Small open economies

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[International Macroeconomics]

Outline

An endowment economy

SOE : a definition

- A country for which international trade / and or international capital flows are an important part of the economy
- In relation to the rest of the world the country is small and has little influence by itself on world-level variables
- Some prices are taken as given : prices of goods (terms of trade), the international interest rate, commodity prices.
- Akin to an atomic agent in a perfectly competitive world : demand at price p infinite
- Often used as a conduit for partial equilibrium analysis as some variables (terms of trade, interest rate) are held exogenous to the system.

A setup of a standard SOE model : an endowment economy

- Infinitely-lived representative agent maximizing expected utility of consumption (c) receiving some stochastic endowment (y) with an access to internationals debt markets at an exogenous rate r.

$$\max_{\{c_t, d_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

$$d_t = (1 + r_t) d_{t-1} + c_t - y_t \tag{2}$$

$$\lim_{j \to \infty} \frac{d_{t+j}}{(1 + r_t)^j} \leq 0 \tag{3}$$

Short comments

- The interest rate is exogenous as is the supply / demand for the only consumption good
- Saving through the trade balance the only way to adjust intertemporal consumption / obtaining risk sharing
- There is no default
- And hence no financing constraints in the current period or in the future
- There are no prices, and no exchange rate issues.

Consumption dynamics

- Suppose $r_t = r$ and that $\beta(1+r) = 1$
- Then the Euler equation is

$$u'(c_t) = \beta(1+r)E_t u'(c_{t+1}) \tag{4}$$

- For quadratic preferences $u(c_t) = -\frac{1}{2}(c_t - \bar{c})^2$ this reduces to

$$c_t = E_t c_{t+1} \tag{5}$$

The Intertemporal Budget Constraint

- Iterating forward the budget constraint (2) at time t for j periods forward we can find the intertemporal resource constraint.

$$(1+r)\, d_{t-1} = \frac{d_{t+j}}{(1+r)^j} + \sum_{s=0}^{j} \frac{y_{t+s} - c_{t+s}}{(1+r)^s} \tag{6}$$

- Apply the conditional expectations at time t, take the limit $j \to \inf$ and use the transversality condition (3)

$$(1+r)\, d_{t-1} = E_t \sum_{s=0}^{\infty} \frac{y_{t+s} - c_{t+s}}{(1+r)^s} \tag{7}$$

- Interpretation : The initial net foreign debt position is equal to current and discounted future savings.
- With quadratic preferences, this can be rearranged as

$$rd_{t-1} + c_t = \frac{r}{1+r} E_t \sum_{s=0}^{\infty} \frac{y_{t+s}}{(1+r)^s} \tag{8}$$

Example : AR(1) endowment process

- Suppose y_t is an AR(1) process with $\rho < 1$ and ε i.i.d. :

$$y_t = \rho y_{t-1} + \varepsilon_t \tag{9}$$

- Then, consumption is :

$$c_t = \frac{r}{(1+r) - \rho} y_t - r d_{t-1} \tag{10}$$

- The trade balance $TB_t = y_t - c_t$:

$$TB_t = \frac{1-\rho}{(1+r) - \rho} y_t + r d_{t-1} \tag{11}$$

Example : AR(1) endowment process II

- The current account $CA_t = TB_t - rd_{t-1} = -(d_t - d_{t-1}) = y_t - y_t^p$:

$$CA_t = \frac{1-\rho}{(1+r)-\rho} y_t \tag{12}$$

- while the evolution of debt :

$$d_t = d_{t-1} - \frac{1-\rho}{(1+r)-\rho} y_t \tag{13}$$

A positive shock to production in the AR(1) model

Suppose that the shocks are not persistent ($\rho < 1$) (like a transitory income shock)

- Consumption increases less than output
- The Trade Balance and the Current Account improve (are procyclical)
- NFA increases (debt falls)

The shocks are persistent ($\rho = 1$) (a “permanent” income shock)

- Consumption increases one-to-one with output
- So the Trade Balance and the Current Account do not change
- Debt is constant

So it depends on the interplay between the “permanent” and current income.

How does it square with data ?

	Emerging Economies	Developed Markets
$\sigma(Y)$	2.74	1.34
$\sigma(\triangle Y)$	1.87	.95
$\rho(Y)$	.76	.75
$\sigma(C)/\sigma(Y)$	1.45	.94
$\sigma(I)/\sigma(Y)$	3.91	3.41
$\rho(TB/Y, Y)$	-51	-.17
$\rho(C, Y)$	.72	.66

Notes: Averages for 13 emerging and 13 developed economies. Quarterly data detrended using Hodrick-Prescott filter. The emerging economies are: Argentina, Brazil, Ecuador, Israel, Korea, Malaysia, Mexico, Peru, Philippines, Slovak Republic, South Africa, Thailand, Turkey. Developed countries: Australia, Austria, Belgium, Canada, Denmark, Finland, Netherlands, New Zeleand, Norway, Portugal, Spain, Sweden, Switzerland. Source: Aguiar and Gopinath (2007)

Can the AR(2) model deliver data-consistent behavior?

Suppose that the shocks are AR(2) instead

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \varepsilon_t \tag{14}$$

and the process is mean-reverting.

If $\rho_1 > 1$, $\rho_2 < 0$ and $\rho_2 > (1+r)(1-\rho_1)$ then a positive endowment shock causes that

- Permanent income increases more than current income
- The Trade Balance and the Current Account is countercyclical

Imposing stationarity (Schmitt-Grohe and Uribe, 2003)

The endogenous variables of the model with quadratic preferences are random walks. The following has been used to force stationarity :

- Endogenous discount factors : $\beta(c_t)$, with $\beta_c < 0$
  - In the steady state, $\beta(c)(1+r) = 1$
  - Example (Mendoza, 1991) : $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and $\beta(c_t) = (1+c_t)^{\phi}$
- Debt-elastic interest rate premium : $r(d_t)$ with $r_d > 0$
  - Example : $r(d_t) = r + \phi(e^{d-\bar{d}} - 1)$
- Convex portfolio adjustment costs : a penalty from deviating from desired steady-state debt level
  - Example : $d_t = (1+r_t)\, d_{t-1} + c_t - y_t + \frac{\phi}{2}(d_t - \bar{d})^2$

Other cases

Simpler examples :

- $y_t = \bar{y} + \varepsilon$?
- $y_t = (1+g)\, y_{t-1}$?

More involved : nonstationarity

- $\Delta y_t = \rho \Delta y_{t-1} + \varepsilon_t$
- Income now permanently higher after a positive shock : TB, CB countercyclical, as in data.

Perhaps modeling production will i) make the models more realistic and ii) allow for solving our issues without making a stand on the endowment processes ?

A production economy

Adding capital

- In reality, agents can affect the future output level by investing in productive capital domestically
- This introduces a possibility of intertemporal saving domestically as well : and the current account is now NOT equal to saving but $S = I + CA$ or $CA = S - I$.
- Perhaps we will be able to match the data better ?
- Allows for discussing growth and the behavior of the trade balance and the current account
- And thinking about the role of productivity shocks, etc.

Redefining the CA and TB

$$CA_t = rb_{t-1} + y_t - c_t - i_t \tag{15}$$

$$TB_t = y_t - c_t - i_t \tag{16}$$

while

$$s_t = rb_{t-1} + y_t - c_t \tag{17}$$

Then

$$CA_t = IB_t + TB_t \tag{18}$$

$$CA_t = s_t - i_t \tag{19}$$

A non-stochastic economy with capital

$$\max_{\{c_t, i_t, b_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{20}$$

subject to

$$b_t = (1+r) b_{t-1} + y_t - c_t - i_t \tag{21}$$

$$y_t = A_t F(k_t) \tag{22}$$

$$k_{t+1} = k_t + i_t \tag{23}$$

$$\lim_{j \to \infty} \frac{b_{t+j}}{(1+r)^j} \geq 0 \tag{24}$$

Steps to a solution

Substitute (22) and, after eliminating i_t from (23) into (21) and write the problem (20) as a Lagrangian with $\lambda_t{}_{t=0}^{\infty}$ as multipliers. The FOCs become

$$\lambda_t = \lambda_{t+1}\beta\left(1+r\right) \tag{25}$$

$$\lambda_t = \lambda_{t+1}\beta\left(A_{t+1}F_k + 1\right) \tag{26}$$

$$u'\left(c_t\right) + \lambda_t = 0 \tag{27}$$

So, if $\beta\left(1+r\right) = 1$ then $c_t = c_{t+1}$. Also, we get the equilibrium conditions

$$r = A_{t+1}F_k \tag{28}$$

$$c_t = rb_{t-1} + \frac{r}{1+r}\sum_{s=0}^{\infty}\frac{A_{t+s}F\left(k_{t+s}\right) - \left(k_{t+1+s} - k_{t+s}\right)}{\left(1+r\right)^s} \tag{29}$$

A steady state

There is no depreciation - and in the steady state investment will be zero. The steady state is determined by $\bar{A}$ and $\bar{r}$.

$$c_{ss} = \bar{c} = r\bar{b} + \bar{y},\ i_{ss} = 0,\ y_{ss} = \bar{A}F\left(\bar{k}\right),\ b_{ss} = \frac{\bar{c}-\bar{y}}{r},\ k_{ss} = F_k^{-1}\left(\frac{r}{\bar{A}}\right),$$
$$TB_{ss} = -rb_{ss},\ CA_{ss} = 0$$

Comparative statics : a permanent productivity shock

Suppose that suddendly in period 0, $\widetilde{A} > \bar{A}$ forever

- At the new steady state, since r is constant, $\widetilde{A} > \bar{A}$ and $F_k > 0$, $\widetilde{k_{ss}} > k_{ss}$ so capital stock should increase, and investment at time 0 jumps (capital fixed in the short run) so that the steady state level is achieved immediately in the next period (no frictions).
- Consumption increases permanently.

$$c_0 = r\bar{b} + \frac{r}{1+r}\left[\tilde{A}F\left(\bar{k}\right) - \left(\tilde{k} - \bar{k}\right)\right] + \frac{\tilde{A}F\left(\tilde{k}\right)}{(1+r)} \tag{30}$$

$$TB_0 = -r\bar{b} - \frac{1}{1+r}\left[\tilde{A}F\left(\tilde{k}\right) - \tilde{A}F\left(\bar{k}\right) + \left(\tilde{k} - \bar{k}\right)\right] \tag{31}$$

Interpretation

$$c_0 = r\bar{b} + \tilde{A}F\left(\bar{k}\right) + \frac{r}{1+r}\left[\frac{\tilde{A}\left(F\left(\tilde{k}\right) - F\left(\bar{k}\right)\right)}{r} - \left(\tilde{k} - \bar{k}\right)\right]$$

where $\left[\frac{\tilde{A}(F(\tilde{k})-F(\bar{k}))}{r} - \left(\tilde{k} - \bar{k}\right)\right] > 0$ because $\frac{\tilde{A}(F(\tilde{k})-F(\bar{k}))}{(\tilde{k}-\bar{k})} > r$ (the average return is higher than marginal return)

$$s_0 = -\frac{r}{1+r}\left[\frac{\tilde{A}\left(F\left(\tilde{k}\right) - F\left(\bar{k}\right)\right)}{r} - \left(\tilde{k} - \bar{k}\right)\right]$$

$$TB_0 = -r\bar{b} - \frac{r}{1+r}\left[\frac{\tilde{A}\left(F\left(\tilde{k}\right) - F\left(\bar{k}\right)\right)}{r} - \left(\tilde{k} - \bar{k}\right)\right] - \left(\tilde{k} - \bar{k}\right) < -r\bar{b}$$

Current consumption increases more than current output : TB countercyclical.

Comparative statics : a temporary productivity shock

Suppose that suddenly in period 0, $\widetilde{A} > \bar{A}$ for one period only.

- At the new steady state, since r and is constant $\bar{A}$ are the same, the capital stock is the same, so there is no investment.
- The output gain is partly consumed, and partly invested in foreign assets :

$$c_0 = c_{-1} + \frac{r}{1+r}\left[\tilde{A} - \bar{A}\right] F\left(\bar{k}\right) \tag{32}$$

$$s_0 = TB_0 - TB_{-1} = \frac{1}{1+r}\left[\tilde{A} - \bar{A}\right] F\left(\bar{k}\right) \tag{33}$$

Comparative statics : on the persistence of shocks

The more persistent the shocks, the more likely that the trade balance deteriorates.

- If there are adjustment costs to capital investment or international investment frictions, the initial effect can be attenuated and partially offset (investment will increase slowly)
- So are the shocks persistent ? Or temporary but because of different frictions they are as if persistent ?
- What are the adjustment costs ?

Quantitative calibration !

A standard RBC SOE

A standard SOE RBC model (Mendoza 1991)

$$\max_{\{c_t, i_t, b_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{34}$$

$$d_t = (1 + r_t) d_{t-1} - y_t + c_t + i_t + \Phi(k_{t+1} - k_t) \tag{35}$$

$$\ln A_{t+1} = \rho \ln(A_t) + \varepsilon_{t+1} \tag{36}$$

$$k_{t+1} = (1 - \delta) k_t + i_t \tag{37}$$

while

$u(c_t) = \frac{\left(c - \frac{h^{\omega}}{\omega}\right)^{1-\gamma} - 1}{1-\gamma}$, $y = Ak^{\alpha} h^{1-\alpha}$, $r_t = r + \psi\left(e^{d_t - \bar{d}} - 1\right)$,

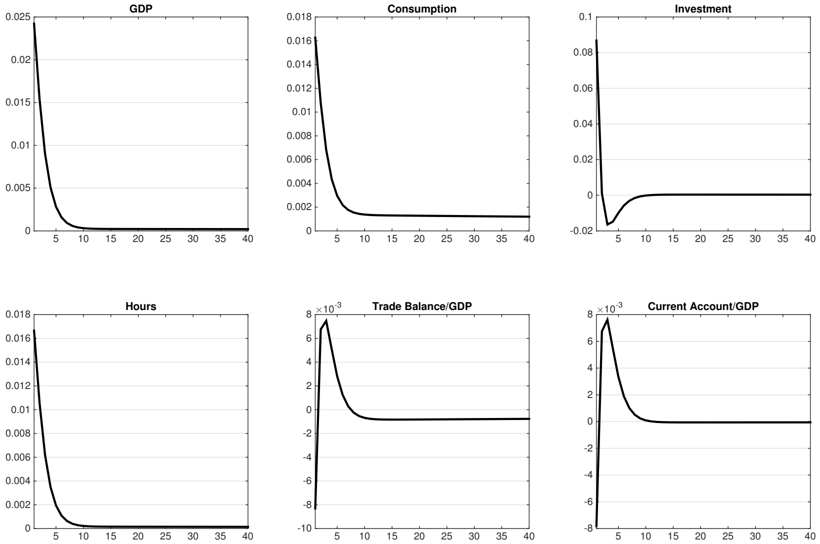
$\Phi(k_{t+1} - k_t) = \phi \frac{(k_{t+1} - k_t)^2}{2}$

Calibration

Parameter	Value	Description
γ	2	Inter-temporal elasticity of substitution
ω	1.45	Labor Supply Elasticity
ψ_2	.000742	Discount Factor Parameter
α	.32	Capital Share
ϕ	0.028	Adjustment Cost
β	0.96	Real Interest Rate
δ	.1	Depreciation Rate
ρ	.42	Persistence TFP shock
σ_ϵ	0.0129	Volatility Innovations TFP
$\bar{d}$	.7442	Debt

Notes: Source: Mendoza (1991)

Impulse response functions

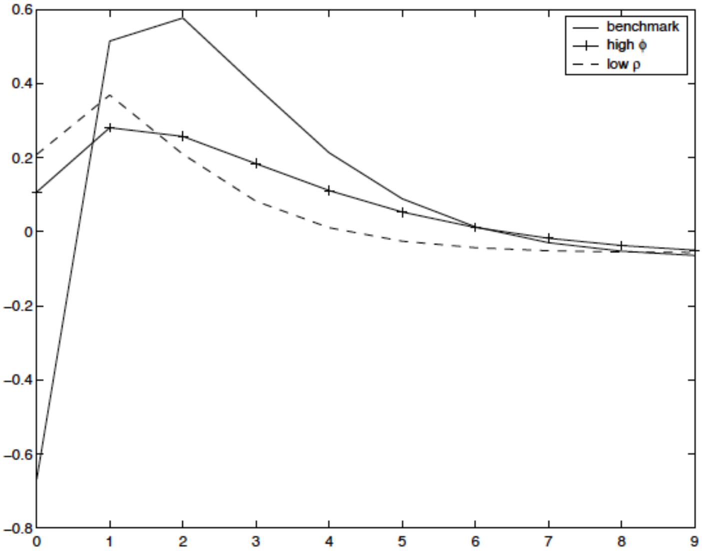


Fit of the model to Canada

	Canada			Model		
	Volatility	σ_x/σ_y	$\rho_{x,y}$	Volatility	σ_x/σ_y	$\rho_{x,y}$
y	2.8	1	1	3.08	1	1
c	2.5	0.89	0.59	2.71	0.88	0.84
i	9.8	3.50	0.64	9.04	2.94	0.66
h	2	0.71	0.8	2.12	0.69	1
$\frac{tb}{y}$	1.9	0.68	-0.13	1.78	0.58	-0.04

Source : Calculations by Jose Lopez and Mendoza (1991)

Adjustment costs, shock persistence and TB/Y reaction to shocks



Differences in business cycles

Business cycle differences among SOEs (Aguiar and Gopinath, 2007)

- There is in general a different pattern of business cycle properties between high/middle income countries (i.e., Canada and Mexico)
- Groupings "developed" vs. "emerging" countries based on per-capita income levels.
- But for many countries the distinction is blurred : Portugal - Spain - Greece - Chile or South Korea ?
- Some other classifications based on Institutional Investor rankings (>73) ; OECD membership.

Data on the different business cycle properties

	Emerging Economies	Developed Markets
$\sigma(Y)$	2.74	1.34
$\sigma(\triangle Y)$	1.87	.95
$\rho(Y)$	.76	.75
$\sigma(C)/\sigma(Y)$	1.45	.94
$\sigma(I)/\sigma(Y)$	3.91	3.41
$\rho(TB/Y, Y)$	-51	-.17
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Notes: Averages for 13 emerging and 13 developed economies. Quarterly data detrended using Hodrick-Prescott filter. The emerging economies are: Argentina, Brazil, Ecuador, Israel, Korea, Malaysia, Mexico, Peru, Philippines, Slovak Republic, South Africa, Thailand, Turkey. Developed countries: Australia, Austria, Belgium, Canada, Denmark, Finland, Netherlands, New Zeleand, Norway, Portugal, Spain, Sweden, Switzerland. Source: Aguiar and Gopinath (2007)

Aguiar and Gopinath (2007)

- Perhaps the shocks to the trend of productivity can explain the differences in the business cycle properties between the different sets of countries
- Developed countries : are shocks rather transitory ?
- Emerging countries : are shocks rather permanent ?
- How to discern between transitory and permanent shocks ?

Aguiar and Gopinath (2007) : model I

- Utility

$$u(C_t) = \frac{\left[C_t^{\gamma}(1-L_t)^{1-\gamma}\right]^{1-\sigma}}{1-\sigma}$$

- Production

$$Y_t = e^{z_t} K_t^{1-\alpha} (\Gamma_t L_t)^{\alpha}$$

where $\Gamma_t = \prod_{s=0}^{t} e^{g_t}$, $z_t = \rho_z z_{t-1} + \varepsilon_t^z$, $g_t = (1-\rho_g)\mu_g + \rho_g g_{t-1} + \varepsilon_t^g$

- Budget constraint

$$C_t + K_{t+1} = Y_t + (1-\delta) K_t - \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - e^{\mu_g}\right)^2 K_t - D_t + q_t D_{t+1}$$

Aguiar and Gopinath (2007) : model (II)

- A debt-elastic interest rate :

$$\frac{1}{q_t} = 1 + r_t = 1 + r^* + \psi \left[\exp\left(\frac{D_{t+1}}{\Gamma_t} - \bar{d} \right) - 1 \right]$$

- To force stationarity, one needs to detrend variables. So, for example let $\hat{x} = x_t / \Gamma_{t-1}$.

The recursive problem

The problem can be written then recursively, as

$$V = \max_{\{\hat{C}, L, \hat{K}', \hat{B}'\}} \frac{\left[\hat{C}^{\gamma}(1-L)^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} + \beta e^{g\gamma(1-\sigma)} E V'$$

subject to

$$\hat{C} + e^{g}\hat{K}' = \hat{Y} + (1-\delta)\hat{K} - \frac{\phi}{2}\left(e^{g}\frac{\hat{K}'}{\hat{K}} - e^{\mu_g}\right)^{2}\hat{K} - \hat{D} + e^{g} q \hat{D}'$$

The steady state (no hats)

$$\beta\left(1+r\right)e^{\mu_g[\gamma(1-\sigma)-1]}\left[\left(1-\alpha\right)\frac{Y}{K}+\left(1-\delta\right)\right]=1$$

$$\beta\left(1+r\right)e^{\mu_g[\gamma(1-\sigma)-1]}=1$$

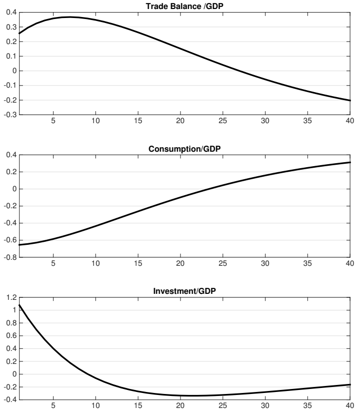
$$C+K\left(e^{\mu_g}+\delta-1\right)=Y+D\left(e^{g}q-1\right)$$

Fit of the model to Mexico

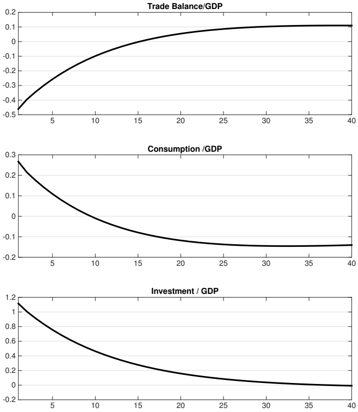
	Mexico	**Model**
σ_y	2.40	2.39
$\sigma_{\triangle(y)}$	1.52	1.72
σ_c/σ_y	1.26	1.27
σ_i/σ_y	4.15	2.59
$\sigma_{tb/y}/\sigma_y$	0.90	0.71
ρ_y	0.83	0.78
$\rho_{y,tb/y}$	-0.75	-0.66
$\rho_{c,y}$	0.92	0.95
$\rho_{c,y}$	0.91	0.92

Source : Calculations by Jose Lopez and Aguiar and Gopinath (2007)

Different shocks and impulse response functions



Transitory (z)



Permanent (g)

But what about other shocks ? Garcia-Cicco, Pancrazi and Uribe (2010)

- Small open economies may face many different shocks that can be analyzed.
  - Interest rate shocks
  - Preference shocks (changes in the marginal rate of intertemporal substitution)
  - Public spending shocks
- Which ones are really relevant then ?
- And, given the financial frictions that firms face in developing countries : are these important in explaining the differences in business cycles ?

Some specifics

- A debt-elastic interest rate allowing for shocks :

$$r_t = r^* + \psi \left[\exp\left(\frac{D_{t+1} - \bar{d}}{\bar{y}} \right) - 1 \right] + \exp(\mu - 1) - 1$$

with $ln\mu_{t+1} = \rho_\mu ln\mu_t + \varepsilon^{\mu}_{t+1}$

- Working-capital constraints in the spirit of Uribe and Yue (2006) :

$$M_t \geq \eta W_t h_t$$

(fraction η of the payroll has to be held as a noninterest-bearing asset)

Variance decomposition

Table 5.6 Variance Decomposition

Shock	g^Y	g^C	g^I	TB/Y
Nonstationary technology	2.6	1.1	0.2	0.1
Stationary technology	81.8	42.4	12.7	0.5
Preference	6.8	27.7	29.1	6.2
Interest rate	6.1	25.8	52.0	92.1
Spending	0.0	0.3	0.3	0.1
Measurement error	0.4	0.7	5.2	0.4

Note: Median of 1 million draws from the posterior distribution of the unconditional variance decomposition.

The role of financial constraints

- High debt-elasticity of interest rates at the steady state : 1.3 !

  $\rightarrow$ helps to explain the cyclicality of the trade balance, in particular its autocorrelation.

  $\rightarrow$ households cannot run persistent trade balances for too long...

- But, firm-level financial frictions are not that important : they do not introduce wedges that drive output growth variances.

Concluding observations

- Feldstein and Horioka (1980) puzzle : with perfect capital mobility marginal productivity of capital should be equalized across countries ; so there should be no correlation between domestic investment and domestic saving for SOEs. But this correlation is high !
  - But in Mendoza (1991), in response to a positive (and especially prolonged) shock to productivity saving may increase along with investment (and with desired magnitudes a in data).
- If you arrive at a negative asset position, the SOE may want to default on its debt... (Eric Mengus will deal with this)

Further reading

- Textbooks :
  Martin Uribe and Stephanie Schmitt-Grohe, Open Economy Macroeconomics, Princeton University Press, 2017
  Carlos A. Vegh, Open Economy Macroeconomics in Developing Countries, Chapter 1, MIT Press, 2013
- Aguiar M. and G. Gopinath, 2007. Emerging market business cycles : The cycle is the trend. Journal of Political Economy 115 : 69-102.
- Feldstein M. and C. Horioka, 1980. Domestic saving and international capital flows. Economic Journal 90 : 314-329.
- Mendoza E.G., 1991. Real business cycles in a small open economy. American Economic Review 81 : 797-818.
- Neumeyer P.A. ad F. Perri, 2005. Business cycles in emerging economies : the role of interest rates. Journal of Monetary Economics 52 : 345-380.
- Schmitt-Grohe S. and M. Uribe, 2003. Closing small open economy models. Journal of International Economics 61 : 163-185.