HEC Paris – Financial Markets – Fall 2022 Final Exam – Solution Key

Problem 1 [40 points]:

Consider firm A that is located in the euro area and has a beta of 1.5. The risk-free rate is r_f = 5% per year in the euro area and the expected return on the market portfolio is 10% per year. Firm A is expected to pay a dividend of 5 Euros per share every year in perpetuity. The next dividend will be paid in exactly one year from today. Assume that the CAPM holds and the market is in equilibrium.

a) Show that firm A's required rate of return is equal to 12.5%?

$k = r_f + beta * (r_m - r_f) = 0.05 + 1.5 * (0.10 - 0.05) = 0.125$

b) Calculate today's price for one share of company A.

$P^A_0 = 5 / 0.125 = 40$

Suppose that firm A's management has the opportunity to reinvest part or the totality of this year company's earnings into new projects that offer an $ROE^{new\ projects}$ of 10% per year.

c) Which fraction of the earning should be consistently reinvested into the firm to have dividends grown at a constant rate of 5% starting from next year?

g= ROE *b hence 5%=10%*b hence b=0.5

d) Suppose the management adopt earning redistribution policy described in question c). What will be the market reaction to the announcement of this new dividend policy. Explain whether the stock price will go up or down and why. No calculations are necessary.

ROE_new < k ---> PVGO < 0. Hence, for what concerns market cap, it is better to distribute all earnings to shareholders. Thus the stock price will decrease if the company retains 50% of the earnings.

Consider now company B, and Company C also located in the euro area, both companies are expected to pay a dividend of 5 euros per share each year. The beta of company B is equal to 1 whereas the beta of company C is equal to 1.5.

Suppose now that the European Central Bank unexpectedly increases its key interest rate by 100 basis points such that the risk free rate now becomes r_f = 6% for all maturities. But the expected rate of return of the market portfolio as well as company B's and C's betas do not change.

Following the European Central Bank's announcement the stock price of company B remains unchanged whereas the stock price of company C increases.

e) Briefly explain whether or not, and why, such market reaction to the ECB new policy is contradiction with the CAPM assumption and the efficient market hypothesis

This is not in contradiction with CAPM and Market Efficiency. According to the CAPM, the policy of the ECB decreases the required return of company C, whose beta>1 , but not the required return of company B, whose beta =1. Hence the stock market reaction.

Problem 2 [50 points]:

Consider the following bonds

	Time to Maturity	Coupon	Face (Par) Value	Coupon payment frequency	Credit rating	Bond Price at t=0
Bond A	2.5 Years	60	1000	1 year	AAA	1077.80
Bond B	2.5 Years	0	1000	N/A	AAA	906.60
Bond C	2.5 Years	60	0	1 year	AAA	
Bond D	3 years	40	1000	1 year	BB	

Note that Bond C is in effect an annuity.

a) What are the cash-flows of bonds A, B, C and D? For each bond, draw a timeline with the dates of the coupon and face (par) value payments and the associated cash-flows.

Bond A: 60 in 0.5 years, 60 in 1.5 years and 1060 in 2.5 years
Bond B: 1000 in 2.5 years
Bond C: 60 in 0.5 years, 60 in 1.5 years and 60 in 2.5 years
Bond D: 40 in 1 year, 40 in 2 years, and 1040 in 3 years

b) What is the 2.5-year interest rate (i.e., $r_{2.5\text{ years}}$) for AAA-rated bonds on a per year basis?

$(1000 / 906.62)^{1/2.5} – 1 = 0.0400 = 4\%$ per year

c) Rank bonds A, B and C from the highest to the lowest duration. No calculation is necessary if you explain how a bond's coupon rate and its maturity affect its duration.

Duration increases with maturity and decreases with the coupon rate: hence B > A > C

d) What is the no arbitrage price of Bond C?

Bond C can be replicated with a long position in bond A and a short position in bond B. Hence $P^C_0 = P^A_0 - P^B_0 = 171.20$

e) Suppose that bond C trades for 151.20 Euros in the market. If there is an arbitrage strategy, first clearly identify it. Then, fill the arbitrage table below indicating clearly each position you want to take as of date t=0 resulting from your arbitrage strategy as well as all the cash-flows associated with these positions, at the dates (when relevant) indicated in the table:

Transactions at t=0	Cash flows at t=0	Cash flows at t=0.5 yr	Cash flows at t=1 year	Cash flows at t=1.5 yrs	Cash flows at t=2 yrs	Cash flows at t=2.5 yrs
Buy C in the market	**–151.20**	**+60**		**+60**		**+60**
Sell the replicating portfolio						
Short-sell Bond A	**+1077.80**	**–60**		**–60**		**–1060**
Buy Bond B	**–906.60**	**0**		**0**		**+1000**
Replicating p/f total	+171.20	-60		-60		-60
Arbitrage total	**+20**	**0**		**0**		**0**

In this case the Bond C is underpriced so buy 1 bond C for 150.20 sell its replicating portfolio, that is short-sell 1 bond A and buy 1 bond B. I gain 171.20 - 151.20 = 20 at t=0.

Suppose the yield curve of BB-rated bond is flat at r_t^{BB} = 10% per year for all maturities *t* whereas the yield curve of B-rated bonds is flat at r_t^{B} = 13% per year for all t.

f) Which one of the following is the duration of bond D?
0.76 years, 1.3 years, 2.86 years, 3 years, 3.47 years

2.86 Years as most of the cash-flows are paid in year 3

g) Company XYZ has a credit rating of BB. The company plans to issue 100 000 (one hundred thousand) bonds with a maturity of 5 years and face value 1 000 (one thousand) Euros each.

Which coupon rate should Company XYZ set on its bond at the primary market if it wants to raise exactly 100 000 000 (one hundred million) Euros from the issuance of its 5-year bonds?

Set the coupon rate = 10% so that the bond sells at par

h) Suppose that company XYZ sets a coupon rate for its bond as in your answer to question g) above. However, unexpectedly, a few minutes before the issuance of XYZ bonds, but after bond's coupon rate is decided upon (i.e. the coupon rate is the same as the one you found in question f), XYZ credit rating is downgraded from BB to B. Would the proceeds from the issuance of XYZ bonds (that is, the overall value of these bonds) be greater or smaller than 100 000 000 (one hundred million) Euros? Briefly explain (no calculations needed).

Proceeds are less than 100 000 000 as the bond market participants discount the bonds' cash-flows by more than their coupon rate of 10%. Indeed, the bonds' cash flows are now discounted at the interest rate 13% for B-rated bonds.

<u>Problem 3 [50 points]:</u>

Company XYZ is expected to pay its first dividend of 11.5 €/share exactly 2 years from now (at date *t*=2). The growth rate of dividends will be 5% per year forever. The required rate of return on the shares of XYZ is 15% per year. The risk-free yield curve is flat at 5% per year for all maturities, i.e., the risk-free interest rate is constant at 5% per year for all dates in the future.

a) Using the Constant Growth Dividend Discount Model, show that the XYZ's stock price today (t=0) is equal to 100 €/share.

$$S_0 = \frac{\frac{D_2}{k-g}}{1+k} = \frac{\frac{11.5}{0.15-0.05}}{1+0.15} = \frac{115}{1.15} = 100$$

b) Show that the no-arbitrage delivery price at date t=0 (today) of a forward contract F that will deliver one share of stock XYZ in exactly one year from now (at date t=1) is equal to 105 euros/share. That is, $F_{t=0,\, T=1\ year}$ = F(0, 1YR) = 105 €/share.

$$F_{0,\ T=1\,year} = S_0 \times (1+r_1)^1 = 100 \times (1+0.05) = 105 \text{ €/share}$$

c) Suppose that the CAPM holds. Consider the following sentence:

"The fact that $F_{0,\,1\ year}$ = F(0, 1YR) = 105 €/share implies that the investors expect that at time t=1 year the spot price of ABC will be $S_{1\text{-Year}}$ =105."

This sentence is (tick the correct answer's box):

- ☐ True in general
- ☐ True only if XYZ has a beta larger than 0
- ☒ True only if XYZ has a beta equal to 0
- ☐ True only if XYZ has a beta smaller than 0

d) Consider time t = 0.5-year (i.e., exactly 6-months after t=0). The prospects for company XYZ have changed, and the current stock price is now $S_{0.5\text{-year}}$ = 92.71 €/share. The risk-free yield curve is still flat at 5% per year for all maturities, i.e., the risk-free interest rate remains 5% per year for all dates in the future.

What is the no-arbitrage price at date t = 0.5-year of a forward contract G that will deliver one share of XYZ in exactly 6-months from that date (with delivery at date t=1 year) ? That is, calculate $G_{t=0.5\text{-yr},\, T=1\text{-yr}}$.

$$G_{t=\frac{1}{2}year,\ T=1\,year} = S_{\frac{1}{2}-year} \times \left(1 + r_{1/2}\right)^{1/2} = 92.71 \times (1 + 0.05)^{1/2} = 95 \text{ €/share}$$

e) Suppose that at date t=0 you bought (went long) on the 1-year forward contract F on one share of stock XYZ. Now it is date t = 0.5–year and you would like to close out (i.e., get out of) your long F position as of date t = 0.5–year in such a way that the total of your cash flows as of date T = 1 year adds up to zero. In the table below show all the transactions you would need to undertake in order to be able to do so using forward contract G and borrowing or lending at the risk-free rate. The yield curve is still flat at 5% per year for all maturities.

Transactions as of date t = 0.5–year	Cash Flows as of date t = 0.5–year	Cash Flows as of date T = 1–year
Long position on forward F	0	$S_1 - F_{0,1} = S_1 - 105$
Short position on forward G	0	$G_{1/2,1} - S_1 = 95 - S_1$
Lend present value of $(F_{0,1} - F_{1/2,1})$ for 1/2–year, i.e., lend $10 / 1.05^{0.5}$	$- 10 / 1.05^{0.5} = - 9.759$	$9.759 \times 1.05^{0.5} = +10$
Total	– 9.759	0

f) Based on the table above, very briefly explain whether you get paid or do you have to pay on date t = 0.5–year to close out (get out of) your position on forward contract F (which you had bought at date t = 0) ? (No calculation is needed.)

You would need to pay 9.76 € to be able to close out (get out of) your position in F as of t=0.5.

Problem 4 [60 points]:

Consider stock ABC which does not pay any dividends for the coming five years. The stock price at time t=0 (i.e., today) equals S_0 = S_0 = 100 €/share. The yield curve for default risk free bonds is flat at r=5% for all maturities in the future.

a) Suppose you own one share of ABC and you plan to sell it in exactly 2 years from now. If you want to be sure that, at t=1 (i.e., at exactly two years from now), you will be able to sell your ABC share for at least of 110 €/share, which of the following option positions would you get into? Check the box for correct answer:

☐ buy a European call on one ABC share with K_C = 110 €/share and T = 2 years

☐ sell a European call on one ABC share with K_C = 110 €/share and T = 2 years

☐ buy an American call on one ABC share with K_C = 110 €/share and T = 2 years

☐ sell an American call on one ABC share with K_C = 110 €/share and T = 2 years

☐ buy a European put on one ABC share with K_P = 110 €/share and T = 2 years

☐ sell a European put on one ABC share with K_P = 110 €/share and T = 2 years

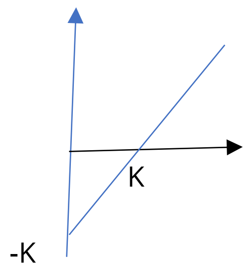
☐ buy an American put on one ABC share with K_P = 110 €/share and T = 2 years

☐ sell an American put on one ABC share with K_P = 110 €/share and T = 2 years

Buy an EU put option with on one share of ABC strike price 110€ and maturity 2 years

Now, consider different European call and put options on one share of stock ABC, both with the maturity of T=1 year and both with the same strike (exercise) price of K. The call option C' has a price C'_0 = 3 €/share and the put option P' has a price P'_0 = 3 €/share at time t=0.

b) Represent in a graph (payoff diagram) the payoff you would get in 1 year with a portfolio composed of one long position in the European call option C' and one short position in the European put option P' as a function of the price of stock ABC at T=1 year (i.e., as a function of $S_{T=1\ year}$). Clearly indicate long call and short put payoffs as well as the slopes of the lines that you draw. Fill-in appropriately where it is marked "…" on the graph:



c) In the absence of an arbitrage opportunity, show how you would calculate the strike price K' of the European call option (with price C'_0 = 3 €/share and T=1 year) and European put option (with price P'_0 = 3 €/share and T=1 year) described above? (Hint: think about the Put-Call Parity). Reminder: ABC's share price today is S_0 = 100 €/share.

$C'_0 + K' / (1+r)^T = P'_0 + S_0$

$K' = (1+r)^T \times (P'_0 + S_0 - C'_0) = (1.05)^1 \times (3 + 100 - 3) = (1.05) \times 100 = 105$

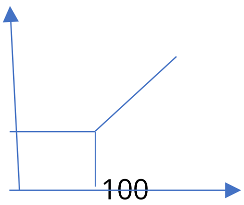
d) Now, together with the European call (C') and European put P' described in part c), consider an American call option C^{AM} on one share of ABC with the same maturity of T=1 year and the same strike price of K' as options C' and P'. Suppose you can buy and sell this American call option for C_0^{AM} = 3.5 €/share. Briefly explain why you should be able to do arbitrage and fill the arbitrage table below (hint: you don't need to know the answer to K' from c) to find the solution to d)):

Transactions initiated at t=0	Cash flows as of t = 0	Cash flows as of T = 1 year	
		$S_1 < K'$	$S_1 > K'$
Buy European Call	$- C'_0 = - 3$	0	$S_1 - K$
Sell American Call	$+ C^{AM}_0 = + 3.50$	0	$K - S_1$
Arbitrage total	+0.50	0	0

It is never optimal to exercise an American call on a non-dividend paying stock before its maturity, it should have the same market price than the European call with the same K and same maturity. Hence arbitrage is to buy European call and sell American call with the same K' and T on the same share.

The same company ABC has a default-risk rating of AAA. This company has issued two bonds at t=0. A zero-coupon bond Z with maturity in 1 year and face value of 100€, and a convertible zero-coupon bond ZC with maturity 1 year. At the maturity date of T=1 year, the owner of the convertible bond can choose between receiving 100 €/bond or receiving one share of ABC valued at S_1.

e) In a graph, represent the value at date t=1 year to the holder of a convertible bond ZC as a function of $S_1 = S_{1Year}$, i.e., the market price of share of ABC in 1 year:



Today (at t=0) the AAA-rated bond Z sells for Z_0 = 95.238 €/bond, whereas the AAA-rated convertible bond ZC trades for ZC_0 = 101 €/bond. The risk-free yield curve is still flat at 5% per year.

f) What is the no-arbitrage premium (price) C''_0 for the European call option C'' that is on one share of ABC with maturity T=1 year and strike price K=100?

$$ZC_0 = Z_0 + C''_0 \quad \Rightarrow \quad C''_0 = ZC_0 - Z_0 = 5.762$$

g) What is the no-arbitrage premium (price) P''_0 today (at t=0) for the European put option P'' that is on one share of ABC with maturity of T=1 year and strike price K=100?

$ZC_0 = S_0 + P''_0$ implying $P''_0 = ZC_0 - S = 1$ €

or equivalently use the put-call parity to get the same result:

$C''_0 + K / (1+r)^T = P''_0 + S_0 \Rightarrow P''_0 = C''_0 + K / (1+r)^T - S_0 = 5.762 + 95.238 - 100 = 1$ €