Financial Markets
4: Options

Stefano Lovo

HEC, Paris

Guaranteed capital

Today you buy a stock index at $S_0 = Eu\ 1,000$.
You will sell the index in T years at $\tilde{S}_T$.

You would like to make a profit from an increase in the stock index price (i.e., when $\tilde{S}_T > S_0$).

But you do not want to lose money from a decline in the price (i.e., when $\tilde{S}_T < S_0$).

You would like to buy an insurance that pays

$$S_0 - \tilde{S}_T \text{ iff } S_0 > \tilde{S}_T$$

Guaranteed capital

Option

Definition

An **option** is a security which:

- gives the right (not the obligation) to its owner to
  - Buy (or sell) another asset: the **underlying asset**.
  - At a pre-specified price K: the **strike price**.
  - At, or until, a specified date T: the **expiration date** or **maturity**.
- The seller of the option has the obligation to sell (or to buy) the underlying asset if the owner of the option exercises his right.

Example: European put option

Example

The owner of an European put option on

- Underlying asset: 20 shares of Apple with
- Strike price is $K = USD\ 500$,
- Maturity is $T = 1$ year,

Example: European put option

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- Underlying asset: 20 shares of Apple with
- Strike price is $K = USD\ 500$,
- Maturity is $T = 1$ year,

in 1 year has the right to sell 20 shares of Apple at price USD 500 each.

at T=1 Year:

	Owner of the Eu put option	Seller of the Eu put option
The owner exercises the option	-20 AAPL +USD 10,000	+20 AAPL -USD 10,000
The owner does not exercises the option	nothing	nothing

Option: an example

Example

The owner of a European call option on

- Underlying asset: 20 shares of Apple with
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Option: an example

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- Underlying asset: 20 shares of Apple with
- Strike price is $K = USD\ 500$,
- Maturity is $T = 1$ year,

by 1 year has the right to buy 20 shares of Apple at price USD 500 each.

	Owner of the call option	Seller of the call option
The owner exercises the option	+20 AAPL -USD 10,000	-20 AAPL +USD 10,000
The owner does not exercises the option	nothing	nothing

An option's features

An option is characterized by the following features:

- The **underlying asset**: the financial asset or commodity that can be exchanged at maturity.
- The **maturity** (T): the date at (or until which) the exchange can occur.
- The **size** of the contract (q) : The amount of the underlying asset that can be exchanged.
- The **strike price** (K): The delivery price that will be paid -per unit- if the option is exercised.
- The **type** of the option.
  - Call option = Right to buy the underlying asset.
  - Put option = Right to sell the underlying asset.

Different types of options

	Call	Put
European	Right to buy at maturity	Right to sell at maturity
American	Right to buy until maturity	Right to sell until maturity

Example: American put option

Example

The owner of an American put option on

- Underlying asset: 20 shares of Apple with
- Strike price is $K = USD\ 500$,
- Maturity is $T = 1$ year,

has time until 1 year to sell 20 shares of Apple at price USD 500 each.

at any time t $\leq$ 1 Year:

	Owner of the Eu put option	Seller of the Eu put option
The owner exercises the option	-20 AAPL +USD 10,000	+20 AAPL -USD 10,000
The owner does not exercises the option	nothing	nothing

An option can be exercised at most once

Options are derivatives

Just like forward/futures contracts, option contracts are derivative securities that ...

1. exist on many types of underlying assets: individual stocks, stock indices, bonds, commodities, currencies, etc.

2. may be settled physically or in cash depending on the option contract specifics;

3. may be traded over-the-counter (OTC) markets or on exchanges;

Example

- Euro Stoxx 50 options
  - Traded on Eurex
  - Calls and puts
  - European style
  - Underlying asset: Euro Stoxx 50 index
  - One maturity date per month
  - Many strike prices
  - Cash settlement, with daily settlement to prevent counterparty risk

When is it worth exercising a call option?

Example

You own an European call option on 1 share of Apple with
Strike price is K = USD 500,
Maturity is T = 1 year,
In 1 year, you have the right to buy one shares of Apple at price USD 500.

Scenario 1: $S_{1Y} = USD\ 400$

Payoff from not exercising the call option	0
Payoff from exercising the call option	$400 - 500 < 0$

Scenario 2: $S_{1Y} = USD\ 600$

Payoff from not exercising the call option	0
Payoff from exercising the call option	$600 - 500 > 0$

When is it worth exercising an call option?

General rule: The owner of an European call option with maturity T and strike price K will exercise the option if and only if

$$S_T \geq K$$

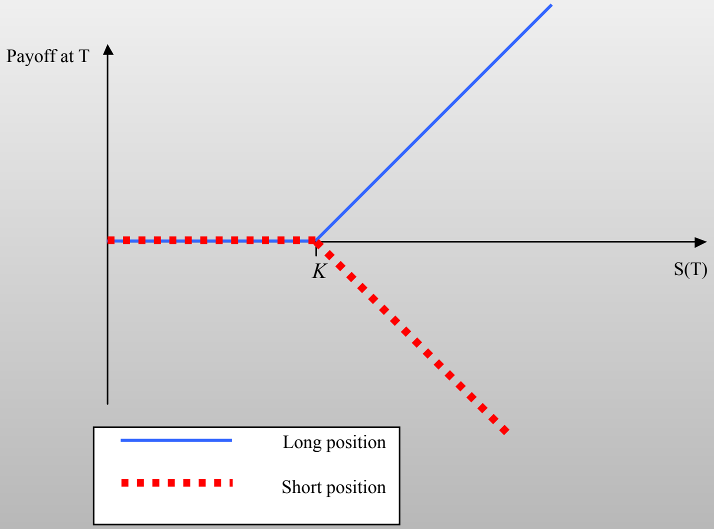
At time T, the payoff from a long position in this option is

$$max\{S_T - K, 0\}$$

At time T, the payoff from a short position in this option is

$$-max\{S_T - K, 0\} = min\{K - S_T, 0\}$$

Payoffs from a call option



Difference between a Call option and forward/futures

- long position on a forward contract on 1 Apple share, maturity T and Forward price $F_{0,T} = 400USD$
- long position on a European Call option on 1 Apple share, maturity T and Strike price $K = 400USD$

What is the difference?

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- long position on a European Call option on 1 Apple share, maturity T and Strike price $K = 400 USD$

What is the difference?

- Freedom of choice:
  - Long forward: at T you MUST buy 1 share of Apple and pay USD 400 for it.
  - Long Call: at T you CHOOSE WHETHER to buy or NOT 1 share of Apple and pay USD 400 for it.

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- Freedom of choice:
  - Long forward: at T you MUST buy 1 share of Apple and pay USD 400 for it.
  - Long Call: at T you CHOOSE WHETHER to buy or NOT 1 share of Apple and pay USD 400 for it.
- Freedom has a price:
  - At t=0, when you enter the long forward contract, you pay nothing.
  - At t=0, to enter the long Call option, you have to pay a strictly positive price C to the party whose entering the short call option.
- Counterparty risk:

  Which side(s) o is subject to counterparty risk?

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  Which side(s) o is subject to counterparty risk?

  The long position only for a call option, Both the long an short position in a Futures contract

  Which side(s) of an option contract is subject to margin requirements?

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  Which side(s) o is subject to counterparty risk?

  The long position only for a call option, Both the long an short position in a Futures contract

  Which side(s) of an option contract is subject to margin requirements?

  The short position only in a call option, both sides in a Future contract

Difference between a Call option and forward/futures

- short position on a forward contract on 1 Apple share, maturity T and Forward price $F_{0,T} = 400USD$
- short position on a European Call option on 1 Apple share, maturity T and Strike price $K = 400USD$

What is the difference?

Difference between a Call option and forward/futures

- short position on a forward contract on 1 Apple share, maturity T and Forward price $F_{0,T} = 400USD$
- short position on a European Call option on 1 Apple share, maturity T and Strike price $K = 400USD$

What is the difference?

- No freedom of choice:
  - Short forward: at T you MUST sell 1 share of Apple for USD 400.
  - Short Call: at T you MUST sell 1 share of Apple for USD 400 for it, **IF AND ONLY IF** the party who owns the long position in the Call exercise his/her right to buy the Apple share.

Difference between a Call option and forward/futures

- short position on a forward contract on 1 Apple share, maturity T and Forward price $F_{0,T} = 400USD$
- short position on a European Call option on 1 Apple share, maturity T and Strike price $K = 400USD$

What is the difference?

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- Freedom has a price:
  - At t=0, when you enter the short forward contract, you pay nothing
  - At t=0, to when you enter the short Call option, you are paid a strictly positive price C by the party whose entering the long call option.
- Counterparty risk:

  The long position only for a call option, faces counterparty risk, hence margin account are required only for short position. Both the long an short position in a Futures contract fces counterparty risk and are required to set margin accounts

When is it worth exercising a Put option?

Example

You own an European put option

- Underlying asset: 1 share of Apple with
- Strike price is K = USD 500,
- Maturity is T = 1 year,

In 1 year, you have the right to sell one shares of Apple at price USD 500.

Scenario 1: $S_{1Y} = USD\,400$

Payoff from not exercising the call option	0
Payoff from exercising the call option	$500 - 400 > 0$

Scenario 2: $S_{1Y} = USD\,600$

Payoff from not exercising the call option	0
Payoff from exercising the call option	$500 - 600 < 0$

When is it worth exercising an put option?

General rule: The owner of an European put option with maturity T and strike price K will exercise the option if and only if

$$S_T \leq K$$

At time T, the payoff from a long position in this option is

$$max\{K - S_T, 0\}$$

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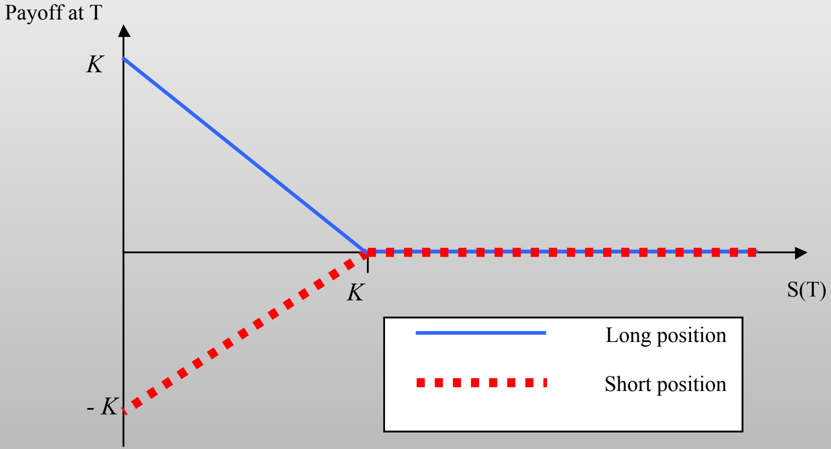
$$-max\{K - S_T, 0\} = min\{S_T - K, 0\}$$

Payoffs from a put option



Exercise or not?

At time $t = 0$ you pay $C = Eu$ 10 to buy an European call option on ABC stock with strike price $K = Eu$ 50 and maturity T.
At time T the spot price of an ABC stock is $S_T = Eu$ 55.

Do you exercise the option or not?

Exercise or not?

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At time T the spot price of an ABC stock is $S_T = Eu\ 55$.

Do you exercise the option or not?

	$t = 0$	$t = T$
Payoff from not exercising the call option	-10	0
Payoff from exercising the call option	-10	$55 - 50 > 0$

The 'intrinsic value of an option'

Definition

The intrinsic value of this option at date $t < T$ is what you could have gained if you had the right to exercise the option at t.

- The intrinsic value of a call option is $max\{S_t - K, 0\}$.
- The intrinsic value of a put option is $max\{K - S_t, 0\}$.

At some time $t \leq T$,

- An option is said to be in the money if its intrinsic value is strictly positive.
- An option is said to be at the money if $K = S_t$.
- An option is said to be out of the money if its intrinsic value is 0 and exercising the option at t would generate a loss.

Speculating with options

XYZ stock is selling for $S_0 =$ Euros 100. You believe that the stock price will go up.

1. Suppose you buy the stock:
   What is your holding period return if $S_{1mo} =$ Euros 105 in one month?

Speculating with options

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   $HPR_{1mo} = \frac{S_{1mo} - S_0}{S_0} = \frac{105-100}{100} = +5\%$

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   What if $S_{1mo} =$ Euros 95?

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   What if $S_{1mo} =$ Euros 95?
   $HPR_{1mo} = -5\%$
2. You buy a call option on XYZ stock with exercise price $K =$ Euros 100, expiring in one month, at price $C_0 =$ Euros 1.
   What is your return if $S_{1mo} =$ Euros 105?

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   What is your return if $S_{1mo} =$ Euros 105?
   $HPR_{1mo} = \frac{\max(S_{1mo} - K, 0) - C_0}{C_0} = \frac{(105 - 100) - 1}{1} = 400\%$ if $S_1 = 105$
   What is your return if $S_{1mo} =$ Euros 95?

Speculating with options

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   What is your return if $S_{1mo} =$ Euros 105?
   $HPR_{1mo} = \frac{\max(S_{1mo} - K, 0) - C_0}{C_0} = \frac{(105-100)-1}{1} = 400\%$ if $S_1 = 105$
   What is your return if $S_{1mo} =$ Euros 95?
   $HPR_{1mo} = \frac{\max(S_{1mo} - K, 0) - C_0}{C_0} = \frac{0-1}{1} = -100\%$
   $\Rightarrow$ Like futures, option returns are leveraged (levered) compared to spot position returns for the underlying asset.

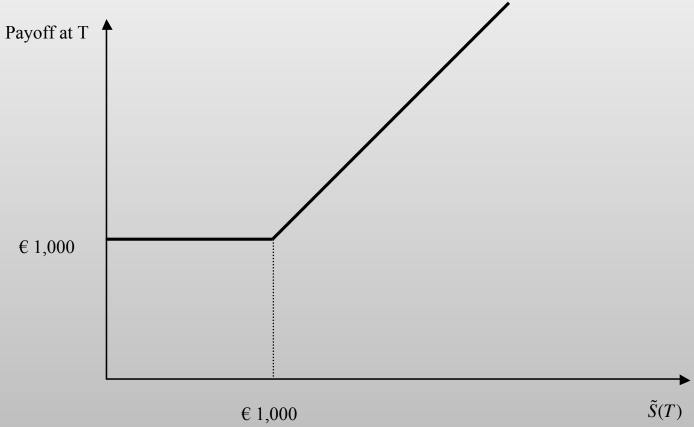
Portfolios of options: 1) guaranteed capital

Today you buy a stock index at $S_0 = Eu\ 1,000$. You will sell the index in T years at $\tilde{S}_T$. You would like to make a profit from an increase in the stock index price (i.e., when $\tilde{S}_T > S_0$). But you do not want to lose money from a decline in the price (i.e., when $\tilde{S}_T < S_0$).

	Time T	
	$S_T < 1,000$	$S_T \geq 1,000$
1 stock index	S_T	S_T
Long 1 put with K=1,000	$1,000 - S_T$	0
	$1,000$	S_T

Hedging: guaranteed capital

One underlying asset + 1 put option



Portfolios of options: Put-Call Parity

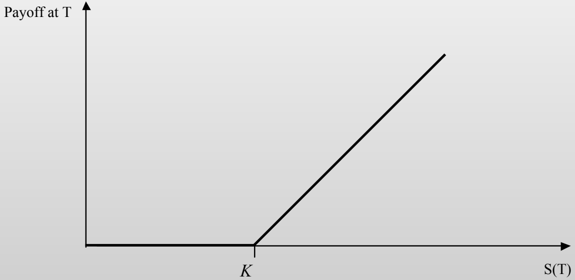
Consider a portfolio that contains

- One long position in a put option with maturity T and strike price K.
- One underlying asset.
- A borrowing of $\frac{K}{(1+r_T)^T}$ until T.

Let P be the price of the put option

	Today	Time T	
		$S_T < K$	$S_T \geq K$
Long 1 put option	$-P$	$K - S_T$	0
Long 1 underlying asset	$-S_0$	S_T	S_T
Borrowing	$\frac{K}{(1+r_T)^T}$	$-K$	$-K$
Total	$\frac{K}{(1+r_T)^T} - P - S_0$	0	$S_T - K$

1 underlying +1 put option + borrowing for $\frac{K}{(1+r_T)^T}$



Let C be the no arbitrage price of an European call option with maturity T and strike price K. Then,

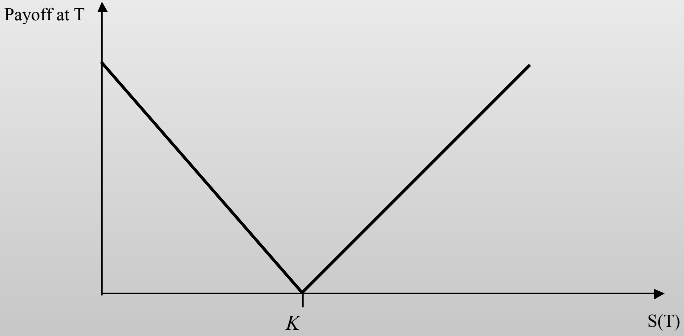
$$C = -\frac{K}{(1+r_T)^T} + P + S_0$$

Portfolios of options: Bottom Straddle

You expect the price of the stock to change in one week but you do not know whether the price will rise or decrease. What is your trading strategy?

	Today	Time T	
		$S_T < K$	$S_T \geq K$
Long 1 put option K, $T = 1$ week	$-P$	$K - S_T$	0
Long 1 call option K, $T = 1$ week	$-C$	0	$S_T - K$
Total	$-P - C$	$K - S_T$	$S_T - K$

1 put option +1 call option with the same strike price K



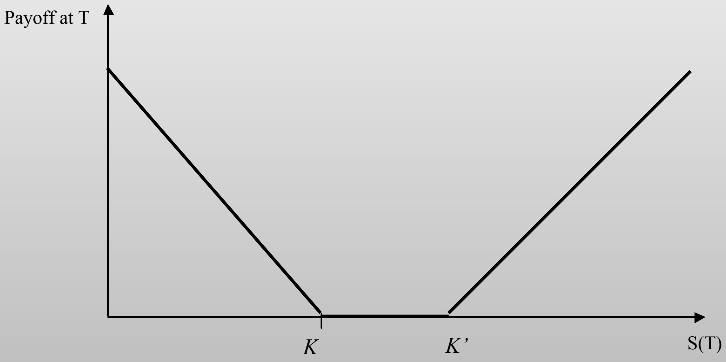
Portfolios of options: Bottom vertical combination

You expect that at time T the price of the stock will change dramatically, but you do not know whether the price will increase or decrease. What is your trading strategy?

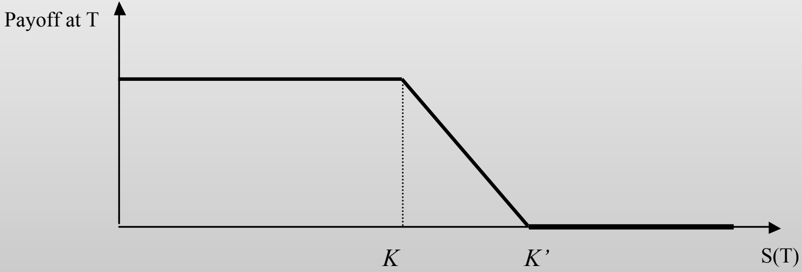
	Today	Time T		
		$S_T < K$	$K \leq S_T < K'$	$S_T \geq K'$
Long 1 put option K, T	$-P$	$K - S_T$	0	0
Long 1 call option $K' > K$, T	$-C'$	0	0	$S_T - K'$
Total	$-P - C'$	$K - S_T$	0	$S_T - K'$

Portfolios of options: Bottom vertical combination

One put option with strike price K, + one call option with strike price $K' > K$

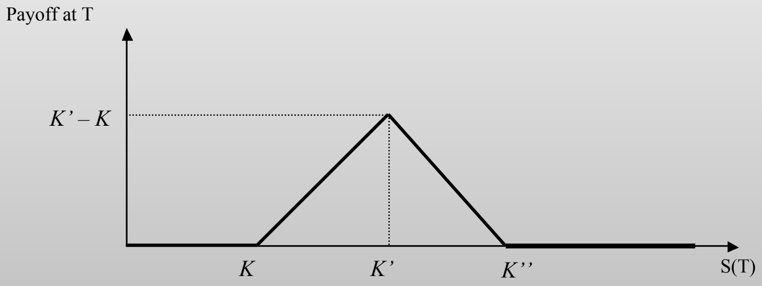


Portfolios of options: Bearish spread



How can you obtain this profile of payoff?

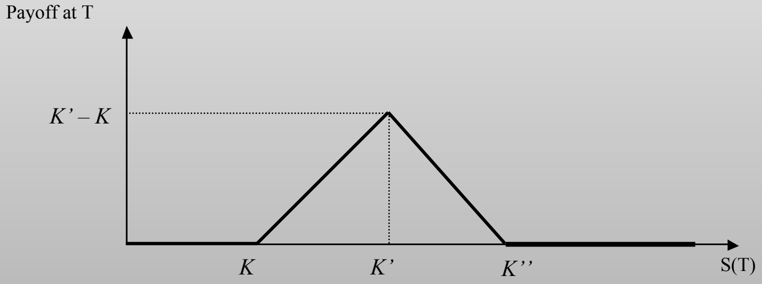
You expect that at time T the price of the stock will not change. What is your trading strategy?



Portfolios of options: 5) Butterfly spread

Let $K < K' < K''$ and $K'' = 2K' - K$

	Today	Time T			
		$S_T < K$	$K \leq S_T < K'$	$K' \leq S_T < K''$	$S_T \geq K''$
Long 1 Call option, K	$-C$	0	$S_T - K$	$S_T - K$	$S_T - K$
Short 2 Call options, K'	$2C'$	0	0	$2(K' - S_T)$	$2(K' - S_T)$
Long 1 Call option, K''	$-C''$	0	0	0	$S_T - K''$
Total	$2C'$-C''-C	0	$S_T - K$	$2K' - K - S_T$	0



Option pricing

- How to determine options' price (premium) at t=0: C_0, P_0?
- Unlike with forwards/futures, no simple formula. Instead we'll study:

1. The qualitative determinants of option prices
2. A relation between call price and put price (put-call parity)
3. Upper and lower bounds on option prices (arbitrage bounds)
4. An option pricing method relying on additional assumptions (binomial model)

Factors influencing option prices

... holding all else constant		C_0	P_0
Price of the underlying asset	$S_0 \nearrow$	$\nearrow$	$\searrow$
Strike (or Exercise) price	K (or X) $\nearrow$	$\searrow$	$\nearrow$
Time until maturity	$T \nearrow$	$\nearrow$	*
Volatility of underlying asset's price	$\sigma \nearrow$	$\nearrow$	$\nearrow$
Risk-free rate	$r_f \nearrow$	$\nearrow$	$\searrow$

* Effect of maturity is (+) for calls, but ambiguous for puts.

Two of these dimensions (namely, K (X) $\nearrow$, or $T \nearrow$) can be seen rather easily (whereas changes in other dimensions are less obvious to trace) in one day's option prices (see Tesla Inc. Options:).

Options pricing

1. We derive the price of a call option (C) and we will use the put-call parity to compute the price (P) of a put option.
2. We will find a lower bound for C.
3. We will find an upper bound for C.
4. We will find the no arbitrage level of C, introducing some assumptions on the probability distribution of $\tilde{S}_T$.

Lower bound for a call option price C

Consider the put-call parity:

$$C = -\frac{K}{(1+r_T)^T} + P + S_0 \Rightarrow P = \frac{K}{(1+r_T)^T} + C - S_0$$

that is a put option can be replicated with a call option, investment of $\frac{K}{(1+r_T)^T}$ in ZCB with matuirty T and short selling the underlying asset.

Because $P > 0$ we must have:

$$C > \max\left\{S_0 - \frac{K}{(1+r_T)^T}, 0\right\}$$

Example

Today, 1 share of ABC trades for $S_0 = Eu\ 60$; the 1 year interest rate is $r_{1Y} = 2\%$; an European call option on 1 share of ABC, with maturity $T = 1$ year and strike price $K = Eu\ 50$ trades for $C = Eu\ 5$.

Identify an arbitrage strategy.

Example

Today, 1 share of ABC trades for $S_0 = Eu\ 60$; the 1 year interest rate is $r_{1Y} = 2\%$; an European call option on 1 share of ABC, with maturity $T = 1$ year and strike price $K = Eu\ 50$ trades for $C = Eu\ 5$.

Identify an arbitrage strategy.

$$5 = C < S_0 - \frac{K}{(1 + r_T)^T} = 60 - \frac{50}{1.02} = 10.98$$

Trade	Today	Time T	
		$S_T < 50$	$S_T \geq 50$
Short sell 1 ABC	60	$-S_T$	$-S_T$
Long 1 Call option	-5	0	$S_T - 50$
Lend $\frac{50}{1.02}$	$-\frac{50}{1.02}$	50	50
Total	5.98	$50 - S_T > 0$	0

Upper bound for a call option price C

Consider the following portfolio

Trade	Today	Time T	
		$S_T < K$	$S_T \geq K$
Buy 1 underlying	$-S_0$	S_T	S_T
Short 1 Call option	C	0	$K - S_T$
Total	$C - S_0$	S_T	K

At time T this portfolio produces only strictly positive cash flows, Thus, by no arbitrage, it must result:

$$C < S_0$$

Example

Today, 1 share of ABC trades for $S_0 = Eu\ 60$; the 1 year interest rate is $r_{1Y} = 2\%$; an European call option on 1 share of ABC, with maturity $T = 1$ year and strike price $K = Eu\ 4$ trades for $C = Eu\ 61$.

Identify an arbitrage strategy.

Example

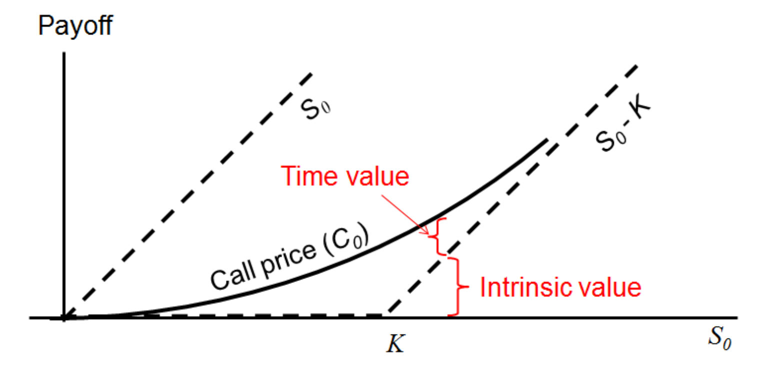
Today, 1 share of ABC trades for $S_0 = Eu\ 60$; the 1 year interest rate is $r_{1Y} = 2\%$; an European call option on 1 share of ABC, with maturity $T = 1$ year and strike price $K = Eu\ 4$ trades for $C = Eu\ 61$.

Identify an arbitrage strategy.

Trade	Today	Time T	
		$S_T < 4$	$S_T \geq 4$
Buy 1 ABC	-60	S_T	S_T
Short 1 Call option	61	0	$4 - S_T$
Total	1	S_T	4

Arbitrage bounds

- $S_0 - K$ is the intrinsic value of the call: it is the payoff you would get by exercising the option at date 0
- The difference between the price of a call and its intrinsic value is the time value of the option



(1) intrinsic value = what you would get if you exercised (not possible because European)
(2) call price = what you get because you have to wait

Upper and lower bounds for a call option price C and for put option price P

- Call option:

$$\max\left\{S_0 - \frac{K}{(1+r_T)^T}, 0\right\} \leq C \leq S_0$$

- Put option: from the put call parity

$$P = C - S_0 + \frac{K}{(1+r_T)^T}$$

Hence, we have

$$\max\left\{\frac{K}{(1+r_T)^T} - S_0, 0\right\} \leq P \leq \frac{K}{(1+r_T)^T}$$

Some exercises

1. Today, 1 share of ABC trades for $S_0 = Eu\ 60$; the 1 year interest rate is $r_{1Y} = 2\%$; an European put option on 1 share of ABC, with maturity $T = 1$ year and strike price $K = Eu\ 40$ trades for $P = Eu\ 40$. **Identify an arbitrage strategy.** Short the put option and invest P in a 1-year ZCB.
2. Prove that it is never optimal to exercise an American call option before maturity (provided the underlying asset pays no cash-flows before the maturity of the call). It is better to sell the option rather than exercise it.
3. Prove that it can be optimal to exercise an American put option before maturity.