Quote Driven Market: Static Models

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The Financial market participants

- **Traders**
  - **Customers:** Agent that are willing to trade the security
    - Institutional investors: (pension funds, mutual funds, foundations) Hold and manage the majority of assets; account for the bulk of trading volume; trade large quantities.
    - Individual Investors: (retail traders, household, banks) Account for the bulk of trades; trade smaller quantities.
  - **Dealers:** Large professional traders who do trade for their own account and provide liquidity to the market .
- **Intermediaries**
  - **Brokers**
  - **Specialist**
  - **Market Makers**

The Financial market participants

- **Traders**
  - **Customers**
    - Institutional investors:
    - Individual Investors:
  - **Dealers**
- **Intermediaries**
  - **Brokers**: Match costumer orders if possible and if not they transmit traders' orders to the market. Brokers do not trade for their own account.
  - **Specialist:** (NYSE) Agents that are responsible for providing liquidity and smoothing trade on given securities.
  - **Market Makers:** Agents that stand ready to buy and sell the security at their bid and ask prices respectively. Liquidity suppliers.

Type of Markets

- **Order Driven Markets** (or Pure Auction Markets): Investors are represented by brokers and trade directly without the intermediation of market-makers. (Island, Paris Bourse)
  - Call Auction Markets: Occur at specific time (ex. at the opening and or at the fixing); investors place orders (prices and quantities) that are executed at a single clearing price that maximizes the volume of trade.
  - Continuous Auction Markets: Investors trade against resting orders placed earlier by other investors (and the "crowd" of floor brokers if available). (Euronext, Toronto Stock Exchange, ECNs)

- **Quote Driven Markets**
- **Hybrid Markets**

Type of Markets

- **Order Driven Markets**
  - Call Auction Markets
  - Continuous Auction Markets
- **Quote Driven Markets**: Dealers post bid and ask quotes at which public investors can trade. (Bond Markets MTS, FX markets, London Stock Exchange) in this case dealers are also called market makers.
- **Hybrid Markets**:Call auction markets opened to dealers and specialists (NYSE). Dealer markets where traders' limit orders are posted (Nasdaq)

Quote Driven Market

MB Trader ELTR

Name	Bid	Size	Name	Ask	Size
MLCO	18 1/4	10	LEHM	18 3/8	20
GSCO	18 1/4	20	MASH	18 1/2	10
PRUS	18 1/4	30	AGED	18 5/8	17
BEST	18 1/8	10	SHWD	18 5/8	10
TSCO	18 1/16	10	TSCO	18 3/4	50

- **Bid Price**: Price at which a market maker is willing to buy a given amount of the asset.
- **Ask Price**: Price at which a market maker is willing to sell a given amount of the asset.
- **Inside spread**: Difference between the smallest ask and the largest bid.
- **Market deepness**: Maximum volume of trade that can be traded at the same price.
- **Tick price**: Minimum difference between prices that can be quoted.
- **Round lot**: Normal unit of trading.

A few questions we'll try to answer in this course

- What are the factors that affect the price level?
- When is the bid price different from the ask price?
- What affects the inside spread?
- What affects the market deepness?
- How informed speculators can best exploit their private information?
- What is the information content of trading prices?
- What is the information content of trading volume?

Normal-CARA math preliminaries

Let $\tilde{x} : N(m_x, \sigma_x^2)$
Let $\tilde{y} : N(m_y, \sigma_y^2)$ and $Cov(\tilde{x}, \tilde{y}) = \sigma_{xy}$
Then

$$E[\tilde{x}|\tilde{y} = y] = m_x + (y - m_y)\frac{\sigma_{xy}}{\sigma_y^2} \tag{1}$$

Let $\tilde{z}$ such that $f(\tilde{z}|x)$ is $N(x, \sigma_z^2)$. Then,

$$f(\tilde{x}|\tilde{z} = z) : N\left(\frac{\frac{m_x}{\sigma_x^2} + \frac{z}{\sigma_z^2}}{\frac{1}{\sigma_x^2} + \frac{1}{\sigma_z^2}}, \frac{1}{\frac{1}{\sigma_x^2} + \frac{1}{\sigma_z^2}}\right) \tag{2}$$

Let $U(w) = -e^{-\gamma w}$ (CARA utility function with risk aversion coefficient γ). Then,

$$E[U(x)] = E\left[-e^{-\gamma\tilde{x}}\right] = -e^{-\gamma\left(m_x - \gamma\frac{\sigma_x^2}{2}\right)} \tag{3}$$

Rodamp: Quote driven markets Static models

- Inventory model
- Informed traders
  - Non-strategic informed traders (Gloseten and Milgrom (1985))
  - Strategic informed trader (Kyle (1985))
- Informed market makers

A simple way to model Quote Driven Markets

- We consider a market where a risky asset (security) is exchanged for a risk-less asset (money). The fundamental value of the risky asset is represented by the random variable $\tilde{v}$.

- **Trading rules**:
  1. Market makers simultaneously post bid and ask prices at which they are willing to buy or sell, respectively, an institutionally given amount q of the security.
  2. Traders decide either to buy or to sell the risky asset (submit market orders):
     - if they want to sell, they will sell q shares of the risky asset to the MM who posts the highest bid;
     - if they want to buy, they will buy q shares of the risky asset from the MM who posts the lowest ask;

The price competition among MMs can be seen as two first-price-sealed-bid auctions within a common value framework:

- The bid prices are the outcome of bidding strategy in a first price auction to buy the risky asset.
- The ask prices are the outcome of bidding strategies in an first price procurement auction to sell the risky asset.

Benchmark: Risk Neutral MM, No asymmetries of information

Lemma

If market makers are risk neutral then

Ask price = Bid price = $E[\tilde{v}]$

Proof: Bertrand competition.

Implications: When market makers are risk neutral and there is agreement on the distribution of $\tilde{v}$, the inside spread is nil and transaction cost is minimized.

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The economy

- **Asset fundamental value:** $\tilde{v} : N(V, \sigma^2)$
- **Liquidity traders:** are hit by liquidity shocks that consist in an urgent need to sell q units of the risky asset or buy q units of the risky asset.
- **Market-Makers:** Agents clearing traders' market orders. MM i's post trade utility is:

$$u(C_i + I_i\tilde{v} - q(P - \tilde{v}))$$

  - CARA uitlity $u(w) = -e^{-\gamma w}$
  - MM i's cash endowment: C_i
  - MM i's initial asset inventory: I_i
  - Trading price: P
  - Traded quantity q

Reservation quotes

Definition

- The **bid reservation quote** rb_i for MM i is the maximum price that he is willing to pay for q additional units of the risky asset:

$$E[u(C_i + I_i\tilde{v})] = E[u(C_i + I_i\tilde{v} - q(rb_i - \tilde{v}))]$$

- The **ask reservation quote** ra_i for MM i is the minimum price at which he is willing to sell q units of the risky asset:

$$E[u(C_i + I_i\tilde{v})] = E[u(C_i + I_i\tilde{v} + q(ra_i - \tilde{v}))]$$

Reservation quotes

Lemma

Given the CARA Normal set up:

$$rb_i = V - \frac{\gamma\sigma^2}{2}(q + 2I_i)$$
$$ra_i = V + \frac{\gamma\sigma^2}{2}(q - 2I_i)$$

- *Reservation quotes are increasing in V.*
- *Reservation quotes are decreasing in I_i.*
- $rb_i < ra_i$.
- $ra_i - rb_i = \gamma\sigma^2 q$ *increases with γ and σ.*

Proof: Recall that for $\tilde{x} : N(m_x; \sigma_x^2)$ we have,

$$E\left[U(x)\right] = E\left[-e^{-\gamma\tilde{x}}\right] = -e^{-\gamma\left(m_x - \gamma\frac{\sigma_x^2}{2}\right)}$$

Fragmented / anonymous markets

- Each MM knows his initial portfolio (C_i, I_i) but not the other's.
- MM's inventories are i.i.d.

Computing the optimal quoting strategy

$$\begin{aligned} W_i(0) &:= C_i + I_i\tilde{v} \\ W_i(b) &:= W(0) + q(\tilde{v} - b). \end{aligned}$$

Note that

$$\begin{aligned} E[u(W_i(b)] &= E[u(W_i(0))e^{-\gamma(rb_i - b)q}] \\ &\simeq E[u(W_i(0))] + E[u(W_i(0))]\gamma q(b - rb_i) \end{aligned}$$

In the bid auction MM i sets b_i^* so that :

$$b_i^* \in \arg\max_b (E[u(W_i(b)] - E[u(W_i(0)])\Pr(b_{-i} < b_i)$$

$$\simeq \arg\max_b (rb_i - b)\Pr(b_{-i} < b_i)$$

$$b_i^* \simeq E[\tilde{rb}_{-i}^{(1)} | \tilde{rb}_{-i}^{(1)} \leq rb_i]$$

Note that rb_i is decreasing in I_i.

Lemma

In a symmetric equilibrium MM i sets its bid and ask quotes at:

$$b(I_i) \simeq E[rb_i(\tilde{I}_{-i}^{(l)})|\tilde{I}_{-i}^{(l)} > I_i]$$
$$a(I_i) \simeq E[ra_i(\tilde{I}_{-i}^{(h)})|\tilde{I}_{-i}^{(h)} < I_i]$$

respectively. Where $\tilde{I}_{-i}^{(l)}$ *and* $\tilde{I}_{-i}^{(h)}$ *are the lowest and highest among MM i's competitor's inventory, respectively.*

Some empirical implications

- The MM who wins the bid auction is the one with the smallest inventory.
- The MM who wins the ask auction is the one with the largest inventory.
- The largest MM's inventories, the smaller will be the bid and ask quotes.
- Transaction cost measured as the inside spread:
  - Decreases with the number of MM.
  - Increases with MM's risk aversion γ
  - Increase with the intrinsic asset risk σ^2

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- Informed Intermediaries
  - Informed brokers
  - Informed market makers

Informed non-strategic traders (Glosten and Milgrom JFE 1985)

The economy: Agents from three groups trade the security for money:

- **Risk neutral market makers**: provide liquidity to the market: they trade the risky asset with the other agents at their bid and ask prices.
- **Risk neutral informed speculators (informed traders)**: risk neutral speculators with some private information about the liquidation value of the risky asset.
- **Liquidity (or noise) traders**: come to the market for reason other than speculation (liquidity, portfolio hedging, etc.); their excess demand is exogenous and random.

The population of traders is composed of a proportion μ of informed traders and $(1-\mu)/2$ buyer liquidity traders and $(1-\mu)/2$ seller liquidity traders .

Security fundamentals and information structure

- **Security fundamental value:**

$$\tilde{v} = \tilde{V} + \tilde{\varepsilon}$$

with $\tilde{V} \in \{V_1, V_2\}$, $\Pr(\tilde{V} = V_2) = \pi$, $V_1 < V_2$, $E\left[\tilde{\varepsilon}|\tilde{V}\right] = 0$, $Var(\tilde{\varepsilon}|\tilde{V}) \geq 0$.

- **Informed traders' information**
Each informed trader receives private signal $\tilde{s} \in \{l, h\}$ with

$$\Pr(\tilde{s} = l|V_1) = \Pr(\tilde{s} = h|V_2) = r \in \left(\frac{1}{2}, 1\right)$$

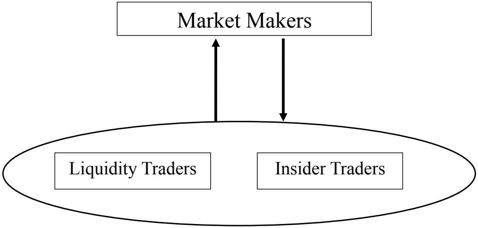
The signals are not correlated with $\tilde{\varepsilon}$ and conditionally i.i.d. across informed traders.

$$V_1 \leq E\left[\tilde{v}|l\right] < E\left[\tilde{v}\right] < E\left[\tilde{v}|h\right] \leq V_2$$

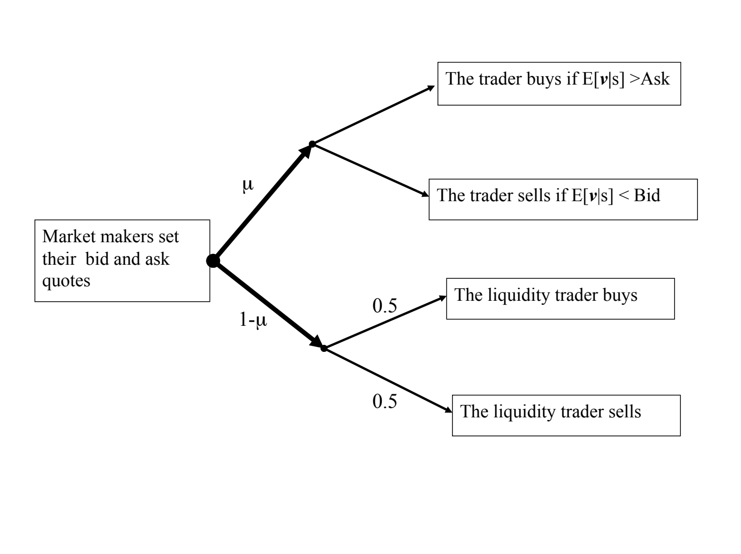
- Market makers have no private information regarding $\tilde{v}$.

Trading Mechanism

1. Market makers submit bid and ask prices
2. Insider traders and liquidity traders submit anonymous order.



Trading Mechanism



Equilibrium

Theorem

In equilibrium all market makers bid and ask prices satisfy:

$$b^* = E[\tilde{v} | \textit{trader sells at } b^*] = V_1 + \pi^S(\pi, \mu, r)(V_2 - V_1) \tag{4}$$

$$a^* = E[\tilde{v} | \textit{trader buys at } a^*] = V_1 + \pi^B(\pi, \mu, r)(V_2 - V_1) \tag{5}$$

where

$$\pi^S(\pi, \mu, r) = \frac{(\mu(1-r) + (1-\mu)/2)\pi}{(1-\mu)/2 + \mu((1-r)\pi + r(1-\pi))}$$

$$\pi^B(\pi, \mu, r) = \frac{(\mu r + (1-\mu)/2)\pi}{(1-\mu)/2 + \mu(r\pi + (1-r)(1-\pi))}$$

Empirical Implications

- If $\mu > 0$ and $r > 1/2$, then bid-Ask spread is positive despite market makers are risk neutral.
- Bid-Ask spread increases with μ that is the proportion of informed traders in the economy.
- Bid-Ask spread increases with r that is the quality of informed traders's signal.
- information Bid-Ask spread increases with the relevance of traders information:

$$Var[\tilde{V}] = \pi(1-\pi)(V_2 - V_1)^2$$

A Generalization of Glosten and Milgrom

Asset fundamental value: $\tilde{v} = \tilde{V} + \tilde{\varepsilon}$,

$\tilde{V} \in [V_1, V_2]$ with density $f(.)$ and $E[\tilde{\varepsilon}|\tilde{V}] = 0$, with $Var(\tilde{\varepsilon}) \geq 0$

- Risk neutral market makers.
- Informed risk averse investors who buy with probability $D(a, V)$ and sell with probability $S(b, V)$.
  - $D(.)$ continuous and increasing in V.
  - $S(.)$ continuous and decreasing in V.

Bid Side

Market maker payoff:

$$W^b_{MM}(b) := E\left[(\tilde{V} - b)S(b, \tilde{V})\right]$$

Note that if for some $\delta > 0$, $S(V - \delta, V) > 0$, then

$$W^b_{MM}(V_1 - \delta) > 0$$

$$W^b_{MM}(E[\tilde{v}]) = E\left[(\tilde{V} - E[\tilde{v}])S(E[\tilde{V}], \tilde{V})\right]$$

$$= \left(\int_{V_1}^{V_2} vf(v)S(E[\tilde{V}], v)dv - E[\tilde{V}]\int_{V_1}^{V_2} f(v)S(E[\tilde{V}], v)dv\right) < 0$$

iff

$$E[\tilde{V}] = \int_{V_1}^{V_2} vf(v)dv > \int_{V_1}^{V_2} vf(v)\left(\frac{S(E[\tilde{V}], v)}{\int_{V_1}^{V_2} f(z)S(E[\tilde{V}], z)dz}\right)dv$$

Generalized GM: Equilibrium

Theorem

An equilibrium always exits. Market makers' expected profit is nil and bid and ask prices satisfy

$$b < E[\tilde{V}] < a$$

The spread $a - b$ *is increasing in* $\frac{\partial(D(a,V)-S(b,V))}{\partial V}$

Generalized GM: Example

- Investors have CARA utility function with risk aversion γ.
- Investors have an initial inventory I of the risky asset.
- $G(I) = \Pr(\text{Investor } i\text{' inventory is less than } I)$
- Fundamental value of the asset is $\tilde{v} = \tilde{V} + \tilde{\varepsilon}$ with

$$\tilde{V} : N(V, \sigma_V^2)$$
$$\tilde{\varepsilon} : N(0, \sigma_\varepsilon^2)$$

- Investors know the realization of $\tilde{V}$ but not the reaization of $\tilde{\varepsilon}$.

Exercise: Show that:

$$
\begin{aligned}
D(a, \tilde{V}) &= G\left(\frac{\tilde{V} - a}{\gamma\sigma_\varepsilon^2} - \frac{1}{2}\right) \\
S(b, \tilde{V}) &= 1 - G\left(\frac{\tilde{V} - b}{\gamma\sigma_\varepsilon^2} + \frac{1}{2}\right)
\end{aligned}
$$

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The economy:

Security liquidation value

$$\tilde{v} : N(w, \sigma^2)$$

Agents from three groups trade the security for money:

- **Noise traders**: Trade for liquidity reasons. Their aggregate market order is a random variable $\tilde{m}$ orthogonal to $\tilde{v}$, with

$$\tilde{m} : N(0, \sigma_m^2)$$

- **One informed speculators** : risk neutral speculator who privately knows $\tilde{v}$ but not $\tilde{m}$.
- **Risk neutral dealers**: provide liquidity to the market. They do not know neither $\tilde{v}$ nor $\tilde{m}$.

Trading protocol

1. Liquidity traders and the informed trader simultaneously choose the quantity they want to trade.
   No restriction on volume of trade
   - m: quantity traded by the liquidity traders.
   - x: quantity traded by the informed trader.

2. Dealers observe the aggregate order $q = x + m$ and set a price p at which they clear the market.

Informed trader's payoff:

$$\Pi_I = x(\tilde{v} - p)$$

Dealer's payoff:

$$\Pi_D = (x + m)(p - \tilde{v})$$

Equilibrium requirements

1. Competition among dealers lead to set p so that they make nil expected profits:

$$E[\Pi_D] = E\left[(x+m)(p-\tilde{v})|\, x+m\right] = 0 \Rightarrow$$

$$p^*(x+m) = E[\tilde{v}|x+m]$$

   Remark: Conditional expectation depends on the informed trader's strategy $x^*(\cdot)$.

2. The informed trader chooses the quantity x^* maximizing his expected pay-off:

$$x^*(v) \in \arg\max_x E\left[x(v - p^*(x+\tilde{m}))\right]$$

   Remark: The optimal x^* depends on the dealers' strategy $p^*(.)$.

Linear Equilibrium

Theorem

There is a linear equilibrium where

$$x^*(v) = (v - w)\beta \qquad (6)$$

$$p^*(x+m) = w + (x+m)\lambda \qquad (7)$$

with $\beta := \sqrt{\frac{\sigma_m^2}{\sigma^2}}$ *and* $\lambda := \frac{1}{2}\sqrt{\frac{\sigma^2}{\sigma_m^2}}$.
The informed trader ex-ante expected payoff is

$$E[\Pi_I] = \sigma_m \sigma / 2$$

Ex-ante transaction cost for trading z*:*

$$z(\tilde{v} - E[\tilde{p}|z]) = \lambda z^2$$

Empirical implications

- The informed trader strategy is more (less) aggressive as σ_m^2 (resp. σ^2) increases.
- Price sensitiveness to trading volume decreases (increases) with σ_m^2 (resp. σ^2) increases.
- Informed trader's profit increases with the amount of noisy trading σ_m^2 and the precision of his signal σ^2.
- Transaction cost decreases with the amount of noisy trading σ_m^2 and increases with the precision of the informed trader's signal σ^2.

Proof 1/2

- $\tilde{x} + \tilde{m} = x^*(\tilde{v}) + \tilde{m} = \beta(\tilde{v} - w) + \tilde{m} : N(0, \beta^2\sigma^2 + \sigma_m^2)$
- $Cov(\tilde{v}, \tilde{x} + \tilde{m}) = \beta\sigma^2$.
- Recall taht: if $\tilde{y} : N(m_y, \sigma_y^2)$, $\tilde{z} : N(m_z, \sigma_z^2)$ and $Cov(\tilde{y}, \tilde{z}) = \sigma_{yz}$, then

$$E\left[\tilde{y}|\tilde{z} = z\right] = m_y + (z - m_z)\frac{\sigma_{yz}}{\sigma_z^2}$$

1 Given the informed trader strategy $x^*(\tilde{v}) = \beta(\tilde{v} - w)$, dealer's price is

$$\begin{aligned} p^*(x+m) &= E[\tilde{v}|\tilde{x} + \tilde{m} = x + m] \\ &= w + (x+m)\underbrace{\left(\frac{\beta\sigma^2}{\beta^2\sigma^2 + \sigma_m^2}\right)}_{\lambda} \end{aligned}$$

Proof 2/2

1. Informed trader maximization problem given $p^*(x+m) = w + \lambda(x+m)$:

$$\begin{aligned} \max_x \quad & E\left[x(v - (w + \lambda(x + \tilde{m}))\right] = \max_x x(v-w) - \lambda x^2 \\ \Rightarrow \quad & x^*(v) = \underbrace{\left(\frac{1}{2\lambda}\right)}_{\beta}(v-w) = \beta(v-w) \end{aligned}$$

2.

$$\lambda = \frac{\beta\sigma^2}{\beta^2\sigma^2 + \sigma_m^2}$$

$$\beta = \frac{1}{2\lambda}$$

That is satisfied for

$$\beta = \sqrt{\frac{\sigma_m^2}{\sigma^2}}$$

and

$$\lambda = \frac{1}{2}\sqrt{\frac{\sigma^2}{\sigma_m^2}}$$

Main implications from Glosten and Milgrom, and Kyle

1. Uninformed dealers compete in quotes to attract traders' market orders.

2. Traders can be either informed or uninformed about $\tilde{V}$

3. Semi-strong form informational efficiency: Transaction price at which an order x is executed is

$$p(x) = E[\tilde{V}|x]$$

$$E[p(\tilde{x})] = E[\tilde{V}]$$

4. Price sensitivity to volume increases with
   - Adverse selection: Probability of facing an informed trader.
   - $Var(\tilde{V})$: relevance of informed trader's information

5. Market-Makers/ dealers expected profit is nil.

- Inventory model
- Informed traders
  - Non-strategic informed traders
  - Strategic informed trader
- Informed market makers

Informed Market Makers (Calcagno Lovo REStud 2006)

Economy:

- Asset fundamental value: $\tilde{v} = \tilde{V} + \tilde{\varepsilon}$, with $\tilde{V} \in \{V_1, V_2\}$, $\Pr(\tilde{V} = V_2) = \pi$ and $E[\tilde{\varepsilon}|\tilde{V}] = 0$.
- Market Participants:
  - Risk neutral market makers.
    - MM1: one MM who knows $\tilde{V}$.
    - MM2: one or many uninformed MM.
  - μ informed speculators who know $\tilde{V}$.
  - $1 - \mu$ liquidity (or noise) traders who buy and sell with probability $1/2$.

Trading rules: Quote Driven Markets

1. Market makers simultaneously post bid and ask prices at which they are willing to buy or sell, respectively, an institutionally given amount $q = 1$ of the security.

2. Traders decide either to buy or to sell the risky asset (submit market orders):
   - if they want to sell, they will sell $q = 1$ shares of the risky asset to the MM who posts the highest bid;
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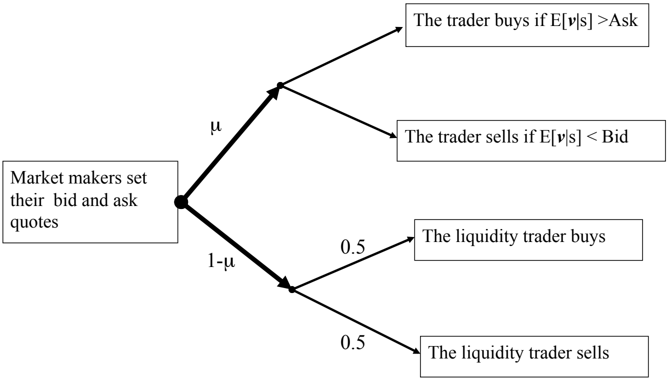
Trading Mechanism



Market Makers' payoff functions

First price sealed-bid and ask auctions in a common value framework with one informed bidder and adverse selection from traders.

- MM1' expected payoff for a given $\tilde{V} \in \{V_1, V_2\}$

$$
\begin{cases}
(a_1 - V_1)\Pr(a_2 > a_1)\frac{1-\mu}{2} + (V_1 - b_1)\Pr(b_2 < b_1)\frac{1+\mu}{2} & , \quad \tilde{V} = V_1 \\
(a_1 - V_2)\Pr(a_2 > a_1)\frac{1+\mu}{2} + (V_2 - b_2)\Pr(b_2 < b_1)\frac{1-\mu}{2} & , \quad \tilde{V} = V_2
\end{cases}
$$

- MM2's expected payoff

$$
\begin{aligned}
&(1-\pi)(a_2 - V_1)\Pr(a_1 > a_2|V_1)\frac{1-\mu}{2} + \pi(a_2 - V_2)\Pr(a_1 > a_2|V_2)\frac{1+\mu}{2} \\
&+(1-\pi)(V_1 - b_2)\Pr(b_1 < b_2|V_1)\frac{1+\mu}{2} + \pi(V_2 - b_2)\Pr(b_1 < b_2|V_2)\frac{1-\mu}{2}
\end{aligned}
$$

Equilibrium quoting strategy

Lemma

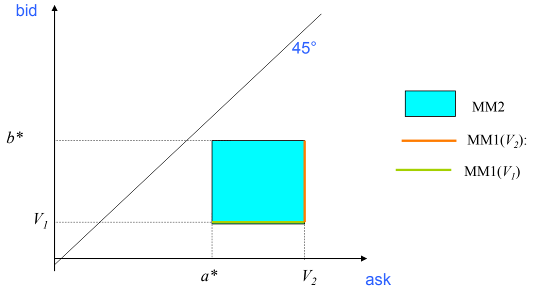
The euilibrium quoting strategy is in mixed strategies, Fully revealing (MM1's quotes reveal $\tilde{V}$) and unique.

- If $\tilde{V} = V_2$ it is profitable to **buy**:
  1. MM1 randomizes its bid in $]V_1, b^*]$
  2. MM1 sets $a_1 = V_2$.
- If $\tilde{V} = V_1$ it is profitable to **sell**:
  1. MM1 sets $b_2 = V_1$.
  2. MM1 randomizes its ask in $]a^*, V_2]$

- MM2 does not know whether it is profitable to buy or to sell:
  1. MM2 randomizes its bid in $[V_1, b^*]$
  2. MM2 randomizes its ask in $[a^*, V_2]$

a^* and b^* are the ask and bid in Glosten and Milgrom

Equilibrium Supports



Some equilibrium properties and implications

- MM2's equilibrium payoff is nil.
- MM1's equilibrium payoff is
  - $(a^* - V_1)\frac{1-\mu}{2}$ of $\tilde{V} = V_1$
  - $(V_2 - b^*)\frac{1-\mu}{2}$ of $\tilde{V} = V_2$
- Bid-ask spread is strictly larger than $a^* - b^*$.
- Bid and ask prices do not reflect the expected value of the asset conditional on trade.
- Equilibrium distribution of quotes:

$$\Pr(a_2 > x) = \frac{a^*-V_1}{x-V_1} \quad , \quad \Pr(b_2 < x) = \frac{V_2-b^*}{V_2-x}$$

$$\Pr(a_1 = V_2|V_2) = 1 \quad , \quad \Pr(b_1 < x|V_2) = \frac{(1-\pi)(1+\mu)(x-V_1)}{\pi(1-\mu)(V_2-x)}$$

$$\Pr(a_1 > x|V_1) = \frac{(1-\pi)(1+\mu)(x-V_1)}{\pi(1-\mu)(V_2-x)} \quad , \quad \Pr(b_1 = v_1|V_1) = 1$$

Sketch of the proof

1. Observe that MM2 equilibrium must be nil.
2. Observe that MM1 will stay away from the non profitable side (bid or ask).
3. Use MM2 (1) and (2) expected payoff expression to derive MM1 mixed strategies.
4. Use (3) to determine the maximum bid and ask.
5. Use MM1(V_1) payoff expression and (4) to determine MM2's mixed strategy for the ask.
6. Use MM1(V_2) payoff expression and (4) to determine MM2's mixed strategy for the bid.

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