Financial Market Microstructure

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The Market Microstructure Approach

Some definitions (the term is originally due to Garman (1976))

- **O'Hara (1995):** "The study of the process and outcomes of exchanging assets under explicit trading rules..."
- **Madhavan (2000):** "The process by which investors' latent demands are ultimately translated into transactions."
- **Biais, Glosten, and Spatt (2005):** "The investigation of the economic forces affecting trades, quotes, and prices."

Main Questions

1. **Liquidity:** What are the determinants of market liquidity? Liquidity risk? Measures (bid-ask spreads and depth)?
2. **Price Discovery:** How and to what extent do prices impound new information? At what speed?
3. **Volatility:** What are the determinants of price changes at high frequency? How do volatility and liquidity interact?

Definition

An **asset's liquidity** is a measure of:

- The **speed** at which an asset can be bought and sold. (The faster, the more liquid.)
- The **price impact** of the traded quantity. (The smaller the impact, the more liquid.)
- The **cost** of a "round-trip." (The cheaper, the more liquid.)

Why Do We Care?

- **Illiquidity means lower returns on portfolios:** (i) Portfolio managers care about market liquidity, and (ii) the brokerage industry devises trading strategies to minimize costs due to illiquidity.
- **Illiquidity affects asset prices/cost of capital:** (i) Illiquidity acts as a "tax" on asset payoffs and (ii) is a source of risk.
- **Price discovery affects the allocation of capital in the economy.**

Actual Markets: Does Price Reflect Fundamentals?

MB Trader ELTR

Name	Bid	Size	Name	Ask	Size
MLCO	18 1/4	10	LEHM	18 3/8	20
GSCO	18 1/4	20	MASH	18 1/2	10
PRUS	18 1/4	30	AGED	18 5/8	17
BEST	18 1/8	10	SHWD	18 5/8	10
TSCO	18 1/16	10	TSCO	18 3/4	50

An example on liquidity and cost of capital


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Mid quote = $(18.25 + 18.375)/2 = 18.3125$
Transaction price $\simeq 18.3125 \pm 0.0625$
Transaction cost $\simeq 0.34\%$ of 18.3125

Small transaction cost, large price impact

Example

Stock A is an immediate perpetuity paying \$1 per year. The required return on capital is $r = 5\%$ per year. What is the current price for stock A? Answer \$21

Suppose that at resale transaction costs are $s = 0.34\%$ of the price. What is the current price for stock A?

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Example

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Suppose that at resale transaction costs are $s = 0.34\%$ of the price. What is the current price for stock A?
Suppose each investor plans to keep the perpetuity for one year and then resell to another investor..

$$P = \frac{1 + P(1 - 0.0034)}{1.05} \Rightarrow P = \frac{1.05}{0.0534} = 19.66 = 21(1-6.4\%)$$

Market Informational Efficiency

Does the price of an asset reflect all information about the asset's fundamentals?

Definition

- **Weak form efficiency**: Trading prices incorporate all past public information.
- **Semi-Strong form efficiency**: Trading prices incorporate all present and past public information.
- **Strong form efficiency**: Trading prices incorporate all public and private information available in the economy.

Roadmap

- Key concepts
- Auctions
- Quote-driven markets: static models
  - Inventory models
  - Information models
- Quote-driven markets: dynamic models
  - Market efficiency and herd behavior
  - Robust price formation
- Limit Order Markets

The Market Microstructure Approach

It recognizes the role of:

- **Heterogeneity among Market Participants.** Participants in security markets have various objectives (e.g., dealers differ from final investors; hedgers differ from speculators, etc.).
- **Institutional framework.** Market design and market regulation matter.
- **Private Information.** Informational asymmetries among market participants are prevalent in securities markets.

Market Participants

- **Customers**
- **Dealers**
- **Intermediaries**

Market Participants

- **Customers:** Agents willing to trade the security:
  - Institutional investors (pension funds, mutual funds, foundations): Hold and manage the majority of assets; account for the bulk of trading volume; trade large quantities.
  - Individual Investors (retail traders, households, banks): Account for the bulk of trades; trade smaller quantities.
- **Dealers**
- **Intermediaries**

Market Participants

- **Customers**
- **Dealers:** Large professional traders who trade for their own accounts and provide liquidity to the market.
- **Intermediaries**

Market Participants

- **Customers**
- **Dealers**
- **Intermediaries:**
  - Brokers: Match customer orders but do not trade for their own account.
  - Specialists (NYSE): Are responsible for providing liquidity and smoothing trade on given securities.
  - Market Makers: Agents that stand ready to buy and sell the security at their bid and ask prices respectively, providing liquidity.

Institutional Framework: Market Types

- **Call Auction Markets:** Occur at specific times (e.g., at the opening or at the fixing); investors place orders that are executed at a single clearing price that maximizes the volume of trade.
- **Continuous Auction Markets (or limit order markets):** Investors trade against resting orders placed earlier by other investors (e.g., Euronext, Toronto SE, ECNs).
- **Dealer Markets (or quote-driven markets):** Market-makers post bid and ask quotes at which investors can trade (e.g., Bond Markets MTS, FX markets).
- **Alternative Trading Systems (ATS)**
  - **Electronic Communication Networks (ECN):** Continuous order-driven anonymous markets (e.g., Island, Instinet, Archipelago, Redibook).
  - **Crossing Networks (CN):** Cross multiple orders at a single price determined in a base market (e.g., POSIT, Xtra XXL).

Most Common Orders

- **Buy limit order:** Order to buy up to a quantity q for a price not larger than p.
- **Sell limit order:** Order to sell up to a quantity q for a price not smaller than p.

- **Buy market order:** Order to buy a quantity q at the best current market conditions.
- **Sell market order:** Order to sell a quantity q at the best current market conditions.

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A game theoretical approach

The market for **one** financial asset:

- Market Participants: set of players N
- Institutional framework:
  - Set of actions X_i available to market participant i
  - Set of action profiles $X := \times_{i \in N} X_i$
  - No trade action: $x_\emptyset \in X$
  - Asset allocation rule $Q : X \to \mathbb{R}^N$
  - Cash allocation rule $P : X \to \mathbb{R}^N$
  - $\forall x \in X$:

$$\sum_{i \in N} Q_i(x) = \sum_{i \in N} P_i(x) = 0$$

- Asset fundamental value $\tilde{v}$
- Participant i's monetary payoff from transaction x:

$$\tilde{v} Q_i(x) + P_i(x)$$

- Participant i's utility after transaction x, given initial wealth $\tilde{W}_i$:

$$U_i(\tilde{W}_i + \tilde{v} Q_i(x) + P_i(x))$$

Model of uncertainty

- Ω: set of all possible states of Nature (finite).
- $\mathcal{A}$: Set of all subsets of Ω.
- $\pi : \mathcal{A} \to [0, 1]$: probability measure of $\mathcal{A}$.

- $v : \Omega \to \mathbb{R}$: Value of the asset ($\tilde{v}$).
- $W_i : \Omega \to \mathbb{R}$: Value of agent i initial portfolio ($\tilde{W}_i$).
- $U_i : \Omega \to$ Set of possible utility functions: Utility of agent i .

Partitions

Definition

- A partition $\mathcal{P}$ of Ω is a collection of nonempty, pairwise disjoint subsets of Ω whose union is Ω.
- $\mathcal{P}(\omega)$ denotes the element of $\mathcal{P}$ that contains the state ω.

Modelling incomplete information using partitions

- All agents start with common prior π over Ω.
- Agent *i* receive private information that we described as a partition $\mathcal{P}_i$ over Ω:
  if the true state is ω, then agent *i* is informed that the true state belongs to the set $\mathcal{P}_i(\omega)$.
- If the true state is ω, the *type* of agent *i* is $\theta_i = \mathcal{P}_i(\omega)$.

An example

- $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$, $\pi(\omega) = 0.2$
- $v(\omega_1) = 1$; $v(\omega_2) = 2$, $v(\omega_3) = 3$, $v(\omega_4) = 4$, $v(\omega_5) = 5$
- $\mathcal{P}_1 = \{\{\omega_1, \omega_2, \}, \{\omega_3, \omega_4\}, \{\omega_5\}\}$
- $\mathcal{P}_2 = \{\{\omega_1, \omega_3, \}, \{\omega_2, \omega_4\}, \{\omega_5\}\}$
- $\mathcal{P}_3 = \{\{\omega_1\}, \{\omega_2, \}, \{\omega_3\}, \{\omega_4\}, \{\omega_5\}\}$
- $\mathcal{P}_4 = \{\{\omega_1, \omega_2, \omega_3, \omega_4\}, \{\omega_5\}\}$

Agent 1 and 2 receive different partial information; Agent 3 is perfectly informed; Agent 4 is the least informed.

If for example $\omega = \omega_3$, then
for agent 1 $E[\tilde{v}|\mathcal{P}_1(\omega_3)] = 3.5$
for agent 2 $E[\tilde{v}|\mathcal{P}_2(\omega_3)] = 2$
for agent 3 $E[\tilde{v}|\mathcal{P}_3(\omega_3)] = 3$
for agent 4 $E[\tilde{v}|\mathcal{P}_4(\omega_3)] = 2.5$

Joining and Meeting Partitions...

Let $\mathcal{P}$ and $\mathcal{P}'$ be two partitions of Ω.

Definition

Partition $\mathcal{P}$ is said to *refine* partition $\mathcal{P}'$ if every element θ of $\mathcal{P}$ is contained in some element θ' of $\mathcal{P}'$.

Definition

The meet of the partitions $\mathcal{P}_i$ and $\mathcal{P}_j$, that we will denote $\mathcal{M}_{ij}$, is the finest partition that is refined by both $\mathcal{P}_i$ and $\mathcal{P}_j$.

Definition

The join of the partitions $\mathcal{P}_i$ and $\mathcal{P}_j$, that we will denote $\mathcal{J}_{ij}$, is the least fine partition that refines both $\mathcal{P}_i$ and $\mathcal{P}_j$.

If the state is ω_3...

- What could agent 1 and 2 know if they share their information?
- What could agent 1 and 4 know if they share their information?
- What is that agent 1 and 2 commonly know?

Common Knowledge: Informally

Something is common knowledge if we both know that it's true:

1. and I know that you know it's true;
2. and you know that I know it's true;
3. and I know that you know that I know it's true;
4. and so on, for any string of beliefs we put together.

Common Knowledge Formally

Lemma

If the true state is ω, then what is common knowledge for player i and j is $\mathcal{M}_{ij}(\omega)$.

Sharing Information Formally

Lemma

If the true state is ω, then if player i and j share their information they both know $\mathcal{J}_{ij}(\omega)$.

Bayesian Equilibrium

- Let $\mathcal{P}_i$ be the set of possible types for agent *i*.
- Let $\mathcal{P}_i^t$ be the set of agent *i*'s possible information about past actions at time t.
- A *strategy* for player *i* is a mapping:

$$\sigma_i : \mathcal{P}_i \times_{t \geq 0} \mathcal{P}_i^t \to \Delta X_i$$

- A Bayesian Nash equilibrium is a strategy profile $\{\sigma_i, \ldots, \sigma_N\}$ such that for all player *i* and all histories $h \in H$ and all *t*

$$\sigma_i(\theta_i, h) \in \arg\max_{x_i \in X_i} E[U_i(\tilde{W}_i + \tilde{v} Q_i(x_i, \sigma_{-i}) + P_i(x_i, \sigma_{-i}))|\theta_i, h_i^t(h)]$$

where $h_i^t(\mathrm{h})$ represents what agent *i* has observed at time t if the history of action profile is h.

Why do people trade in the financial market?

There are two possible reasons for trading:

- Speculating on private information about $\tilde{v}$.
- Hedging, when U_i is concave.

Trading based on information

- $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$, $\pi(\omega) = 0.2$
- $v(\omega_1) = 1$; $v(\omega_2) = 2$, $v(\omega_3) = 3$, $v(\omega_4) = 4$, $v(\omega_5) = 5$
- $\mathcal{P}_1 = \{\{\omega_1, \omega_2, \}, \{\omega_3, \omega_4\}, \{\omega_5\}\}$
- $\mathcal{P}_2 = \{\{\omega_1, \omega_3, \}, \{\omega_2, \omega_4\}, \{\omega_5\}\}$
- $\mathcal{P}_3 = \{\{\omega_1\}, \{\omega_2, \}, \{\omega_3\}, \{\omega_4\}, \{\omega_5\}\}$
- $\mathcal{P}_4 = \{\{\omega_1, \omega_2, \omega_3, \omega_4\}, \{\omega_5\}\}$

If for example $\omega = \omega_3$, then
for agent 1 $E[\tilde{v}|\mathcal{P}_1(\omega_3)] = 3.5$
for agent 2 $E[\tilde{v}|\mathcal{P}_2(\omega_3)] = 2$

Can we say that in state ω_3 agent 1 and 2 could agree on a trade where agent 1 buys the asset from agent 2 at a price of 2.9?

No trade theorem

If

- Initial allocation is ex-ante Pareto optimal
- At $\omega \in \Omega$ it is common knowledge that a transaction x is acceptable to both parties

Then,

- Each market participant is indifferent between x and the no-trade action $x_{\emptyset}$.

Rational agents starting from common prior cannot trade solely because they have different information.

No trade theorem (Milgrom and Stokey 1982)

Theorem

If traders start from common priors and it is common knowledge that all traders are rational and the current allocation is ex-ante Pareto efficient, then new asymmetric information will not lead to trade, provided that traders are strictly risk averse.

Corollary

If traders start from common priors and have no reason to trade a priori, then they will not trade based on the arrival of new private information.

An example of the no trade theorem with two Traders

- Two traders: **Alice** (owns an asset) and **Bob** (potential buyer).
- The asset's true value is uncertain:
  - Value is either **$100** or **$200**, each with probability 50%.
- Both traders are risk neutral, **rational** and know each other is rational, and agree on the 50% probability.

Step 1: Common Prior

- Before receiving any private information, both traders believe:

$$E[\tilde{V}] = 0.5(100) + 0.5(200) = 150.$$

- So any transaction can only occur at price 150, and at this price Alice is indifferent between selling or not trading and Bob is indifferent between buying or not trading.

Step 2: Bob Receives Private Information

- Bob gets a **private signal** suggesting the value is likely higher.
- Based on his information, Bob offers Alice **$160** for the asset.

Step 3: Alice's Rationality Check

- Alice knows Bob has **private information.**
- If the true value were only **$100**, Bob would never offer $160.
- The fact that Bob is willing to pay **signals that the true value is high**.
- Alice **updates her belief** and realizes that selling might be a bad idea.

Step 4: No Trade Occurs

- Alice **rejects the offer** because Bob's willingness to trade signals a high value.
- No trade happens **purely due to asymmetric information.**
- This illustrates the **No-Trade Theorem**:

Key Insight

If traders are rational and start with a **common prior**, no trade will occur based solely on **asymmetric information.**