Financial Market Microstructure

Stefano Lovo

HEC, Paris

Some definitions (the term is originally due to Garman (1976))

- **O'Hara (1995) :** "The study of the process and outcomes of exchanging assets under explicit trading rules..."
- **Madhavan (2000) :** "The process by which investors' latent demands are ultimately translated into transactions".
- **Biais, Glosten and Spatt (2005) :** "The investigation of the economic forces affecting trades, quotes and prices"

Main Questions

1. **Liquidity:** What are the determinants of market liquidity? Liquidity risk? Measures (bid-ask spreads and depth)?
2. **Price Discovery:** How and to what extent do prices impound new information? At which speed?
3. **Volatility:** What are the determinants of price changes at the high frequency? How do volatility and liquidity interact?

Market liquidity

Definition

An **asset's liquidity** is a measure of

- The **speed** at which an asset can be bought and sold. (the faster, the more liquid)
- The **price impact** of the traded quantity. (the smaller impact, the more liquid)
- The **cost** of a "round-trip". (the cheapest, the more liquid)

Why do we care?

- **Illiquidity means lower returns on portfolios:** (i) portfolio managers care about market liquidity and (ii) the brokerage industry devises trading strategies to minimize costs due to illiquidity.
- **Illiquidity affects asset prices/cost of capital:** (i) illiquidity = "tax" on asset payoffs + (ii) source of risk.
- **Price discovery affects the allocation of capital in the economy.**

Illiquidity and Price Impact

- Law of one price: Two identical assets must trade for the same price. The same asset must trade for the same price in all locations.

Actual markets: low of one price?

US treasury bills: bonds with maturity of less than 2 years.
US treasury notes: bonds with maturity of more than 2 years.

At any time there are coexist bill and notes with only one residual payment at the same future date.

Actual markets: low of one price?

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Amihud and Mendelson (1991): On average notes trade at a discount

notes' average annual yield $\simeq$ bills' average annual yield $+0.43\%$

Actual markets: price reflects fundamentals?

MB Trader ELTR

Name	Bid	Size	Name	Ask	Size
MLCO	18 1/4	10	LEHM	18 3/8	20
GSCO	18 1/4	20	MASH	18 1/2	10
PRUS	18 1/4	30	AGED	18 5/8	17
BEST	18 1/8	10	SHWD	18 5/8	10
TSCO	18 1/16	10	TSCO	18 3/4	50

An example on liquidity and cost of capital

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Mid quote $=(18.25 + 18.375)/2 = 18.3125$
Transaction price $\simeq 18.3125 \pm 0.0625$
transaction cost $\simeq 0.34\%$ of 18.3125

Example

Stock A is an immediate perpetuity paying \$1 per year. The required return of capital is $r = 5\%$ per year. What is the current price for stock A? Answer \$21

Suppose that at resale transaction costs are $s = 0.34\%$ of the price. What is the current price for stock A? Answer $\$19.66 \simeq \$21(1 - 6.4\%)$

Market Informational Efficiency

Dos the price of an asset reflect all information about the asset's fundamentals?

Definition

- **Weak form efficiency**: Trading prices incorporate all past public information.
- **Semi-Strong form efficiency**: Trading prices incorporate all present and past public information.
- **Strong form efficiency**: Trading prices incorporate all public and private information available in the economy.

Roadmap

- Key concepts
- Auctions
- Quote driven markets: static models
  - Inventory models
  - Information models
- Quote driven markets: dynamic models
  - Market efficiency and herd behavior
  - Robust price formation
- Limit Order Markets

It recognizes the role of:

- **Heterogeneity among Market Participants.** Participants to security markets have various objectives (e.g. dealers are different from final investors; hedgers different from speculators etc...).
- **Institutional framework.** Market design and market regulation matter.
- **Private Information.** Informational asymmetries among market participants are prevalent in securities markets.

Market Participants

- **Costumers**
- **Dealers**
- **Intermediaries**

Market Participants

- **Costumers:** Agents that are willing to trade the security:
  - Institutional investors (pension funds, mutual funds, foundations): Hold and manage the majority of assets; account for the bulk of trading volume; trade large quantities.
  - Individual Investors (retail traders, household, banks): Account for the bulk of trades; trade smaller quantities.
- **Dealers**
- **Intermediaries**

Market Participants

- **Costumers**
- **Dealers:** Large professional traders who do trade for their own account and provide liquidity to the market.
- **Intermediaries**

Market Participants

- **Costumers**
- **Dealers**
- **Intermediaries:**
  - Brokers: Match costumer orders but do not trade for their own account.
  - Specialists (NYSE): Are responsible for providing liquidity and smoothing trade on given securities.
  - Market Makers: Agents that stand ready to buy and sell the security at their bid and ask prices respectively. Liquidity suppliers.

Institutional Framework: Market types

- **Call Auction Markets:** Occur at specific time (ex. at the opening and or at the fixing); investors place orders that are executed at a single clearing price that maximizes the volume of trade.
- **Continuous Auction Markets (or limit order markets):** Investors trade against resting orders placed earlier by other investors (Euronext, Toronto SE, ECNs).
- **Dealer Markets (or quote driven markets) :** Market-makers post bid and ask quotes at which investors can trade. (Bond Markets MTS, FX markets).
- **Alternative Trading Systems (ATS)**
  - Electronic Communication Networks (ECN): Continuous order driven anonymous markets (Island, Instinet, Archipelago, Redibook)
  - Crossing Networks (CN): Cross multiple orders at a single price determined in a base market. (POSIT, Xtra XXL)

Most common orders

- Buy limit order: Order to buy up to a quantity q for a price not larger than p.
- Sell limit order: Order to sell up to a quantity q for a price not smaller than p.

- Buy market order: Order to buy a quantity q at the best current market conditions.
- Sell market orders: Order to buy a quantity q at the best current market conditions.

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A game theoretical approach

The market for **one** financial asset:

- Market Participants: set of players N
- Institutional framework:
  - Set of actions X_i available to market participant i
  - Set of action profiles $X := \times_{i \in N} X_i$
  - No trade action: $x_\emptyset \in X$
  - Asset allocation rule $Q : X \to \mathbb{R}^N$
  - Cash allocation rule $P : X \to \mathbb{R}^N$
  - $\forall x \in X$:

$$\sum_{i \in N} Q_i(x) = \sum_{i \in N} P_i(x) = 0$$

- Asset fundamental value $\tilde{v}$
- Participant i's monetary payoff from transaction x:

$$\tilde{v} Q_i(x) + P_i(x)$$

- Participant i's utility after transaction x, given initial wealth $\tilde{W}_i$:

$$U_i(\tilde{W}_i + \tilde{v} Q_i(x) + P_i(x))$$

Model of uncertainty

- Ω: set of all possible states of Nature (finite).
- $\mathcal{A}$: Set of all subsets of Ω.
- $\pi : \mathcal{A} \to [0, 1]$: probability measure of $\mathcal{A}$.

- $v : \Omega \to \mathbb{R}$: Value of the asset ($\tilde{v}$).
- $W_i : \Omega \to \mathbb{R}$: Value of agent *i* initial portfolio ($\tilde{W}_i$).
- $U_i : \Omega \to$ Set of possible utility functions: Utility of agent *i* .

Definition

- A partition $\mathcal{P}$ of Ω is a collection of nonempty, pairwise disjoint subsets of Ω whose union is Ω.
- $\mathcal{P}(\omega)$ denotes the element of $\mathcal{P}$ that contains the state ω.

Modelling incomplete information using partitions

- All agents start with common prior π over Ω.
- Agent *i* receive private information that we described as a partition $\mathcal{P}_i$ over Ω:
  if the true state is ω, then agent *i* is informed that the true state belongs to the set $\mathcal{P}_i(\omega)$.
- If the true state is ω, the *type* of agent *i* is $\theta_i = \mathcal{P}_i(\omega)$.

An example

- $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$, $\pi(\omega) = 0.2$
- $v(\omega_1) = 1$; $v(\omega_2) = 2$, $v(\omega_3) = 3$, $v(\omega_4) = 4$, $v(\omega_5) = 5$
- $\mathcal{P}_1 = \{\{\omega_1, \omega_2, \}, \{\omega_3, \omega_4\}, \{\omega_5\}\}$
- $\mathcal{P}_2 = \{\{\omega_1, \omega_3, \}, \{\omega_2, \omega_4\}, \{\omega_5\}\}$
- $\mathcal{P}_3 = \{\{\omega_1\}, \{\omega_2, \}, \{\omega_3\}, \{\omega_4\}, \{\omega_5\}\}$
- $\mathcal{P}_4 = \{\{\omega_1, \omega_2, \omega_3, \omega_4\}, \{\omega_5\}\}$

Agent 1 and 2 receive different partial information; Agent 3 is perfectly informed; Agent 4 is the least informed.

If for example $\omega = \omega_3$, then
for agent 1 $E[\tilde{v}|\mathcal{P}_1(\omega_3)] = 3.5$
for agent 2 $E[\tilde{v}|\mathcal{P}_2(\omega_3)] = 2$
for agent 3 $E[\tilde{v}|\mathcal{P}_3(\omega_3)] = 3$
for agent 4 $E[\tilde{v}|\mathcal{P}_4(\omega_3)] = 2.5$

Joining and meeting partitions...

Let $\mathcal{P}$ and $\mathcal{P}'$ be two partitions of Ω.

Definition

Partition $\mathcal{P}$ is said to *refine* partition $\mathcal{P}'$ if every element θ of $\mathcal{P}$ is contained in some element θ' of $\mathcal{P}'$.

Definition

The meet of the partitions $\mathcal{P}_i$ and $\mathcal{P}_j$, that we will denote $\mathcal{M}_{ij}$, is the finest partition that is refined by both $\mathcal{P}_i$ and $\mathcal{P}_j$.

Definition

The join of the partitions $\mathcal{P}_i$ and $\mathcal{P}_j$, that we will denote $\mathcal{J}_{ij}$, is the less fine partition that refines both $\mathcal{P}_i$ and $\mathcal{P}_j$.

Sharing information and common knowledge

If the state is ω_3...

- What could agent 1 and 2 know if they share their information?
- What could agent 1 and 4 know if they share their information?
- What is that agent 1 and 2 commonly know?

Common knowledge: informally

Something is common knowledge if we both know that it's true;

1. and I know that you know it's true;
2. and you know that I know it's true;
3. and I know that you know that I know it's true; ...
4. and I know that you know that I know that you know that I know that you know it's true;

   ...
5. and so on, for any string of beliefs we put together.

Common knowledge formally

Lemma

If the true state is ω, then what is common knowledge for player i and j is $\mathcal{M}_{ij}(\omega)$.

Sharing information formally

Lemma

If the true state is ω, then if player i and j share their information they both know $\mathcal{J}_{ij}(\omega)$.

Bayesian Equilibrium

- Let $\mathcal{P}_i$ be the set of possible types for agent *i*.
- Let $\mathcal{P}_i^t$ be the set of agent *i*'s possible information about past actions at time t.
- A *strategy* for player *i* is a mapping

$$\sigma_i : \mathcal{P}_i \times_{t\geq 0} \mathcal{P}_i^t \to \Delta X_i$$

- A Bayesian Nash equilibrium is a strategy profile $\{\sigma_i, \ldots, \sigma_N\}$ such that for all player *i* and all histories $h \in H$ and all *t*

$$\sigma_i(\theta_i, h) \in \arg\max_{x_i \in X_i} E[U_i(\tilde{W}_i + \tilde{v}Q_i(x_i, \sigma_{-i}) + P_i(x_i, \sigma_{-i}))|\theta_i, h_i^t(h)]$$

where h_i^t(h) represents what agent *i* has observed at time t if the history of action profile is h.

Why do people trade in the financial market?

There are two possible reasons for trading:

- Speculate on private information about $\tilde{v}$.
- Hedging, when U_i is concave.

Trading based on information

- $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$, $\pi(\omega) = 0.2$
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If for example $\omega = \omega_3$, then
for agent 1 $E[\tilde{v}|\mathcal{P}_1(\omega_3)] = 3.5$
for agent 2 $E[\tilde{v}|\mathcal{P}_2(\omega_3)] = 2$

Can we say that in state ω_3 agent 1 and 2 could agree on a trade where agent 1 buys the asset from agent 2 at a price of 2.9?

No trade theorem

If

- Initial allocation is ex-ante Pareto optimal
- At $\omega \in \Omega$ it is common knowledge that a transaction x is acceptable to both parties

Then,

- Each market participant is indifferent between x and the no-trade action $x_{\emptyset}$.

Rational agents stating from common prior cannot trade solely because they have different information.

No trade theorem (Migrom and Stokey 1982)

Theorem

If traders start from common priors and it is common knowledge that all traders are rational and the current allocation is ex-ante Pareto efficient, then new asymmetric information will not lead to trade, provided that traders are strictly risk averse.

Corollary

If traders start from common priors and have no reason to trade a priori, then they will not trade based on the arrival of new private information.

Trading based on hedging

- $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$, $\pi(\omega) = 0.2$
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- $\mathcal{P}_4 = \{\{\omega_1, \omega_2, \omega_3, \omega_4\}, \{\omega_5\}\}$

If for example $\omega = \omega_2$, then
for agent 1 $E[\tilde{v}|\mathcal{P}_1(\omega_2)] = 1.5$
for agent 4 $E[\tilde{v}|\mathcal{P}_2(\omega_2)] = 2.5$

Suppose that:

- agent 1 owns the asset and is risk averse
- agent 2 does not own the asset and is risk neutral.

Can we say that in state ω_1 agent 1 and 4 could agree on a trade where agent 1 sells the asset to agent 4 at a price of $1.5 - \epsilon$?

Summary

1. Financial markets display frictions and illiquidity.
2. Market microstructure: The investigation of the economic forces affecting trades, quotes and prices.
3. Market Participants.
4. Market Mechanism.
5. Asymmetric information.
6. No trade theorem: if agents are rational trade cannot purely due to speculation on private information.