Financial Markets 2022/23 – Quiz 2 – Solution

$$P_0 = \frac{C}{(1+y)} + \frac{C}{(1+y)^2} + \ldots + \frac{C+N}{(1+y)^T} = \frac{C}{y}\left(1 - \frac{1}{(1+y)^T}\right) + \frac{N}{(1+y)^T}$$, where y = yield-to-maturity

$$P_0 = \frac{C}{(1+r_1)} + \frac{C}{(1+r_2)^2} + \ldots + \frac{C+N}{(1+r_T)^T}$$, where r_t = t-year interest rate

$$(1+r_T)^T = (1+r_t)^t \times (1+f_{t\to T})^{(T-t)}$$ where T > t, and $f_{t\text{->}T}$ = forward rate from year t to year T

Question 1:
Consider a government coupon-paying bond with a maturity of exactly T=3 years from now, annual coupon payments of 14 euros once a year, a par (face) value of 1000 euros. The relevant term-structure of interest rates is such that r_1 =3 percent (per annum) for year 1, r_2=5 percent (per annum) for year 2, and r_3=8 percent (per annum) for year 3. The bond has face value N = 1000 euros. Determine the price of this bond today (i.e., at date t=0). If your answer is 1234.56789, enter it as 1234.57 with two digits after the decimal point. Round your answer to 2 decimal places.

Answer:

$$P_0 = \frac{C}{1+r_1} + \frac{C}{(1+r_2)^2} + \frac{C+N}{(1+r_3)^3} = \frac{14}{1+0.03} + \frac{14}{(1+0.05)^2} + \frac{14+1000}{(1+0.08)^3} = 831.24$$

Question 2:
A government zero-coupon bond Z has a par (face) value of N = 1000 euros and it is exactly 14-years away from maturity. What is the yield-to-maturity (y) of this zero-coupon bond if its market price today (at t=0) is equal to Z_0 = 524 euros ? If your answer is, for ex., 0.0500, then enter your answer with <u>four</u>-digits after the decimal point as 0.0500 (but do <u>not</u> enter 5.00%, or 5% or 5,00% or 0,0500, or "five percent", etc.) Round your answer to 4 decimal places.

Answer:

$$Z_0 = \frac{1000}{(1+r_T)^T} \Longrightarrow r_T = \left(\frac{1000}{Z_0}\right)^{\left(\frac{1}{T}\right)} - 1 = \left(\frac{1000}{524}\right)^{\left(\frac{1}{14}\right)} - 1 = 0.0472$$

Question 3:
The yield curve based on government zero-coupon bonds is completely flat. The price of a government coupon-paying bond B with a face (par) value of 1000 euros, coupon-payment frequency of one per year, and with exactly 50 years until maturity, is equal to 1000 euros today. If this bond B has an annual coupon with a rate of 6 percent per annum, what must be the 21-year government zero-coupon bond rate on a per annum basis? If your answer is 1 percent per year, enter your answer as 0.0100 with four digits after the decimal point. Round your answer to 4 decimal places.

Answer: flat yield curve at 6% ➔ $r_1 = r_2 = \ldots = r_{21} = \ldots = r_{50} = 6\%$ ➔ $r_{21} = 6\%$

Question 4:

Today (at t=0) the 1-year spot rate is 4 percent (per annum), while the 7-year spot rate is 6 percent (per annum). What is, as of date t=0, on a per-annum basis, the forward interest rate f that is expected to prevail between dates 1 and 7 ? If your answer is 12.34 percent, enter it as 0.1234 using four digits after the decimal point. Round your answer to 4 decimal places.

Answer:

$$(1+r_7)^7 = (1+r_1)^1 \times (1+f_{1\to 7})^6$$

$$\Rightarrow f_{1\to 7} = \left(\frac{(1+r_7)^7}{(1+r_1)^1}\right)^{\left(\frac{1}{6}\right)} - 1 = \left(\frac{(1+0.06)^7}{(1+0.04)^1}\right)^{\left(\frac{1}{6}\right)} - 1 = 0.0634$$

Question 5:

Find the price at t=0 of a government coupon-paying bond that matures in exactly 15-years from today (t=0), with a coupon rate of 7 percent per year, annual coupon payments, and a yield-to-maturity of 5 percent per year. Par (face) value is 1000 euros. If your answer is 1234.56789, enter it as 1234.57 with two digits after the decimal point. Round your answer to 2 decimal places.

Answer:

$$P_0 = \frac{C}{y}\left(1 - \frac{1}{(1+y)^T}\right) + \frac{N}{(1+y)^T} = \frac{70}{0.05}\left(1 - \frac{1}{(1+0.05)^{15}}\right) + \frac{1000}{(1+0.05)^{15}} = 1207.59$$