Financial Markets
1: Stocks

Stefano Lovo

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Objective of this course

1. Get familiar with the most common financial assets.
2. Recognize the payments that these assets make to their holders.
3. Determine the price at which these asset are traded in the financial market.

What financial assets we study in this course?

- Stocks (Part 1)
- Bonds (Part 2)
- Derivatives
  - Forwards and futures (Part 3)
  - Options (Part 4)

Financial markets' sizes

- Values in trillion US $

	GDP	Stocks	Bonds	Derivatives
World	81	69	90	530*
USA	19.4	26.0	39.3	
Euro area	12.6	6.5	17.0	
China	12.2	7.9	11.8	
Japan	4.8	4.9	12.7	
Germany	3.7	1.7	3.7	
France	2.6	2.2	4.6	
UK	2.6	3.2	6.0	

* notional value

Pre-requisite for this Course

- Time value of money:
  - Interest rate
  - Discount rate
  - Future value
  - Present value
  - Annuities
- CAPM
  - Market portfolio and its return.
  - Beta of an asset.
  - Security Market Line: $E[\tilde{r}_i] = r_f + \beta_i(E[\tilde{r}_M] - r_f)$

Housekeeping 1/2

Material

–Slides + Reader

–Problem sets + Practice quizzes + Practice exam

–Textbook (Optional) *Investments*, by Bodie, Kane and Marcus

–Tutorials (Optional), dates on the syllabus.

-Discussion forum

Evaluation

(1) Best 4 out of 5 quizzes = 1/3 of final grade

- 10 min at beginning of class, open book
- Bring a calculator
- If you take the quiz, you also take the rest of the class.

(2) Final exam = 2/3 of final grade

- 2 hours, open book

Housekeeping 2/2: **COVID 19 manners**

- If you attend class face to face it is compulsory to
  1. wear a mask on mouth and nose
  2. respect distancing when sitting in the room
  3. If you have any symptoms of flu or cold, please do not come to class.

- If you attend class on-line
  - Keep your camera on and your microphone off
  - Be prepared to answer questions
  - Feel free to unmute your microphone to ask and answer questions
  - You can ask questions using the chat
  - Answer zoom polls wen launched

Part 1: Stocks & Market Efficiency

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Overview

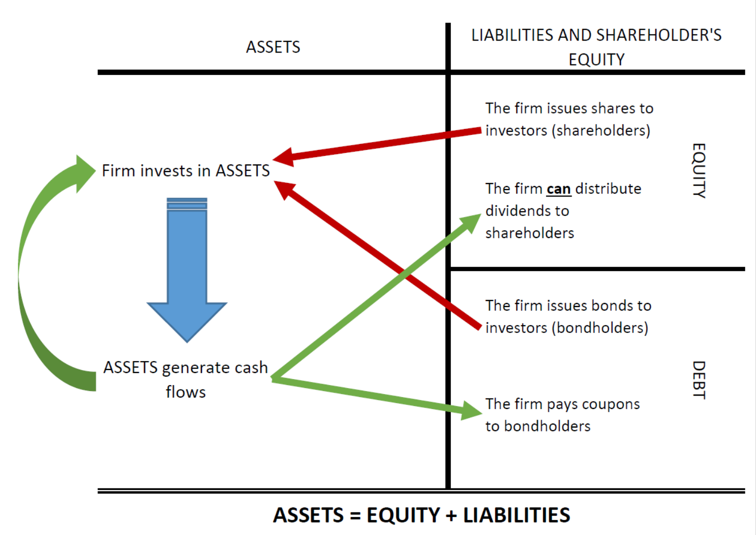
- Today

  1. Stock basics
  2. Dividend Discount Model
  3. Present Value of Growth Opportunities
  4. Price-Earnings ratio

- Next class

  5. Market efficiency
  6. Law of one price

Balance sheet of a firm



Stock definition

Definition

A share of common stock (also referred to as stock or equity) is a financial contract that represents ownership of a specific portion of the company that has issued it.

Stockholders' rights

(stockholder = shareholder = equity holder)

1. Ownership rights
   - The firm belongs to stockholders (unless it is bankrupt)
   - Stockholders approve the firm's important decisions
   - Stockholders hire and fire managers of the management board.
2. Residual cash-flow rights
   - The firm pays suppliers, employees, tax authorities first

     ...then creditors (banks, bondholders)

     ...whatever is left **can** be distributed as dividends to stockholders
   - Stockholders have limited liability

Stock Cash-flows

Time	year 1	year 2	...	year n	...
Cash flow	$\tilde{D}_1$	$\tilde{D}_2$	...	$\tilde{D}_n$	...

where $\tilde{D}_t$ is the dividend that each shareholder will receive at time t.

Important Remark: Today, future dividends are not known. For this reason we treat them as random variables.

Market capitalization vs. book value

- Example: Apple

  book value = \$70 billion

  market cap = \$2 000 billion

- Book value = equity capital booked on the balance sheet $\rightarrow$ determined by accounting rules
- Market capitalization = number of shares $\times$ price of one share $\rightarrow$ what determines how much stock market investors are willing to pay for a stock?

Valuing a stock

Example A stock is selling today for $P_0 = €30$. The analysts expect that the company will pay a dividend of $D_1 = €2$ in exactly one year. You expect to sell the stock right after the dividend payment in one year at a price of $P_1 = €33$.

- What is your expected holding period return?

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- What is your expected holding period return?

$$HPR = \frac{\text{expected profit}}{\text{initial investment}} \quad \underbrace{\frac{E[D_1]}{P_0}}_{\text{dividend yield}} + \underbrace{\frac{E[P_1] - P_0}{P_0}}_{\text{cap. gain (or loss)}} =$$

$$0.0667 + 0.1 = 16.667\%$$

- **Remark:** Neither capital gain (or loss) nor dividend yield is guaranteed! Your realized return may be different from your expected return

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- Should you buy this stock?

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- What is your expected holding period return?

  16.667%

- Should you buy this stock?
  Cannot answer, depends on the riskiness of the stock: need to compare this expected HPR to the required rate of return.

Stock valuation problem

Short term investor: an investor who plans to buy a stock, hold it for 1 year, cash-in the dividend (if any) and sell the stock.

Time:	today	year 1
Action:	buy the stock	cash-dividend and resell
Cash-flow:	$-P_0$	$\tilde{D}_1 + \tilde{P}_1$

$$\text{Expected Net present Value} = E[NPV] = -P_0 + \frac{E\left[\tilde{D}_1 + \tilde{P}_1\right]}{1+k}$$

where k is **the opportunity cost of capital**, i.e., the interest rate the investor could gain in an alternative investment with the same risk factor.

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where k is **the opportunity cost of capital**, i.e., the interest rate the investor could gain in an alternative investment with the same risk factor. According to the CAPM...

$$k = r_f + \beta_i(E[\tilde{r}_M] - r_f)$$

Valuing a stock; CAPM Refresher:

In the Capital Asset Pricing Model (CAPM), the expected return is given by

$$k = r_f + \beta(E[r_M] - r_f)$$

- r_f: risk-free rate
- β: systematic risk = part of the risk that cannot be eliminated by holding a diversified portfolio
- $E[r_M] - r_f$: equity risk premium = expected excess return on the market portfolio (remember "excess return" means return minus risk-free rate)

A simple problem

Consider a 1 year investment in stock ABC with $\beta = 1.1$. You have the following information:

$$\begin{aligned} P_0 &= €30, E[\tilde{D}_1] = €2, E[\tilde{P}_1] = €33 \\ r_f &= 2\%, E[\tilde{r}_M] = 25\% \end{aligned}$$

Suppose CAPM holds. Would you buy or short sell the asset?

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$$OCC = k = 2\% + 1.1(25\% - 2\%) = 27.3\%$$

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$$E[NPV_{buy}] = -P_0 + E\left[\frac{\tilde{D}_1 + \tilde{P}_1}{1+k}\right] = -30 + \frac{2+33}{1+27.3\%} = -2.51$$

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$$\begin{aligned} E[NPV_{buy}] &= -P_0 + E\left[\frac{\tilde{D}_1 + \tilde{P}_1}{1+k}\right] = -30 + \frac{2+33}{1+27.3\%} = -2.51 \\ E[NPV_{short}] &= +P_0 - E\left[\frac{\tilde{D}_1 + \tilde{P}_1}{1+k}\right] = +30 - \frac{2+33}{1+27.3\%} = 2.51 \end{aligned}$$

A simple problem: conclusion

Consider a 1 year investment in stock ABC with $\beta = 1.1$. You have the following information:

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There is equilibrium between demand and supply for this stock only if

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$$

There is equilibrium between demand and supply for this stock only if

$$
P_0 = V_0 := E\left[\frac{\tilde{D}_1 + \tilde{P}_1}{1 + k}\right] = \frac{2 + 33}{1 + 27.3\%} = 27.49
$$

- I f there is equilibrium between demand and supply for a given stock, how the stock opportunity cost of capital compares to the stock expected holding period return?

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  $OCC = HPR$

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- What is the fair value of the stock (i.e., the stock price such that the expected return is as given by the CAPM)?

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$$V_0 = \frac{33 + 2}{1.273} \simeq €27.49$$

Dividend Discount Model (DDM)

Consider an investor that plans to take a position (long or short) into a stock for n years. Let k be the annual required return given the risk of this stock (opportunity cost of capital).

Excess demand for the stock

$$P_0 < \frac{E[\tilde{D}_1]}{(1+k)^1} + \frac{E[\tilde{D}_2]}{(1+k)^2} + \cdots + \frac{E[\tilde{D}_n + \tilde{P}_n]}{(1+k)^n}$$

Excess supply for the stock

$$P_0 > \frac{E[\tilde{D}_1]}{(1+k)^1} + \frac{E[\tilde{D}_2]}{(1+k)^2} + \cdots + \frac{E[\tilde{D}_n + \tilde{P}_n]}{(1+k)^n}$$

Equilibrium price only if

$$P_0 = \frac{E[\tilde{D}_1]}{(1+k)^1} + \frac{E[\tilde{D}_2]}{(1+k)^2} + \cdots + \frac{E[\tilde{D}_n + \tilde{P}_n]}{(1+k)^n}$$

Poll 1: investing horizon

Consider the following three investors willing ton buy stock ABC

1. Investor 1 plans to resell the stock after one year
2. Investor 2 plans to resell the stock after two year
3. Investor 3 plans on holding the stock forever

Which investors values the stock the most?

Poll 1: investing horizon

Investor 1 plans to resell the stock after one year

Investor 2 plans to resell the stock after two year

Investor 3 plans on holding the stock forever

Fundamental value of a stock

Definition

The fundamental value of a stock V_0 is the present value of the expected infinite stream of future dividend that the stock will pay, using the opportunity cost of capital given the risk of the stock:

$$V_0 = \frac{E[\tilde{D}_1]}{(1+k)^1} + \frac{E[\tilde{D}_2]}{(1+k)^2} + \cdots + \frac{E[\tilde{D}_n]}{(1+k)^n} + \cdots = \sum_{t=1}^{\infty} \frac{E[\tilde{D}_t]}{(1+k)^t}$$

Fundamental value of a stock

Theorem

Under the CAPM assumptions, the equilibrium price of a stock is equal to its fundamental value.

- The fundamental value V_0 of a stock does not depend on your holding period (e.g., whether you hold it for 1 year, 2 years, or forever)
- To calculate V_0 you need to specify a scenario for the dividend schedule.

Constant Growth DDM

- Simplest scenario: dividends grow at constant rate g
- The DDM becomes

$$V_0 = \frac{(1+g)D_0}{1+k} + \frac{(1+g)^2 D_0}{(1+k)^2} + \ldots + \frac{(1+g)^T D_0}{(1+k)^T} + \ldots$$

- And if $k > g$, using the annuity formula with infinite horizon

$$\boxed{V_0 = \frac{D_1}{k-g} = \frac{(1+g)D_0}{k-g}}$$

The DDM with constant growth rate is called the Gordon model

- $g \geq k$ cannot happen. Why?

Example

Hi5 Inc. has just paid an annual dividend of €2.5 per share. You expect the dividend to grow at 5% per year indefinitely. Given its riskiness, you require an expected return of 12% per year on this stock.

- What is the value of Hi5's stock?

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- What is the value of Hi5's stock?

$$D_0 = €2.5 \quad g = 0.05 \quad k = 0.12 \quad \Rightarrow V_0 = \frac{(1+g)D_0}{k-g} = \frac{1.05 \times 2.5}{0.12 - 0.05} = 37.50€$$

- The company is financed with 1 million shares outstanding. What is the market capitalization of Hi5?

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$$37.5 \times 1 \text{ million} = 37.5 \text{ million } €$$

- An investor is offering to sell Hi5 shares at €35 per share.What do you think?

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Poll 2: Companies that paid no dividend

Google and Faceboock never paid any dividend. Nevertheless their shares have a positive price in the stock market

Is this in contradiction with the discounted dividend model?

DDM: life cycle considerations

Many firms do not pay dividends Facebook Google

- What is the value of their equity?

DDM: life cycle considerations

Many firms do not pay dividends Facebook Google

- What is the value of their equity?
  Assumption of constant growth rate of dividends not appropriate for these stocks. They have positive value because (investors expect that) they **will eventually** pay dividends. Need to consider different scenarios of dividend schedules for these stocks $\rightarrow$ multi-stage growth DDM.

[See problem 2 in problem set on stocks.]

Present Value of Growth Opportunities

- Growth opportunities arise if the company retains some of its earnings and invests them in new projects
- The retention ratio or plowback ratio (b) is the proportion of earnings per share (EPS, or E_t) that is reinvested in new projects
- The dividend payout ratio ($1 - b$) is the proportion of earnings per share paid out as dividend
- If new projects have returns on investment (or returns on equity) of $\text{ROE}^{\text{new project}}$, then the growth rate of the company's earnings is

$$\boxed{g = b \times \text{ROE}^{\text{new project}}}$$

- Why?

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- Why?

$$E_t \quad =$$

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- Why?

$$\begin{aligned} E_t &= E_{t-1} + b \times E_{t-1} roe = (1 + b \times ROE) E_{t-1} \\ D_t &= \end{aligned}$$

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- Why?

$$E_t = E_{t-1} + b \times E_{t-1} roe = (1 + b \times ROE)E_{t-1}$$

$$D_t = (1 - b)E_t = (1 - b)(1 + b \times ROE)E_{t-1} = (1 + b \times ROE)D_{t-1}$$

- What is P_0?

Present Value of Growth Opportunities

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- Why?

$$\begin{aligned} E_t &= E_{t-1} + b \times E_{t-1} roe = (1 + b \times ROE) E_{t-1} \\ D_t &= (1 - b) E_t = (1 - b)(1 + b \times ROE) E_{t-1} = (1 + b \times ROE) D_{t-1} \end{aligned}$$

- What is P_0?

$$P_0 = \frac{E_1 (1 - b)}{k - b \times roe}$$

Poll 3: Example

FatCat Co. has earnings per share E_1 = €4 and a required rate of return of 10% per year

Q1 If FatCat pays out all earnings as dividends forever, what is its fundamental value?

Poll 3: Example

FatCat Co. has earnings per share E_1 = €4 and a required rate of return of 10% per year

Q1 If FatCat pays out all earnings as dividends forever, what is its fundamental value? Ans. €40

Now, suppose FatCat increases its retention ratio to 25% to undertake new projects that generate returns on investment of 16% per year

Q2 What is $\frac{D_t}{E_t}$, i.e., FatCat's new dividend payout ratio?

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Q2 What is $\frac{D_t}{E_t}$, i.e., FatCat's new dividend payout ratio?Ans.75%

Q3 What is FatCat's growth rate?

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Q2 What is $\frac{D_t}{E_t}$, i.e., FatCat's new dividend payout ratio?Ans.75%

Q3 What is FatCat's growth rate?Ans. 4%

Recap about stock pricing formula so far...

- In general:

$$V_0 = \frac{E[\tilde{D}_1]}{(1+k)^1} + \frac{E[\tilde{D}_2]}{(1+k)^2} + \cdots + \frac{E[\tilde{D}_n]}{(1+k)^n} + \cdots = \sum_{t=1}^{\infty} \frac{E[\tilde{D}_t]}{(1+k)^t}$$

- If dividends grow at a constant rate $g < k$, then

$$V_0 = \frac{D_1}{k-g}$$

- If the growth of dividends is obtained by consistently reinvesting at interest rate ROE a fraction b of earnings, then

$$V_0 = \frac{E_1(1-b)}{k - bROE}$$

where: k=OCC; g= dividend rate of growth; b= plowback ratio; ROE= return on equities.

Present Value of Growth Opportunities

If the growth of dividends is obtained by consistently reinvesting at interest rate ROE a fraction b of earnings, then

$$V_0 = \frac{E_1(1-b)}{k - bROE}$$

Example: ThinDog Co. has earnings per share E_1 = €4 and a required rate of return of k=10% per year.

- what is ThinDog stock price if 100% of earning are consistently distributed to shareholders (i.e., $b = 0$)

Present Value of Growth Opportunities

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Example: ThinDog Co. has earnings per share E_1 = €4 and a required rate of return of k=10% per year.

- what is ThinDog stock price if 100% of earning are consistently distributed to shareholders (i.e., $b = 0$) Ans. €40
- what is ThinDog stock price if 60% of earning are reinvested into the firm at $ROE = 5\%$?

Present Value of Growth Opportunities

If the growth of dividends is obtained by consistently reinvesting at interest rate ROE a fraction b of earnings, then

$$V_0 = \frac{E_1(1-b)}{k - bROE}$$

Example: ThinDog Co. has earnings per share E_1 = €4 and a required rate of return of k=10% per year.

- what is ThinDog stock price if 100% of earning are consistently distributed to shareholders (i.e., $b = 0$) Ans. €40
- what is ThinDog stock price if 60% of earning are reinvested into the firm at $ROE = 5\%$? Ans. €22.86

Present Value of Growth Opportunities

- We can decompose the firm's stock price P_0 into two components

$$\boxed{P_0 = P_0^{\text{no growth}} + \text{PVGO}}$$

- $P_0^{\text{no growth}}$ is the price that would prevail if the company paid out all its earnings and would not grow ($b = g = 0$)

$$P_0^{\text{no growth}} = \frac{E_1}{k}$$

- PVGO is the Present Value of Growth Opportunities: it is the difference between the actual value of the stock and its hypothetical value if the firm did not grow

$$PVG0 = \frac{E_1(1-b)}{k - b \times ROE} - \frac{E_1}{k}$$

Example

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Now, suppose FatCat increases its retention ratio to 25% to undertake new projects that generate returns on investment of 16% per year

Q2 What is FatCat's new dividend payout ratio?Ans.75%

Q3 What is FatCat's growth rate?Ans. 4%

Q4 What is FatCat stock's new fundamental value and PVGO (assuming next year's earnings are still 4€ per share)? Ans. €50 and €10

Growth opportunities and capital budgeting

- When is it the case that $PVGO > 0$?

Growth opportunities and capital budgeting

- When is it the case that $PVGO > 0$?

- How do you relate this to capital budgeting decisions?

Price-Earnings ratio

- The price-earnings ratio P/E is a commonly used financial indicator
- It gives some information on growth rates expected by the market

$$\boxed{\frac{P_0}{E_1} = \frac{1-b}{k-g}}$$

- A higher P/E is indicative of higher PVGO

$$\boxed{\frac{P_0}{E_1} = \frac{1}{k} \times \left[1 + \frac{\text{PVGO}}{P_0^{\text{no growth}}}\right]}$$

- Which company has the highest P/E? Amazon, Apple or Facebook?

Practical use of P/E ratio

1. Another method to value a stock: the “comparables approach“
   - You observe P/E for listed company A and want to value the stock of unlisted company B in the same industry $\Rightarrow$ (P/E of A) $\times$ (Earnings of B)
   - What are the underlying assumptions of the comparables approach?
     Assumption: A and B have same g and k. That's why we choose A in same industry as B; in practice use several comparable firms A in same industry. See corporate finance course.
2. Time variation in market P/E
   - Historical P/E ratio

Informational efficiency

- Fama (1970): A market is informationally efficient if all publicly available information is instantaneously reflected in security prices

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Definition

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Definition

- **Weak-form efficiency:** Trading prices incorporate all past common information.

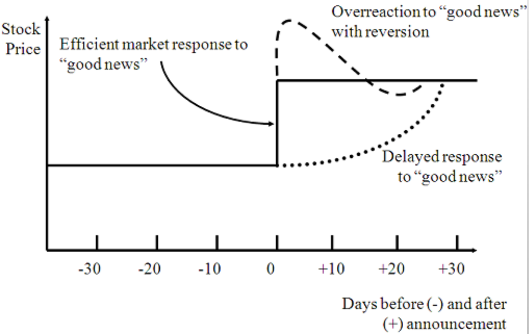
  You can't beat the market knowing the past.
- **Semi-strong-form efficiency:** Trading prices incorporate all public information (past and present).

  You can't beat the market using publicly available information
- **Strong form efficiency:** Trading prices incorporate all information available in the economy (public and private).

  No information of any kind can be used to beat the market.

Informational efficiency

- Example: Stock price reaction to good news



- NB: Informational efficiency is about whether security prices accurately reflect fundamental value, not whether capital markets optimally allocate resources

Why may financial markets be informationally efficient?

1. "Wisdom of the crowd"

   - The market aggregates information disseminated among many investors (even if each single investor has very little info about the fundamental value)

How many dry half green peas are there in this jar?

To answer, please send an e-mail to lovo@hec.fr with subject "beans"

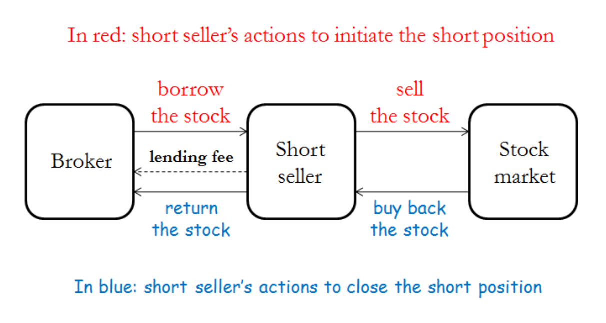
Why may financial markets be informationally efficient?

2. Sophisticated investors

   - If price is too low, sophisticated investors buy the stock, pushing the price up
   - Conversely, if price is too high, sophisticated investors sell the stock, pushing the price down.
     But what if they don't own the stock?

Short selling

- A short sale is the sale of a security you don't own



Stock return (un)predictability

- If the market is informationally efficient, it is impossible to predict future returns based on available information

Poll: Informational efficiency – Examples

Is market efficiency contradicted in the following situations?

Q1 Through the introduction of a complex computer program analyzing past stock price changes, a brokerage firm is able to predict price movements well enough to earn a consistent 3% profit, adjusted for risk, above normal market return.

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Ans.: Yes

Q2 Crook Inc. was facing a lawsuit brought forward by disgruntled clients. The result of the lawsuit has just become public this morning before markets open and the company is liable of a €100 million fine. The stock price of Crook Inc. goes up by 10% today.

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Q3 On average, investors in the stock market this year are expected to earn positive returns on their investment. Some will earn considerably more than others.

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Market Informational Efficiency: Empirical evidence

- Financial market is weak form efficient (no use of technical analysis)
- Financial market is semi-strong form efficient
  - Market reaction to news
- Financial market is not strong form efficient.
- It is hard to beat the market!

Pricing by Arbitrage

Any financial asset can be represented as a **list of cash flows with labels** that describe when and under which conditions each cash flow will be paid.

Question: Is it possible to find the market price of an asset once we have its complete description?

Pricing by Arbitrage: example 1

Asset name	current price	in 1 year	in 2 years, iff DJ < 20,000
Asset A	P_A=???	€1,000	€2,000

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Asset name	current price	in 1 year	in 2 years, iff DJ < 20,000
Asset A	P_A=???	€1,000	€2,000
ZCB	P_{ZCB} = €98	€100	0
BEAR	P_{Bear} = €600	0	€1,000

What is the fair/equilibrium price of Asset A?

Pricing by Arbitrage: example 1 continued

Consider a portfolio R that contains 10 assets ZCB and 2 assets Bear.
What is the stream of cash flows generated by this portfolio?

Pricing by Arbitrage: example 1 continued

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Asset name	current price	in 1 year	in 2 years, iff DJ < 20,000
ZCB	P_{ZCB} = €98	€100	0
BEAR	P_{Bear} = €600	0	€1,000
Portfolio R	€$(10 \times 98 + 2 \times 600)$ = €2180	€10×100 = €1,000	€$2 \times 1,000$ = €2,000

Pricing by Arbitrage: example 1 continued

Consider a portfolio R that contains 10 assets ZCB and 2 assets Bear.
What is the stream of cash flows generated by this portfolio?

Asset name	current price	in 1 year	in 2 years, iff DJ < 20,000
Asset A	P_A=???	€1,000	€2,000
Portfolio R	€2180	€1,000	€2,000

Remark: As holding Portfolio R is perfectly equivalent to holding asset A, we say that Portfolio R is the replicating portfolio of Asset A.

Replicating portfolio and arbitrage strategy

Definition

A replicating portfolio of a given financial asset A is a portfolio R composed of other assets such that portfolio R generates **exactly the same cashflow in exactly the same circumstances** as asset A.

Definition

An arbitrage strategy is a way to make 'money for nothing', i.e., a portfolio that have 0 or negative cost when you buy it, and whose only cashflows are strictly generates positive.

Pricing by Arbitrage: example 1 continued

Case i): Suppose that $P_A = 2,000 < P_R = 2,180$
Then we have an arbitrage strategy:

Trade	Today	In 1 Year	In 2 Years iff DJ < 20,000
Buy Asset A	€-2,000	€1,000	€2,000
Short Portfolio R	€2,180	€-1,000	€-2,000
	€180	**€0**	**€0**

This arbitrage strategy increases my current wealth by €180 without affecting:

- the cash flows I will have to pay or receive in the future;
- the risk of my portfolio.

Pricing by Arbitrage: example 1 continued

If $P_A < P_R$, then

1. There exists an arbitrage strategy consisting of buying Asset A and short selling the Replicating portfolio R.
2. Anybody who likes money will implement this strategy no matter his/her risk attitude.
3. There is excess demand of Asset A and excess supply of Portfolio R, i.e., assets ZCB and Bear.

$$\Downarrow$$

The price of asset A increases whereas the prices of assets ZCB and Bear decrease.

Pricing by Arbitrage: example 1 continued

Case ii): Suppose that $P_A = 2,500 > P_R = 2,180$
Then we have an arbitrage strategy:

Trade	Today	In 1 Year	In 2 Years iff DJ < 20,000
Short Sell Asset A	€2,500	€-1,000	€-2,000
Buy Portfolio R	€-2,180	€1,000	€2,000
	€ 320	**€0**	**€0**

This arbitrage strategy increases my current wealth by €320 without affecting:

- the cash flows I will have to pay or receive in the future;
- the risk of my portfolio.

Pricing by Arbitrage: example 1 continued

If $P_A > P_R$, then

1. There exists an arbitrage strategy consisting of short selling Asset A and buying the Replicating portfolio R.
2. Anybody who likes money will implement this strategy no matter his/her risk attitude.
3. There is excess supply of Asset A and excess demand of Portfolio R, i.e., assets ZCB and Bear.

$$\Downarrow$$

The price of asset A decreases whereas the prices of assets ZCB and Bear increase.

Pricing by Arbitrage: example 1 Conclusion

Theorem

Law of one price: *In equilibrium, i.e., in the absence of arbitrage opportunities, the price of Asset A is equal to the price of its replicating portfolio.*

Arbitrage portfolio and strategy

Definition

An arbitrage portfolio is a portfolio whose price is not positive (or strictly negative), that produces not negative (or some strictly positive) cash flows in the future.

Example

Trade	Today	In 1 Y.	In 2 Y. iff DJ < 20,000
Short Asset A	€2,500	€-1,000	€-2,000
Buy Portfolio R	€-2,180	€1,000	€2,000
	€ 320	**€0**	**€0**

Arbitrage portfolio and strategy

Definition

An arbitrage portfolio is a portfolio whose price is not positive (or strictly negative), that produces not negative (or some strictly positive) cash flows in the future.

Example

Trade	Today	In 1 Y.	In 2 Y. iff DJ < 20,000
Short Asset A	€2,500	€-1,000	€-2,000
Buy 1.147 of R	€-2,500	€1,147	€2,294
	€ 0	**€147**	**€294**

Pricing by arbitrage: the method

1. Provide a complete description of the asset we want to price.
2. Build its replicating portfolio starting from assets with known prices.
3. The price of the asset is equal to the price of its replicating portfolio.

Pricing by arbitrage: the general framework

The ingredients:

- There are n financial assets $1, 2, \dots n$.
- Let p_i be the current price of asset i.
- Let $CF_i(\omega_j)$ be the cash flow that asset i pays to its holders when ω_j happens.

Example

ω_1= In 1 year time;
ω_2= In 2 years, iff DJ < 25,000;
ω_3= in 3 weeks iff the price of oil in 2 weeks is USD 100
$\dots$

Pricing by arbitrage: the general framework

The dishes:
Consider portfolio R that contains assets $1, 2, 3, \ldots n-1$, in quantities $y_1, y_2, \ldots, y_n$ respectively. The market price of this portfolio is:

$$P_R = p_1 y_1 + p_2 y_2 + \ldots p_{n-1} y_{n-1} = \sum_{i=1}^{n-1} p_i y_i$$

The cash-flow that portfolio R pays in event ω_j is

$$CF_R(\omega) = CF_1(\omega)y_1 + CF_2(\omega)y_2 + \cdots + CF_{n-1}(\omega)y_{n-1} = \sum_{i=1}^{n-1} CF_i(\omega)y_i$$

Pricing by arbitrage: the general framework

Definition

A portfolio R is the replicating portfolio of Asset n if and only if for all possible ω

$$CF_R(\omega) = CF_n(\omega)$$

Pricing by arbitrage: central result

Theorem

The no-arbitrage price of an asset is equal to the the price of its replicating portfolio.

Poll: Pricing by arbitrage: Example 2

Asset name	current price	in 1 year	in 2 years, iff DJ < 20,000
Asset A	P_A =€2,180	€1,000	€2,000
ZCB	P_{ZCB} = €98	€100	0
BEAR	P_{Bear} =???	0	€1,000

- What is the replicating portfolio of asset Bear?

Poll: Pricing by arbitrage: Example 2

Asset name	current price	in 1 year	in 2 years, iff DJ < 20,000
Asset A	P_A =€2,180	€1,000	€2,000
ZCB	P_{ZCB} = €98	€100	0
BEAR	P_{Bear} =???	0	€1,000

- What is the replicating portfolio of asset Bear?

  buy 0.5 of asset A and short 5 asset ZCB
- What is the fair/equilibrium price of asset Bear ?

Poll: Pricing by arbitrage: Example 2

Asset name	current price	in 1 year	in 2 years, iff DJ < 20,000
Asset A	P_A =€2,180	€1,000	€2,000
ZCB	P_{ZCB} = €98	€100	0
BEAR	P_{Bear} =???	0	€1,000

- What is the replicating portfolio of asset Bear?
  buy 0.5 of asset A and short 5 asset ZCB
- What is the fair/equilibrium price of asset Bear ?
  €600

Pricing by arbitrage: Example 3

Asset name	current price	in 1 Y.	in 2 Y., iff DJ < 20,000	in 2 Y., iff DJ $\geq$ 20,000
Asset A	P_A =?	€1,000	€2,000	0
ZCB	P_{ZCB} = €98	€100	0	0
B	P_B = 90	0	€100	€100

- What is the replicating portfolio of asset A?
- What is the fair/equilibrium price of asset A ?

Pricing by arbitrage: Example 3 continued

It s impossible to replicate Asset A using Assets ZCB and B:

$$\begin{aligned} \textit{Year } 1 : 1,000 &= y_Z CB * 100 + y_B * 0 \\ \textit{Year } 2 \textit{ iff } DJ < 20,000 : 2000 &= y_{ZCB} * 0 + y_B * 100 \\ \textit{Year } 2 \textit{ iff } DJ \geq 20,000 : 0 &= y_{ZCB} * 0 + y_B * 100 \end{aligned}$$

There is no y_{ZCB} and y_B solving this system.

When this happens we say that *markets are incomplete* and the pricing-by-arbirage method **cannot** be used.