Problem Set 4: Options

Problem 1

An American investor buys one call option on the euro at an exercise price of \$1.1 per euro. The option premium is \$0.01 per euro.

a. For what range of exchange rates will the investor exercise the call option at expiration?

b. For what range of exchange rates will the investor realize a net profit, taking the original cost into account and using a risk-free interest rate of 0%?

c. If the investor had purchased a put with the same exercise price and premium instead of a call, how would your answers to the two questions above change?

Problem 2

You think LVMH's stock price is going to appreciate substantially in the next six months. LVMH's current price is equal to €100, and a European call option on LVMH with a six-month maturity and an exercise price €100 has a price of €10. You have €10,000 to invest, and you can:

(i) Invest €10,000 in the stock.

(ii) Invest €10,000 in 1,000 call options (one option gives you the right to buy one share of Peugeot in six months).

(iii) Buy 100 options and invest the remaining €9,000 in a risk-free money market fund paying 4% over six months (4% is the effective semi-annual rate, that is, the effective annual rate is 8.16%).

What is your holding period return if LVMH's stock price six months from now is (a) €80, (b) €100, (c) €110, (d) €120?

Problem 3

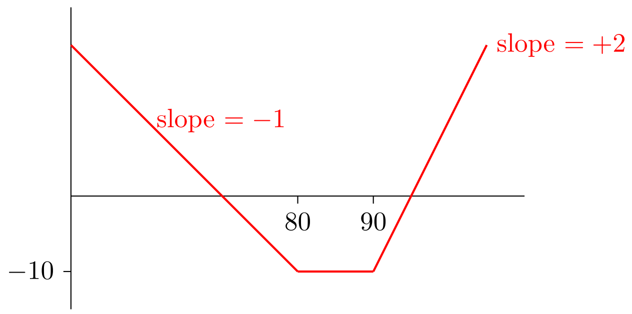
The following financial derivatives have the same underlying asset and the same maturity date:

- A European Call option with exercise price 80
- A European Call option with exercise price 90
- A forward contract with delivery price 70

a. Draw the payoff diagram of the forward contract (that is, plot the payoff at maturity of a long position in the forward as a function of the spot price of the underlying asset at maturity).

b. Draw the payoff diagram of a portfolio composed of one long position in the forward and one short position in the call option with exercise price 90.

c. Find a portfolio that contains some units of the three derivatives above and with a payoff at maturity as follows:



d. The underlying asset trades at 80 today. An investor decides to invest massively in the portfolio of question c). Which is most likely: (i) the investor thinks that the market under-estimates the volatility of the underlying asset; or (ii) the investor thinks that the market over-estimates the volatility of the underlying asset? Explain.

Problem 4

Suppose that you bought one forward contract (forward price $F_0 = 100$) and two European puts (strike $K = 100$) and sold one European call (strike $K = 120$) on the same underlying asset and the same maturity.

a. Draw carefully the payoff diagram of your portfolio at date T, i.e., your cash flows CF_T as a function of the underlying asset spot price S_T.

b. Which one(s) of the following four explanations is most likely to justify the purchase of this portfolio:

   (i) You think that the market underestimates the true value of the underlying asset.
   (ii) You think that the market overestimates the true value of the underlying asset.
   (iii) You think that the market underestimates the price volatility of the underlying asset.
   (iv) You think that the market overestimates the price volatility of the underlying asset.

c. Which one(s) of the following three reasons would best explain why you decided to sell the call with $K = 120$:

   (i) You wanted to reduce the overall initial cost of the portfolio.
   (ii) You wanted to benefit from a drop in the price of the underlying asset.
   (iii) You wanted to limit your downward exposure in case of an increase in the price volatility of the underlying asset.

Problem 5

You have the following information on two European options that have the same underlying asset:

Put option: maturity 2 years; strike price 100; put price 8

Call option: maturity 2 years; strike price 100; call price 5

Furthermore, you know that the underlying asset currently trades at a price 90, that the term structure of interest rates is flat, and that the annual risk-free rate is 5%.

a. Assume in this question only that the underlying asset does not pay any dividend between now and t=2 years. Is there an arbitrage opportunity? If yes, show the arbitrage strategy in a cash flow table.

Assume from now on that the underlying asset will pay a dividend of 15 at t=1 year.

b. Show that the put-call parity now writes $C_0 + \frac{K}{(1+r_f)^2} = P_0 + S_0 - \frac{D}{1+r_f}$. [Hint: find a portfolio containing a long position in the call and in the risk-free asset and another portfolio containing a long position in the put and in the stock and a short position in the risk-free asset, that have the same cash flows and apply the law of one price.]

c. Is there an arbitrage opportunity? If yes, show the arbitrage strategy in a cash flow table.

Problem 6

Stock XYZ is selling for €100 today (t=0). Two states of the economy are possible in one year (t=1). In the good state of the economy, the stock will sell for €120. In the bad state of the economy, it will sell for €90. XYZ will not pay any dividend in the years to come. The risk-free interest rate is 3%.

We consider the following European options:

	Underlying asset	Expiry	Strike price	Price at t=0
Call option 1	Stock XYZ	T=1	€100	
Call option 2	Stock XYZ	T=1	€110	€4.21
Put option 1	Stock XYZ	T=1		

Put 1 will pay €20 at date 1 in the bad state of the economy.

a. How much will Put 1 pay at date 1 in the good state of the economy?

b. What is the price of Put 1 today?

c. What is the price of Call 1 today?

d. What would be the price of Call 2 today if Call 2 was American instead of European?

Assume in questions e) and f) only that you can only trade stock XYZ and Put option 1 (you cannot trade the call options). We consider security ABC, which will pay €12 in the good state of the economy and €19 in the bad state of the economy at date 1.

e. Show how you can replicate the payoffs of this security (using only Put 1 and stock XYZ and assuming that you can buy or sell fractions of any security.) What is the no-arbitrage price of security ABC?

f. Assume that security ABC sells for €16 today. Find an arbitrage strategy and provide a table that contains the detail of your trades and transactions at dates 0 and 1.

Problem Set 4: Options

Solutions

Problem 1

a. Exercise the call at maturity if USD/EUR > 1.1

b. Profit will be positive when USD/EUR – 1.1 > 0.01, i.e., when USD/EUR > 1.11

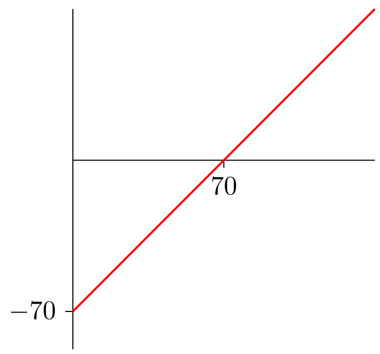
c. Exercise put if USD/EUR < 1.1. Profit will be positive when 1.10 – USD/EUR > 0.01, i.e., when USD/EUR < 1.09.

Problem 2

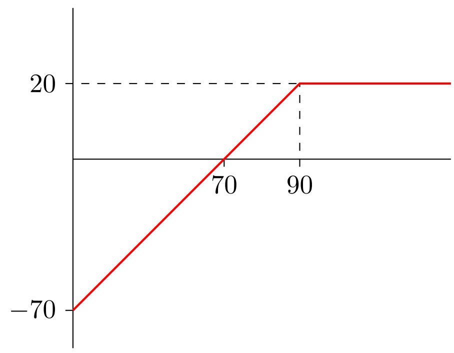
Stock price =	80	100	110	120
	CF in six months			
(i) All stocks (100 shares)	8,000	10,000	11,000	12,000
(ii) All options (1000 options)	0	0	10,000	20,000
(iii) Bills + Options	9,360	9,360	10,360	11,360
	Rate of return			
(i) All stocks (100 shares)	−20%	0%	10%	20%
(ii) All options (1000 options)	−100%	−100%	0%	100%
(iii) Bills + Options	−6.4%	−6.4%	3.6%	13.6%

Problem 3

a.



b.

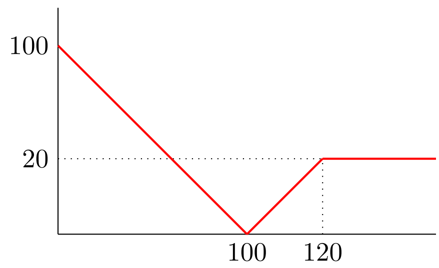


c. Sell 1 forward contract, buy 1 call option with $K = 80$, and buy 2 call options with $K = 90$.

d. That the market under-estimates volatility. Once we take into account the cash-flows at date 0, the strategy makes money if the price moves a lot and loses some if the price does not move too much.

Problem 4

a.



b. (ii) and/or (iii)

The strategy pays off if the price moves far away from 100, therefore (iii).

The payoff is the larger when the price approaches 0, therefore (ii).

c. (i)

Selling the call generates a positive payoff at t=0, therefore (i).

Problem 5

a. There is no arbitrage opportunity if the put-call parity holds:

$$C_0 + \frac{K}{(1+r_f)^T} = P_0 + S_0$$

Since instead $C_0 + \frac{K}{(1+r_f)^T} = 5 + \frac{100}{1.05^2} = 95.70$ and $P_0 + S_0 = 8 + 90 = 98$, the call is underpriced relative to put. The arbitrage strategy is the following:

	$t = 0$	$t = T$ if $S_T < K$	$t = T$ if $S_T > K$
Buy the call	-5	0	$S_T - 100$
Invest $\frac{100}{1.05^2} = 90.70$ at the risk-free rate	-90.70	100	100
Sell the put	8	$S_T - 100$	0
Short-sell underlying asset	90	$-S_T$	$-S_T$
Total payoff	2.30	0	0

b. We follow the same logic leading to the put-call parity without dividends that we studied in class. The first portfolio is:

	t=1	$t = 2$ if $S_T < K$	$t = 2$ if $S_T > K$
Long position in the call	0	0	$S_T - K$
Invest $\frac{K}{(1+r_f)^2}$ at the risk-free rate at t=0 for 2 years	0	K	K
Total payoff	0	K	S_T

The second portfolio is:

	t=1	$t=2$	
		if $S_T < K$	if $S_T > K$
Long position in the put	0	$K - S_T$	0
Long position in the stock	D	S_T	S_T
Borrow $\frac{D}{1+r_f}$ at the risk-free rate at t=0 for 1 year	$-D$	0	0
Total payoff	0	K	S_T

where we borrow $\frac{D}{1+r_f}$ at t=0 for 1 year in order to offset the dividend payment at t=1.

The price of the first portfolio at t=0 is $C_0 + \frac{K}{(1+r_f)^T}$. The price of the second portfolio at t=0 is $P_0 + S_0 - \frac{D}{1+r_f}$. Since the two portfolios have the same cash flows at all futures dates, they should have the same price today:

$$C_0 + \frac{K}{(1+r_f)^T} = P_0 + S_0 - \frac{D}{1+r_f}$$

c. $C_0 + \frac{K}{(1+r_f)^T} = 95.70$ and $P_0 + S_0 - \frac{D}{1+r_f} = 83.71$, so the call is over-valued relative to the put. There is an arbitrage strategy:

	$t=0$	$t=1$	$t=T$	$t=T$
			if $S_T < K$	if $S_T > K$
Sell the call	5		0	$100 - S_T$
Borrow $\frac{100}{1.05^2}$ at the risk-free rate at t=0 for 2 years	90.70		-100	-100
Buy the put	-8		$100 - S_T$	0
Buy the underlying asset	-90	15	S_T	S_T
Borrow $\frac{15}{1.05}$ at the risk-free rate at t=0 for 1 year	14.29	-15		
Total payoff	11.99	0	0	0

Problem 6

a. The strike price of Put 1 is €110. Therefore it pays nothing in the good state.

b. Put-call parity with Call 2 and Put 1: $4.21 + \frac{110}{1.03} = 100 + P$ implies $P = 11$ €.

c. Call 1 pays 0 in the bad state and 20 in the good state, i.e., its payoff is twice the payoff of Call 2. Its price is $2 \times 4.21 = 8.42$ €.

d. Same price.

e. Replicating portfolio with x_1 units of stock XYZ and x_2 units of Put 1:

$$\begin{cases} 12 = 120\ x_1 + 0 \\ 19 = 90\ x_1 + 20\ x_2 \end{cases}$$

which gives $x_1 = 0.1$ and $x_2 = 0.5$. The no-arbitrage price is $0.1 \times 100 + 0.5 \times 11 = 15.5$ €.

f. Security ABC is overpriced. Arbitrage strategy: sell short 1 security ABC + buy 0.1 share of XYZ + buy 0.5 Put

	Payoff at t=0	Payoff at t=1	
		Bad state	Good state
Short 1 security ABC	+16	-19	-12
Long 0.1 share of stock	-10	+9	+12
Long 0.5 put option	-5.5	+10	0
Total payoff	+0.5	0	0