ANALYTIC NARRATIVES: WHAT THEY ARE AND HOW THEY CONTRIBUTE TO HISTORICAL EXPLANATION[*]

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Abstract

The expression "analytic narratives" is used to refer to a range of quite recent studies that lie on the boundaries between history, political science, and economics. These studies purport to explain specific historical events by combining the usual narrative approach of historians with the analytic tools that economists and political scientists draw from formal rational choice theories. Game theory, especially of the extensive form version, is currently prominent among these tools, but there is nothing inevitable about such a technical choice. The chapter explains what analytic narratives are by reviewing the studies of the major book *Analytic Narratives* (Bates et al., 1998), which are concerned with the workings of political institutions broadly speaking, as well as several cases drawn from military and security studies, which form an independent source of the analytic narratives literature. At the same time as it gradually develops a definition of analytic narratives, the chapter investigates how they fulfil one of their main purposes, which is to provide explanations of a better standing than those of traditional history. An important principle that will emerge in the course of the discussion is that narration is called upon not only to provide facts and problems, but also to contribute to the explanation itself. The chapter distinguishes between several expository schemes of analytic narratives according to the way they implement this principle. From all

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the arguments developed here, it seems clear that the current applications of analytic narratives do not exhaust their potential, and in particular that they deserve the attention of economic historians, if only because they are concerned with microeconomic interactions that are not currently their focus of attention.

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1. Introduction

The expression "analytic narratives" refers to studies that are located at the academic boundaries between history, political science, and economics. These studies purport to explain specific historical states of affairs by combining the usual narrative approach of historians with the analytic approach that is familiar to economists and political scientists. Being specific, and indeed often highly specific, the historical situations, events or actions they cover rarely overlap from one study to another. If there is any unity to analytic narratives, it does not lie in the objects but in the method of explanation, and from this angle, they have two broad principles in common. The first is that analytic narratives jointly exploit the resources of narration and analysis, the presumption being that this can result in better solutions to explanatory problems than if either technique were used in isolation. The second principle is that the analytic component is drawn from the theories of rational decision-making, prominent among which is game theory; the presumption here is that the tools they offer do fit the purpose of combining narration and analysis. More needs to be said to characterize analytic narratives, but these two features are part and parcel of their definition.

The two principles come out most clearly in *Analytic Narratives* (Bates et al., 1998), an important collective book that popularized the expression and provided the approach with a manifesto as well as illustrative case studies. These studies belong to the historical branch of political science, and to get the full range of the genre one must turn to the historical parts of those other fields – to wit, military studies, security studies and international relations (IR) studies – in which analytic narratives have also undergone autonomous development. Proximate forms of analytic narratives had circulated there before the eponymous book came out. Besides giving structure to these significant, albeit unconscious, past attempts, *Analytic*

Narratives pursues a specific programme on political institutions, which it proposes to reconstruct as equilibria of individual interactions, these generally being modelled by game theory. The chapter is concerned with the connections between analytic narratives and history; and although it will mention the connection they also have with theoretical political science, this will not be developed here.

We will explain what analytic narratives (AN) are by surveying, first, the five cases in the eponymous book, and then five further cases drawn from military and security studies, to which we append a case that is again borrowed from political history, but uses the same techniques as those in the latter group. In general, we follow a bottom-up approach, first summarizing the cases, and then attempting to capture their methodological features. As we journey along this inductive road, we will identify a third guiding principle of AN, which is less transparent than the first two, to the effect that the narrative component does not simply provide the data against which explanatory hypotheses are to be tested, *but also contributes to the explanation as such*. The chapter takes the third principle to be definitional, just like the first two, which amounts to defining AN more precisely than is usually done.

The emphasis on the third principle, and the upgrading of the narrative component more generally, is common to this chapter and other accounts by the same author, where this component receives even more emphasis. Along the same line of analysis, this chapter draws an internal distinction between different forms of AN. The crucial observation here is that some AN give the final explanatory word to a narrative, while others state their explanatory conclusions in theoretical language. Thus, although we regard it as being definitional, we take the third principle to be implementable in quite different ways from one AN to another.

Besides providing a definition, the chapter assesses the extent to which AN contribute to historical explanation. For this, it uses the scheme of deductive explanation, which Hempel (1965) and other philosophers of science proposed to clarify the structure of scientific explanation. This scheme is popular among some AN contributors: however, we will argue that the AN themselves conform only very roughly and imperfectly to it. Here, however, the discovery of deductive failures in the explanatory arguments functions as a positive feature, as it prepares us for the claim that the narrative component of AN complements deduction in structuring their explanations. This is how the chapter connects its two topics, i.e., the definition of AN and the account of their explanatory capacity.

The chapter develops as follows. Section 2 summarizes the five studies by Bates, Greif, Levi, Rosenthal and Weingast collected in *Analytic Narratives*. Section 3 exploits them to make some progress with the definition of AN. In particular, it argues that AN should involve proper formalism, but that this formalism does not have to be limited to the game theory employed in the book. This section merely makes explicit what the authors themselves suggest. Section 4 extends the sample with two military studies by Haywood and Mongin, a group of studies on the Cuban crisis, which are included here only to facilitate comparison, several security studies by Zagare (in particular taken from his 2011 reference book *The Games of July*), and a study of post-communist political transitions by Nalepa. Section 5 discusses how AN contribute to historical explanation, by reference to the deductive scheme of scientific explanation. Section 6 deals with the narrative element of AN, arguing that it can make up for some of the failures of the deductive scheme that the previous section pointed out. This concludes our assessment of the contribution AN make to historical explanation, at the same time as establishing our proposed definition for this genre. Section 6 also sets out a taxonomy of AN based on the way narratives enter the exposition, the three categories being alternation (of the narrative with the model), local complementation (of the narrative by the model) and analyzed narratives (in which the model and the narrative are merely juxtaposed). Section 7 briefly concludes, suggesting that AN may be a tool for economic history.

2. The five studies of *Analytic Narratives*

Here we simply review the five case studies presented in *Analytic Narratives*, following the chronological order adopted in that book. The next section will use this major sample to introduce a more general discussion of AN.

Case 1: Middle-Ages Genoa (Greif). In the Middle Ages, the city-state of Genoa was first governed by elected consuls (1096-1194) and then by an appointed magistrate, the *podestà*, who was chosen from outside the city (1194-1334). Under the consulate, civil peace prevailed from 1096 to 1164 (period I), and then there was civil war lasting from 1164 to 1194 (period II). Under the *podesteria*, civil peace prevailed throughout (period III). Genoa's main economic activity was long-distance trade in the Mediterranean, and this activity was prosperous concomitantly with civil peace, i.e., for periods I and III, but with a noticeable peak at the end of the former. The main actors of economic and political life were the clans,

which appear to have kept their identity and relative influence fixed for much of the period under study. In view of this fact, the time sequence emerges as problematic. Why did the clans first cooperate and then fight under the politically unchanged conditions of the consulate? Why did they cooperate most efficiently at the end of the first period of civil peace? How did the institutional move to the *podesteria* contribute to reestablishing the civil peace that prevailed henceforth and why did it occur when it did? These are Greif's main explanatory questions. He notes that the historians' work fails to answer them satisfactorily, or even to raise them in full clarity.

Greif responds by constructing two classes of extensive-form games of perfect information. He then investigates their subgame perfect equilibria, as is classically done for such games.[1] The first class, which relates to the consulate regime, has two games, both of which involve the clans as players; the difference between them hinges on whether the number of maritime possessions of Genoa is taken to be exogenous or endogenous. We will report only on the simpler of the two, which is the game with an exogenous number of possessions.[2] This game explains the changes from period I to II by using the external threat posed by the German Emperor as a variable parameter. Depending on whether the threat is absent or present, Greif retains a different subgame perfect equilibrium – here relabelled as *mutual deterrence equilibrium* (MDE). The presence of this threat pushes the clans towards *mutually advantageous* MDE by the following mechanism. In general, clans compete to gain control over the consulate, which would guarantee them a higher share of trade benefits, and this competition stabilizes peacefully only because they spend on deterrence resources they could more profitably spend on joint trade; this is what MDE formally captures. Now, the controlling clan also incurs the burden of external wars when they happen, so that the external threat changes the clans' *ex ante* net benefits of conquering the consulate; this is why MDE with fewer resources spent on deterrence, and more on joint trade, arise when there is such a threat. The chapter presents his first class of games only informally; a full treatment appears in Greif (2006, ch. 8).

[1] For the game theory discussed in this chapter, see the texts by Morrow (1994) and Harrington (2009), and at a more advanced level, by Myerson (1991) and Osborne and Rubinstein (1994).
[2] Only the second game makes it possible to investigate the clans' trade-off between fighting a civil war to gain control of Genoa and peacefully collaborating to get more maritime possessions.

The second class has a single game, which is intended for period III and has the *podestà* as a third player. Among others, it captures a clan's two strategic possibilities of accepting the *podestà*'s authority or attempting to take control of Genoa, at the risk of starting a war against the *podestà* and the other clans. For relevant parametric conditions, this game has a subgame perfect equilibrium that explains the stabilizing effect of the *podesteria*. At this equilibrium, clan 1 (which makes the first move) abstains from challenging clan 2; clan 2 (which reacts to clan 1's move) fights if challenged; and the *podestà* (who reacts to the two clans' moves) joins forces with clan 2 against clan 1 in case of a fight, but colludes with clan 1 otherwise. That the *podestà* can possibly collude with clan 1 motivates clan 2 to fight, and that the *podestà* can possibly support a fighting clan 2 motivates clan 1 not to challenge in the first place. As entailed by subgame perfect equilibrium, these two threats are credible. The parameters on which the existence of this equilibrium depends are the players' probabilities of victory and defeat and the accompanying payoffs. Most important are the parameter values for the *podestà* since they ought to match his reward scheme and military means, as described by historians. Greif's chapter presents the *podesteria* model in complete detail (see also Greif, 2006, ch. 8).[3]

Case 2: Ancien Régime finances (Rosenthal). A classic historical problem concerns understanding why the pace of institutional change differed between France and England in the 17th and 18th centuries, with one country keeping the absolutist monarchy until its final disruption, while the other moved gradually towards representative government. Rosenthal reconsiders the problem in the light of the two countries' difference in fiscal structure. Given that the product of taxes was mostly spent on wars, this leads him also to raise another question, i.e., how a country's style of warfare relates to its political regime.

The examination is carried out in terms of an informally stated model that an appendix makes formal. There are two actors, the King and the elite (an abstraction representing the parliaments in France and England, and the provincial estates in France, where they existed), who enjoy separate fiscal resources and try to make the best of them in fighting profitable wars. By assumption, the King alone has the power to launch a war, and if he exerts it the elite decides whether or not to participate financially. Since most wars need joint funding, there is a free rider problem, which, the model shows, is more acute when the fiscal resources

[3] Critics of Greif's approach to the *podesteria* have complained that this institution originated in a decision made by the German Emperor, not by the Genoese. See however, fn 4.

are shared between the King and the elite than when they are in one player's hands. This translates into the prediction that wars are more frequent, the higher the King's share of fiscal resources. For Rosenthal, France's absolutism was a case of sharing, whereas England's representative government was one of near control by the elite. Hence, he has a rough prediction to test on the two countries, and can address the issue of how their warfare relates to their regimes. His model also implies the correct prediction that the overall level of taxation was higher in England than in France. However, it is unclear how it addresses the initial question of the different pace of political change in the two countries.

Case 3: 19th century conscription (Levi). In the 19th century, several Western states changed their regulations of military service, moving from conscription with provisions for buying out of the duty to more or less universal conscription. Historians have usually emphasized democratization and military efficiency as being the two likely reasons for this. However, the latter is technically doubtful (a professional army would have dominated all other arrangements), and the former is objectionable in view of the timing of reforms (they often took place either before or after universal suffrage prevailed). Starting from these objections, Levi compares the changes in France, the United States, and Prussia, paying attention not only to the chronological pattern but also to the variable pattern of buying out (there were three distinctive forms, i.e., substitution, replacement, commutation). She does not mean to displace the previous explanations entirely, but rather to subsume them under her own.

To do so, Levi develops an informal analysis in the spirit of formal political economy, whereby three main actors contribute to shape national decisions on the conscription regime. They are the army, which wants only military efficiency; the government, which balances it against social and economic considerations, such as employing the population efficiently; and the legislature, which aligns itself with the coalition among three social groups (traditional elites, middle class, and workers) that make up the constituent body. With this construction at hand, the pattern of reform in each country can be explained by hypothesizing changes in the actors' motivations. Levi proposes two such changes, i.e., the increased demand from army and government for troops, and the legislature's evolving preferences, both of which push in the direction of universal conscription. She relates the latter change to a reshuffle within the politically influential coalition (the pivotal middle class turning away from the traditional elites and becoming allied with the workers), as well as to an increased taste for equality

among the social groups. The two main hypotheses from the historical literature appear again, though included within a systematic explanation scheme. The study draws on Levi's (1997) thorough antecedent work on the same topic and thus includes rich historical evidence.

Case 4: Antebellum Federation (Weingast). Historians of the United States have long been puzzled by the relative stability of the federation through the decades that preceded the Civil War. Classically, they argue that slavery was at first not as divisive an issue as it would become, and that the Democratic Party after Jackson successfully managed a coalition of southern and northern interests. Others have emphasized the role of local political issues and changing economic conditions. Weingast includes these factors in a narrative that stresses explicit political arrangements, especially the following *rule of balance*: slave states should remain equal in number with free states, so as to provide the South with a veto power in the Senate. The narrative records the crises that the Union underwent each time the admission of a new state threatened the balance. The first crisis led to the emergence of the compromise rule, which helped resolve the second, but did not work with the third. This ultimate failure depended on an admixture of economics and politics: to keep an effective balance despite the continuing expansion to the West, the slave economy would have had to develop beyond its feasible limits.

Weingast includes three formal models in his narrative, the first of which is of the spatial brand of voting theory. This model aims at weighing the political influences on the politics of the Union of the agrarian South, commercial Northeast and intermediate Northwest, respectively. When these three actors differ only on the economic dimension, the Northwest acts as an electoral pivot, and the Union as whole inclines in the direction of agrarianism, because the Northwest is closer to the South than the Northeast on this particular dimension. However, if slavery enters the political debate, the previous conclusion does not necessarily hold because the Northwest is closer to the Northeast than the South on that dimension. The spatial model makes it possible to clarify coalitional possibilities when a political debate has two dimensions. Following a related treatment, Riker (1982) had famously claimed that it was to some Northeast politicians' advantage to introduce slavery on the electoral agenda, and it became a political issue after 1830 precisely for that reason (the so-called *Riker thesis*). The other two models used in Weingast's study are extensive form games of a straightforward sort. Comparison of their subgame perfect equilibria shows that giving the South a veto power has the effect of blocking the compromise between the Northeast and the Northwest

that would otherwise prevail. This reinforces the general point that the rule of balance was an important component of federal stability in antebellum America.[4]

Case 5: ICO (Bates). From 1962 to 1989, the International Coffee Organization (ICO) regulated the international prices of coffee by setting quotas on the exports of its members, notably Brazil and Colombia, which were the main producers. Bates accounts for the birth, regular functioning and final collapse of this institution. This involves him using, or at least mentioning, various game-theoretic tools, but the study nonetheless follows a classic narrative structure, with a beginning, an end, and intervening steps that exactly reproduce the objective sequence. While the other studies state their explanatory problems in advance and subject their narrative parts to the solution of these problems, this one lets its explanatory puzzles and answers emerge as the story unfolds.

The birth of the ICO raises one such explanatory puzzle. As early as the 1950s Brazil and Colombia had a cartel policy of restricting quantities and boosting prices, and tried to attract other coffee producers to this policy. However, it is only at the beginning of the 1960s that they succeeded in doing so and thus became able to establish the ICO. Bates explains why success was delayed by arguing that the proximate cause of its establishment was that the United States became favourable to the cartel policy. This is a paradoxical answer because the United States was on the consumption, not the production side. Bates makes this answer plausible by relating how Brazil and Colombia, having failed in their first attempt, turned to the US State Department brandishing the communist threat and the long-term advantages of a cartel organization, and eventually met with success when some large US coffee-selling companies decided to support their lobbying. Each of these small narrative segments is followed by an allusive formal argument that clarifies the strategic situation.

3. Some defining characteristics of analytic narratives

[4] Perhaps because its topic has been heavily researched, Weingast's study seems to have aroused special attention from readers of *Analytic Narratives*. Some have complained that it is not clear whether the rule of balance in the Senate was central to the stability of the Federation, and accordingly how much its collapse contributed to the civil war. A quick answer may be that the study selects one particular sequence of actions and events for investigation, and this provides a partial, but real explanatory argument (which incidentally comes out most clearly at the post-modelling narrative stage, see section 6 below).

The five studies of *Analytic Narratives* have a common focus on institutions, and more precisely on institutions that either belong to the internal state organization (cases 1 to 4) or indirectly depend on it (case 5, which deals with international relations). They also embody a common approach to the understanding of institutions. The key idea is that institutions operate not only through the formal rules that overtly define them, but also through implicit rules of behaviour that guarantee their function given the way participating agents respond to them. As an example, the *podesteria* can be viewed either by considering the official terms of employment of the *podestà*, or, more relevantly for an explanatory purpose, as a system of mutual threats between the clans and the *podestà* that made it possible for the latter to fulfil his role effectively. This heuristic is implemented by representing institutions in terms of equilibria of interactive processes, and it is at this juncture that game theory comes into play. Thus, Greif devises a game in which the existence of an equilibrium demonstrates that mutual threats can credibly balance each other, which secures compliance of the clans with the *podesteria* institution.

Conceptualizing institutions, and more specifically political institutions, as *equilibria of interactive processes*, whether by game-theoretic or other means, is a significant contribution to the neo-institutionalist school of thought. Elsewhere, Greif (2006, ch.1) clarifies the differences between this "self-enforcement" conception and those of previous neo-institutionalists. These writers had already discussed institutions from the perspective of the agents' interests, but in a somewhat more naïve fashion, often simply assuming that institutions are imposed "top-down" on the agents (what Greif calls the "institutions as rules" conception).[5] Two other major differences, even with historically oriented neo-institutionalists such as North (1981, 1990), have to do with the method of case studies adopted in *Analytic Narratives* and the specific attention this work pays to the role of narratives.

More crucially for our purposes, the five studies illustrate the two principles stated at the opening of this chapter. Each exhibits a collaboration between narrative writing and the employment of analytic tools, and each borrows these tools from the theories of rational

[5] An answer to the objection echoed in fn 3 is forthcoming along this line. The *podesteria* may well have been imposed "top-down" upon the Genoese from the outside, yet the question arises nonetheless of why they made its functioning possible, a question that the "institutions as rules" conception addresses. Clark's (2007) otherwise critical account of Greif (2006) clearly recognizes this.

decision-making. In the rest of this section, we expand on the second principle, using the studies as reference material. We defer an examination of the first principle to section 6, where the third principle will also be considered.[6]

Formal models of varying precision and sophistication occur in cases 1, 2, 4 and 5, though not in case 3. One may thus wonder whether or not AN in general must involve a formalism. As for the studies that do employ one, they primarily rely on extensive form games of complete information, and one may thus wonder how much this choice of models matters to AN. This section answers these two questions by arguing that (i) *AN do require formal models* and (ii) *AN can borrow these models from any formal branch of the theories of rational decision-making*. These two answers spell out what the have only suggested (in *Analytic Narratives*) or briefly stated (in their informative rejoinders to critics; see Bates et al., 2000a and 2000b). Thus, they acknowledge that their "restriction of models to extensive form games limits the range of issues [they] address" (2000b, p. 691), and they endorse "the requirement of a formal model" (p. 693). This amounts to making claims (i) and (ii).

One argument for restriction (i) is that ignoring it would take the edge off the AN methodology. Historians already borrow from common-sense ideas on individual rationality to confer explanatory value on their narratives. But they rarely make these ideas explicit, and this may be for two reasons: they may regard them as being too banal to be stated, or they may consider that a fuller statement would break the narrative flow. As they are not subject to the same discursive constraints as ordinary narratives, AN can unfold the terse suggestions made by the latter and thus attempt to enhance their explanatory value. To do that, AN bring in specialized concepts of individual rationality: but if they eschew formalization of these concepts, they may be hardly different from the scholarly expansions that historians append to ordinary narratives in the introductions, conclusions, and appendices of their books. Interestingly, when revisiting *Analytic Narratives*, Levi (2002, p. 109) claims that the essays

[6] *Analytic Narratives* has given rise to a rather large number of discussions, which space reasons prevent us from covering here. The reader may in particular consult *American Political Science Review*, 94, 2000, no 3, and *Social Science History*, 24, 2000, no 4, which contain one or more reviews followed by a rejoinder from the five authors. Some of these discussions express strong scepticism about either the individual contributions or the methodological project itself; among the reasons for this scepticism is the routine complaint that "rational choice theory" (whether formal or not) is either flawed or inapplicable. None of these discussions – even the favourable ones – properly recognizes the special function that narratives, as against other forms of reporting of historical events, fulfil in the AN methodology.

in this book "do not represent a breakthrough". It seems more fruitful, however, to allow that they do introduce something new – for better or worse. With (i) included in their definition, AN create an unusual tension between the narrative and the formal modelling. How this tension can be managed is the most exciting problem AN raise for the methodologically minded social scientist.[7]

There is actually a more direct reason for supporting restriction (i). The AN methodology crucially relies on the concept of equilibrium, understood one way or another, and the full development of this concept plainly requires a formalism. Levi rightly observes that *Analytic Narratives* makes extensive use of comparisons of equilibria: "the emphasis is on identifying the reasons for the shift from an institutional equilibrium at one point in time to a different institutional equilibrium at a different point in time" (2002, p. 111). This is the method of *comparative statics*, which economists implemented and made famous before passing it on to other social scientists. However, the method will remain at a heuristic level as long as one does not select a formal theory – e.g., that of extensive form games of complete information – and specify a model within that theory – e.g., by fixing a set of players, a set of strategies and preference orderings that fully define the game to be studied. The comparative statics exercise, which is quantitative by essence, is possible only if some of the data of the modelling stage – e.g., the preference data of the game - are stated parametrically, with a range of numerical values set for each parameter. The exercise then consists in deducing how the equilibria change as a consequence of the parameters varying within their ranges. Even more clearly than in *Analytic Narratives*, this will be illustrated in Zagare's (2011) *The Games of July* (see cases 9 and 10 in next section).[8]

Having defended restriction (i), we move to the generalization proposed in (ii). A common thread between cases 1, 2, 4 and 5 is their reliance on non-cooperative games of extensive form. In their introductory manifesto, the authors defend this particular form on the ground that they focus on "sequences of actions, decisions, and responses that generate events and outcomes" (Bates et al., 1998, p. 9; see also Levi, 2002, p. 111). The implicit claim is that the sequence of moves in the game is capable of paralleling a concrete sequence of actions and

[7] One of the first works to explore this tension is the collection by Grenier et al. (2001); it does not yet refer to AN.

[8] In this paragraph, we imply the familiar conception of a *model* as a construct mediating between theories and real objects. The alternative conceptions canvassed in recent philosophy of science could also be brought into relation to AN.

reactions by historical actors. One may however doubt that such nice parallelism really takes place. Consider the *podesteria* game; it involves moves such as "to challenge", "to fight", "to prevent", which can hardly represent genuine actions by the clans or the *podestà*. If the moves are idealizations, their sequential ordering cannot represent the passing of historical time; and indeed, if the *podestà* plays last rather than first in the game, this is for theoretical convenience, not descriptive accuracy. Of course, this semantic observation does not entail that extensive form games are unimportant to AN, but only that this genre has no privileged association with them. Cases 6 and 7 in the next section will show that games in normal form can be equally relevant. On a different note, cases 1, 2, 3 and 5 share a limitation by only considering extensive form games of *perfect information* (or possibly extensions of these to exogenous uncertainty). This conveniently guarantees that solutions to the games can be found by backward induction, as the subgame perfect equilibrium requires in this case. As next section will also show (cases 9, 10 and 11), AN can support the more sophisticated formalism of extensive form games of *imperfect information*, along with the perfect Bayesian equilibrium concept, which is commonly used to solve these games.

AN can be developed in other technical directions than noncooperative game theory. Cooperative game theory may provide appropriate models when it comes to analyzing the formation of coalitions, as in the ICO case.[9] Furthermore, not every historical state of affairs that involves multiple individuals calls for a game-theoretic analysis: individual decision theory, whether of the expected utility form or others, may be sufficient for the modelling purpose. This is the case by definition when the multiple individuals face natural uncertainty, but also and more subtly when social uncertainty can acceptably be represented *as if it were natural*. When commenting on Clausewitz's military narratives, Mongin (2009) argues that some of his judgments can plausibly be reconstructed as expected utility comparisons; this is so despite the fact that the situations are strategic in an intuitive sense. There seems to be no rule to determine when game theory is indispensable to the analysis of interaction and when it is not. It is useful to register this indeterminacy, and then avoid restricting the technical apparatus of AN beforehand.[10]

[9] Bates suggests considering the Shapley value for this purpose. For this and other concepts of cooperative game theory, see Myerson (1991) or Osborne and Rubinstein (1994).
[10] Schiemann (2007) promotes a further extension of AN to behavioural economics, and illustrates this by a study of an event from the Yugoslav civil wars in the 1990s.

3. Analytic narratives from military and security studies

While the previous AN belong to the historical part of political science, those covered in this section mostly belong to the historical parts of military studies (cases 6 and 7) and security studies (cases 8, 9 and 10). We distinguish between the last two fields as follows: military studies is concerned with actual war actions – battles, campaigns, guerillas, information wars and the like – and security studies with actions taken under the shadow of war, i.e., facing the possibility of wars that may or may not break out. As two game-theoretic contributors to security studies write, "the games we analyze are not war games as such, but the choice that players make may precipitate conflict that leads to war" (Brams and Kilgour, 1988, p. 3). This basic distinction is sometimes overlooked, which is unfortunate because military and security studies have different conceptual orientations.[11]

Case 6: World War II battles (Haywood, 1954). Shortly after World War II, Haywood, then a colonel in the US Air Force, discovered von Neumann and Morgenstern's (1944) work and made the first ever application of game theory to war events, publishing a sketch in 1950 and a detailed version in 1954. For this application, he selected a 1943 naval battle in the Pacific War and a strategic turning point in the 1944 Normandy campaign. His main concern was to connect the US military doctrine of decision, which was prescriptive, with von Neumann and Morgenstern's Min-Max solution for 2-person zero-sum games, which he also regarded as being prescriptive. He argued that the military doctrine of decision was right in prescribing officers to act on an estimate of the enemies' capabilities, not on a guess of their intentions – the argument being that if the game has no Min-Max solution in pure strategies, guessing intentions leads to an infinite regress of strategic calculations. Despite this prescriptive orientation, Haywood's inquiry has some bearing on historical explanation.[12]

In the Bismarck Sea battle of February 1943, the US Air Forces destroyed a naval Japanese convoy that was sailing from Rabaul on New Britain Island to Lae on the New Guinea coast. Of the two possible routes, north of New Britain and south of it, the Japanese commander had chosen the former. Unaware of this, the US general in charge had to choose between concentrating his reconnaissance flights on one route or the other, and he actually took the northern option, whence his crushing victory. Haywood argues for the rationality of both

[11] See Betts (1997) for a more thorough discussion that includes a history of security studies.
[12] Haywood has rather mysteriously disappeared from the academic scene, despite Brams's (1975) and Harrington's (2009) supportive reviews of his contribution.

moves, including the Japanese one. Having pointed out that the northern route was mistier than the southern one, he computes the number of bombing days associated with each of the four possible outcomes and solves the resulting 2x2 zero sum game using von Neumann and Morgenstern's solution. Here Min-Max reasoning on the two sides leads to an outcome that fit the facts, which confers some explanatory value on this reasoning. The other application is the Avranches battle fought between US General Bradley and the German general von Kluge in August 1944. Haywood analyzes it in terms of a 3x2 matrix, which this time has no pure strategy solution. As he does not quantify the payoffs, he cannot exhibit mixed strategies solutions, and this leaves one in doubt about what he achieves in terms of explanation.[13]

Surprising though it seems, game theory rarely enters military studies as properly defined. Applications of a prescriptive or instrumental nature certainly exist, the best known being those pursued in the 1960s and 1970s for the RAND Corporation and some US military agencies (see, e.g., Erickson, 2015). But it seems as if the historical part of military studies has no genuine game-theoretic application to offer between Haywood's and the next case to be reviewed. This is not to say that scholars in this area have no interest in history. To the contrary, there is a long tradition among military strategists, which dates back to Jomini and Clausewitz, of basing their thinking on a careful examination of past battles and campaigns. However, this tradition is almost entirely narrative in the ordinary sense; so much so that it acted as a foil to the anti-narrative position epitomized by the *Annales* school historians in the middle of 20th century, notably Braudel (1969).[14] By revisiting the Waterloo campaign, Mongin (2008, 2018) attempts to show that it is possible to turn even a worn-out example of military narrative into an AN.

Case 7: The Waterloo Campaign (Mongin, 2008). As is well known, Napoleon's return to power in 1815 ended with his resounding defeat by Wellington and Blücher on the battlefield of Waterloo in Belgium. On June 16, the campaign began favourably for him, with the French beating the Prussians at Ligny, near Charleroi. On June 17, Napoleon decided to send a large detachment under Marshal Grouchy against the defeated Prussians, and he took the rest of his army to Waterloo, near Brussels, where the English and Dutch were ready for a

[13] At any rate, a later historical discovery showed that Bradley had in fact been cognizant of the orders received by von Kluge, as the Allies had broken the German Enigma code (see Ravid, 1990).
[14] Clark (1990) conveniently summarizes the position taken by post-war *Annales* historians; see also Stone's (1979) critical discussion.

defensive battle. On June 18, the French failed to break through the enemy lines and were eventually crushed when the Prussians came as additional help. Despite innumerable histories, an explanatory gap remains in this sequence: why did Napoleon decide to send out Grouchy's detachment? By doing so, he ran the risk of not having it on his side when he faced Wellington, or, much worse, Wellington and Blücher together if they managed to join forces, a possibility that effectively materialized.

To make progress with this explanatory question, Mongin proposes a zero-sum game in normal form with two players, Napoleon and Blücher, allowing for uncertainty in several ways. First, both Napoleon and Blücher are uncertain of which battles will result from their independent decisions. Specifically, Napoleon can keep his army united, dispatch Grouchy for a pursuit of Blücher, or dispatch it for interposition between him and Wellington; and on his part, Blücher may either retreat to Germany or try to join Wellington at Waterloo. This is nothing but standard strategic uncertainty. However, a second form of uncertainty enters, since Napoleon does not know Blücher's type – here whether or not the latter was badly weakened after Ligny – and this means that the game is one of incomplete information. Third, external circumstances matter besides the players' decisions, and both are *ex ante* uncertain of the issue of each given battle. This is nonstrategic information, which is treated here as if it were objective and amenable to common expected utility calculations by Napoleon and Blücher.

Given suitable parameter restrictions, von Neumann and Morgenstern's Min-Max solution delivers a unique equilibrium, which involves pure strategies. As in Haywood's example, this arguably delivers not only an equilibrium, but also rational choice recommendations. Napoleon should choose to dispatch Grouchy for interposition, and Blücher should try to join Wellington. That Napoleon effectively chose interposition, rather than mere pursuit, can only be conjectured from the historical record, but the game reinforces this hypothesis. The *ex post* failure is not an objection since it could result from an unfavourable resolution of objective uncertainty and from Grouchy misapprehending the plan – some historical evidence points in these two directions. Overall, the study exemplifies how an analytic narrative can be both formal and interpretive, since assumptions and conclusions are assessed in terms of evidential reports that are always incomplete, equivocal, and, given the high stakes, unavoidably biased. The conclusions adjudicate among existing positions, indeed by reinforcing classic pro-Napoleonic arguments against equally classic anti-Napoleonic ones.

Case 8: The Cuban Missile Crisis (various authors). Few diplomatic events have raised more scholarly interest than the crisis that took place from October 16 to 28, 1962, between the United States and the USSR. On 16 October, 1962, President Kennedy was shown U2 photographs demonstrating that the Soviet Union was building missile bases in Cuba. Kennedy and his advisers pondered over several options, which included doing nothing, making a diplomatic move, bombing the missile sites, and blockading Cuba with the US Navy. Deliberation and further investigation led to the blockade decision of 22 October. Khrushchev was notified and the blockade was then publicly announced to the nation. In the ensuing days the crisis deepened, with some secret diplomacy nonetheless taking place. It was eventually resolved on 28 October, when Kennedy and Khrushchev managed to coordinate on a compromise solution. Essentially, in return for the USSR removing its missile systems from Cuba, the United States would lift the blockade, pledge not to invade Cuba, and – this was a later and secret part of the deal – remove missiles from Turkey.

Innumerable accounts of this famous sequence have circulated, with the flow being sustained by the appearance of declassified secret material (see, e.g., Allison's 1999 revision of his classic 1971 study). Among the accounts based on, or inspired by, game theory, none seems to us sufficiently rich in narrative content to qualify as an AN. Rather, they treat the Cuban Missile Crisis as a mere application of theoretical ideas, and, if we include this topic here, this is because it offers a touchstone of *deterrence models*, which recur in the AN literature. As Zagare (2014) has explained, the game-theoretic literature on the Crisis has gone through three essentially different stages. While the first authors, like Schelling (1960), gestured towards game theory rather than actually used it, a second wave from the mid-1970s onwards exploited 2x2 normal form games such as the Prisoner's Dilemma and Chicken, sometimes adding ingenious variations to them. A third wave, which began in the mid-1980s, adopted extensive-form games, whether of complete or incomplete information, to analyze deterrence. Examples of the second wave appear in Brams's (1975) *Game Theory and Politics*, and at a more advanced level, in his *Superpower Games* (1985).[15] For the third wave, which he associates with a "sea change", Zagare (2014) mentions an early model by Wagner (1989)

[15] In the latter work, Brams introduces a 2x2 game that schematizes the American and Soviet choices (Blockade and Air Strike, Withdraw and Maintenance, respectively) and applies his "theory of moves" to find that the Compromise issue (Blockade, Withdraw) emerges as a "non-myopic" equilibrium.

and his own "perfect deterrence theory" (as developed in Zagare and Kilgour, 2000), both of which involve extensive form games of incomplete information. This leads us now to a specific examination of Zagare's contributions to AN.

Case 9: The Moroccan Crisis of 1905-1906 (Zagare, 2015). After they had overcome their conflict over Sudan in 1898, France and Britain moved towards an alliance that materialized in formal agreements in April 1904 (the so-called *Entente Cordiale* agreements). The most important of them involved a trade of influences, with France supporting Britain's leading role in Egypt, and Britain supporting France's freedom of action in Morocco (with some role conferred on Spain in the northern part). Having not been consulted at all over Morocco, and also responding to the Sultan's wish to counter France's threatening influence on his country, Germany championed its sovereignty and an "open door" policy for foreign trade and investments. The German diplomatic pressures on the French government led in 1905 to the resignation of foreign minister Delcassé and the reluctant acceptance by Président du Conseil Rouvier of an international conference. Officially devoted to the economic and administrative reforms that Morocco needed, the conference took place from January to April 1906 in Algeciras. While Germany hoped to drive a wedge into the Entente Cordiale and score a diplomatic victory over France, Britain supported its ally and Germany ended up almost entirely isolated, getting only limited concessions that would not suffice to curb France's colonial activism.

Zagare (2015) revisits the 1905-1906 events by appealing to the *Tripartite Crisis Game*, which belongs to his more general "perfect deterrence theory". This is an extensive form game of incomplete information with three players acting sequentially as follows. Challenger can either keep to the *status quo* or make a demand on Protégé, who can either concede or hold firm, in which case Defender enters the stage by either supporting or not supporting Protégé. If Defender has supported Protégé, Challenger plays again by either backing down or accepting a conflict, and if Defender has not supported Protégé, the latter plays again by either backing down or realigning on Challenger's side. Information is incomplete in that each player has two possible types: Challenger may be "determined" or "hesitant", Protégé "loyal" or "disloyal", and Defender "staunch" or "perfidious". This figurative terminology captures the fact that, for each nation player, some of its preferences over the terminal nodes are unknown to the other two players. To deal with the 1905-1906 crisis, Zagare specializes the Triple Crisis Game by assuming that Challenger – here Germany – is "determined"; this

technically means that, at the last stage, Challenger prefers to accept a conflict rather than to back down. Thus, incomplete information is limited to Protégé and Defender – here France and Great Britain respectively. Technically, Protégé is "loyal" if, at the last stage, it prefers backing down to realigning, and Defender is "staunch" if it prefers reaching the node where Challenger accepts the conflict to reaching the node where Protégé realigns. Fixing initial probability values for Protégé being "loyal" and Defender being "staunch", Zagare shows how they get revised at the perfect Bayesian equilibria he computes. In the Moroccan study as well as in other recent articles and in his 2011 book, Zagare explicitly claims to be using the AN methodology.

Case 10: The July 1914 Crisis (Zagare, 2011). *The Games of July* (2011) investigates the diplomatic events that decisively contributed to the outbreak of World War I, particularly emphasizing four historical turning points. The first deals with a remote, but influential decision made by Bismarck in 1879 to offer a military alliance to Austria, despite the tension this created with Russia, which was the main target of this arrangement. The second relates to the unqualified support – or "blank check" – Austria obtained from Germany in early July 1914 to crush Serbia, and the third to the escalation of conflict with the other powers once Austria began taking action. The fourth is devoted to the British decision to maintain an ambiguous policy during the July crisis, a decision that may have misled Germany in believing in its neutrality and thus may have contributed to the outbreak of the war. Each turning point raises specific explanatory problems that a brief narrative and review of historical literature helps locate. The book answers them through the instrumentality of game-theoretic modelling, in accordance with a methodology that the author distils in preliminary chapters and identifies with that of AN (see also Zagare, 2009).

Each case relies on a game of its own, although all are taken from the common shelf of "perfect deterrence theory". The first, second and fourth sequences are handled by means of relevant variations of the Tripartite Crisis Game, and the third by means of the *Asymmetric Escalation Game*, which also belongs to "perfect deterrence theory". We focus on the fourth case, which is concerned with British policy, because this permits comparisons with the Moroccan case, in which this policy had already played a crucial role. The Liberal Grey, who had succeeded the Conservative Lansdowne at the Foreign Office in the midst of the Moroccan crisis, essentially pursued his predecessor's policy of supporting France without making any military commitment to it. The persisting problem for the British was to secure

peace on the continent by combining deterrence (of the Germans) and restraint (of the French, especially in their support to the Russians), and this led them to foster ambiguity on their final intentions. Whether it was to the point to maintain this carefully balanced policy in July 1914, as Grey did, is a major historical question. As is well known, it eventually needed Germany's invasion of Belgium on August 4 for Great Britain to engage militarily on the side of France and Russia.

As in case 9, the game-theoretic treatment proceeds from the Tripartite Crisis Game. Germany, France and Great Britain still occupy the roles of Challenger, Protégé and Defender, but this time Defender is "staunch" and Challenger may be either "determined" or "hesitant" (the latter means that, at the last stage, Challenger prefers to back down rather than to accept a conflict). As the "blank check" issued to Austria was not known to the other players, endowing Germany with two types appropriately represents this uncertainty, but it is not obvious that the staunch type describes how Britain was perceived in July 1914.[16] However, interesting mixed equilibria occur even under this limiting assumption, and they are consistent with Grey's "straddle strategy, as Zagare (2011, p. 160) aptly designates it, thus providing the British diplomacy with a rationale; (see also Zagare and Kilgour, 2006). As these equilibria seem compatible with the strategic situation more broadly, they might serve to capture the protagonists' effective interaction. Supposing they are indeed the historically relevant ones, the war would have broken out not because of Britain's ambiguity, which had a serious intent even though it was a gamble, but because in 1914, unlike in previous earlier crises such as the Moroccan one, the gamble turned out badly.

We close this section with another case that does not belong to either military or security studies, but rather to historical political science. Despite this thematic discrepancy, we include it here because it involves an extensive form game of incomplete information and the use of perfect Bayesian equilibrium as in cases 9 and 10. The game belongs to the class of deterrence models that bridges the work in security studies with some of the work in political science.

[16] The assumption made regarding Britain's type appears to be connected with a mathematical difficulty. The Triple Crisis Game can currently be solved only in limiting cases. Concerning the Moroccan crisis, the restriction was that Challenger was "determined", and here it is that Britain is "staunch" (Zagare (2015, p. 335, fn 7).

Case 11 (Nalepa, 2010). During the transition from communism to democracy experienced by eastern European countries, communist officials had to choose, roughly, between opposing the trend as much as they could, or seeking a deal with the democratic opposition such that they would retreat from power in return for a promise that they would not later be banned from public functions. Nalepa (2010) starts from the observation that, in some countries, the communists reached such a compromise with the democrats and the democrats kept their promise (at least by and large and for some period of time). This is puzzling because the democrats had every reason to renege on it. However, as the author argues, the communists did have a way of avoiding this. The communist secret police had once infiltrated opposition movements, so the democratic parties, which were heir to these movements, were themselves in some danger of falling prey to a ban under similar "transitional justice" measures. Only the communists knew the extent of the infiltration, and this gave them an informational advantage over their opponents. An explanation of the historical puzzle is forthcoming along these lines, and Nalepa, who claims to be using the AN methodology, substantiates it by game-theoretic modelling.

The first model, which she attributes to Przeworski, is a perfect information extensive form game in which the communists anticipate the democrats' disavowal and choose opposition rather than compromise. This model is of course a strawman, since it never allows for the possibility of compromise. The second model, which is Nalepa's, introduces asymmetric uncertainty and comes close to a signaling game. By assumption, the communists know exactly the percentage of infiltration among democrats, who, being entirely ignorant, form a uniform probability on this parameter. If the communists choose to compromise rather than oppose, democrats read this move as a signal that they are infiltrated to a significant extent and revise their probability accordingly. This informational exchange is captured in terms of the perfect Bayesian equilibrium concept, which we have already encountered in cases 9 and 10. The study closes by comparing some of the available equilibria obtained from this concept with historical situations. Unlike in Czechoslovakia, where the communists opposed the democrats until they collapsed, compromises prevailed in Poland and Hungary, and for relevant parameter values there exist equilibria related to these situations.

5. Analytic narratives and deductive explanation

The studies covered in sections 2 and 4 suggest some generalizations regarding the explanatory potential of AN. First, with the exception of case 5, which belongs to recent history, and case 6, which similarly belonged to recent history when it was written, they rely on an extensive scholarly record they use not simply to determine the factual data, but also to suggest problems to be solved. The record is usually of the traditional narrative brand, and they identify the problems by noticing explanatory gaps within it. For example, case 1 revisits the alternation of civil war and peace in Genoa with a view to explaining it, which had not really been done before. Case 3 revisits the establishment of universal conscription with a view of synthesizing explanations that hitherto had only been partial; and case 7 revisits the Waterloo campaign with a view to arbitrating a classic disagreement among historians. Moreover, the problems appear to have been selected by carefully considering what the importation of analytic tools could add to the more traditional treatment. As Bates et al. (1998, p. 13) write, "our cases selected us, rather than the other way around". The only exception here appears to be case 2, the topic of which – the Ancien Régime finances in comparative perspective – is arguably too wide for the analysis to get much grip on it. Mongin (2008, 2018) goes as far as to claim that starting from the extant historical literature, defining problems based on the lacunas therein, and restricting the models to limited fragments of it, are preconditions for one's adopting the AN methodology.

One may observe, however, that the problems are not exclusively of an explanatory nature.[17] The Waterloo study aims not only at ranking competing explanations but also at substituting some missing data – what instructions Napoleon gave to Grouchy – with a deduction from the model. Here the gaps in the earlier narratives concern *the facts of the matter*, and not the explanation by itself. Less ambitiously than this substitutive role, though, AN can orient factual research in novel directions, as do other forms of problem-inspired history, like that promoted by the *Annales* school. However, original scholarship has thus far been exceptional among AN contributors, and they do not seem yet to have moved existing scholars towards new agendas.

From the angle of the philosophy of explanation, AN seem naturally to connect with the *deductive scheme* proposed by Hempel (1965), Nagel (1961) and many others.[18] Both the *Analytic Narrative*s team and Zagare mention this well-known scheme (though without

[17] Discussions of AN rarely make this point; see however Downing (2000, p.91).
[18] Useful critical summaries appear in Salmon (1992) and Bird (1998).

detail).[19] Broadly speaking, it postulates that to explain a particular fact is to deduce the statement that the *explanandum* fact occurs from statements that other facts occurred, along with statements of generalities so as to effect a connection – these facts and generalities being offered as the *explanans*. Besides requiring the logical correctness of the deduction, the scheme places epistemic requirements on the constitutive statements, and philosophers of science have dissenting formulations here. However, they all agree on the two basic points that the *explanandum* statement must be known to be true and the *explanans* statements must be, if not necessarily known to be true, at least empirically well supported. The rest of this section discusses the extent to which AN explanations fit with the deductive scheme; we will exhibit significant discrepancies, and thus prepare our ultimate claim that narratives are an essential component of these explanations. This discussion first singles out the *deductive requirement* of the scheme (not to be confused with the scheme itself), and then proceeds to the *epistemic requirements* that the scheme also involves. Since we mean to follow the existing literature, we focus on the use of game theory; but our conclusions can to a degree be generalized to the other formal theories discussed in section 3.

If the deductive requirement is to apply to AN effectively, it needs to be adapted to the distinction between *statics* and *comparative statics* that runs across them. One may consider a game either *specifically*, i.e., for fixed values of its parameters, or *generically*, i.e., by not restricting the parameters or (more commonly) restricting them minimally.[20] Although not entirely sharp, this distinction points towards two different possibilities for deduction. What can be deduced in the case of a specific game is that given outcomes occur as equilibria of that game, and in the case of a generic game, that different outcomes occur as equilibria when the instantiation of that game changes with the parameter values. This is the statics versus comparative statics distinction, as it emerges from the use of game theory.[21] Correspondingly, one may either explain a given historical fact by associating it with an equilibrium of a specific game, or explain a change in historical facts by associating it with a change in the equilibria of a generic game. Comparative static explanations are logically more powerful than static explanations and should be preferred in principle. However, the AN literature makes it plain that comparative static explanations are not easy to come by. An exceptionally clear example appears with Zagare's multiple versions of the Triple Crisis Game, each of

[19] See Bates et al. (1998, p. 12 and 2000a, p. 697) and Zagare (2011, p. 5-7).
[20] Game theory has another technical sense for the word "generic"; this will not be considered here.
[21] Other formal theories would specify the distinction somewhat differently.

which is investigated in terms of comparative statics.[22] Haywood's simple analysis of two specific games represents the polar opposite case of a merely static explanation. Although more sophisticated, Greif's, Rosenthal's and Mongin's analyses fall more on the static side, since they let parameters vary only in relation to given equilibria, so as to secure sufficient conditions for the existence of these equilibria, and they do not study the dependence of equilibria on parameter values across the full range of these values.

Having clarified this preliminary distinction, we can explicate two difficulties AN must face in seeking to satisfy the *deductive requirement*. The first has to do with the *multiple equilibria* occurring under most equilibrium concepts used by AN. When the multiplicity occurs in a specific game or – for some fixed values in the parameter range – in a generic game, the game-theoretic assumptions do not suffice for a definite conclusion, and the deductive machine needs supplementing by some external selection procedure. When the multiplicity occurs in a generic game simply because the parameters change, the situation turns out for the better; now the deductive machine works autonomously. Then the next step will be to compare the equilibria and their underlying parameter values with the available historical evidence. Contributors who claim that the deductive scheme is relevant to AN seem to have this favourable case in mind.[23]

Second, there is the troubling problem of deciding what in the games plays the role of the *generalities* that the *explanans* must contain if the deductive requirement is to come into effect. In a static exercise, the natural candidate for this role is the chosen equilibrium concept, e.g., subgame perfect equilibrium, the Min-Max solution, perfect Bayesian equilibrium. In a comparative static exercise, it makes sense to consider as generalities not only the equilibrium concept but also the generic game (or, at a higher level, the class of generic games, the Triple Crisis Game being such a class). So we do find general statements in AN explanations. However, they are general only in the sense of being expressible as logically universal statements, not necessarily in the deeper sense of being *nomological*, that is of counting as putative laws of nature. Hempel's (1965) paradigmatic version of the deductive scheme proposes various conditions, besides the logical form, for a generality to be

[22] On the definitions given above, the Triple Crisis Game is not a generic game, but rather *a set* of such games. For instance, the specialized version for the Moroccan crisis is one such generic game, and the specialized version for Grey politics is another.
[23] See Bates et al. (1998, p. 15) and Zagare (2011, p. 16).

nomological.[24] The question arises whether the game-theoretic statements singled out above meet these conditions, but we will not try to answer it here, being content with stressing its relevance. AN contributors who allude to Hempel (Zagare, 2011, approvingly; Bates et al., 1998, disapprovingly) do not seem to have gone far into it. The common view among them is, more loosely, that the game-theoretic pattern uncovered in one study can be transferred with some success to other studies. Some go farther and claim to have at their disposal theories that apply across a significant range of historical states of affairs. This is the case with Greif (whose "theory of endogenous institutional change" includes the Genoa study as a particular application) and Zagare (whose "perfect deterrence theory" encompasses most studies relative to World War I and is also applied to some contemporary events).

We now consider how AN satisfy the *epistemic requirements* of the deductive scheme of explanation. As noted above, AN typically draw their problems from the extant historical literature and use little more than this corpus for checking their solutions empirically. To pass this empirical test, they need to answer three questions in the affirmative. (i) Do the equilibria of the games approximate what historians have observed concerning the *explanandum*? (ii) Do the game-theoretic assumptions that constitute the *explanans* draw support from what historians have observed concerning the circumstances of the *explanandum*? (iii) Is the *explanans* independently supported, i.e., does it also draw support from what historians have observed concerning other states of affairs than those under current investigation? We will review these questions in turn.

Regarding (i), there appears to be a gap between cases 1-4 of section 2, and cases 6-10 of section 4. The *explananda* of the second group are narrowly circumscribed in time and space, directly bear on interactive decisions, and often if not always involve designated individuals, such as Bradley, Napoleon or Grey. By contrast, the *explananda* of the first group extend rather widely across time, space or both, bear on institutional or organizational facts rather than interactive decisions as such, and without exception involve collective actors, such as clans, political elites or regions. To be linked to game-theoretic equilibria, the observable *explananda* of the first group need to undergo a more thorough abstraction process than those of the second group. This makes their explanations *prima facie* more debatable than the others are. Pushing this line, Mongin (2018) recommends applying the AN methodology

[24] These and other conditions have been thoroughly discussed in the philosophy of science; see, e.g., Bird (1998, ch. 1).

preferentially to *explananda* that share with military and security *explananda* the convenient properties of being spatio-temporally well defined, and involving recognizable decisions made by recognizable historical actors. However – as also pointed out by Mongin – such recommendations threaten to trivialize AN. The explanations of the first group are more challenging than those of the second, which may remain too close to the historians' accounts to bring much illumination. It seems as if a balance needs to be struck between the two dangers of arbitrariness and pedestrianism.

When it comes to (ii), the question of the identity of players arises again, and there are now the further questions of endowing them with relevant *strategy sets* and *preference orders*. AN keep the number of players to a bare minimum. This may be easier to accept when players are hypothetical constructs, as in the first group of studies, than when they are identifiable historical figures, as in some studies of the second group. Indeed, somewhat shockingly, Grouchy does not enter the Waterloo game, and the games for the July crisis never include all major powers together.[25] Technical convenience explains these lacunas: thus reduced, the Waterloo game can accommodate some informational complexity, and the July 1914 games can be resolved despite their rich informational structure. For similar reasons, AN tend to rely on rather small sets of pure strategies. To allow for mixed strategies enlarges the players' possibilities, but like much of game-theoretic economics, the AN literature is reluctant to take this option; only cases 9 and 10 make a significant exception. Historians will no doubt complain that the definition of both players and strategies in AN impoverishes or distorts the historical evidence.

The definition of the players' preferences is even more problematic. A modestly sized set of strategies, and hence of outcomes, is already enough to turn preferences into complex objects. Thus, *podesteria* with seven outcomes, and Triple Crisis with six, induce 7! and 6! possible orderings, not small numbers. Moreover this computation assumes there are no indifferences. Zagare (2015, p. 332) contrasts two inferential methods to define preferences sensibly: one can try to infer them either from the historical actors' observable choices, or from plausible general assumptions (such as the standard monotonicity and dominance assumptions of decision theory). Preferences are said to be "revealed" in the former case (which is loosely reminiscent of the revealed preference method in economics) and "posited"

[25] There were five at the time: Britain, France, Germany, Austria and Russia. Zagare and Kilgour (2006, p. 635) address this objection.

in the latter. We understand these two ways as being complementary rather than exclusive. Observed choices, even repeated under different historical circumstances, can hardly provide enough data, since preferences typically make counterfactual comparisons, and general assumptions are unlikely to be sufficient either. Whatever the chosen balance between "revealed" and "posited" preferences, historians will still no doubt complain – this time, though, not by arguing that AN *subtract* too much from the available evidence, but rather that they *add* too much to it.

Question (iii) plays an essential role in all formulations of the deductive scheme of explanation. As Nagel (1961, p. 43-43) writes, for instance, the point here is "to eliminate explanations that are in a sense circular and therefore trivial because one or more of the premises is established (and perhaps can be established) only by way of the evidence used to establish the [*explanandum* statement]." To require that *all* statements in an *explanans* be tested independently would be exacting, but even the mild form of the requirement with "some" instead of "all" turns out to be challenging. Statements referring to historical particulars are the most recalcitrant, because of the paucity of historical data. Thus, one of the games in case 1 postulates that clans strike a trade-off between the benefits of gaining control of Genoa and the costs of becoming responsible for its external security, but the sparse historical record does not contain any independent evidence for this assumption.

Explanans statements that are akin to generalities have a better chance of being tested independently. The Triple Crisis Game of cases 9 and 10 illustrates this possibility. It underlies the *explanantia* proposed for no less than four different historical *explananda* (Germany's choice of an alliance with Austria in 1878, its diplomatic failure at Algeciras in 1906, its "blank check" to Austria in 1914, and finally, Britain's ambiguous policy in 1914). As Zagare and Kilgour (2000 and 2003) argue, what is central to the Triple Crisis Game is *the assumption that Protégé can realign with Challenger at the final stage*. Strategically, this gives Protégé leverage over Defender while enlarging the room for maneuver of Challenger, and the Triple Crisis Game thus acquires a flexibility that makes it applicable across various historical situations. This assumption constrains the four *explanantia* above, and thus provides a way of testing any of these individual *explanantia* by the empirical success or failure of its neighbour. This establishes that Zagare's explanations meet the independent testability condition at least in part. However, one should of course not confuse independent testability with successful independent testing. The central assumption of the Triple Crisis

Game runs into the historical problem that it applies more convincingly to the 1878 and 1906 contexts, in which Protégé's threat to realign was plausible, than to the 1914 contexts, where this threat made limited sense.[26]

To sum up this section, we have borrowed the classic deductive scheme from the philosophy of explanation and used it as a thread to investigate how AN contribute to historical explanation. This scheme recommended itself because game theory has a deductive machinery, and also because contributors to the field of AN often lay claim to it. We found that AN do not always involve proper deductions, and that they meet the epistemic conditions of the deductive scheme only imperfectly. The next section shows that the narrative component of AN can alleviate these failures. More generally, it considers the role of this component in fuller detail.

6. The role of narration in analytic narratives

Consider first the deductive failure connected with the *multiplicity of equilibria*. Authors of AN are aware of this difficulty, and typically resolve it by appealing to their narratives to decide among the possible equilibria.[27] This sketch of an answer needs to be refined by distinguishing between *different kinds of multiplicity*, as we did in section 5. Suppose the author of an AN wishes to devise an explanation in terms of some generic game. Narrative information has already established what the *explanandum* consists of, and is now expected to say what parameter values of the generic game actually prevailed in the circumstances of the *explanandum*. If the generic game associates a unique equilibrium with these values, a dichotomy straightforwardly follows: either the equilibrium agrees with the *explanandum*, and the explanation can proceed further, or there is no agreement, and the explanation has failed. But now consider the case in which the generic game associates several equilibria with the historically relevant parameter values, and exactly one of these equilibria agrees with the *explanandum*. It is not clear whether one may still hope for an explanation. A standard move in applied game-theoretic work, for instance in industrial organization, is to check whether

[26] Zagare and Kilgour (2006) and Zagare (2011, p. 161-162) show awareness of this problem. Indeed, it would have been extraordinary if in July 1914 France had threatened Britain that it might align with Germany.

[27] See Bates et al. (1998, p. 15): "Repeated games, for example, can yield a multiplicity of equilibria. To explain why an outcome occurred rather than another, the theorist must ground his or her explanation in empirical materials". It is for "the narrative" to provide these "materials".

the unsuitable equilibria can be discarded on intuitive grounds. This informal procedure can sometimes lead to new developments in game theory. However, AN contributors can regiment it less technically *by letting the narrative speak*. The pieces of narrative information to use for the selection task may overlap, but should not be identical with, those which have already served to determine the *explanandum*; otherwise a gross circularity would result.

The last move illustrates how the recourse to the narrative may complement an imperfect deductive explanation. The selection it operates is conceptually different from that which consists in fixing parameter values. However, writers of AN are not always clear about which kind of multiplicity, and hence which kind of selection, they are concerned with. The reason for this seems to be that extensive form games of incomplete information entered the field only belatedly. Under perfect information, backward induction provides the extensive form game with an essentially unique equilibrium once the parameters are fixed. Under imperfect information, backward induction is no longer available, and subjective beliefs are part of the definition of equilibria, which tends to make them non-unique even for fixed parameter values. The *Analytic Narratives* contributors were not yet in a position to clarify this necessary distinction, which, by contrast, comes out well in the introductory comments to *The Games of July*.

Let us now return to the problem of preference assumptions. Although it seems a good strategy to combine "revelation" (from choices) and "position" (of commonsensical comparisons), this will not always be sufficient to determine the players' preferences, and here again the narrative can help. For one thing, by granting that the historical actors have some internal stability, it enlarges the set of choice data on which "revelation" depends; for another, again granting stability, it offers a means of cross-checking what "position" suggests. To illustrate, Napoleon's preferences in June 1815 cannot be guessed only from his choices at the time plus the notion that he preferred victory to defeat. His preferences included his risk attitudes, and to assess the latter it is best to adopt some temporal distance and remember that he had been a bold and generally lucky gambler throughout his career. Thus, enlarging the narrative beyond the initial scope limits the arbitrariness of the preference assumptions in AN. This illustrates how the narrative can facilitate compliance with the deductive scheme – this time, when an epistemic, not a logical requirement, is concerned.

Still in the epistemic sphere, consider the problem of independently testing a generic game. One way AN contributors address this problem is by applying the game to spatio-temporally disconnected historical states of affaires, so as to explain them jointly. (This actually exceeds the usual demand for an independent test, which demands that one resort to control cases, but not to the point of devising a full-fledged explanation for these cases.) Besides Zagare's repeated employment of the Triple Crisis Game, we can exemplify the procedure with Nalepa's generic game, which she uses to account for three different democratic transitions (case 11). Interestingly, Nalepa (2010) reinforces her joint explanation by selecting new facts from the narrative. She mentions that the communists began negotiating with the democrats at an early stage in Poland and Hungary, and belatedly in Czechoslovakia. This suggests that the communists believed in their bargaining power more strongly in Poland and Hungary, a suggestion that connects with another fact of the matter: they had infiltrated democrats more deeply in these two countries. Thus, the temporal pattern of negotiations indirectly supports the main explanatory point, which is that the degree of infiltration was crucial to the communists' success and failure in defending their position. Revisiting the narrative with an eye on independent testing has turned out to be productive.

Although it points in a clear direction, the previous analysis is not sufficiently specific, because it does not make clear why it is *narration*, rather than any other way of presenting historical evidence, that helps fill explanatory gaps. This suggests a more general question: why do AN contributors so strongly value this particular mode of exposition? The primary reason seems to be that they are concerned with *interactions*, whether directly or indirectly, and historical reports of actions typically comes under narrative guise.[28] Now, it is still another question whether AN should *themselves* preserve the form of their existing sources. Arguably, by doing so, they are more objective than they would be if they reshaped these sources in non-narrative form. Presumably, reshaping would sometimes add and sometimes suppress too much information. Mongin (2008, 2018) illustrates this point with the sources on the Waterloo campaign. They all consist of narratives, from the witnesses' unelaborated testimonies to the military strategists' highbrow accounts, with a number of contextual variations in between, such as those of popular military history. To summarize this evidence in any other way than narration would distort it. Moreover, the problem addressed in the

[28] The novelist Philip Roth is said to have made this pronouncement: "Everything that matters comes to us in the form of a narrative". At least, every *action* that matters comes to us in this form.

Waterloo study precisely consists of a gap found in earlier narratives, and a natural way to make this clear is to devise a summary narrative in which the gap is shockingly visible.

Related to these points is the fact that the formal theories of rational choice have privileged connections with the narrative mode of exposition. Models formulated with these theories typically come with *stories* to make their technical points salient or even simply intelligible. In the case of AN, these stories may or may not be realistic to the point of mapping onto parts of a historical narrative. In section 3, we argued that the actions in extensive form games were idealizations of concrete actions. Although complete similarity is beyond reach, the stories behind rational choice models, and in particular games, have sufficient common ground with genuine histories for relevant interchanges to take place between the two.[29]

There are many other reasons why it may be justified to value the narrative mode of expression, but they are not specific to the methodology of AN and belong rather to that of history in general, so we will not review them here.[30] However, one of these reasons deserves to be singled out. Philosophers of history often argue that narratives, properly understood, encapsulate causal claims in their reports of temporal succession. Here is a famous didactic example: "The king died, and then the queen died of grief". Whether causal claims such as that made by this sentence are satisfying from an explanatory viewpoint is a matter of dispute. Some philosophers, like Danto (1985), think that narratives are explanatory by themselves, while others, like Dray (1971), think that narratives are only occasionally so. An intermediary position, which is probably White's (1984, 1987), is that the causal content of a narrative can always be extracted and subjected to separate scrutiny, so that the narrative will or will not be explanatory, depending on how the examination of the content turns out. This intermediary position seems promising for the methodology of AN. By stressing the possibility of extraction, it opens the door to the modelling stage of AN, and by making this extraction relative to causality, it reorients their assessment from their deductive towards their causal performance, an enrichment of the current discussion of AN.

[29] More on this in Grenier et al. (2001) and Mongin (2008).
[30] See in particular Roberts's (2001) collection, with classic pieces by Dray, Mink, White and others, and the 1985 collection of Danto's works in the philosophy of history, *Narration and Knowledge*.

We now complete our examination of the narrative component of AN by paying special attention to their *expository feature*s; for the main, this is borrowed from Mongin (2016). We will recognize three distinctive expository patterns for AN.

Case 1, concerning Genoa, follows a chronological order extending from the consulate period, with its succession of civil peace and war under the consulate, to prolonged civil peace under the *podesteria*.[31] The exposition of the consulate period follows a remarkable pattern. First, a standard narrative records the main facts and introduces the *explananda*; then comes a game-theoretic model with relevant variations, which suggests the *explanans* hypotheses; and finally a narrative consolidates the explanation. Unlike the first, this narrative borrows theoretical terms from the modelling part, e.g., "mutual deterrence equilibrium", and serves to clarify and empirically support the *explanans* hypotheses, thus assuming the function of problem-solver. Despite its special features, this is a narrative all right, so we do have an *alternation* pattern. This pattern also appears, though a little less transparently, in the rest of case 1, as well as (albeit with some differences) in cases 4 and 5.

In case 6, on Waterloo, the exposition begins with a campaign narrative in the style of military history, which introduces the main facts and the (here unique) *explanandum*. Then, a game-theoretic model delivers the explanatory hypothesis, and a discussion follows that introduces more historical evidence. As a distinctive feature, this study considers the initial narrative as being essentially satisfactory, except for the explanatory gap it draws attention to. Thus, the model and its discussion are parenthetical, and the initial narrative can be resumed once the gap is filled. This pattern of *local supplementation* differs from alternation in being less ambitious, since it does not involve creating a new narrative. However, the two patterns locate the final explanation in a narrative, and this feature is more important than the difference between them.

Case 11, on "transitional justice", goes through the following expository steps. It introduces the historical problem of "transitional justice" in the early post-communist years, puts forward a theoretical hypothesis both informally and formally, proceeds to a narrative history of transitions, and finally compares facts from this narrative with the theoretical hypothesis.

[31] One may note the dramatic quality of this sequence, which reminds one of the triadic plot structure in many dramas or fictional stories: an initially stable situation, a conflict between the characters, and a positive or negative resolution of this conflict (see Freytag, 1863, elaborating on Aristotle's *Poetics*).

Although well developed, the narrative here is only a provider of data, and the whole study obeys a standard hypothesis-testing scheme. What customizes it is that it gives a narrative form to its empirical evidence. We will call this expository pattern *analyzed narrative*. Unlike in the first two patterns, it does not entrust the final explanation to a narrative, whether ordinary or revised, but rather states it abstractly and theoretically.[32]

Starting from this contrast, one may conceive of AN in two different ways. In a *restrictive* view, they count as such only if they follow the alternation or local supplementation patterns; in a *liberal* view, they may also follow the analyzed narrative pattern. A reason for preferring the former is that it seems best to emphasize what is most specific about AN; we have already used such an argument in section 3. The thought-provoking move is to *make* narratives analytic, and this necessitates the return of the narrative at the end of the study. As they simply *juxtapose* the analytic and narrative components, analyzed narratives are less novel. For two reasons, however, this argument may be too stringent.

First, as we have mentioned, the three patterns share the feature of bringing in historical evidence narratively, and this is by itself an important specification, since not every work in economic or political history does that. There have even been voices in these fields, as well as in history more generally, calling for narration to be downgraded, an attitude that conflicts with the way it enters analyzed narratives. Well-known representatives of this anti-narrative stance are the members of the *Annales* school, who championed "problem-oriented" against "narrative-oriented" history, and the intransigent "new economic historians" whose flags were economic modelling and econometric techniques.[33] One reason for preferring the liberal view of AN is that they clearly illustrate the opposite stance of the "revival of narrative", to borrow a famous phrase by Stone (1979). Second, the beginning of this section has pointed

[32] Crettez and Deloche's (2018) treatment of Cesar's death further illustrates the subgenre of analyzed narratives. Following the general AN methodology, they carefully review the historical evidence and extract from it a problem they solve with the aid of a formal model. How plausible is the suggestion made by Suetonius and others that Cesar was aware of the plot to murder him when he went to the Ides of March meeting of the Senate? The authors' two-person game of normal form has a single Nash equilibrium that is mixed, which in their view suggests a negative answer to this question. Here the narrative provides both the evidence and the problem, but the solution is stated in theoretical, non-narrative terms.

[33] The firm contrast that *Annales* postulates between narrative- and problem-oriented history appears among others in Furet (1981). The anti-narrative stand is also popular among some new economic historians, e.g., Kousser (1984), who defends "quantitative social scientific history" against a "revivalism" of narrative. Not every cliometrician has adopted this stance; witness the open attitude of the editors of this *Handbook*.

out several means by which narration can rescue flimsy explanations, and these means are also available in the third pattern. In particular, case 11 selects equilibria from narrative information in a manner no different from cases 9 and 10. In terms of the principles stated in the introduction, the third one, whereby the narrative actively contributes to historical explanation, appears to be common to AN broadly understood, although the alternation and local supplementation patterns apply it more systematically and, as it were, more interestingly than the analyzed narrative pattern does.

These reasons can tilt the balance in favour of the liberal view of AN, and we will adopt this view here, thus completing our attempt at defining the AN genre. To make this definition more transparent, we may cite two groups of studies it does *not* cover. (i) Some studies are concerned with specific historical events, involve a significant amount of narrative information, and base their explanations on the outcome of complex interactions, but refrain from adopting a formalism and thus provide only promising sketches of explanation. Besides case 3 above, Myerson's (2004) discussion of the Weimar disaster is a good example – and all the more so given that his informal comments are evidently made with a possible modelling in view.[34] Works like these are *proto-analytic narratives*. (ii) Other studies are also concerned with specific historical events, base their explanations on the outcome of complex interactions, and do develop these explanations by means of properly formalized models, but do not confer an explanatory function on the narrative, nor even prioritize it among the sources of historical information. Two studies by Greif that antedate his adherence to AN methodology can serve as examples. Greif (1993) investigates the community of Maghribi Jewish traders who operated in maritime commerce in the 11th and 12th centuries, and Greif et al. (1994) investigate the connection between the merchant guilds of medieval Europe and long-distance trade. These studies focus on the commitment and coordination problems that traders faced in their dealings with official rulers or other traders, and they use game-theoretic models to show that well-designed informal (in the Maghribi example) or formal (in the guild example) institutions could overcome these problems. Their exposition mixes theoretical elements with historical evidence, which is only occasionally narrative, in a dialogue that clearly differs from the alternation pattern implemented in the Genoa case.

[34] For instance, Myerson (2004) suggests treating the events of 1930-1933 in terms of a signalling game between the Allies and the German conservative leaders. To get rid of the reparations burden, the latter would try to impress the former by pushing forward Nazism as a political force (a dangerous game if ever there was one).

Despite their emphasis on interactions and game theory, which likens them to AN, they are closer to other formalized works in historical political science or economics. Let us designate them *analytic non-narrative histories*.[35]

7. Conclusion

This chapter has defined AN in terms of three principles, the most intriguing of which is that AN call upon a narrative also at the explanatory stage. We have pursued our definitional investigation at the same time as making progress with the other topic of the chapter, i.e., how AN contribute to historical explanation. In this latter discussion, we have selected the deductive scheme of scientific explanation as a benchmark: overall, analytic narratives exhibit more deviation from, than conformity to, the deductive scheme, and this is precisely why they call upon the narrative for help. A more complete account of their explanatory performance would have clarified the kind of causal connections they can hope to establish, and this would have led us also to investigate the kind of counterfactual history they develop. The necessary brevity and thematic unity of this chapter made it impractical to go in these directions. Similarly, we refrained from explicitly defining what a narrative consists of. This would have required us to compare the narrative mode of discourse with the other modes, such as exposition, argumentation and description, which historians also use, and thus to delve in the recent work of narratologists as well as the more traditional concerns of rhetoricians and literature teachers.

Thus far, political scientists have paid more attention to AN than other social scientists. This is easily explained by the fact that the two main currents that have shaped the development of AN, i.e., the equilibrium approach to institutions and the deterrence approach to national security, are primarily of concern in political science. But these disciplinary associations are in part a matter of contingency, and it is anyhow the case that "analytic narratives should have no boundaries with respect to subject or evidence" (Bates et al., 2000b, p. 690). In particular, there is no reason why AN could not also have a significant place in economic history. What might restrict their use therein is that they are concerned with fine patterns of actions and events, like the formal theories of rational choice they borrow from, and are thus unable to handle long-term historical processes, such as Britain's Industrial Revolution, or

[35] More examples could be found in Greif's (2002) survey of game-theoretic economic history.

large-scale sets of social and economic relations, such as slavery in 19th century USA. But these wide topics are of course the bread and butter of today's economic historians; and if AN can teach them anything, it would be precisely by directing their attention towards the fact that it is possible to approach some microscopic structures no less rigorously than these topics, albeit by different formal means.

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