Problem Set 2: Bonds

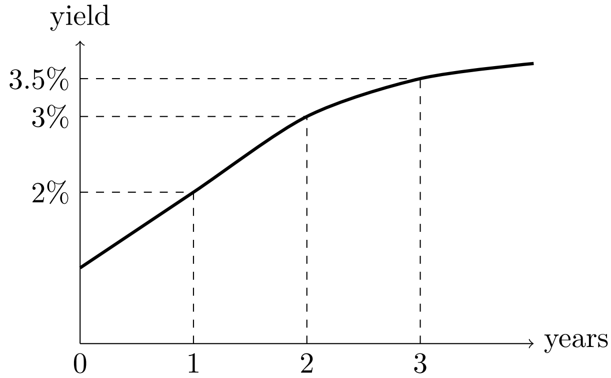
Problem 1

a. What is the price of a coupon-paying bond with maturity 3 years, par value €1,000, coupon rate 5%, and yield 6%?

b. Why does the bond trade at a discount to par value?

Problem 2

a. What is the price of a coupon-paying bond with maturity 3 years, face value €1,000, coupon rate 5%, and priced by the following yield curve:



b. Why does the bond trade at a premium to par value?

Problem 3

A bond with a 2-year maturity has annual discount rates $r_1 = 3\%$ and $r_2 = 4\%$.

a. Can the bond's yield be equal to 2.8%?

b. Can the bond's yield be equal to 3.8%?

c. Can the bond's yield be equal to 4%?

Problem 4

Please find below details of four default-free bonds:

	Annual coupon rate	Face Value	Maturity	Price at t=0
Bond 1	0%	50	1 year	48.08
Bond 2	0%	50	2 years	45.35
Bond 3	0%	50	3 years	41.98
Bond 4	10%	1000	3 years	1104.6

a. Find the term structure of interest rates for t=1 year, 2 years, and 3 years.

b. Find the composition of the portfolio formed by zero-coupon bonds 1, 2, and 3 that replicates coupon paying bond 4.

c. Is there an arbitrage opportunity? Show all of your calculations to justify your answer. If there is an arbitrage opportunity, create a detailed arbitrage table.

d. Assume you buy bond 4 today and sell it in one year (right after the coupon payment). What is the return of your investment if one year from now, the term structure is flat at 5%?

e. What is the forward rate between year 1 and year 3 equal to?

Problem 5

A bond has been issued with a sinking-fund provision: the first half of the issue will be reimbursed in two years and the other half in three years. The bond also pays an annual coupon equal to 10% of the remaining principal (face value), that is, 10% of the initial principal in years 1 and 2 and 5% of the initial principal in year 3. You hold for 100 million of face value of this bond.

a. Write the three future annual cash flows.

b. The term structure is currently flat at 9%. What is the value of the bond and its yield-to-maturity?

c. How much do you stand to lose if the term structure moves uniformly from 9% to 9.1% within one day?

Problem 6

Please find below the prices and characteristics of three bonds, which you will assume are default-free:

<u>Bond 1:</u> Coupon rate = 5% (one coupon a year); Maturity = 3 years; Face value = 1000 euros; Price at date 0 = 1001.8 euros

<u>Bond 2:</u> Coupon rate = 7% (one coupon a year); Maturity = 3 years; Face value = 1000 euros; Price at date 0 = 1056.986 euros

<u>Bond 3:</u> Zero-Coupon; Maturity = 2 years; Face value = 1000 euros; Price at date 0 = 924.556 euros

a. What should the prices of zero-coupon bonds of face value 1000 euros and of, respectively, maturity 1 and 3 years be equal to so that there are no arbitrage opportunities?

We now consider a new bond issued by JunkBond Inc. The bond has face value 1000 euros and a coupon rate equal to 10%. The maturity date is 2 years from now. The bond has a default risk for its payoffs in year 2. With probability 0.8, the bond will pay off 1100 euros (as expected) while with probability 0.2, the bond will pay off nothing (the issuer defaults). At date 1, there is no default risk and the coupon will be paid as expected with probability 1.

Consider also the following derivative asset (called a Credit Default Swap, or CDS): the asset pays off 100 euros at date t=2 if JunkBond defaults and 0 euro otherwise. The derivative asset pays no cash flow at date t=1. The current price of that derivative asset is 15 euros.

b. In the absence of arbitrage opportunities, what should the price of the bond issued by JunkBond Inc. be equal to?

Problem 7

Consider a coupon bond paying a coupon rate of 7% over 4 years and with a face value of 100. The yield-to-maturity of this bond is equal to 4%.

a. What is the duration of this bond?

b. Using the bond's duration, give an estimate of the capital loss of this bond following a sudden 30 basis point increase in the bond's yield.

Problem Set 2: Bonds

Solutions

Problem 1

a. $P_0 = \dfrac{50}{1.06} + \dfrac{50}{1.06^2} + \dfrac{1050}{1.06^3} = 973.27$ €

b. The bond trades at a price below par value because its yield is above its coupon rate.

Problem 2

a. $P_0 = \dfrac{50}{1.02} + \dfrac{50}{1.03^2} + \dfrac{1050}{1.035^3} = 1043.19$ €

b. The bond trades at a price above par value because its yield is below its coupon rate. Recall that the bond's yield is a weighted-average of the discount rates for the bond's cash flows at the different dates, so the bond's yield is below 3.5%.

Problem 3

a. No. The bond's yield must be a weighted-average of the discount rates for the bond's cash flows at the different dates. Thus, the yield must be between 3% and 4%.

b. Yes.

c. Yes, only if the bond is a zero-coupon bond. Since a zero-coupon bond makes a payment at maturity only, the yield is equal to the discount rate associated with the bond's maturity.

Problem 4

a. Using the prices of bond 1, bond 2, and bond 3:

$$r_1 = \frac{50}{48.08} - 1 = 4\%$$

$$r_2 = \left(\frac{50}{45.35}\right)^{1/2} - 1 = 5\%$$

$$r_3 = \left(\frac{50}{41.98}\right)^{1/3} - 1 = 6\%$$

b. 2 × (Bond 1) + 2 × (Bond 2) + 22 × (Bond 3)

c. Replicating portfolio price $= 2 \times 48.08 + 2 \times 45.35 + 22 \times 41.98 = 1110.42 > 1104.6$, thus there is an arbitrage opportunity.

Arbitrage strategy: short 2 bonds 1 + short 2 bonds 2 + short 22 bonds 3 + long 1 bond 4

	t=0	t=1	t=2	t=3
short 2 bonds 1	96.16	-100	0	0
short 2 bonds 2	90.70	0	-100	0
short 22 bonds 3	923.56	0	0	-1,100
long 1 bond 4	-1104.6	100	100	1,100
Total	+5.82	0	0	0

d. The price of bond 4 in one year right after the coupon payment will be $\dfrac{100}{1.05} + \dfrac{1100}{1.05^2} = 1092.97$

Therefore, return $= \dfrac{100 + 1092.97 - 1104.6}{1104.6} = 8\%$

e. $(1+r_1)(1+f_{1\to 3})^2 = (1+r_3)^3$ implies $f_{1\to 3} = \left(\frac{(1+r_3)^3}{1+r_1}\right)^{1/2} - 1 = 7\%$

Problem 5

a. $CF_1 = 10\%$ interest payment on 100 million $=$ 10 million

$CF_2 = 10\%$ interest payment on 100 million $+$ repayment of half the issue (i.e., 50 million) $=$ 60 million

$CF_3 = 10\%$ interest payment on 50 million (what is left), i.e., 5 million $+$ repayment of the second half of the issue (i.e., 50 million) $=$ 55 million

b. $P = \dfrac{10}{1.09} + \dfrac{60}{1.09^2} + \dfrac{55}{1.09^3} = 102.1452$ million

Yield-to-maturity $= 9\%$

c. $P = \dfrac{10}{1.091} + \dfrac{60}{1.091^2} + \dfrac{55}{1.091^3} = 101.9275$ million

Loss $=$ 102.1452 million $-$ 101.9275 million $=$ 217,700

Problem 6

a. We solve for the discount factors d_1, d_2, and d_3, using the prices of the three bonds:

$$\begin{cases} 1001.8 &= 50 \times d_1 + 50 \times d_2 + 1050 \times d_3 \\ 1056.986 &= 70 \times d_1 + 70 \times d_2 + 1070 \times d_3 \\ 924.556 &= 1000 \times d_2 \end{cases}$$

Solving the system, we find: $d_1 = 0.9709$; $d_2 = 0.9246$; $d_3 = 0.8638$. Thus, the price of the zero coupon with T=1 is 970.9 € and that of the zero coupon with T=3 is 863.8 €.

b. We price the bond by arbitrage, that is, we look for the replicating portfolio of the risky bond. We denote n_1, n_2, and n_D respectively the number of zero coupons of maturity 1, zero-coupons of maturity 2, and of credit derivatives in the portfolio. The portfolio should have the same cash flows as the risky bond at date t=1, at date t=2 if the bond does not default, and at date t=2 if the bond defaults. That is, we must have:

$$\begin{cases} t = 1: & 100 &= n_1 \times 1000 + n_2 \times 0 + n_D \times 0 \\ t = 2 \text{ if no default}: & 1100 &= n_1 \times 0 + n_2 \times 1000 + n_D \times 0 \\ t = 2 \text{ if default}: & 0 &= n_1 \times 0 + n_2 \times 1000 + n_D \times 100 \end{cases}$$

where the LHS of the equations are the CF of the risky bond at each date and in each scenario and the RHS are the CF of the portfolio.

Solving the system of equations, we $n_1 = 0.1$; $n_2 = 1.1$; $n_D = -11$.

Thus, in absence of arbitrage Junk Bond is worth $0.1 \times 970.9 + 1.1 \times 924.6 - 11 \times 15 = 949.15$€

Note that the true probability of default plays NO ROLE in this formula. A similar property will hold in models of option prices.

Problem 7

a. Bond price: $P = \dfrac{7}{1.04} + \dfrac{017}{1.04^2} + \dfrac{7}{1.04^3} + \dfrac{7}{1.04^4} = 110.89$

Bond duration: $D = \dfrac{1 \times \frac{7}{1.04} + 2 \times \frac{7}{1.04^2} + 3 \times \frac{7}{1.04^3} + 4 \times \frac{017}{1.04^4}}{110.89} = 3.65$ years

b. $\Delta P = -D \times \dfrac{\Delta r}{1+r} \times P = 3.65 \times \dfrac{0.003/1.04}{\times} 110.89 = -1.17$ so the capital loss is 1.17