Further issues: International business cycles and puzzles

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[International Macroeconomics]

Outline

International business cycles

Two country business cycle models

- SOE models cannot answer several questions at the heart of international economics such as :
  - How are business cycles transmitted internationally ?
  - International risk sharing and the extent of the home bias.
  - Real exchange rate movements and their real effects.
- ...especially when we are interested in general equilibrium effects (so when countries are not small).

Backus, Kehoe and Kydland 1992 : one-good model

- A canonical paper to analyze international business cycles
- Extends Kydland and Prescott (1982) to a two-country setup
- Questions :
  - How well can simple models fit the data ?
  - What should we add to improve the fit ?
  - To what extent the same productivity shock can explain the (co- ?) movement of various variables ?

Setup (abridged)

- Two countries with many infinitely-lived representative households
- In the home country, the problem is

$$\max_{\{c_t, l_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t^{\mu} \left(1 - l_t\right)^{1-\mu}\right)^{1-\sigma}}{1-\sigma} \tag{1}$$

subject to

$$y_t = Z_t k_t^{\alpha} l_t^{1-\alpha} \tag{2}$$

$$nx_t = y_t - c_t - k_{t+1} + (1 - \delta) k_t \tag{3}$$

The planner's problem

- Start with the planner's problem that assigns weights Ψ

$$\max_{\left\{c_t, c_t^*, l_t, l_t^*, k_{t+\mathbf{1}}, k_{t+\mathbf{1}}^*\right\}} \left[\Psi U\left(c_t, l_t\right) + (1-\Psi)\, U\left(c_t^*, l_t^*\right)\right]$$

subject to

$$y_t + y_t^* = c_t + c_t^* + i_t + i_t^*$$

- The first order conditions are

$$\Psi U_c\left(c_t, l_t\right) = (1-\Psi)\, U_c\left(c_t^*, l_t^*\right)$$

$$\frac{1-\mu}{\mu}\frac{c}{1-l} = (1-\alpha)\frac{y}{l}$$

$$\lambda = \lambda' \beta\left(\alpha \frac{y'}{k'} + (1-\delta)\right)$$

Decentralized problem (home country)

- You can decentralize this problem with state-contingent Arrow-Debreu securities ($b(s^t)$) - a complete markets economy.

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) \frac{\left[c\left(s^t\right)^{\mu}\left(1-l\left(s^t\right)\right)^{1-\mu}\right]^{1-\sigma}}{1-\sigma} \tag{4}$$

subject to

$$\begin{aligned} & w\left(s^t\right) l\left(s^t\right)+r\left(s^t\right) k\left(s^t\right)+b\left(s^t\right) \\ = \; & c\left(s^t\right)+k\left(s_{t+1}\right)+(1-\delta) k\left(s^t\right) \\ & +\sum_{s^{t+1}} q\left(s^t, s_{t+1}\right) b\left(s^t, s_{t+1}\right) \end{aligned}$$

- Therefore, the interpretation of results will be through the lens of economies in which claims can be traded on any conceivable state of the world - an idealistic benchmark.

Optimality conditions

$$U_c\left(s^t\right) = \lambda\left(s^t\right) \tag{5}$$

$$q\left(s^t, s_{t+1}\right) = \beta \frac{\pi\left(s^t, s_{t+1}\right)}{\pi\left(s^t\right)} \frac{U_c\left(s^t, s_{t+1}\right)}{U_c\left(s^t\right)} \tag{6}$$

$$\left[r\left(s^t\right) + 1 - \delta\right] \beta \sum_{s_{t+1}} \frac{\pi\left(s^t, s_{t+1}\right)}{\pi\left(s^t\right)} \frac{U_c\left(s^t, s_{t+1}\right)}{U_c\left(s^t\right)} = 1 \tag{7}$$

$$\frac{1-\mu}{\mu} \frac{c\left(s^t\right)}{1 - l\left(s^t\right)} = w\left(s^t\right) \tag{8}$$

$$\frac{U_c\left(s^t, s_{t+1}\right)}{U_c\left(s^t\right)} = \frac{U_c^*\left(s^t, s_{t+1}\right)}{U_c^*\left(s^t\right)} \tag{9}$$

Calibration

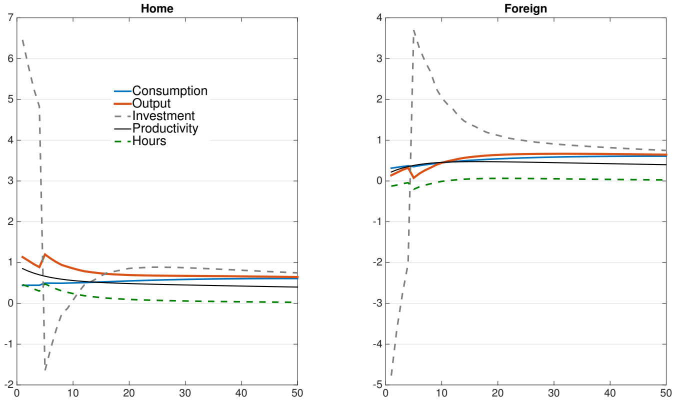
Parameter	Value	Description
β	0.99	Time Preferences
μ	0.34	Consumption Share
σ	2	Intertemporal Elasticity
α	0.36	Capital Share
δ	0.5	Depreciation Rate

$$\begin{bmatrix} Z_t \\ Z_t^* \end{bmatrix} = A \begin{bmatrix} Z_{t-1} \\ Z_{t-1}^* \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_t^* \end{bmatrix}$$

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_2 \end{bmatrix}$	$\begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}$	Productivity Shocks
$\rho_{\epsilon,\epsilon^*}$	0.258	Correlation Shocks
$\sigma_\epsilon^2 = \sigma_{\epsilon^*}^2$	0.00852	Variance Shocks

Notes: Backus, Kehoe, and Kydland (1992)

Impulse response functions



In percentage deviations from steady state values.

Business cycle facts (domestic variables)

	U.S. data		Europe		Model	
	σ_x/σ_{GDP}	$\rho_{x,GDP}$	σ_x/σ_{GDP}	$\rho_{x,GDP}$	σ_x/σ_{GDP}	$\rho_{x,GDP}$
GDP	1	1	1	1	1	1
C	0.75	0.82	0.83	0.81	0.42	0.77
I	3.27	0.94	2.09	0.89	10.99	0.27
L	0.61	0.88	0.85	0.32	0.50	0.93
Z	0.68	0.96	0.98	0.85	0.67	0.89
nx	0.27	-0.37	0.49	-0.25	2.51	0.01

Notes: Backus, Kehoe, and Kydland (1995)

BKK 92 fit (International co-movement)

	US and Europe	Model
	International Correlations	
GDP	0.66	-0.21
C	0.51	0.88
I	0.53	-0.94
L	0.33	-0.78
Z	0.56	0.25

Source : Backus, Kehoe and Kydland (1995).

Backus, Kehoe and Kydland 1995 : two traded intermediate goods

- Each country produces now a specialized intermediate good

$$a_{1,t} + a_{2,t} = y_t = Z_t k_t^{\alpha} l_t^{1-\alpha}$$

$$b_{1,t} + b_{2,t} = y_t^* = Z_t^* k_t^{*\alpha} l_t^{*1-\alpha}$$

- that are used to produce the final consumption- and investment-good with a CES (Armington) aggregator.

$$c_t + i_t = \left[\omega a_{1,t}^{1-\theta} + (1-\omega)\, b_{1,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

- Let $p_{1,t}^a$ and $p_{1,t}^b$ be the prices of the domestic and foreign good in terms of the units of the domestic final good. Then, the terms of trade are $tot_t = \frac{p_{1,t}^b}{p_{1,t}^a} = \left(\frac{a_{1,t}}{b_{1,t}}\right)^{\theta} \frac{1-\omega}{\omega}$
- while the trade balance to GDP ratio $nx_t = \frac{p_{1,t}^a a_{2t} - p_{1,t}^b b_{1,t}}{y_t}$ and the real exchange rate is $rer_t = \frac{p_{1,t}^a}{p_{2,t}^a} = \frac{p_{1,t}^b}{p_{2,t}^b}$.

Business cycle facts and the BKK 95 model

	Data			**Model**		
	σ_x/σ_{GDP}	$\rho_{x,GDP}$	ρ_{x,x^*}	σ_x/σ_{GDP}	$\rho_{x,GDP}$	ρ_{x,x^*}
GDP	1	1	0.58	1	1	0.18
C	0.81	0.86	0.36	0.53	0.96	0.65
I	2.84	0.95	0.30	2.74	0.96	0.29
L	0.66	0.87	0.42	0.31	0.97	0.14
nx	0.27	-0.49		0.43	-0.64	
tot	1.79	-0.24		0.61	0.65	
rer	4.02	0.13		0.45	0.65	

Notes: Heathcote and Perri (2002)

Source : Heathcote and Perri (2002).

The role of elasticity of substitution

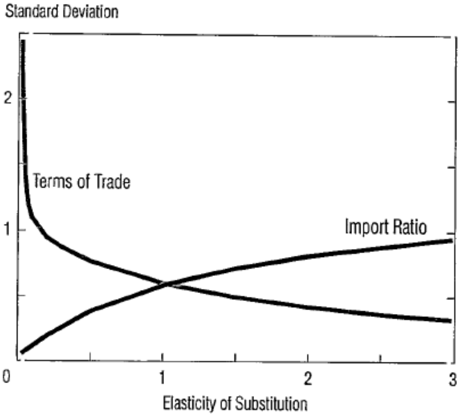


Figure : Changes in the elasticity of substitution $\frac{1}{\theta}$ between foreign and domestic goods on the volatility of the terms of tot and import ratio. Source : Backus, Kehoe and Kydland (1995).

Financial structure : Heathcote and Perri, 2002

- Can assumptions on different financial structures can help improving fit of the models with the data ?
- This affects only the menu of financial assets agents can invest in (shows up in the budget constraint)
- A "bond" economy only :

$$\begin{aligned} & p_1^a\left(s^t\right)\left[w\left(s^t\right)l\left(s^t\right)+r\left(s^t\right)k\left(s^t\right)+b\left(s^t\right)\right] \\ = \; & c\left(s^t\right)+i\left(s^t\right)+p_1^a\left(s^t\right)q\left(s^t\right)b\left(s^{t+1}\right) \end{aligned}$$

- Financial autarky :

$$\begin{aligned} & p_1^a\left(s^t\right)\left[w\left(s^t\right)l\left(s^t\right)+r\left(s^t\right)k\left(s^t\right)\right] \\ = \; & c\left(s^t\right)+i\left(s^t\right) \end{aligned}$$

Equilibrium conditions with different assumptions

- Complete markets :

$$q\left(s^t, s_{t+1}\right) = \beta \frac{\pi\left(s^t, s_{t+1}\right)}{\pi\left(s^t\right)} \frac{U_c\left(s^t, s_{t+1}\right)}{U_c\left(s^t\right)} \frac{p_1^a\left(s^t, s_{t+1}\right)}{p_1^a\left(s^t\right)}$$

$$rer\left(s^t, s_{t+1}\right) = \chi \frac{U_c^*\left(s^t, s_{t+1}\right)}{U_c\left(s^t, s_{t+1}\right)}$$

with $\chi = rer\left(s_0\right) \frac{U_c(s_0)}{U_c^*(s_0)}$

- The "bond" economy :

$$q\left(s^t\right) = \frac{1}{1 + r\left(s^t\right)} = \beta \sum_{s_{t+1}} \frac{\pi\left(s^t, s_{t+1}\right)}{\pi\left(s^t\right)} \frac{U_c\left(s^t, s_{t+1}\right)}{U_c\left(s^t\right)} \frac{p_1^a\left(s^t, s_{t+1}\right)}{p_1^a\left(s^t\right)}$$

- Financial autarky : $nx_t = 0$

Performance of different models on domestic variables

(A) Volatilities[a]

Economy	% std. dev.	% std. dev. / % std. dev. of y			% std. dev.			
	y	c	x	n	ex	im	nx	ir
US data	1.67	0.81	2.84	0.66	3.94	5.42	0.45	4.07
Complete markets	1.21	0.53	2.74	0.31	0.99	0.99	0.20	0.70
Bond economy	1.21	0.52	2.73	0.32	0.96	0.96	0.19	0.76
Financial autarky	1.18	0.51	2.04	0.28	1.29	1.18	0.00	1.51

(B) Correlations with output[b]

Economy	correlation between c, y	x, y	n, y	ex, y	im, y	nx, y	p, y	rx, y
US data	0.86	0.95	0.87	0.32	0.81	− 0.49	− 0.24	0.13
Complete markets	0.96	0.96	0.97	0.55	0.89	− 0.64	0.65	0.65
Bond economy	0.95	0.96	0.97	0.59	0.86	− 0.65	0.65	0.65
Financial autarky	0.92	0.99	0.99	1.00	0.15	0.00	0.65	0.65

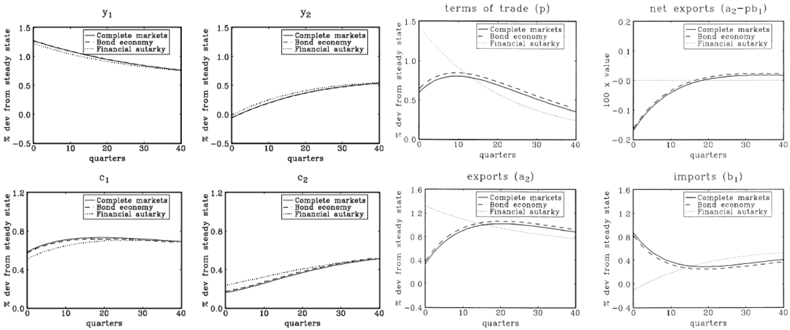
Performance of different models : cross country correlations

(C) Cross country correlations and international relative price volatility

Economy	correlation between y_1, y_2	c_1, c_2	x_1, x_2	n_1, n_2	% std. dev. p	rx
Data	0.58	0.36	0.30	0.42	2.99	3.73
Complete markets	0.18	0.65	0.29	0.14	0.78	0.55
Bond economy	0.17	0.68	0.29	0.17	0.84	0.59
Financial autarky	0.24	0.85	0.35	0.14	1.68	1.18

Source : Heathcote and Perri (2002).

Some impulse response functions



Source : Heathcote and Perri (2002).

Readings

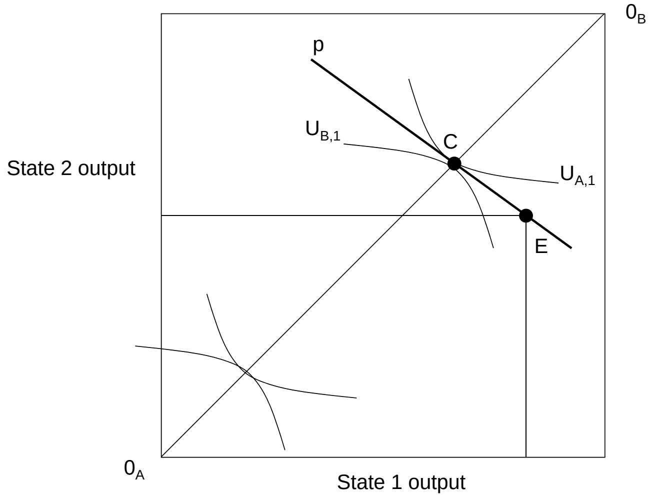
- Backus D., P. Kehoe and F. Kydland, 1992. International real business cycles. Journal of Political Economy 100 :
- Backus D., P. Kehoe and F. Kydland, 1995. International business cycles : Theory vs. evidence. Thomas F. Cooley (ed.) Frontiers of Business Cycle Research, Princeton University Press.
- Heathcote J., and F. Perri, 2002. Financial autarky and international business cycles. Journal of Monetary Economics 49 : 601-627.

International risk sharing

International risk sharing

- What is the extent of risk sharing between countries ? I.e., how far are we from first best allocations (in a Pareto sense) ?
  - What are the preferences ?
  - What is known when ? (ex-ante vs. ex-post efficiency)
  - Frictions ? Some can be "natural".
  - Crucial problem : what is the "true" model from which to benchmark/assess efficiency ?
- Asking the question differently : what should we add to our models to account for the observed facts ? (but... we asked that before ?)
- But, importantly : what could be the welfare gains from changing this environment ?
  - The role of different (asset) market structures
  - The role of different frictions...
- Related to the existence of the home bias in asset holdings.

Risk sharing in an Edgeworth box



Cole and Obstfeld, 1991

- Can there be perfect risk sharing without complete markets ?
- Two countries with endowments ; each country produces a differentiated good indexed $1, 2$.
- Cobb-Douglas preferences

$$C = (c_1)^{\alpha} (c_2)^{1-\alpha}$$

- result in demand

$$c_1 = \frac{\alpha Y}{p_1}, c_2 = \frac{(1-\alpha) Y}{p_2}, c_1^* = \frac{\alpha Y^*}{p_1}, c_2^* = \frac{(1-\alpha) Y^*}{p_2}$$

- with the resource constraints $c_1 + c_1^* = Y, c_2 + c_2^* = Y^*$
- and prices $p_1 = \frac{\alpha(Y+Y^*)}{Y}, p_2 = \frac{(1-\alpha)(Y+Y^*)}{Y^*}$
- and the allocations are $\frac{C}{C+C^*} = \frac{p_1 Y}{p_1 Y + p_2 Y^*} = \alpha; \frac{C}{C^*} = \frac{\alpha}{1-\alpha}$

International Risk Sharing (Quantities)

- Output, investment and employment comove strongly across countries.
- But, the correlation of consumption, while positive, is lower than that of output (already BKK, 1992).
- Net exports are not very volatile (one third of GDP) and are strongly countercyclical.

An endowment BKK economy (Heathcote and Perri, 2014)

$$\max \left(\kappa \frac{c_1^{1-\gamma}}{1-\gamma} + (1-\kappa) \frac{c_2^{1-\gamma}}{1-\gamma} \right) \tag{10}$$

subject to

$$a_1 \left(s^t\right) + a_2 \left(s^t\right) = z_1 \left(s^t\right)$$

$$b_1 \left(s^t\right) + b_2 \left(s^t\right) = z_2 \left(s^t\right)$$

$$c_1 = \left(\omega a_1^{\frac{\sigma-1}{\sigma}} + (1-\omega)\, b_1^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

$$c_2 = \left((1-\omega)\, a_2^{\frac{\sigma-1}{\sigma}} + \omega b_2^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

Optimality conditions

- The FOC are

$$\kappa\omega a_1^{-\frac{1}{\sigma}} c_1^{-\gamma+\frac{1}{\sigma}} = (1-\kappa)(1-\omega)\, a_2^{-\frac{1}{\sigma}} c_2^{-\gamma+\frac{1}{\sigma}} \tag{11}$$

$$\kappa\omega b_1^{-\frac{1}{\sigma}} c_1^{-\gamma+\frac{1}{\sigma}} = (1-\kappa)(1-\omega)\, b_2^{-\frac{1}{\sigma}} c_2^{-\gamma+\frac{1}{\sigma}} \tag{12}$$

- Solve a log-linearized system around the steady state with $\widehat{x}(s^t) = \ln(x(s^t)) - \ln \overline{x}$:

$$s\hat{a}_1 + (1-s)\,\hat{a}_2 = \hat{z}_1$$

$$(1-s)\,\hat{b}_1 + s\hat{b}_2 = \hat{z}_2$$

$$\left(-\gamma + \frac{1}{\sigma}\right)\hat{c}_1 - \frac{1}{\sigma}\hat{a}_1 = \left(-\gamma + \frac{1}{\sigma}\right)\hat{c}_2 - \frac{1}{\sigma}\hat{a}_2$$

$$\left(-\gamma + \frac{1}{\sigma}\right)\hat{c}_1 - \frac{1}{\sigma}\hat{b}_1 = \left(-\gamma + \frac{1}{\sigma}\right)\hat{c}_2 - \frac{1}{\sigma}\hat{b}_2$$

Solution

- Efficient allocations have the following properties iff

$$\sigma < \tilde{\sigma}(s,\gamma) = \frac{1}{\gamma} - \frac{(1-\gamma)}{2s\gamma}$$

  - The pass-through from relative output to relative consumption

$$\frac{(\hat{c}_1 - \hat{c}_2)}{(\hat{y}_1^c - \hat{y}_2^c)} > 1$$

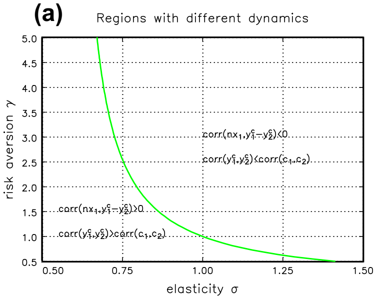
  - Net exports are countercyclical

$$corr\left(\left(\hat{y}_1^c - \hat{c}_1\right), \left(\hat{y}_1^c - \hat{y}_2^c\right)\right) < 0$$

  - The cross-country output correlation exceeds the cross-country allocation.

$$corr\left(\hat{y}_1^c, \hat{y}_2^c\right) > corr\left(\hat{c}_1, \hat{c}_2\right)$$

Intuition on the difference between one and two-good models



Source : Heathcote and Perri (2014)

Full BKK 1995 model with production

	International Correlations				Domestic Statistics		
	(y_1, y_2)	(c_1, c_2)	(x_1, x_2)	(n_1, n_2)	% sd y	% sd $\frac{nx}{y}$	corr$(\frac{nx}{y}, y)$
1. Data	0.55	0.31	0.51	0.57	1.54	0.44	−0.51
Complete markets models							
2. BKK (see Table 9.5)	0.55	0.93	−0.07	−0.01	1.54	0.23	−0.43
3. No spillovers: $\rho = 0.91$, $\psi = 0$	0.55	0.71	0.35	0.56	1.54	0.19	−0.40
4. Separable utility: $\gamma = 1$	0.55	0.94	0.02	0.15	1.54	0.23	−0.43
5. Low elasticity: $\sigma = 0.6$	0.55	0.88	−0.08	0.10	1.54	0.28	−0.47
6. All: $\rho = 0.91$, $\psi = 0$, $\gamma = 1$, $\sigma = 0.6$	0.55	0.35	0.39	0.71	1.54	0.47	−0.46
Bond economy model							
7. BC: $\rho = 1$, $\psi = 0$, $\sigma = 5$	0.55	0.29	−0.39	0.92	1.54	0.82	−0.39

Notes: All data are from the OECD Quarterly National Accounts (GDP and components) and Main Economic Indicators (employment). The sample for the data statistics is 1960.1–2012.2. The variable y denotes real GDP, c denotes real consumption (both private and public), n denotes civilian employment, x denotes real gross fixed capital formation, and nx/y denotes net exports over GDP (all nominal). All variables except net exports are in logs. All variables are HP filtered with a smoothing parameter of 1600. Statistics from the model are produced by simulating the model for the same numbers of periods as the data and taking averages over 20 simulations. In lines 2 through 7 the standard deviation and correlation of shock innovations are calibrated to replicate the standard deviation of output and the international correlation of GDP. BKK: Backus et al. (1994); BC: Baxter and Crucini (1995).

Source : Heathcote and Perri (2014)

Home bias : a related problem

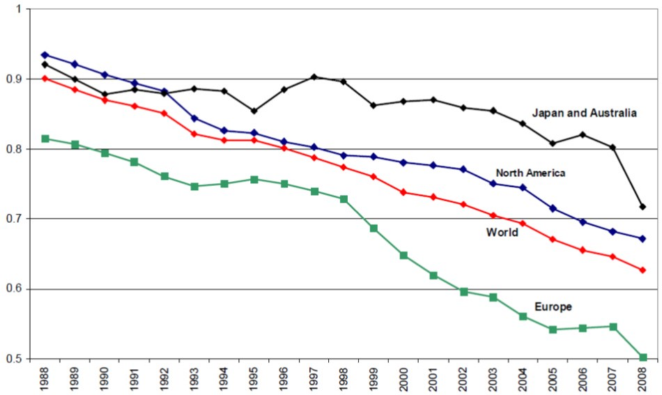
- Investors' portfolios are less internationally diversified than what CAPM (and other models) predict (or... it seems so).
- This tendency to underweight foreign securities in portfolios is called the "home bias".
- Typically the talk about "equity home bias" but a "bond home bias" is also observed.
  - In simplest models all investors should hold exactly the same portfolios (Lucas 1982).
  - Home bias found also for domestic securities ! (Coval and Moskowitz, 2001)
- Imperfect measurement of the home bias $HB_{i,j}$:

$$HB_{i,j} = 1 - \frac{Sh_{i,j}}{Sh_j} \tag{13}$$

where

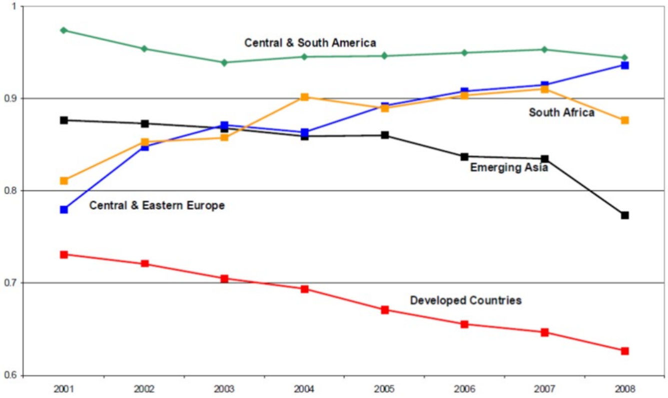
- $Sh_{i,j}$ is the share of country j in country i portfolio
- Sh_j is the country's j value of equity (stock market) share in world equity

The Home bias in developed countries across time



Source : Couerdacier and Rey (2011).

The Home bias in developing countries across time

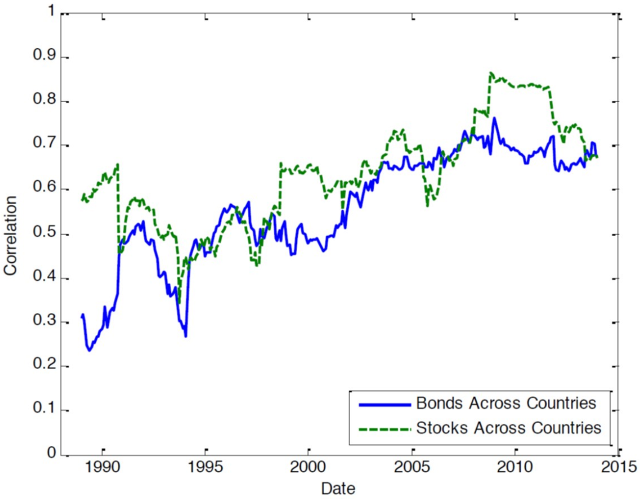


Source : Couerdacier and Rey (2011).

Frictions

- Transaction costs ? (Tesar and Werner 1995 but Banggui et al. (2013)).
- Trade costs ? (Obstfeld and Rogoff, 2001)
- Information costs (Cheo et al., 2005,...) ? Ambiguity aversion (Uppal et al., 2003), / familiarity (Solnik and Zuo, 2012), information acquisition (van Nieuwburgh and Veldkamp 2009)
- Limited contract enforcement (Kehoe and Perri, 2002)
- High foreign trade/GDP : lower need for diversification (but then, there is more knowledge about foreign markets/assets)
- Multinational companies replicate diversification (Mitra-Stiff, 1995 but Andrade et al., 2010).
- High covariances of the business cycles and asset markets : lower gains from diversification given exchange rate risk...
- Large domestic asset markets
- Is diversification really beneficial ? (Newbery and Stiglitz 1984)

Average correlations of stock and bond returns



Source : Viceira et al. (2016).

Backus and Smith puzzle

Focusing on the real exchange rate : the Backus and Smith (1993) puzzle

- The correlation between the consumption ratio $\frac{c}{c^*}$ and the real exchange rate $Q = \frac{sP^*}{P}$ is close to zero or negative (Backus and Smith, 1993)
- In the simplest of the models, however, it is close to 1.

The Backus-Smith puzzle

The optimality conditions (11)-(12) are of the sort

$$\kappa U_{c_1}\left(s^t\right) G_{a_1}\left(s^t\right) = (1-\kappa) U_{c_2}\left(s^t\right) G_{a_2}\left(s^t\right)$$

$$\kappa U_{c_1}\left(s^t\right) G_{b_1}\left(s^t\right) = (1-\kappa) U_{c_2}\left(s^t\right) G_{b_2}\left(s^t\right)$$

implying

$$\frac{G_{a_1}\left(s^t\right)}{G_{a_2}\left(s^t\right)} = \frac{G_{b_1}\left(s^t\right)}{G_{b_2}\left(s^t\right)} = \frac{p_2}{p_1} \equiv e\left(s^t\right) \tag{14}$$

Since in equilibrium, marginal products of intermediate goods are set equal to their prices (expressed in the domestic final good), this ratio is equivalent to the price of foreign consumption to domestic consumption - the real exchange rate !

The Backus-Smith puzzle II

So

$$e\left(s^t\right)=\frac{G_{a_1}\left(s^t\right)}{G_{a_2}\left(s^t\right)}=\frac{\left(1-\kappa\right)U_{c_2}\left(s^t\right)}{\kappa U_{c_1}\left(s^t\right)}$$

$$\ln e\left(s^t\right)=\ln\frac{1-\kappa}{\kappa}+\gamma\ln\left(\frac{c_1\left(s^t\right)}{c_2\left(s^t\right)}\right)$$

If a country's marginal utility is high (= low consumption), then it must be that the prices of its consumption basket is high !

The Backus-Smith puzzle III

	% sd e	% sd $\frac{c_1}{c_2}$	corr$(\frac{c_1}{c_2}, e)$
1. Data	6.39	0.97	-0.21
Baseline parameters: $\rho = 0.91$, $\psi = 0$, $\gamma = 1$, $\sigma = 0.6$			
2. Efficient allocations	0.47	0.47	1
3. Bond Economy	0.73	0.36	0.99
4. Financial Autarky	3.15	0.02	0.79
Very low elasticity: $\rho = 0.91$, $\psi = 0$, $\gamma = 1$, $\sigma = 0.38$			
5. Efficient allocations	0.54	0.54	1
6. Bond Economy	2.88	0.15	-0.17
High elasticity and pers. shocks: $\rho = 1$, $\psi = 0$, $\gamma = 1$, $\sigma = 5$			
7. Efficient allocations	0.14	0.14	1
8. Bond Economy	0.23	1.28	-0.69

Source : Heathcote and Perri (2014)

Asset pricing view of exchange rates

Utility-based asset pricing

Let the problem of the investor be

$$\max_S u(C_t) + E_t[\beta u(C_{t+1})] \tag{15}$$

subject to

$$C_t = Y_t - P_t S \tag{16}$$

and

$$C_{t+1} = Y_{t+1} + X_{t+1} S \tag{17}$$

where S is the amount of assets the investor buys, P_t is the price of a payoff X_{t+1} and Y is the income (endowment).

The Euler equation and Stochastic Discount Factors

Substituting for the constraints and taking the FOCs we get

$$P_t u'(C_t) = E_t[\beta u'(C_{t+1}) X_{t+1}] \tag{18}$$

This Euler equation can be rewritten as

$$1 = E_t[\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{X_{t+1}}{P_t}] = E_t[M_{t+1} R_{t+1}] \tag{19}$$

where M_{t+1} is the stochastic discount factor (SDF, also called the pricing kernel) used to price assets and R_{t+1} is the return on the asset.

Real Exchange Rates in the Asset-pricing framework

Suppose there exists a unique SDF in the space of traded assets (law of one price + convex combinations of assets required).

Consider a return in the foreign currency $R^{\star}_{t+1}$. The Euler equations for the foreign and domestic agents are respectively

$$E_t[M^{\star}_{t+1}R^{\star}_{t+1}] = 1 \tag{20}$$

and

$$E_t[M_{t+1}\frac{Q_{t+1}}{Q_t}R^{\star}_{t+1}] = 1 \tag{21}$$

where Q_t is the real exchange rate (prices of the home/foreign good).

Then, since there is a unique SDF,

$$\frac{Q_{t+1}}{Q_t} = \frac{M^{\star}_{t+1}}{M_{t+1}} \tag{22}$$

(One can also express this in nominal terms.)

Real Exchange Rates in the Asset-pricing framework

The above can be written as

$$\ln Q_{t+1} - \ln Q_t = \ln M^*_{t+1} - \ln M_{t+1}$$

- Brandt et al. (2006) : since the observed variance of the RERs is high but that of SDFs much higher, the $Cov\left(\ln M^*_{t+1}, \ln M_{t+1}\right)$ must be high as well !

$$\begin{aligned} Var\left(\ln M^*_{t+1} - \ln M_{t+1}\right) &= Var\left(\ln M^*_{t+1}\right) + Var\left(\ln M_{t+1}\right) \\ &\quad -2Cov\left(\ln M^*_{t+1}, \ln M_{t+1}\right) \end{aligned}$$

- Colacito and Croce (2011) : How is this possible even if we cannot see this in consumption co-movement ?
  - Epstein-Zin preferences and Bansal-Yaron long-run risks (that can be highly correlated).
  - while in the short run the cross-country correlation is driven by transitory shocks...
  - However, the long-run risk preference system is called into question...

Application : the Unbiasedness hypothesis

Let the **forward rate** be

$$F_t = S_t \frac{(1+i_t)}{(1+i_t^*)} \tag{23}$$

Unbiasedness hypothesis :
The forward rate is an unbiased predictor of the future spot rate.

- Given the current information, the average errors in forecasting should be zero.
- There is no systematic pattern in the errors.

Assumptions that we maintained :

- No transaction costs
- Risk-neutral investors
- Identical assets in terms of security

Evidence on the unbiasedness hypothesis

- A problem : conditional expectations of future exchange rates are unobservable.
- Auxiliary assumption : rational expectations.
- This leads to a regression

$$s_{t+1} - s_t = \alpha + \beta(f_{t+1} - s_t) + \varepsilon_t \tag{24}$$

where we expect $\alpha = 0$ and $\beta = 1$.

Comparison of test means, 1976-2006

Exchange rate	Rate of Appreciation	Forward Premium	Difference
DEM / USD	-1.09	-1.57***	0.48
	(2.07)	(0.27)	(2.12)
GBR / USD	0.82	2.24***	-1.42
	(1.99)	(0.24)	(2.05)
JPY / USD	-2.39	-3.37***	0.98
	(2.20)	(0.25)	(2.29)
DEM / JPY	1.98	1.81***	0.17
	(2.07)	(0.20)	(2.11)
DEM / GBR	-1.58	-3.80***	2.22
	(1.67)	(0.26)	(1.70)
JPY / GBR	-2.65	-5.60***	2.95
	(2.22)	(0.17)	(2.27)

Statistics for tests shown whether coefficient different from zero. ***, **, and * denote statistical significance at the 1 %, 5 %, and 10 % levels.

Source : Bekaert and Hodrick (2009).

Regression tests, 1976-2006

Exchange rate	α	β	R2
DEM / USD	-2.59	-0.96**	0.006
	(2.48)	(0.83)	
GBR / USD	4.17*	-1.50***	0.013
	(2.20)	(0.84)	
JPY / USD	-10.99***	-2.55***	0.033
	(2.79)	(0.67)	
DEM / JPY	4.60**	-1.45**	0.007
	(2.30)	(0.89)	
DEM / GBR	-3.69	-0.55**	0.003
	(2.56)	(0.64)	
JPY / GBR	-20.40***	-3.17***	0.026
	(5.18)	(0.88)	

Statistics for tests shown whether $\alpha \neq 0$ and $\beta \neq 1$. *, **, and * denote statistical significance at the 1 %, 5 %, and 10 % levels.**

Source : Bekaert and Hodrick (2009).

Interpretation

- Popular interpretation : there is a carry trade !
- However, one should take into account also the intercept in the interpretation...
- For example, for the JPY / USD we need to take

$$\widehat{\alpha} + \widehat{\beta}(\overline{f - s}) = -10.99 - 2.55 \times (-3.37) = -2.39\,\% \qquad (25)$$

- Then, the expected forward market return on average was

$$(-2.39) - (-3.37) = 0.98\,\% \qquad (26)$$

First explanations

- Irrational expectations ? (could be also... ambiguity aversion : Ilut 2010)
- Peso problems ? (Burnside...)

Risk premia ?

- Forward rate contracts are assets.
- If investors are risk-averse, and the returns on them covary with some market portfolio, there could be risk premia associated with them !
- It is not easy to embed portfolio decisions in general equilibrium, multiperiod models, with many different assets.
- Evidence : Lustig and Verdelhan (2007), Lustig and Verdelhan (2011).

Currency premia and the UIRP [I]

Consider the UIRP strategy in the asset pricing framework. The currency excess return r^e_{t+1} is given (in log-terms) by

$$r^e_{t+1} = \Delta q_{t+1} + r^\star_t - r_t \tag{27}$$

The properties of the economy (consumption streams... shocks...) are going to give us the properties of the risk free rates and the real exchange rates.

Currency premia and the UIRP [II]

Example : if the pricing kernels and returns are log-normal, then the risk-free rates are

$$r_t = -ln(E_t M_{t+1}) = -E_t m_{t+1} - \frac{1}{2} Var_t(m_{t+1}) \tag{28}$$

$$r_t^\star = -ln(E_t M_{t+1}^\star) = -E_t m_{t+1}^\star - \frac{1}{2} Var_t(m_{t+1}^\star) \tag{29}$$

I also know that given (22)

$$E_t(\Delta q_{t+1}) = -E_t(m_{t+1}) + E_t(m_{t+1}^\star)$$
$$= -r_t^\star + r_t - \frac{1}{2} Var_t(m_{t+1}^\star) + \frac{1}{2} Var_t(m_{t+1})$$

So that

$$E_t(r_{t+1}^e) = \frac{1}{2} Var_t(m_{t+1}) - \frac{1}{2} Var_t(m_{t+1}^\star) \tag{30}$$

Three classes of models that can replicate the UIRP puzzle

- Habit formation : Verdelhan (2010).
- The "long run risk model" of Bansal and Yaron (2004) : Bansal and Shaliastovich (2008), Colacito (2009).
- Disaster risk : long tails. (Farhi and Gabaix, 2011).

Conclusions and future research

- UIRP : no longer a puzzle ?
- How to link better the SDFs and economic fundamentals ? SDFs should be heteroskedastic and countries should be heterogenous. What are the driving forces ?
- So what are the links with other asset prices - like long term bonds ?
- Criticism : empirically, the factors explaining exchange rate excess returns do not necessarily explain other returns ! So is there a unified risk-based explanation in these markets or not ?