Additional Problems to solve in class during the 20 mins freed-up by the Quiz2:

Problem 1 (60 points): In this problem parts {a}, {b&c}, {d}, {e&f}, {g} can be solved independently
Consider the following risk-free bonds with annual coupon payments:

	Coupon rate	Face value (par value)	Maturity	Price
Bond A	0%	1,000 €	2 years	
Bond B	2%	1,000 €	2 years	1,000.00 €
Bond C	0%	100 €	1 year	98.04 €

You will assume that it is possible to buy or sell any fraction of the three bonds. All bonds have just been issued and they pay their coupons at the end of each year.

a) What is the yield-to-maturity of Bond B (justify your answer in 1 line).

b) Find the composition of the portfolio formed by Bonds B and C that replicates Bond A.

c) Deduce from question b) the no-arbitrage price of Bond A.

You will now assume that the term structure of risk-free interest rates is flat at 2% per year.

d) Find the yield-to-maturity of Bond A (justify your answer in 1 line).

e) Compute the duration of Bond A and the duration of Bond B.

Consider the following bond with annual coupon payments:

	Coupon rate	Face value (par value)	Maturity	Price
Bond D	0%	100 €	1 year	98.54 €

Bond D is expected to default with a 2% probability, whereas Bond C is still default-risk free.

f) Is there an arbitrage opportunity? If yes, find an arbitrage strategy and show the exact composition of your portfolio (you are not asked to show the arbitrage table with the cash flows). If not, explain why there is no arbitrage opportunity.

Solution Key:

a) 2% because trades at par (P0=N=1000)
(5 points): full points for correct result with justification; 0 if no justification

b) Solve system 20*nB+1020*nC=0 gives nB= 0.9804
1020*nb = 1000 nC=- 0.1961
=> Long 0.9804 bond B + Short 0.1961 Bond C

(10 points): full points if final result correct (whatever the method) ; half the points if write correctly system of equations but incorrect numerical result

c) P0(bond A) = 0.9804*1000-0.1961*98.04 = 961.17 €
full points for correct formula (give the points even if use wrong number from question b)

d) 2% because flat yield curve
(5 points): full points for correct result with justification; 0 if no justification

e) D(bond A) = 2 years
D(bond B) = (20/1.02+2*1020/1.02^2)/1000 = 1.98 years
(10 points): full points = half for duration of bond 1 + half for duration of bond 2

f) Yes: risky bond D should have lower price than risk-free bond C. Arbitrage strategy = buy bond C + short-sell bond D.
full points = half to say there is an arbitrage opportunity with a reasonable explanation + half for correct arbitrage strategy

Problem 2 (50 points)

All the bonds in this problem are default-risk-free. All bonds have just been issued and they pay their coupons at the end of each year.

	Maturity	Coupon	Face (par) value	Price
Bond A	5 years	2 €	100 €	100.00 €
Bond B	1 year	0 €	100 €	99.01 €
Bond C	2 years	2 €	100 €	101.97 €

a) Determine the yield-to-maturity of bond A (y_A).

b) Determine the yield-to-maturity of bond B (y_B).

c) Determine the risk-free interest rates at maturities of 1 year (r_1) and 2 years (r_2).

d) Consider the following bond D:

	Maturity	Coupon	Face (par) value	Price
Bond D	2 years	1 €	100 €	101.00 €

Find an arbitrage strategy that uses bonds B, C, and D (assuming that you can buy and short-sell any fraction of bonds B, C, and D). You will present your answer in an arbitrage table showing the composition of your portfolio and the cash flows at each date. (Hint: You will need to compute the number of each bond in the portfolio with 4 decimal places.)

e) In this question, you will assume that you hold 1,000,000 € worth of long position in bond B and 1,000,000 € worth of short position in bond C. The net value of your positions is thus equal to 0€. What would happen to the net value of your positions if the risk-free yield curve shifts up in a parallel fashion? No calculations required: Briefly but clearly justify your answer.

Solution

a) Bond A trades at par → y_A = 2%

b) y_B = 100/99.01-1 = 1%

c) Bond B is risk-free and has 1-year maturity → $r_1 = y_B$ = 1%

Using the price of bond C: 101.97 = 2/1.01 + 102/(1+r_2)^2 → r_2 = 1%

d) Bond A given the flat yield curve at 1%, the no-arbitrage price of bond D is 100€.

Therefore at 101€ it is overpriced. Find replicating portfolio:

1 = 100*xB + 2*xC

101 = 102*xC

xB = – 0.0098

xC = 0.9902

	t=0	t=1	t=2	
0.0098 short positions in bond B	+0.97	-0.98		
0.9902 long positions in bond C	-100.97	1.98	101	
1 short position in bond D	+101	-1	-101	
Total				

e) The initial value of your portfolio is 0. Le Delta be the parallel shift in the yield curve, then the new value of your portfolio is about. Duration of bond B is 1 year and it is smaller than duration of bond C, that is a bit less than 2. The value of your long duration in bond B decreases by about DB *Delta, whereas the value of your short position in bond C decrease by about DC *Delta

Hence the value of my portfolio changes by about

+1000 000*(-DB *Delta)-1000 000*(- DC *Delta)= 1000 000* Delta *(DC – DB)>0