_1)(1+f_{1,3})^2 = (1+r_3)^3$ implies $f_{1,3} = 6\%$

c. Bonds 1 and 2 have durations shorter than 2 years; bond 3 has a duration longer than 2 years. Thus, only a combination of bond 3 and of bond 1 or 2 can have a duration of 2 years.

d. $\text{HPR} = \frac{\text{coupon(bond 3)} + P_1\text{(bond 3)} - P_0\text{(bond 3)}}{P_0\text{(bond 3)}}$

Next year's price is equal to the bond's cash flows (15 on year 2 and 115 on year 3) discounted at next year's yield curve (which is the same as this year's yield curve):

$P_1\text{(bond 3)} = \frac{15}{1+r_1} + \frac{115}{(1+r_2)^2} = \frac{15}{1.03} + \frac{115}{1.04^2} = 120.89$

Thus, HPR = 6.3%

Problem 3

a. $F_0 = (100 \times 12 \times 1.03) - (100 \times 0.15 \times 1.03^{0.25}) = 1220.89$€

b. Take a long position in a forward contract on 100 shares and simultaneously short-sell the stock.

	t=0	t=9 months	t=1 year
Take a long position in a forward contract			-1218
Short-sell 100 shares of the underlying stock	1200		
Invest €1200 for 1 year	-1200		1200×1.03
Borrow €15 at t=9 months for 3 months		15	$-15 \times 1.03^{0.25}$
Pay dividends		-15	
Total cash flows at time t	0	0	+2.89

c. To close your position you need to enter a short position in the forward contract with one-year maturity.

By doing this, you now hold a long position in the contract created one year ago (at $t = -1$) and a short position in the contract created today (at $t = 0$), generating a certain cash flow of $1220.89 - 1200 = 20.89$ at $t = 1$.

The value at $t = 0$ of this cash flow is $\frac{20.89}{1.03} = 20.28$€.

Problem 4

a. Put-call parity: $S_0 = C_0 + \frac{K}{(1+r_f)^T} - P_0 = 5 + 50 - 15 = 40$

b. Z is twice a long straddle, that is, it is the combination of two puts and two calls. Z costs 45, whereas the portfolio of two puts and two calls would cost $2 \times 5 + 2 \times 15 = 40$. Hence there is an arbitrage opportunity:

	CF at t=0	CF at t=1	
		if $S_T < 50$	if $S_T > 50$
Sell Z	$+45$	$-2 \times (50 - S_T)$	$-2 \times (S_T - 50)$
Buy 2 Calls	-2×5		$2 \times (S_T - 50)$
Buy 2 Puts	-2×15	$2 \times (50 - S_T)$	
Total	$+5$	0	0

c. Note that you can combine the forward and the put (both long positions) to replicate a long call position:

	CF at t=0	CF at t=1	
		if $S_T < 50$	if $S_T > 50$
Buy 1 Call	-5		$S_T - 50$
Sell 1 Forward		$50 - S_T$	$50 - S_T$
Sell 1 Put	15	$-(50 - S_T)$	
Total	$+10$	0	0