Practice Exam 2

2 hours

Problem 1 (40 points)

The stock of Company XYZ has a required annual rate of return of 20%. The management of Company XYZ considers two investment strategies. They differ according to the flow of expected earnings and expected dividends they yield (next dividend will be paid in exactly one year from now).

	t=1 year	t=2 year	t=3 year	and after t=3 year
Investment strategy 1				
Earnings per share	€ 10	€ 10	€ 10	10 forever
Dividends per share	€ 10	€ 10	€ 10	10 forever
Investment strategy 2				
Earnings per share	€ 10	€ 11	€ 12.1	Continue to grow at an annual rate of 10% for ever
Dividends per share	€ 6	€ 6.6	€ 7.26	Continue to grow at an annual rate of 10% for ever

a) What is the fundamental value of one share of stock XYZ if the management of XYZ selects investment strategy 1?

b) What is the return on equity (or return on investment) of the new projects in which XYZ reinvests its earnings under investment strategy 2?

c) Which investment strategy should the management of XYZ choose to maximize the fundamental value of company XYZ?

d) Assume that the management of XYZ chooses the wrong investment strategy. How much value per share can be created by firing the management team and appointing a new one which would implement the optimal investment strategy?

Problem 2 (60 points)

The term structure of risk-free interest rates is flat at 1% per year. Consider the following risk-free bonds with annual coupon payments:

	Coupon rate	Face value (par value)	Maturity	Price
Bond 1	0%	€ 100	1 year	€ 99.01
Bond 2	5%	€ 100	2 years	€ 107.88

a) What is the yield-to-maturity of Bond 2?

b) Determine the duration of Bond 1 and the duration of Bond 2?

In questions c) and d) you will consider Portfolio P which is composed of a long position in Bond 1 and a short position in Bond 2 such that the value of the portfolio is zero.

c) If the yield curve shifts up, does the value of Portfolio P: (i) increase; (ii) decrease; (iii) does not change; (iv) may either increase or decrease. [Justify your answer in 1 line.]

d) If the 1-year interest rate goes up and the 2-year interest rate goes down, does the value of Portfolio P: (i) increase; (ii) decrease; (iii) does not change; (iv) may either increase or decrease. [Justify your answer in 3 lines.]

e) Assume **for this question only** that a 2-year *risk-free* zero coupon bond with face value €100 is trading at €99. Is there an arbitrage opportunity? If yes, find an arbitrage strategy that uses only Bond 1, Bond 2, and the zero coupon bond. [Clearly show the exact composition of your portfolio, but you are not asked to show the arbitrage table with the cash flows.]

f) Assume **for this question only** that a 2-year zero coupon bond *with default risk* with face value €100 is trading at €99. Is there an arbitrage opportunity? If yes, find an arbitrage strategy. [Clearly show the exact composition of your portfolio, but you are not asked to show the arbitrage table with the cash flows.]

Problem 3 (40 points)

Bond Z is a risk-free zero-coupon bond with a face (par) value of €1,000 and a maturity date in 10 years (t=10). The term structure of risk-free interest rates is flat at 2% per year.

a) Find the price of Bond Z.

b) Determine the no-arbitrage forward price of a forward contract on Bond Z with expiration date in one year time (t=1)?

c) Assume that you take a long position in the forward contract of question b) today (t=0). What will be your profit (or loss) in one year time (t=1) if risk-free interest rates at all maturities increase from 2% to 3% over the course of next year (between t=0 and t=1)?

Problem 4 (60 points)

Stock ABC currently trades at a price of €48 and will pay no dividends in the coming years. The risk-free interest rate is 10% per year. You will consider in this exercise a European call option and a European put option that both have Stock ABC as the underlying asset, expiry in 1 year, and a strike (exercise) price of €55.

a) Assume **for this question only** that the call currently trades at a price (premium) of €3 and that the put currently trades at a price (premium) of €6. Find an arbitrage strategy. [Write the arbitrage table with the positions taken at t=0 and the cash flows at t=0 and t=1 year.]

b) Assume **for this question only** that the put currently trades at a price (premium) of €1 [and the call is not available in this question]. Find an arbitrage strategy. [Write the arbitrage table with the positions taken at t=0 and the cash flows at t=0 and t=1 year.]

c) Assume **for this question only** that the call currently trades at a price (premium) of €2 and that the put currently trades at a price (premium) of €4. You take three long positions in the call and one long position in the put today (t=0). You fund these positions by borrowing €10 at the risk-free interest rate today (t=0) and pay back your loan in one year (t=1). At which condition on ABC stock price in one year (t=1) do you make a positive profit at that date?

d) Assume **for this question only** that in one year time (t=1) the ABC stock price will be either €50 or €60. Determine the no-arbitrage price of the put.

Elements of Answer

Problem 1

a) $V_0 = D_1/(k\text{-}g) = 10/0.2 = €50$

b) ROE = g/b = 0.1/0.4 = 25%

c) Under strategy 2: $V_0 = D_1/(k\text{-}g) = 6/(0.2\text{-}0.1) = €60 > €50$ → Choose strategy 2

d) PVGO = 60-50 = 10 € per share

Problem 2

a) 1% because flat yield curve at 1%

b) D(Bond 1) = 1 year because zero coupon
D(Bond 2) = $(5/1.01 + 2*105/1.01^2) / 107.88 = 1.95$ year

c) (i) because D(bond 1) < D(bond 2) → price of bond 1 decreases by less than price of bond 2 → value of P increases

d) (ii) because P has positive CF at t=1, whose PV decreases when r1 increases, and negative CF at t=2, whose PV increases when r2 decreases → PV of P decreases

e) No-arbitrage price of a 2-year zero = $100/1.01^2 = €98.03$ so there is an arbitrage opportunity.
To find the arbitrage strategy, replicate the zero coupon bond with Bond 1 and Bond 2:
n1*100 + n2*5 = 0
n2*105 = 100
which gives n1 = -0.0476 and n2 = 0.9524
Arbitrage strategy = 1 short position in 2-year zero + 0.0476 short position in Bond 1 + 0.9524 long position in Bond 2

f) Yes, because a risky bond should be even less expensive than a risk-free one. Same arbitrage strategy as in e)

Problem 3

a) $P_0 = N/(1+r_F)^T = 1{,}000/1.02^{10} = 820.35$

b) $F_0 = P_0*(1+r_F) = 820.35*1.02 = 836.76$

c) Price of bond at t=1: $P_1 = 1{,}000/1.03^9 = 766.42$
Profit at t=1: $P_1\text{-}F_0 = 766.42\text{-}836.76 = -70.34$

Problem 4

a) Put-call parity: $C_0+K/(1+r) = P_0 + S_0$?
$3 + 55/1.1 = 6 + 48$?
$53 = 54$?

→ not satisfied → arbitrage strategy:

	CF at 0	CF at T if ST<55	CF at T if ST>55
long call	-3		ST-55
invest €50 in risk-free asset	-50	55	55
short put	6	-(55-ST)	
short stock	48	-ST	-ST
total	1	0	0

b) Put-call parity: C_0+K/(1+r) = P_0 + S_0 ?

K/(1+r) ≤ P_0 + S_0 ?

50 ≤ 1+48 ?

→ not satisfied → arbitrage strategy:

	CF at 0	CF at T if ST<55	CF at T if ST>55
long put	-1	55-ST	
long stock	-48	ST	ST
borrow €50	50	-55	-55
total	1	0	ST-55>0

c) Payoff must exceed €11: achieved if S_1 below 55-11 = €44 or above 55+11/3 = €58.67

d) Replicate the payoff of Put 1 with nS long position in the stock and nB € in the risk-free asset:

50 nS + 1.1 nB = 5

60 nS + 1.1 nB = 0

→ nS = -0.5 and nB = 27.27

price of Put 1: P_0 = 48 nS + nB = 3.27