Real exchange rates and the Redux model

Tomasz Michalski[1]

[1] GREGHEC: HEC Paris

[International Macroeconomics]

Outline

The real exchange rate

The real exchange rate

The ratio of relative prices between two countries denominated in a common currency.

- Example : PPP-consumption based *real* exchange rate : the price of foreign relative to domestic goods and services.

$$q_t = s_t + p_t^\star - p_t \tag{1}$$

- Note the presence of the nominal exchange rate s_t.
- If PPP holds at all times, $q = 0$.
- According to the PPP theory : If $q > 0$ we say the currency is undervalued (and should appreciate). O/w it is overvalued (and should depreciate).

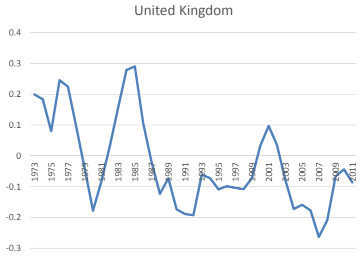
Failures of the PPP

The following may cause that the PPP will not hold either in the short or the long-run.

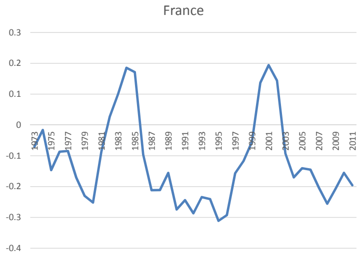
- Transport costs or trade barriers
- The presence of nontraded goods ; they are also included in the nontraded services
- Incomparable baskets of goods
- Measurement errors : a price level is an index
- Local tastes, different wealth levels (that affect consumption) etc.
- Short run : Stickiness of prices ?
- Incomplete pass-through
- Poorer countries have lower prices for nontradables (Balassa-Samuelson, Bhagwati)

PPP and the deviations from it.

Deviations from the PPP 1973-2011



British pound vs. U.S. dollar



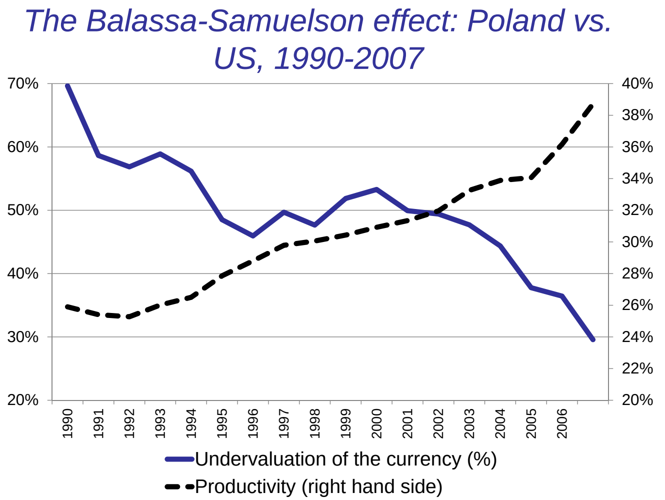
French franc (Euro) vs. U.S. dollar

Source: Penn World Tables

Facts about real exchange rates and the pass-through of exchange rate shocks (Burstein and Gopinath, 2015)

- RERs comove closely with nominal exchange rates at short and medium-term horizons (at both aggregate and product-levels).
- The half-life of deviations for G7 + CHE is long : from 1.6 years for Switzerland to 8.7 years for Canada.
- The scale of fluctuations of CPI-based RER for tradable and overall goods is similar, and a large fraction of RER fluctuations can be ascribed to relative prices of tradeables movements across countries.
- ERPT is incomplete, varies across countries and is lower into consumer (between 0.01 for ITA and 0.36 in FRA in the long run) than border prices (between 0.47 for CHE to 0.97 for FRA).
- There are large deviations from relative PPP for traded goods produced in a common location and sold in multiple locations (so, there is pricing to market).

Harrod-Balassa-Samuelson in action ?



Source: Penn World Tables

The Harrod-Balassa-Samuelson effect

- Suppose that there are tradeable and non-tradeable goods. The constant returns to scale neoclassical production functions are

$$\begin{aligned} Y_T &= A_T F\left(K_T, L_T\right) \\ Y_N &= p A_N F\left(K_N, L_N\right) \end{aligned}$$

  where p is relative price of nontradeables in terms of tradeables

- Assuming perfect competition and factor market clearing, we can derive that for example, for the Cobb-Douglas production functions

$$p = \frac{p_{NT}}{p_T} = \Gamma \frac{A_T}{A_{NT}} \left(\frac{w}{r}\right)^{\alpha-\gamma}$$

  where Γ is a constant.

Harrod-Balassa-Samuelson effect (static)

- What does this imply about the price levels between two countries ?
- Suppose that tradeables have a share $\beta \in (0,1)$ in consumption and that the price index is $P = p^{1-\beta}$
- The price ratio between the home and foreign country is then

$$\frac{P}{P^*} = \left(\frac{p}{p^*}\right)^{1-\beta} = \left(\frac{\frac{A_T}{A_{NT}}\left(\frac{w}{r}\right)^{\alpha-\gamma}}{\frac{A_T^*}{A_{NT}^*}\left(\frac{w^*}{r^*}\right)^{\alpha-\gamma}}\right)^{1-\beta}$$

- Assume that $A_{NT} = A_{NT}^*$. Then

$$\frac{P}{P^*} = \left(\frac{A_T}{A_T^*}\left(\frac{w}{w^*}\right)^{\alpha-\gamma}\left(\frac{r^*}{r}\right)^{\alpha-\gamma}\right)^{1-\beta}$$

- From a perspective of a developing country. So if : $A_T < A_T^*$, $w < w^*, r^* < r$ then $P < P^*$ and the price level in a developing country will be higher than that in a developed one.

Harrod-Balassa-Samuelson effect (dynamic)

$$\widehat{p} = \frac{\mu_{LN}}{\mu_{LT}}\widehat{A}_T - \widehat{A}_N \tag{2}$$

where $\widehat{p} = \frac{dp}{p}$ and $\mu_{LN} > \mu_{LT}$ are the labor intensities in each industry.

- Assuming that $\frac{\mu_{LN}}{\mu_{LT}} = \frac{\mu^*_{LN}}{\mu^*_{LT}}$ and rearranging we get

$$\widehat{P} - \widehat{P}^* = (1-\gamma)\left(\frac{\mu_{LN}}{\mu_{LT}}\left(\widehat{A}_T - \widehat{A}^*_T\right) - \left(\widehat{A}_N - \widehat{A}^*_N\right)\right) \tag{3}$$

- Suppose now that we are comparing a developed country where $\widehat{A}^*_T = \widehat{A}^*_N = 0$. Then

$$\widehat{P} - \widehat{P}^* = (1-\gamma)\left(\frac{\mu_{LN}}{\mu_{LT}}\widehat{A}_T - \widehat{A}_N\right) \tag{4}$$

- The price level in the developing country is growing relatively faster for “noninflationary” reasons !
- Nontradable price developments blur the picture.

The Redux model : layout and solution

The Redux model (Obstfeld and Rogoff, 1995)

- Goal : to analyze the behavior of important aggregates (consumption, price levels, current accounts...) in a rigorous dynamic model.
- A general-equilibrium two-country model with optimizing forward-looking agents with a role for monetary policy and the exchange rate.
- A micro-founded model with intertemporal choice and nominal rigidities.
- A building-block for the entire International Macroeconomics literature : a generalization of the old Mundell-Fleming-Dornbusch framework.
- "New Open Economy Macroeconomics"

A reminder : The old monetary model

Flexible prices, PPP and UIRP hold : Frenkel (1976), Mussa (1976), and Bilson (1978).

$$s = p_t - p_t^\star \tag{5}$$

$$m_t - p_t = l(i_t, y_t) \tag{6}$$

$$m_t^\star - p_t^\star = l(i_t^\star, y_t^\star) \tag{7}$$

then

$$s_t = (m_t - m_t^\star) - (l(i_t, y_t) - l(i_t^\star, y_t^\star)) \tag{8}$$

and linearizing

$$s_t = (m_t - m_t^\star) - c(y_t - y_t^\star) + b(i_t - i_t^\star) \tag{9}$$

To add realism, one can add assumptions : for example "sticky" prices a la Dornbusch : interesting dynamics.

How we shall proceed ?

- Assumptions
- The optimization problem
- A steady-state solution
- How to analyze a shock in this model ?
- Analysis of a permanent, unanticipated monetary shock

Supply [I]

- A continuum of monopolistic producers-consumers $z \in [0, 1]$ that produce each one variety of a differentiated good
- $[0, n]$ producers are located in the Home country (all symmetric).
- Labor is the only factor of production. No capital or investment.
- Pricing above cost, constant markup given the CES demand.
- Since there is monopolistic competition, the level of output in the decentralized economy will be lower than under a benevolent planner.

Agents [I]

- The utility of a Home agent j is

$$U_t^j = \sum_{s=t}^{\infty} \beta^{s-t}[\ln C_s^j + \chi \ln \frac{M_s^j}{P_s} - \frac{\kappa}{2} y_s(j)^2] \tag{10}$$

where

$$C_s^j = [\int_0^1 c_s^j(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{\theta-1}} \tag{11}$$

is the real consumption index (CES aggregator) at time s with $c^j(z)$ being the consumption of good z and $\theta > 1$ being the price elasticity of demand.
The home price level (with $p(z)$ being the home currency price of good z) is

$$P_s = [\int_0^1 p_s(z)^{1-\theta} dz]^{\frac{\theta}{1-\theta}} \tag{12}$$

Agents [II]

- The term $-\frac{\kappa}{2}y_s(j)^2$ captures the disutility from working.
- Compatible with a stylized disutility from effort function $-\phi l$ and the production function

$$y = Al^{\alpha} \tag{13}$$

- with $\alpha \in (0,1)$. Then

$$l = (y/A)^{\frac{1}{\alpha}} \tag{14}$$

- If $\alpha = \frac{1}{2}$ and $\kappa = 2\phi/A^{\frac{1}{\alpha}}$ we get $-\frac{\kappa}{2}y_s(j)^2$.
- The foreign price level is formulated analogously

$$P_s^{\star} = [\int_0^1 p^{\star}(z)^{1-\theta} dz]^{\frac{\theta}{1-\theta}} \tag{15}$$

The Purchasing Power Parity in the model

- Assume there are no impediments to trade (perfect arbitrage). Let S be the *nominal* exchange rate.
- For each good
$$p(z) = Sp^{\star}(z) \tag{16}$$
- Then
$$P_s = [\int_0^n p_s(z)^{1-\theta} dz + \int_n^1 S_s p_s^{\star}(z)^{1-\theta} dz]^{\frac{\theta}{1-\theta}} \tag{17}$$
- so that at any point in time
$$P = SP^{\star} \tag{18}$$
- Why does the PPP hold here ?
  - Law of one price
  - Price indices have identical weights for Home and Foreign goods

Asset markets and the budget constraint

- An internationally traded riskless real bond (denominated in the composite real good).
- The international bond market is perfectly integrated. There is the same *real* interest rate in both countries.
- In each country you have symmetric agents. So borrowing and lending only across countries.

The period budget constraint for a Home individual j at time t :

$$P_t B^j_{t+1} + M^j_t = P_t(1 + r_t)B^j_t + M^j_{t-1} + p_t(j)y_t(j) - P_t C^j_t - P_t \tau_t \quad (19)$$

where B are bond holdings and τ are (lump-sum) taxes.

The government

- Balanced budget (Ricardian equivalence holds).
- For the most basic setup, no government spending and all seignorage revenues are rebated to the population via transfers

$$\tau_t + \frac{M_t - M_{t-1}}{P_t} = 0 \tag{20}$$

Consumer optimization and world demand

- The home individual's j demand for a good z is

$$c^j(z) = (\frac{p(z)}{P})^{-\theta} C^j \tag{21}$$

- Similarly, the foreign agent's demand is then

$$c^{\star j}(z) = (\frac{p^\star(z)}{P^\star})^{-\theta} C^{\star j} \tag{22}$$

- We can then find (from the PPP condition) that world demand for any good z will be

$$y^d(z) = (\frac{p(z)}{P})^{-\theta} C^w \tag{23}$$

with symmetric agents in each country $C^w = nC + (1-n)C^\star$

Solution [I]

We can substitute for the budget constraint into the maximization problem of the agent and take the first order conditions

$$max_{y(j),M^j,B^j} U_t^j = \sum_{s=t}^{\infty} \beta^{s-t} [\ln((1+r_s)B_s^j + \frac{M_{t-1}^j}{P_s} + y_s(j)^{\frac{\theta-1}{\theta}} (C_s^w)^{\frac{1}{\theta}}$$
$$- \tau_s - B_{s+1}^j - \frac{M_s^j}{P_s}) + \chi \ln \frac{M_s^j}{P_s} - \frac{\kappa}{2} y_s(j)^2]$$

and the F.O.C.s are

$$C_{t+1} = \beta(1+r_{t+1})C_t \tag{24}$$
$$\frac{M_t}{P_t} = \chi C_t \frac{1+i_{t+1}}{i_{t+1}} \tag{25}$$
$$y_t^{\frac{\theta+1}{\theta}} = \frac{\theta-1}{\theta\kappa} \frac{(C_t^w)^{\frac{1}{\theta}}}{C_t} \tag{26}$$

where $1 + i_{t+1} = \frac{P_{t+1}}{P_t}(1 + r_{t+1})$ (the Fisher equation)

Solution [II]

- Domestic money supply = domestic money demand
- Global net foreign assets must be zero

$$nB_{t+1} + (1-n)B^{\star}_{t+1} = 0 \tag{27}$$

- and world real consumption equals world real income.

$$C^w_t = nC_t + (1-n)C^{\star}_t = n\frac{p_t(h)}{P_t}y_t(h) + (1-n)\frac{p^{\star}_t(f)}{P^{\star}_t}y^{\star}_t(f) = Y^{\star}_t \tag{28}$$

The steady state [I]

- Suppose that all exogenous variables are constant. Steady state solution.
- The Euler equation pins down the relationship between the real interest rate r and the discount factor. Then

$$\overline{r} = \frac{1-\beta}{\beta} \equiv \delta \tag{29}$$

- Steady state real consumption has to be equal to steady real income in both countries

$$\overline{C} = \delta \overline{B} + \frac{\overline{p}(h)\overline{y}}{\overline{P}} \tag{30}$$

$$\overline{C^{\star}} = -\frac{n}{1-n}\delta \overline{B} + \frac{\overline{p^{\star}}(f)\overline{y^{\star}}}{\overline{P^{\star}}} \tag{31}$$

The steady state [II]

- No simple closed-form solution unless $\overline{B} = 0$.
- Then a symmetric equilibrium with

$$\overline{C} = \overline{C^\star} = \overline{y} = \overline{y}^\star \tag{32}$$

$$\overline{y} = \overline{y^\star} = \left(\frac{\theta - 1}{\theta\kappa}\right)^{\frac{1}{2}} \tag{33}$$

$$\frac{\overline{M}}{\overline{P}} = \frac{\overline{M^\star}}{\overline{P^\star}} = \frac{\chi(1+\delta)}{\delta}\overline{y} \tag{34}$$

The Redux model : applications

Asking questions to the model

- We want to consider how different shocks affect the variables of interest.
- Frictions have to play a role : o/w "jumping" to the new steady state is immediate.
- Method :
  - Log-linearize around the steady state and consider (small) unanticipated shocks
  - What is the new steady state when the effect of frictions stop ? There may be changes in the net assets between countries...
  - Given what the new steady state is, how does the disequilibrium look like ? E.g. what is the change in the net assets between countries just after the shock ? [use the perfect foresight assumption]

Log-linearization around the steady state : prices and demand

Let $\mathrm{p} = \frac{dP_t}{\overline{P}_0}$ etc. Then

$$\mathrm{p}_t = n\mathrm{p}_t(h) + (1-n)(\mathrm{s}_t + \mathrm{p}_t^\star(f)) \tag{35}$$

$$\mathrm{p}_t^\star = n(\mathrm{p}_t(h) - \mathrm{s}_t) + (1-n)(\mathrm{p}_t^\star(f)) \tag{36}$$

$$\mathrm{s}_t = \mathrm{p}_t - \mathrm{p}_t^\star \tag{37}$$

Demands for the representative home and foreign products

$$\mathrm{y}_t = \theta[\mathrm{p}_t - \mathrm{p}_t(h)] + \mathrm{c}_t^w \tag{38}$$

$$\mathrm{y}_t^\star = \theta[\mathrm{p}_t^\star - \mathrm{p}_t(f)^\star] + \mathrm{c}_t^w \tag{39}$$

Log-linearization around the steady state : Euler equations

$$\mathtt{c}_{t+1} = \mathtt{c}_t + \frac{\delta}{1+\delta}\mathtt{r}_{t+1} \tag{40}$$

$$\mathtt{c}^{\star}_{t+1} = \mathtt{c}^{\star}_t + \frac{\delta}{1+\delta}\mathtt{r}_{t+1} \tag{41}$$

Log-linearization around the steady state : money demand and the exchange rate equation

$$\mathtt{m}_t - \mathtt{p}_t = \mathtt{c}_t - \frac{\mathtt{r}_{t+1}}{1+\delta} - \frac{\mathtt{p}_{t+1} - \mathtt{p}_t}{\delta} \tag{42}$$

$$\mathtt{m}_t^\star - \mathtt{p}_t^\star = \mathtt{c}_t^\star - \frac{\mathtt{r}_{t+1}}{1+\delta} - \frac{\mathtt{p}_{t+1}^\star - \mathtt{p}_t^\star}{\delta} \tag{43}$$

Subtracting (43) from (42) and applying (37) we get

$$\mathtt{m}_t - \mathtt{m}_t^\star - \mathtt{s}_t = \mathtt{c}_t - \mathtt{c}_t^\star - \frac{1}{\delta}(\mathtt{s}_{t+1} - \mathtt{s}_t) \tag{44}$$

What is here different from the ordinary monetary model ? There will be asset accumulation !

Log-linearization around the steady state : income = expenditure

Take $\overline{\mathrm{b}} = \frac{d\overline{B}}{\overline{C}_0^w}$ and the barred variables representing approximate changes in the steady state values $\overline{\mathrm{c}} = \frac{d\overline{C}}{\overline{C}_0}$

$$\overline{\mathrm{c}} = \delta\overline{\mathrm{b}} + \overline{\mathrm{p}}(h) + \overline{\mathrm{y}} - \overline{\mathrm{p}} \tag{45}$$

$$\overline{\mathrm{c}}^\star = -\frac{n}{1-n}\delta\overline{\mathrm{b}} + \overline{\mathrm{p}}(f)^\star + \overline{\mathrm{y}}^\star - \overline{\mathrm{p}}^\star \tag{46}$$

One can find that

$$\overline{\mathrm{c}} - \overline{\mathrm{c}}^\star = \frac{1}{1-n}\frac{1+\theta}{2\theta}\delta\overline{\mathrm{b}} \tag{47}$$

Solutions in differences

Terms of trade :

$$\mathrm{y}_t - \mathrm{y}_t^\star = \theta[\mathrm{s}_t + \mathrm{p}_t^\star(f) - \mathrm{p}_t(h)] \tag{48}$$

Labor-leisure choice :

$$\mathrm{y}_t - \mathrm{y}_t^\star = -\frac{\theta}{1+\theta}[\mathrm{c}_t - \mathrm{c}_t^\star] \tag{49}$$

One can find that

$$\overline{\mathrm{c}} - \overline{\mathrm{c}}^\star = \frac{1}{1-n}\frac{1+\theta}{2\theta}\delta\overline{\mathrm{b}} \tag{50}$$

Interpretation of changes in wealth

- A wealth transfer $\overline{\mathrm{b}}$ to Home residents increases their consumption.
- The increase, however, is not equal to $\delta\overline{\mathrm{b}}$ - the per capita interest on the transfer.
- At the same time the Home agents decrease their work effort while the agents in the Foreign do the opposite.
- What happens with the Home's steady-state terms of trade ?

$$\overline{\mathrm{p}}(h) - \overline{\mathrm{s}} - \overline{\mathrm{p}}^{\star}(f) = \frac{1}{1-n}\frac{1}{2\theta}\delta\overline{\mathrm{b}} \tag{51}$$

- They improve for the same reason : the reduction in the supply of the Home good must rise its relative price.

Friction assumed : sticky prices

- There are menu costs - domestic prices don't change one period after a monetary shock - and then the system will reach a steady state.
- In this setup with monopolistically competitive producers, global output is inefficiently lower because of market structure and pricing above cost.
- In such a model, output becomes demand-driven when prices are rigid. With a small demand shock, the producer will be eager to raise output even if he cannot change the price.
- An unanticipated monetary shock will increase welfare because producers will produce more output (as there is demand...).
- The foreign prices move one-to-one with the exchange rate (PPP assumption !)

Effects of a permanent asymmetric monetary shock [I]

- Consider that an unanticipated shock changes money supply permanently. Then, for the first period

$$\overline{\mathtt{m}} - \overline{\mathtt{m}}^{\star} = \mathtt{m} - \mathtt{m}^{\star} \tag{52}$$

- The current account for home is (under no government) in the short run

$$B_{t+1} - B_t = r_t B_t + \frac{p_t(h) y_t}{P_t} - C_t \tag{53}$$

- and after log-linearizing (and noting that prices are fixed in the first period)

$$\overline{\mathtt{b}} = \mathtt{y} - \mathtt{c} - (1-n)\mathtt{s} \tag{54}$$

$$\overline{\mathtt{b}}^{\star} = \mathtt{y}^{\star} - \mathtt{c}^{\star} + n\mathtt{s} = -\frac{n}{1-n}\overline{\mathtt{b}} \tag{55}$$

- $\overline{\mathtt{b}}$ are the new net foreign asset positions in the steady state... and there will be a permanent CA deficit for the Home country...

Effects of a permanent asymmetric monetary shock [II]

- The Euler equations yield

$$\overline{c} - \overline{c}^{\star} = c - c^{\star} \tag{56}$$

- while (44) is now

$$m - m^{\star} - s = c - c^{\star} - \frac{1}{\delta}(\overline{s} - s) \tag{57}$$

- Given the steady state exchange rate determination and (56)

$$\overline{s} = \overline{m} - \overline{m}^{\star} - (c - c^{\star}) \tag{58}$$

- and plugging this back to (57) we get

$$s = m - m^{\star} - (c - c^{\star}) \tag{59}$$

- which implies $s = \overline{s}$ or that the exchange rate jumps immediately to the new steady state value.
- Money supply and consumption differentials are expected to be permanent and constant - so exchange rate needs to be constant between after the shock and the steady state as well.

Effects of a permanent asymmetric monetary shock [III]

- From the asset accumulation equations we get

$$\bar{\mathrm{b}} = (1-n)[(\mathrm{y}-\mathrm{y}^\star) - (\mathrm{c}-\mathrm{c}^\star) - \mathrm{s}] \tag{60}$$

- which can be solved for

$$\mathrm{s} = \Gamma(\mathrm{c}-\mathrm{c}^\star) \tag{61}$$

  with $\Gamma > 0$.

- Home consumption (and income) can increase relative to that of Foreign only if the exchange rate depreciates. This is when Home output rises relative to Foreign country (foreigners buy more of our products).
- One can solve jointly for s and $\mathrm{c}-\mathrm{c}^\star$.

Intuition

- The short run real depreciation rises the home real income relative to Foreign.
- There is expenditure switching thru the exchange rate depreciation (for one period prices are fixed !).
- The home country runs a current - account surplus (intertemporal consumption-smoothing)
- Home producers' higher wealth makes them work less (labor-leisure choice) and causes a fall in the supply of the home goods
- This improves the home's terms of trade
- The nominal exchange rate does not need to depreciate as much as in the flexible price world (so the change is not one-to-one).

What do we learn from this ?

- This exercise :
  - Effects of exchange-rate depreciation on CA and the exchange rate
  - Monetary policy generates an increase in wealth shared among countries.
  - Long run nonneutrality of money (but the effect rather small)
- In general :
  - A workhorse model where you can evaluate various twists and policies
  - An micro-founded extension of the old Mundell-Fleming-Dornbusch framework
  - Interactions of output, consumption, the exchange rate and the CA can be studied.
  - Caution : a very special model with results from unanticipated shocks. The system does not achieve first-best because of monopolistic competition.

Productivity shocks with sticky prices

- Equivalent to a fall in κ disutility of labor.
- A temporary shock : nothing happens ! Just an increase in leisure (demand shocks drive output).
- A permanent shock : the exchange rate appreciates !
  - Output constant in the short run but increases in the long run.
  - To smoothe consumption, home agents borrow and raise current consumption, increasing the demand for the home currency.
  - The exchange rate appreciates.
  - Consumption and the real exchange rate move in separate directions - a mechanism potentially solving the Backus and Smith puzzle.

Extensions

- Pricing to market
- Non-tradeable goods
- Uncertainty
- Government
- Away from the symmetric equilibrium
- Trade costs (failures of PPP)
- Asset markets not perfectly integrated
- More assets
- ...

Pricing to market

- In the Redux model, importers in the Foreign country assumed to have no difficulty in changing prices with the exchange rate. So, various assumptions on pricing to market were investigated.
  - Incomplete pass-through reduces the expenditure-switching effect of exchange rates.
  - Any disturbance requires now larger exchange rate responses : higher volatility.
  - The higher the pricing to market, the higher the correlation between the nominal and real exchange rates.
  - Monetary policy has a beggar-thy-neighbor aspect.

Other extensions

- Non-tradeable goods
  - The higher the share of nontradeables (unaffected by the terms of trade), the more sensitive is the exchange rate to any disturbances.
  - There can be overshooting.
- Uncertainty
  - Results depend on how it is introduced in the model
  - Non-trivial issues related to steady-state analysis