A GAME-THEORETIC ANALYSIS OF THE WATERLOO CAMPAIGN AND SOME COMMENTS ON THE ANALYTIC NARRATIVE PROJECT[*]

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Abstract : The paper has a twofold aim. On the one hand, it provides what appears to be the first game-theoretic modeling of Napoleon's last campaign, which ended dramatically on 18 June 1815 at Waterloo. It is specifically concerned with the decision Napoleon made on 17 June 1815 to detach part of his army against the Prussians he had defeated, though not destroyed, on 16 June at Ligny. Military historians agree that this decision was crucial but disagree about whether it was rational. Hypothesizing a zero-sum game between Napoleon and Blücher, and computing its solution, we show that it could have been a cautious strategy on the former's part to divide his army, a conclusion which runs counter to the charges of misjudgement commonly heard since Clausewitz. On the other hand, the paper addresses methodological issues. We defend its case study against the objections of irrelevance that have been raised elsewhere against "analytic narratives", and conclude that military campaigns provide an opportunity for successful application of the formal theories of rational choice. Generalizing the argument, we finally investigate the conflict between narrative accounts – the historians' standard mode of expression – and mathematical modeling.

Keywords: Napoléon, Blücher, Grouchy, Waterloo, military history, rational choice theories, game theory, zero-sum two-person games, analytical narrative

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1. Introduction

Once launched in economics and its attendant disciplines, theories of rational choice spread abundantly beyond this original realm. An entire branch of experimental psychology is now devoted to them. Schools of sociology and political science align themselves with the theories, against others which reject them. Indeed, the resistance offered by these latter two disciplines has sparked well-known controversies, some of which have gone on to become academic *topoi* (think for instance of that which has wracked the sociology of education for years). But history has escaped this trend, even, so it would seem, in its critical form. Not only has the discipline been reluctant to make use of rational choice theories, but it has also shown itself little inclined to reflective debate on their possible applications. Those who reject cost-benefit analyses are not likely to conduct one to see whether or not it is useful to do so.

A few authors – actually more political scientists or economists than historians – would now break this status quo by promoting "analytic narratives". The term, which has become something of a rallying cry, sounds more abstract than the movement's true objective, which is primarily to apply game theory to traditional topics of political history: municipal conflicts in medieval Genoa, the tax systems of prerevolutionary Europe, conscription laws in the 19th Century, entry of new states to the American federation, regulation of the coffee exchange at the end of the 20th Century. Such an exception is too curious to pass unnoticed. But it has hardly generated a craze. The methodology has been criticized on the grounds that, first, the historical cases have been ill-chosen, second that the models used to explain them do not adhere to the strictest norms of rational explanation, and third that these very norms themselves are dubious.[1]

The present paper also employs game theory to inform the study of history. It is another attempt to implement the "analytic narrative" methodology, but with a shift that its choice

[1] The studies are due, respectively, to Greif, Rosenthal, Levi, Weingast and Bates. They have been brought together in *Analytic Narratives* (1998), which is a manifesto for the group. The three levels of criticisms are found in Elster, to whom the authors have replied; see this important controversy in the *American Political Science Review* (2000, p. 685-702).

of a military topic will make clear. By and large, we would endorse the previous writers against objections of the second and third levels, arguing, as they forcefully did, that there is no obligation to be theoretically sophisticated in applied work, and that it is not very helpful in the present context to rehearse the classic problems of rational choice theories.[2] But objections of the first level are different, and here we certainly agree with the critics that few historical topics are amenable to "analytic narratives". We propose the rule that the selection of cases be justified *explicitly* in terms of the questions the earlier historians have raised and the data they have left to answer them. Game theory should be connected with preexisting accounts of the usual style more tightly than has been done so far. As a companion rule, we propose that the author of an "analytic narrative" state *also explicitly* what he aims at eliciting from the historical material that cannot be obtained by ordinary narratives, and why this material calls for his specific modeling rather than any other. Again, the previous work appears to be somewhat elusive in this respect.

With these guidelines in mind, we tried the "analytic narrative" methodology on Napoleon's last campaign in June 1815 - that which he eventually lost to Wellington and Blücher on the battlefield of Waterloo. One broad reason for this choice is that military studies have often served as a touchstone to rational choice explanation. That they are a fertile terrain for such activity has been suggested a number of times. For example, Pareto (1917-1919, §152) classes them among the few disciplines - along with economics and technology - that embody his concept of "logical action". Among military studies overall, the account of battles and campaigns appears to be more tractable than others. Thus, to illustrate his ideal-type of "instrumental rationality", Weber (1922a, p. 10; Eng. ed. p. 21) mentions the Prussian Moltke and the Austrian Benedek fighting each other at the Sadowa battle. Even more clearly, the authors of campaign narratives, beginning with Jomini and Clausewitz in the 19th Century, have given flesh to the view that their field has a special susceptibility to rational choice explanation.

[2] Unless this serves to argue that rational choice theories are absolutely flawed or irrelevant. See the final comment by Bates and al. in their reply to Elster: "His real opponent is rational choice theory" (2000, p. 702).

This recognized exemplary character of military studies, and above all, campaign narratives should be of interest to the "analytic narrative" writers and their fellow-travelers. From this point of departure, they could hope to pass more easily through the strictures set by the first level of objections, and thus to establish a genuine dialog with professional historians, who seem a little more disposed here than elsewhere to relax their longstanding mistrust of rational choice theories. In this group, game theory enjoys a place of natural support since its technical concepts – beginning with that of the strategy – have intuitive relationships with the military use. Admittedly, von Neumann and Morgenstern (1944) did not pay much attention to military affairs, giving greater importance to parlor games in their examples. But their followers at the RAND Corporation and in US military institutions dealt with nuclear strategy and deterrence at considerable length, and even more relevant to our project, one author in this group, Haywood (1950, 1954), undertook a game-theoretic analysis of some battles of the Pacific War, using the basic tool of von Neumann and Morgenstern, i.e., zero-sum two-person games.[3]

Military applications pay for their didactic facility with a clear disadvantage. Though they may succeed in their ambitions, they have not the same demonstrative consequences as if they had taken on more resistant subjects such as medieval Genoa or the finances of prerevolutionary France. So be it with the present work. Its aim is really to restart the debate on "analytic narratives" from a middle ground that can be accepted by the less passionate critics. We would be entirely satisfied if we made consensual a bare existential point: *there is a class of plausible historical applications for game theory*. The controversy has been so fierce that even such a weak claim was not taken for granted among the participants.

What can be claimed for game theory can also be for the theory of individual decision-making under risk and uncertainty, or *decision theory* in the narrow technical sense. For the latter is usually construed as a special case of the former, and it is anyhow a logical prerequisite for it. Despite these clear connections, military applications with just one

[3] Brams (1975) breathes new life into the distant work of Heywood; see also O'Neill (1994).

agent facing nature are worth exhibiting; we provide one elsewhere by borrowing again from the Waterloo campaign.[4] In contrast, it is an open and very intriguing question whether some use in history may ever be found for *the aggregative theories of social choice*. At any rate, we stop at the classic trio of microeconomics texts. What we are *not* addressing are the "rational choice theories" in the informal sense often favored by social sciences other than economics. The case for rational choice explanation in history has already been made vis-à-vis such informal conceptions with rather plausible examples and arguments, and there would be no point in restating it.[5] By the same token, the only models of interest will be those applying one of the theories above to particular histories – hence they will be mathematical, not informal models. In thus constricting our subject matter, we highlight the most spectacular conflict of all, i.e., that between a liberal (in several senses) discipline and a set of tools so coarse and uncouth as to be perhaps beyond assimilation. Seemingly an oxymoron, the slogan of the "analytic narrative" underlines this tension well.

Among the topics available in military history, ours is scarcely original, but this is perhaps more of an asset rather than a liability, given the previous guidelines. An old chestnut from the strategy courses of staff colleges in the 19th Century and early 20th Century, Napoleon's 1815 campaign has remained an inexhaustible and fascinating subject for war historians up to the 21st century. The rich bibliography in three major languages is an attraction of the case. An even more important reason for selecting it is that, despite so much available evidence, historians have been unable to come to agreement on how to explain Napoleon's stupendous disaster, and what they disagree precisely on is *the rationality* of this prominent actor. The game-theoretic modeling may renew the time-honored controversies, and if it does, the resulting "analytic narrative" may be welcome by even some of the critics.

[4] Mongin (2009) suggests reconstructing several passages in Clausewitz's account of the campaign in terms of individual expected utility maximization. The full game-theoretic perspective is unnecessary to account for the agents' decisions in these passages.
[5] See in particular Hempel's (1965, ch. 9 and 12) classic discussion.

The controversies go back to an account the overthrown emperor dictated at Sainte-Hélène to his companions in exile, which was a plea *pro domo*. Among the texts recording it, we have selected *Mémorial de Sainte-Hélène* by Las Cases because it is the most widely distributed and the most succinct.[6] Napoleon's line is to lay the blame for defeat with marshals Grouchy and Ney, who he claims misjudged their strategic possibilities and did not properly follow his instructions. However, Clausewitz, who had access to *Memorial* as well as further French and German sources, reached the opposite conclusion that the underlings might be pardoned and the hero should not be exculpated. The first genuine scholar of the campaign, the Prussian general is also a passionate critic of Napoleon's handling of it. With various nuances, his position has carried the day in the literature, but the imperial argument, long upheld by French military writers, has not disappeared altogether. One meets it still today, with qualifications that leave it no less worth considering than the other. So historians are in a deadlock, and after so much time, there seems to be little hope that progress will be made by traditional means; this is our best justification for trying the "analytical narrative" perspective.

Clausewitz's interpretation is to be found in a monograph, *The Campaign of 1815 in France*, which the treatise *On War* has regrettably overshadowed, and it is a secondary aim of this paper to draw attention to that part of his work.[7] In it, one can find anticipated use, if not yet the full realization, of Weber's principle of instrumental rationality (henceforth, we simply say "the principle of rationality"). By contrast, the concepts of ends and means that direct the classic definitions of war in the treatise spring from an abstract teleology, detached from acting individuals, which is not the same as that clarified in Weber's methodology. Another contribution of the monograph is that, while following the principle of rationality throughout, it now and then surpasses the level of

[6] Las Cases includes "Relation de la campagne de Waterloo, dictée par Napoléon" in *Mémorial de Sainte-Hélène* under the date of 26 August 1816. The other references are Gourgaud's *La campagne de 1815* and Bertrand's *Cahiers de Sainte-Hélène*. The last work was published long after its author's death and played no role in the Waterloo controversy, contrary to the first two, which came out in 1823 and 1818 respectively.

[7] *Der Feldzug von 1815 in Frankreich*. Posthumous like the others, this work appeared in 1835 in the *Hinterlassene Werke* edited by Marie von Clausewitz; it was written in 1827. Clausewitz's commentators have not spent much time on his campaign narratives. For instance, Aron (1976) hardly mentions them at all and Paret (1992, ch.9) is somewhat quick and derogatory with them.

generality at which historians normally stop, suggesting – even sketching out – *models* in the sense relevant here. These anticipations are all the more intriguing since, contrary to *On War*, *The Campaign* is a historical work; it alternates between traditional narrative and logical argument that opens the way to contemporary formalization. We will actually rebut Clausewitz's substantial interpretation of Waterloo, but praise his general method, and the "analytic narrative" school may be pleased to register him as a glorious precursor.

Section 2 of this paper reviews the main facts and interpretations of the campaign, emphasizing those which matter for the model to come. It will go light on the tactical aspects of the battles, even the major ones of Ligny and Waterloo, and focus on the overall strategy as Napoleon might have conceived of it. This passage intentionally reproduces the standard narrative mode of military historians.

Section 3 changes tone, proposing a model for Napoleon's all-crucial decision, June 17, 1815, the day after his victory over Blücher at Ligny. That day he chose to send more than a third of his forces, under the command of Grouchy, against the retreating Prussians. All the commentators agree that this division of the French army was the key to Wellington's victory, June 18 at Waterloo. Grouchy spent the fateful day at Wavre, baited by Blücher's rear guard, while the advance guard marched unimpeded to join Wellington in the mist of an uncertain battle. The campaign's greatest question, which involves Napoleon's rationality, is whether he could have made better use of Grouchy's detachment. The model we propose to answer this question takes the form of a simple zero-sum game between Napoleon and Blücher. Despite the absence of Grouchy as an autonomous player, it adds precision to the competing hypotheses. In the end, we will side with the proNapoleonic minority against the Clausewitzian majority. Rational choice modeling, which supplies the arguments, will have thus played its customary charitable role vis-à-vis the agent.[8]

[9] Davidson (1980) is famous for emphasizing the principle of charity underlying that of rationality. His argument is that we cannot understand others except rationally, and this requires that we also understand them charitably.

Section 4 subjects the model to the same three levels of criticism brought against the "analytic narrative" methodology, and pursues the argument sketched above for applying it to existing campaign narratives. Section 5 explores the methodological tension that even such cautiously restricted studies cannot avoid. To eventually reconcile the new narrative style with the old one, we will defend the idea that *they support each other by the complementarity of their faults*. A standard historical narrative performs several useful functions at once, but as we argue, does not take any of them to their final stage; in particular, say what the historians might, its explanatory role remains very imperfect. The missing steps can be achieved by models from rational choice theories, which will have the opposite weakness of serving too few purposes at a time. Thus, the strategic game constructed for June 17 goes somewhat farther in explaining the events of that day than the available accounts, but it has the drawback of sacrificing the expressive and evaluative functions that these accounts also supply, and hence should not aim at replacing them. Clausewitz's alternation between standard narrative and (in his case only suggested) technical modeling is the only feasible scheme for an analytical history.

2. The Waterloo Campaign: main facts and interpretations.

In the spring of 1815, a coalition of the European powers was solidifying against France. Napoleon needed to annihilate the two armies already mounted – the English and the Prussian – as quickly as possible. Against Wellington's 93,000 Anglo-Dutch soldiers, who were preparing to meet Blücher's 118,000 Prussians in Belgium before invading France, Napoleon had only the 124,000 troops of Armée du nord; his other forces covered the Rhin or garrisoned fortresses. The only way out was to reproduce his masterstroke from the Italian campaign: first defeat one army, then the other. All historians recognize this plan, and most of them, including Clausewitz, hold that it was the only one conceivable.[9] At first, the execution seemed promising. With his customary swiftness, Napoleon entered Charleroi on June 15, forcing the Prussian advance guard to pull back northeast of the city. The allies had not yet joined forces, and each group alone

[9] *La campagne de France en 1815*, tr. by Niessel, 1973, p. 37-43. From now on, all page references to Clausewitz are to his monograph and this French version, from which we translated the quotations.

left much to be desired. The Anglo-Dutch were deployed widely around Brussels and westward, as Wellington wanted at all costs to maintain communications with Ostend in that direction.[10] And Blücher was headquartered in Sombreffe, some 12 kilometers northeast of Charleroi, with only three corps; a fourth, commanded by Bülow, kept the rear guard and was useless for battle. In taking this forward position Blücher ran the risk of confronting Napoleon with insufficient forces. However, his decision becomes clearer in the light of the agreement he had reached with Wellington on May 3, to the effect that the allies would meet on the Quatre-Bras-Sombreffe line in the case of an offensive by Napoleon. This strategy ran afoul of the classical precept of maximal preliminary grouping, but Blücher could hope that Wellington would play his agreed-on part in the fight.

The Prussians had occupied the hamlet of Ligny, which gave its name to the battle that they ended up pitching alone there, over the afternoon and the beginning of the evening of June 16. Less famous than that of June 18, this battle actually determined the succeeding chain of events, and it is with regards to its interpretation that the main hypotheses square off. *The Campaign*, to quote but one account, gives more space and emphasis to Ligny than it does to Waterloo.

Blücher's risky strategy, which Napoleon immediately recognized, offered him an opportunity to carry out his campaign plan. He won on June 16 following the two standard criteria of victory: lose fewer men than the adversary, and conquer the terrain of the battlefield. While very real, this victory was not yet decisive. Blücher managed to save most of his forces, some 90,000 men, in sufficient order to bring them back to his rear guard. So his initial error – leaving Bülow in reserve – would eventually turn to his and Wellington's advantage. In the *Memorial*, Napoleon implies that the three Prussian corps engaged at Ligny escaped destruction through Ney's fault.[11] In fact, he had sent the marshal away in the north-west direction with the principal objective of holding the road

[10] Hofschröer (1998-1999) stresses that Wellington had weakened himself to prepare for an attack from the west, which there was little reason to expect. The Duke had already faced the charge in his reaction to Clausewitz; see Bassford (1994, p. 42-45, and 2001 for a transcript of Wellington's comments).
[11] *Mémorial de Sainte-Hélène*, Garnier reprint, p. 237 (all page references to this edition).

from Charleroi to Brussels, which Wellington would have to use if he came in support of Blücher. Ney's group - about 25,000 men under his direct command - had the option of either attacking the Anglo-Dutch, or simply holding them back, while taking the Prussians from behind. Napoleon mentions both tasks at once, which is much to ask of poor Ney, considering both his resources and his actual capacity for initiative. At the field of Quatre-Bras, where he met the English forward guard, he carried out the former task slowly and awkwardly, not even considering the latter. The corps of Drouet d'Erlon – 20,000 more men - was to come to Ligny or Quatre-Bras in case of need, but wandered piteously from field to field without engaging; many have seen in this a turning point in the campaign.

Clausewitz defends Ney by arguing that the successive orders that Soult, the campaign's chief of staff, sent him in the name of the Emperor were incompatible. This analysis, which we will not develop here[12], brings out the rationality principle most clearly: "Ney absolutely completed his goals – to block the aid of Wellington. Bonaparte did not come to the idea of having him cooperate in the battle of Ligny until later, after having recognized Blücher's position…. Only today can we see [what Ney could have done], by bringing into our calculations all the fortuitous circumstances that could not be foreseen at the time" (Clausewitz, p. 105). Weber would not distinguish any better than that between *objective* rationality, which can be identified from the perspective of an observer looking back, and the *subjective* rationality of individuals, which is the only pertinent type for explaining their actions.[13]

Starting a movement that would turn out to be decisive, the Prussians did not back up along their natural line of communication, which was the Meuse river valley, but farther northward, in the general direction of Louvain. They regrouped over the course of June 17 at Wavre, a town situated on the river Dyle, mid-way between Ligny and Louvain. This location allowed them to keep as many options open as possible. From there, Blücher could either organize a definitive retreat by reaching Liège by way of Louvain,

[12] More on it in Mongin (2009).
[13] Cf. Weber (1922b, p. 435-439). The distinction between objective and subjective rationality has since become established; see, e.g., Popper's (1967) classic restatement.

or rejoin Wellington, who was a single day's march away. On the same day, Napoleon chose to separate his right wing, which Grouchy had commanded at Ligny, of some 30,000 men. With this detachment, the marshal could either set loose a savage chase against the Prussian rear, without worry of what became of the rest of that army, or keep this army once joined from meeting the Anglo-Dutch, or to carry out the objectives of pursuit and blocking to the extent that they were compatible.

What actually occurred is that Grouchy set off after the Prussians, who intentionally slowed one of their corps, led by Thielemann. On June 18, the marshal pitched battle at Wavre against just this rear guard. Meanwhile, the advance guard, with Bülow and Pirch, marched unobstructed to Waterloo, and it bowled into the French right on the afternoon, early enough to help Wellington, who was not in an easy position.[14] Having missed the chance the first time because of Wellington, the allies succeeded in concentrating their forces the second time thanks to Blücher. It is unlikely that Grouchy would have brought the French victory in a battle including Bülow and Pirch, but if he had been there instead of the two Prussians, he would have given Napoleon the numerical advantage needed to defeat Wellington. The two commanders faced 70,000 men each against one another, and equality favored the latter, who had chosen to fight from a strong defensive position, as he had done to his advantage so often before.

As already indicated, a major problem of the campaign is to decide *what Napoleon intended to achieve with Grouchy's detachment*. It is closely connected with another, which is to decide *how Napoleon interpreted the battle of Ligny.* To what extent did he overestimate the extent of his victory, and misjudge the direction of their retreat? Clausewitz (p. 107-109 and 146-148) claims that he made mistakes on both counts, and this has dominated the literature since. Let us review the evidence available to answer the two questions.

[14] Although Thielemann had finally to surrender Wavre, he had fulfilled his role by holding the enemy for half a day. Clausewitz, then a colonel, served as his chief of staff.

Napoleon's initial orders to Grouchy on June 17 were oral, and neither the *Memorial* nor the marshal's *Memoirs* are reliable enough to permit reconstructing them.[15] Clausewitz, in his chapter XXXVII, claims that Napoleon entrusted Grouchy with *a simple mission of pursuit.* This, together with an error that Napoleon made concerning the direction of the Prussian retreat, would cleanse the marshal of all responsibility in the next day's rout - busy in the east, Grouchy could not at the same time lend a hand to Napoleon. The conclusion is implacable if one accepts the premises, but Clausewitz has little more than hints to establish these, and his supporters to this day have not substantially improved the argument.[16] As it had done with Ney, the *Memorial* (p. 245) charges Grouchy with responsibility for the defeat, claiming that he should have been on the Waterloo battleground. French military writers have often taken this position, while softening it with additional reproaches against Napoleon, and still more against his chief of staff Soult.[17] A good deal of this is transparent apology; however, there are also historians without any emotional stake, such as the 20th Century British general Fuller (1951-1956, ch. 18), who concludes that Grouchy ended up in a place he should not have been.[18] None in the present group of commentators could accept Clausewitz' narrow interpretation of the orders of June 17. Napoleon by their reading, unsure of whether the Prussians had been truly beaten, would have asked Grouchy to protect him from their intrusion into the following battle. He would thus have entrusted Grouchy with a *role of blocking or interposing at the same time as one of pursuit.* This wider interpretation obviates the problem of what Napoleon precisely thought of the direction of the Prussian retreat. There were two possibilities: either the entire Prussian force had moved east, in which case the chase would also serve as interposition; or else the enemy was dispersed, with some forces taking the dangerous way to the west, in which case *Grouchy should prioritize the objective of blocking over that of pursuit*. This line has no more solid proof than the other; what is known of the 17 June does not permit a clear winner in the historical contest.

[15] Compiled by his descendants, Grouchy's *Mémoires* discuss these instructions at length, but the effort at exculpation is so blatant that it is impossible to take them seriously.
[16] Even the careful study by Hofschröer (1998-1999) is far from making Clausewitz's case compelling.
[17] Mauduit (1847) eloquently illustrates the beginning of this line of interpretation, the first of many to incriminate the weakness of Soult and the staff in general.
[18] Here Fuller joins forces with Houssaye (1905-1906), a classic of the French rehabilitation literature.

The first written communication that follows the oral commands of June 17 is a letter dictated to Bertrand, received by Grouchy shortly after he set off. Both interpretations can find something in it, the first because it sends the marshal towards Gembloux, i.e., to the east, and even worse, towards Namur, which distanced him from the Prussians, and the second because it directs him to report on Blücher's maneuvers, and even to warn the staff of any possible intention to join Wellington.[19] From Gembloux, where he did not arrive before the late evening, Grouchy replied to Napoleon with a revealing dispatch. This shows that he had at last understood that Wavre was one of the Prussian destinations, but not yet that it was the only one. Also, Grouchy brings up the possibility of an enemy move towards Wellington, and adds that he would try to prevent it from occurring, which lends some support to the view that the conversation of June 17 had suggested interposition as a goal.[20] Although the marshal's letter arrived at 2:00 in the morning, the staff's reply was not sent before 10:00, in which we can see definite evidence of ill-functioning. On behalf of Napoleon, Soult commanded Grouchy to make all haste to Wavre, pushing back any Prussians he finds as he approached the principal army. "His Majesty desires that you direct your movements to Wavre, in order to come closer to us, and to cooperate with our operations".[21] The minority line uses this sentence to argue that Napoleon wanted to have Grouchy participate in the battle of Waterloo (see, e.g., Fuller, 1951-1956, ch. 18). But Clausewitz countered the charge in advance, underlining that it was too late to send orders to Grouchy; in fact, the marshal did not receive them until the afternoon, by which time he had been stuck at Wavre by Thielemann, and Pirch had nearly reached Mont-Saint-Jean.

Regardless of what can be made of the last dispatch, somewhat confused and certainly too late, *the strategy was clear in itself*. Upon his arrival at Gembloux, Grouchy had to arrange to block the Prussians' move towards Wellington instead of continuing to chase them. Fuller proposes an itinerary consisting of a march to Wavre from the west; thus, the

[19] Cited by Mauduit (1847-2006, p. 142) and subsequent authors, Bertrand's letter is missing from Clausewitz, which weakens his chapter XXXVII.
[20] We use the Mauduit's (1847-2006, p. 160-161) version of this letter. Fuller (p. 285-286) summarizes it accordingly, while Grouchy's *Mémoires* (LV, p. 58-59) distance themselves significantly from the text.
[21] The letter from Soult appears in Clausewitz (p. 141), as do all the subsequent dispatches.

marshal might intercept the first corps heading to Waterloo. Clausewitz (p. 143) also thinks that the westward march was the best strategy, agreeing for once with the *Memorial* (p. 238-240), which first said that very thing.[22] It therefore appears that on the limited level of objective rationality, all the interpreters are in agreement. What divides them is how to apportion subjective rationality between the actors on the basis of their beliefs, and this conclusion is impossible to reach simply from the documents we have surveyed.

We will discuss but briefly the final battle. June 17, after the fights of Quatre-Bras, Wellington withdrew his troops to within about ten kilometers of Brussels, on the Mont-Saint-Jean plateau, whose value for defensive combat he had already spotted.[23] Partly hidden along the crest, the Anglo-Dutch could pepper their opponents almost at leisure, while the attackers were blocked by solid buildings - farms and convents - in the center and on both flanks. On June 18, rain delayed the French attack until 11:30, and hindered the artillery preparation that Napoleon was accustomed to implement before attacking. For this reason and others, the first offensive, directed against the center of the Anglo-Dutch line, was a complete rout. Several historians, including Fuller, conclude that with such a bad start, Napoleon should have given up fighting the moment he heard of the arrival of the Prussians, that is near 3:30 p.m.[24] By moving to the defensive, he might have saved his army and fled with it back to France. But he did not. He tried to settle the outcome with a sequence of thrusts to the enemy's center, while simultaneously trying to close the gaps that the Prussians made in his right wing. The specialists have never seen anything but a constant battle plan, but have judged it simplistic, dangerous given the frail right flank, and above all of stunningly feeble tactical execution. Leaving aside the full succession of attacks, we will single out the last and most famous, that is the engagement, around 7:30, of the Old Guard, which was the last available reserve.

[22] Houssaye (1905-1961, p. 294-295) explains the desirable path. Grouchy would leave Gembloux by the west, marching to Mousty and Ottignies, where he would cross the Dyle and follow the river's left bank.
[23] It would be more accurate to call the battle after Mont-Saint-Jean, where it took place, than after the neighboring village of Waterloo, but Wellington wanted that name to be chosen. The Germans – Clausewitz among them – long preferred to call the battle after the farm of Belle-Alliance, where Blücher and Wellington met in the evening of June 18.
[24] Roberts (2005) puts the best moment for withdrawal even earlier.

Followed by many others, Clausewitz believes that it was an absolutely hopeless move. He goes as far to claim that Napoleon no longer truly knew what he was doing (p. 158).

The moment has arrived for analytically reconsidering the campaign's main junctures. At three key moments – June 17, around mid-day on June 18, and in the final hours of this same day – Napoleon could have departed from the line of events that his previous decisions had set in motion, and he did not. Is this inertia or lack of reflection, in which case he would no longer conform even to subjective rationality? Or is it a failure to correctly appreciate the situation at hand, in which case this form of rationality could be salvaged? Or is it the case that Napoleon assessed the situation correctly from the perspective of objective rationality, simply accepting the immense risks that this assessment made clear? Essentially, Clausewitz interprets the engagement of the Old Guard as pure and simple irrationality, and the dismembering of the army after Ligny as the result from an incorrect belief held in accordance with subjective rationality. He is more cautious in handling Napoleon's decision to continue the battle despite the threatening Prussian advance. At this point, he realizes that a taste for risk exacerbated by the circumstances may be consistent with subjective *and even objective* rationality - the last of the three interpretations we have just sketched out.[25]

The diagnosis is complicated by Napoleon's objectives, which were not of the usual military kind. He needed not just to win the campaign, but to win it *absolutely*; for a weak victory would not have saved France from being invaded and his regime from collapsing. The two goals that Clausewitz usually assigns to war – destruction of the enemy forces and the political advantage that can be taken from the actions, whether victorious or not – were firmly bound together[26]. The Borodino battle of the Russian campaign, as reinterpreted in *On War* (IV, 12), will make this clear by way of contrast. There, Napoleon refused to engage his reserves against Kutuzov, consciously giving up a more complete victory that was otherwise within his reach. He was justified in holding

[25] See Clausewitz, p. 157. This is a brilliant insight for a time when the concept of risk-attitude was not yet separated from those of risk or uncertainty; see Mongin (2009).
[26] The tension between these two goals of war can be seen throughout *On War*, and Aron's (1976, ch. III) commentary brings it even more clearly to light.

back his limited forces, says Clausewitz, because he meant to enter Moscow in such obvious superiority that the tsar would beg peace from him. Borodino illustrates the long-term political objective separating itself from the short-term military objective, a mental situation exactly opposite to that of Waterloo.[27] In 1815, the short term *was* the long term, there was nothing to be gained from restraint, and only by incurring extravagant risks could Napoleon hope to reach his objectives.

Even the brutal sacrifice of the Guard is more ambiguous than it first appears. Recent military analysis permits reconsidering the battle's last phase. The partial fall of the Anglo-Dutch center around 6:30 afforded Napoleon his best chance of the day. Had he launched the Guard at this moment precisely, rather than an hour later, fate, perhaps, would have turned in his favor.[28] This purely tactical reasoning should be contrasted with an interpretation that has sometimes been put forward. Taking the defeat to be certain, Napoleon would have found appropriate to his legend to finish it with some grandiose, desperate gesture. This is a wild suggestion, but it is not incompatible with the other, and what both have in common is that they deepen the actor's goals in order to dispel the impression that he acted irrationally.

The decision to cut the army in two suffers from a difficulty of a spatial and material nature that cannot be overcome by reconsidering the ultimate goals. Since the unexpected northward movement of the Prussians made it impossible for Grouchy to carry out both the blocking and pursuit missions, one can attempt to salvage Napoleon's rationality by emphasizing either his misperception of the retreat (Clausewitz's solution) or his prioritizing interposition over pursuit in the orders to Grouchy (Fuller's). As we have seen, the conflicting hypotheses are loosely formulated and have no firm evidence to rely on. A proper model of the Emperor's choice should help on both scores. Not only will it make each alternative logically more definite, but if it works well, it will discriminate between them, thus acting like a substitute for the missing data.

[27] Herbert-Rothe (2005) also compares the two battles of Waterloo and Borodino.
[28] This idea comes from Roberts (2005, p. 95).

3. A game-theoretic model of the decision of June 17, 1815

In the following, we model only the actions of Napoleon, Grouchy and Blücher, ignoring Wellington, which can be defended on the ground that he remained fixed at Mont-Saint-Jean after bringing his men there on June 17. In a more debatable simplification, we give Blücher only two possible actions:

B_1, march north, then go westward to join Wellington.

B_2, march north, then go eastward to return to Germany.

We omit a third possibility, B_3, which would consist of marching straight east to Germany. This brings the analysis closer to the actual choice of the Prussians, who did not take B_3 into consideration. The omission is more debatable from Napoleon's point of view, since he initially expected B_3 to occur. However, it would be awkward to formalize the revision of beliefs that took place on June 17 and 18, and we will assume that, even on Clausewitz's interpretation, Napoleon is at any time uncertain between B_1 and B_2, instead of reaching this state of mind only after believing in B_3.

No less schematically, two states of the world are possible:

E_1, Blücher is badly weakened,

E_2, Blücher is not badly weakened.

Before knowing which state is realized, Blücher therefore has four strategies at his disposal:

(B_i, B_j) = if E_1, then B_i; if E_2, then B_j, i, j, = 1, 2.

(By definition, a *strategy* is a function that associates actions to states of the world recognizable by the player.)

On the French side, another gross simplification will integrate Grouchy into Napoleon, treating them as though the latter were in fact the sole decider. It is somewhat paradoxical that this is less contestable from the marshal's own view point – his *Memoirs* describe him as a simple executor of orders – than from the point of view of Napoleon and his staff officers, who overloaded him with complex instructions. Not only does it classically facilitate the game-theoretic analysis to bring the number of players to two, but we will

eschew the difficulty of handling *conditional strategies* such as the following: chase the Prussian rear-guard if it does not appear that the advance-guard is moving to join the Anglo-Dutch force, drive westward in the opposite case. Yet the minority position à la Fuller would be best formalized by analyzing Grouchy just in terms of such strategies.

Thus fused with Grouchy, the player Napoleon has three possible actions:
S_1, keep the army together,
S_2, detach Grouchy's forces and send them to block Blücher's path to Wellington,
S_3, detach Grouchy's forces and send them in Blücher's pursuit.
Now technically reinterpreted, *blocking* (or *interposition*) means that the clash between Grouchy and Blücher will occur if Blücher goes west (case S_2B_1) and not if Blücher goes east (S_2B_2), while *pursuit* means that the clash will occur if Blücher goes the latter way (case S_3B_2) and not if he goes the former (S_3B_1). With these definitions, Grouchy's behavior becomes, as intended, mechanical. He rushes where Napoleon commands, and engages in battle or not depending on whether or not he meets Blücher there. Whereas Blücher learns at the interim stage which state of the world is realized, Napoleon does not, and his strategies are therefore constant functions across the states; that is, they are identical to his actions S_1, S_2, S_3.

The following probability parameters represent Napoleon's beliefs:

- k, the probability that Blücher is badly weakened by his defeat at Ligny;
- l, the probability of victory for Napoleon's consolidated army against a united Wellington and Blücher, *supposing Blücher was not badly weakened* (we will take l to be 0 in a simplified variation);
- l', the probability of victory for Napoleon's consolidated army against a united Wellington and Blücher *supposing Blücher was badly weakened*;
- l'', the probability of victory for Napoleon without Grouchy against a united Wellington and Blücher, *supposing Blücher was badly weakened;*
- m, the probability of victory for Napoleon without Grouchy, and against only Wellington, regardless of the state of Blücher's forces.

It is automatic to suppose that $l' > l$ and $l', m > l''$. Other, less obvious inequalities will have to be added to reach a solution.

The model assigns trivial values to all other relevant probability parameters. Thus, it gives a value of 1 to:

- the probability of victory for Napoleon's entire army against Wellington alone,
- the probability of Grouchy's victory against Blücher, *supposing that Blücher was badly weakened.*

And it gives a value of 0 to:

- the probability of victory for Napoleon, without Grouchy, against a united Wellington and Blücher, *supposing that Blücher was not badly weakened;*
- the probability of victory for Grouchy against Blücher, *supposing that Blücher was not badly weakened.*

It seems inelegant to have so many 0 and 1; however, experimenting with more general assumptions, we have found that they did not appreciably change the conclusions. And the *Memorial* - although an obviously suspect source - does suggest taking extreme values here. For example, it claims that Grouchy's detachment was enough to "topple the Prussian rear-guard in whatever position it took" (p. 239). By this token, it is comparatively moderate to assign probability 1 to Grouchy's victory over Blücher conditional on Blücher being *weakened*. Still from the *Memorial*, "if Grouchy had been on field and time had permitted the French army to deploy itself for battle", one after the other the Emperor would have undone the Anglo-Dutch and Prussian armies (p. 245). Again cautiously, we reserve this probability 1 of victory for the case of Napoleon's entire army fighting Wellington *alone.*

The model also includes the following utility values, which reflect Napoleon's evaluations, just as the probability values reflected his beliefs:

- a_1, the utility of victory against Wellington,
- a_2, the utility of victory against Blücher,
- b_1, the utility of defeat against Wellington,

- b_2, the utility of defeat against Blücher,
- c, the utility of no confrontation.

Nothing substantial is added if we assume that victories give positive, and defeats negative, utility:
$a_1 > 0 > b_1$, $a_2 > 0 > b_2$.
However, by a more debatable assumption, we will freely *sum* the numbers thus defined. In particular,
$a_1 + a_2$ = the utility of victory against Wellington and Blücher together
$b_1 + b_2$ = the utility of defeat against Wellington and Blücher together
In other words, Napoleon's victory against his two opponents at Mont-Saint-Jean would have the same value as his beating Wellington alone on this field, accompanied by Grouchy's beating Blücher's forces elsewhere; and similarly, *mutatis mutandis*, for the defeats of the French at the hands of both enemies.

We must still evaluate the situation in which Grouchy and Blücher do not meet. This receives the value $c = 0$ in the case (S_3B_1) where Blücher marches west and Grouchy pursues him in vain, and the value θa_2 - with θ a parameter between 0 and 1 - in the case (S_2B_2) where Blücher marches east and Grouchy engages in a futile block. The second case differs from the first in that a Prussian retreat without combat represents an additional victory for the French, albeit a much lesser one than would have occurred had Blücher been beaten on the field again.

All that remains in order to represent the situation as a *normal or strategic form game* is to define Napoleon's and Blücher's payoffs for the various outcomes. Using the probabilities k, l, l', l'', m and the utilities a_1, a_2, b_1, b_2, c, θa_2, we calculate Napoleon's payoffs by the customary rule of expected utility. Blücher's payoffs will be supposed to be algebraically opposite to Napoleon's. In technical terms, this is a *zero-sum game*, which reflects the nature of this – although not every – military campaign.[29] One might

[29] Following the previous analysis, at Borodino Napoleon did not aim for Kutuzov's total annihilation. Unless the payoffs are redefined, a zero-sum game would therefore not correctly represent the strategic

however argue that the game is not classically zero-sum. Being also a game of incomplete information, it entails opposite values for expected utilities payoffs, which means that an assumption is made on the probabilities as well as the final payoffs.

We now sketch the resolution, leaving the details for the appendix. The argument will emphasize three expected utility payoffs, denoted V_1, V_2, V_3 in the game matrix:

	B_1B_1	B_1B_2	B_2B_1	B_2B_2
S_1	V_1			
S_2			V_2	
S_3	V_3			

The first step is to associate with each of Napoleon's strategies the minimum payoff it can bring, taking account of Blücher's response; this is the strategy's *security payoff*. For S_1, the minimizing strategy is B_2B_1 and the security payment is V_1; for S_2, they are B_2B_1 and V_2; and for S_3, B_1B_1 and V_3. The last two conclusions follow from the automatic assumptions, but the first needs optional ones on the utility values as well as l, l', θ. These boil down to an algebraically precise statement that l and l' are bounded from above.[30]

The second step compares the strategies S_1, S_2, S_3, supposing that each brings in its security payoff. The largest of the three numbers - his *maxmin* - is the greatest amount that Napoleon can guarantee himself, regardless of what Blücher does against him. We will assume that he plays the strategy associated with this value. The comparison between S_1 and S_2 depends on the inequality $V_1 < V_2$, which is equivalent to a joint restriction on k, l, m and the utility values. This restriction is pleasantly simplified when $l = 0$.[31] The

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interaction of the two adversaries. Haywood (1954) also underlines that not every battle is appropriately modeled as a zero-sum game.

[30] B_2B_1 minimizes the payoff of S_1 iff $\dfrac{a_1 + \theta a_2 - b_1 - b_2}{a_1 + a_2 - b_1 - b_2} > l, l'$. As θ grows, the central expression increases towards 1, thus binding l,l' less and less. This is not a very constraining assumption.

[31] We derive $V_1 < V_2$ from $m > k(1 - ld) + ld$, putting $d = (a_1 - b_1 + a_2 - b_2) / (a_1 - b_1)$. This is a substantial and constraining assumption, which is simplified as $m > k$ when $l = 0$. To ensure that the right-hand side is between 0 and 1, we also impose that $ld < 1$ - another bound on l - and that $m > 0$, $k < 1$.

comparison between S_3 and S_2 depends on the inequality $V_3 < V_2$, which follows from a joint restriction on k, m, θ and some utility values.[32] On the basis of these conditions, we conclude that V_2 is Napoleon's maxmin and that he plays S_2.

The third step is to investigate Blücher's strategies, B_1B_1, B_1B_2, B_2B_1, B_2B_2, calculating security payments for each, and finding the highest of these four numbers, i.e., Blücher's maxmin, as well as the corresponding strategy. Without further parameter restrictions, these are V_2 and B_2B_1. Making the same behavioral assumption as for Napoleon, we conclude that Blücher plays B_2B_1.

The resulting outcome (S_2, B_2B_1) satisfies *von Neumann and Morgenstern's solution concept for zero-sum, two-person games*. This is not a genuinely interactive concept; rather, as the previous two paragraphs have illustrated, it applies an individual rationality argument twice over, rationality being identified with prudence (each player protects himself against the opponent's most damaging strategy). However, it is a well known result - holding somewhat more generally than for zero-sum, two-person games - that a solution so defined is also a *Cournot-Nash equilibrium*, i.e., a pair of mutually optimal responses, and conversely.[33] That is to say, S_2 is Napoleon's best response to Blücher's choice of B_2B_1, and B_2B_1 is Blücher's best response to Napoleon's choice of S_2. We could have found the solution (S_2, B_2B_1) just by computing best responses, but this easier method would have been harder to justify in terms of individual rationality strictly speaking.

The chief tool of zero-sum, two-player games, the minimax theorem, was not directly usable here.[34] We obtained the result that Napoleon's maxmin payoff is equal to the algebraic opposite of Blücher's maxmin payoff for the initial – so-called *pure* – strategies of the two players. The theorem would have secured the equality only for the more

[32] A sufficient condition for $V_3 < V_2$ is that $m > k(1-\theta)a_2 / (1-k)(a_1-b_1)$. The right-hand side is less than 1 if $a_1-b_1 > a_2$ and either $k < ½$ or $\theta > k$. The first assumption is fully justified in the historical context of 17 June. Either of last two is conjectural and enters a substantial explanation of the case. There are other sufficient conditions available.

[33] See Nash (1950) and Luce and Raiffa (1957, appendix 2).

[34] Due to von Neumann (1928), this theorem owes its fame to von Neumann and Morgenstern (1944-1947).

numerous – so-called *mixed* – strategies, which amount to randomizing between the pure strategies by means of some probability distribution; hence the need for an *ad hoc* proof.

In limiting case $l = 0$, some of the parametric conditions vanish, while $V_1 > V_2$ becomes equivalent to $m > k$, which is easy to interpret: *the risk that Blücher is not badly weakened is greater than the risk of losing a duel against Wellington.* This assumption is the core of our game-theoretic account of Napoleon's deliberation on 17 June, which goes informally as follows. First, he discarded S_3 because Grouchy could be put to a better use if he adopted S_2. This step is not entirely trivial, because S_3 is not dominated by either S_2 or S_1.[35] Then came the truly difficult choice, that between S_1 and S_2, which he resolved in favor of S_2 after comparing the two risks just said. Had Napoleon really gone through these reflections, he would have acted prudently, not as the inveterate gambler of legend. He faced the unpleasant possibility that Blücher, having weathered Ligny better than expected, would defeat Grouchy, but could exclude the worse possibility that Blücher would join forces with Wellington against him alone.

A passage of *Memorial* (p. 239) suggests the relatively high value for m that we need for the reasoning: the Emperor's remaining forces were enough to "topple the Anglo-Dutch army" despite a slight numerical disadvantage. Unfortunately, it says nothing to suggest that k was small, except perhaps in the following, roundabout way. Had Napoleon believed the Prussians more diminished than we submit, he would have turned different reproaches on Grouchy. In the already cited passage of p. 245, he describes himself beating Wellington and Blücher one after the other, and he keeps the final victory over the Prussian for himself, leaving it to Grouchy to pin him down for some time, while he was finishing the Englishman. Such a chain of events only makes sense if Blücher was not already annihilated by his defeat at Ligny.[36]

[35] One strategy is *dominated* by another if it returns a smaller payoff for all the opponents' responses, in all states of the world. This game does not give dominated strategies to Napoleon, but it does to Blücher; see the appendix.

[36] Inconclusive as they also are, two already discussed staff documents suggest a low k. On June 17, Bertrand warns Grouchy about Blücher's remaining possible maneuvers, and Soult's dispatch of June 18 confirms that Napoleon was concerned about an offensive return of the Prussians.

While distorting Clausewitz' thesis to an extent, the model brings out the main reason for not accepting it. *The thesis maintains that Napoleon dispatched Grouchy for a chase even though the only sensible choice for him was between dispatching Grouchy for interposition and keeping the army together*. As can be checked, the conditions for getting the security payoffs associated with S_1, S_2, S_3 are mild or definitional, and maxmin reasoning excludes S_3 from consideration simply on the basis of a definitional inequality ($V_3 > V_1$ follows from $l' > l''$). So were a Clausewitzian to reject our parametric restrictions, he would in effect support S_1 against S_2, but not S_3 against these strategies. Admittedly, Clausewitz implied specific values for some parameters. According to *The Campaign*, Napoleon believed the Prussians to be badly damaged and was confident to be able to defeat Wellington without Grouchy. This amounts to taking large values for both k and m. One may add the reinforcing assumption that Napoleon highly valued another full victory against Blücher, i.e., that a_2 is large and θ small. This is the Clausewitzian case parametrically expressed, and it does not affect the overall comparison.

What now for the proNapoleonic position? The model allows his proponents to make some order of their probability assignments. Fuller, for example, could agree with a low value for l, fairly large ones for l' and m, and a weak one for k, and he would likely accept a moderate abatement θ. Enriched with such parametric restrictions, the position offers a logical coherence that the other, given its own preferred restrictions, lacks dramatically. This is not to deny that it involves a difficulty that the other does not. For it leaves unexplained the behavior of Grouchy, who, in our simple dichotomy of chase or block, undertook the former instead of the latter, for which he should have received more or less explicit commands. So the model salvages Napoleon's rationality at the cost of wrecking Grouchy's. But the Clausewitzians makes the opposite trade-off, which involves a worse failure, given what can be inferred from each actor's past performances.

We stressed earlier that the two interpretations could only hypothesize what Napoleon's commands to Grouchy truly were. Because of this empirical limitation, we used the model in no less than three functions, all of which involve the same set of parametric restrictions. First, the model permitted evaluating the strategies S_1, S_2, S_3; second, it

established that Napoleon, acting rationally, ordered S_2 rather than S_3; and third, by the same rationality assumption, it explained this alleged fact as well as the observed fact that he did not order S_1. In standard methodological accounts, the explanatory use of rationality assumptions succeeds a straightforward observational stage. It is said, for example, that preference maximization explains the consumers' demand function for a product, while this function results from market data or questionnaires. But in our study like in many other historical works, *what needs explaining is not fully observed*. Equivocal reports (the testimonies and dispatches) stand for the missing pieces of information (the oral commands). This is why we used the model also in the function - numbered two above - of clarifying the *explanandum*. This makes the explanatory process circular, in contradistinction with the consumers' demand case, but not necessarily vicious or inadequate. For there is nothing sinister in a circular reasoning if it makes overall sense of a sufficient amount of sufficiently diverse data (and it is important in this respect that Napoleon's rejection of S_1 can be observed). Still, a reasoning of this style is probably better fitted to assess the comparative value of existing accounts and arguments than to provide a full-fledged explanation of the facts themselves.

4. Response to objections

It is not hard to foresee that the model above will elicit objections similar to those which enlivened the "analytic narrative" controversy. At the most abstract level, game theory itself was called into question. It is well known, for instance, that some games have no solution in any acceptable sense, while others have too many possible solutions, either because there are multiple competing equilibrium concepts, or because the adopted concept - typically the Cournot-Nash equilibrium – allows multiple equilibria to exist.[37] Our game-theoretic analysis escapes these difficulties. It uses the Cournot-Nash equilibrium concept in a case – a two-player, zero-sum game – in which specialists have never questioned its appropriateness, because it coincides there with an intuitive concept of individual rationality, namely prudence. Moreover, our particular game has no other pure strategy equilibrium than the one calculated. But it will no doubt be said that our

[37] Elster (2000) recalls these difficulties, which he has often emphasized elsewhere, e.g., in his 1986 paper.

technical assumptions are *ad hoc*. Thus the criticism passes naturally from abstract objections to those of the intermediate and lowest plane, which are more challenging because they are more specific. *Game-theoretic applications to history risk impoverishing both real events and the theory they draw upon.*

We have anticipated on an objection in this group. Grouchy should count among the strategic actors, alongside with Blücher and Napoleon. If this were done, a distinction between strategies and actions would appear on the French side, paralleling that implemented on the Prussian side. Napoleon would have the choice of either remotely controlling Grouchy, or of delegating him the power to act according to what he would find out on the spot. In a game thus refined, the *ex post* inadequate pursuit might become one of the equilibria instead of being a deviation from the single equilibrium. Although these changes are desirable, the model such as it is will have served at least to illustrate the method and assess the conflicting interpretations.

Another common query in the same group has to do with the actors' objectives, but it does not have much force here. We have already argued for the appropriateness of the zero-sum assumption. The further assumption of additive utility seems defensible on the very same ground, i.e., that nothing short of a crushing victory in the campaign could fulfill the Emperor's objectives. The number and order of the battles mattered little to him as long as he achieved this final result. However, we have not taken the idea to its extreme, since we added only the final, not the expected utilities, which would have altered the conclusions significantly.

There remain the objections of the lowest level of generality, which were the embarrassing ones in the previous controversy. It was said against each selected historical application that it was too ambitious for game theory’s tools. Because our study was aimed specially at avoiding this complaint, a fuller analysis of what is peculiar to military campaigns is now in order; we organize it into six mutually supporting arguments.

First of all, the hierarchical nature of military organization makes it acceptable to concentrate the study on the decisions made by a few key individuals – typically, the general-in-command, his chief-of-staff and the principal lieutenants. In actual fact, the military organization departs from its official definition in a number of ways, and the human material never has the suppleness required to make the leaders' instructions fully effective. As would any model based on rational choice theories, ours integrates these "frictions" – to use Clausewitz's famous term (*On War*, I, VII) – by way of probabilizing consequences for the actions of the irreallistically few decision-makers it selects. The limit of this method is that it rules out some possibly relevant interactions. Thus, to treat the discipline among rank-and-file as a stochastic phenomenon is to forget that it depends on a range of activity on the leaders' part – demonstrations of courage, promises, threats and exhortations – which do not normally enter the model's list of actions. To this, we may reply that the neglected interaction is not always significant to the same degree, and that the empirical material itself should serve as a touchstone. When an army disbands, the modeler must bring forth the relation between the leaders and the troops; when it obeys orders, as it did on June 18 before the tragic denouement, he may pass over it.

In the second place, as a campaign proceeds – and even more clearly once a battle is engaged – each of the general-in-command's decisions is easy to locate in time and space, and so are, in principle, the staff members' and lieutenants' induced decisions.[38] The proximate effects of all these changes are movements of such and such part of the army at a certain hour and in a certain direction, which are, again in principle, ascertainable. The military distinction between *a strategy* and *a tactic* becomes relevant at this juncture. The former organizes the movements of a campaign, which prepare for battle or link multiple battles in view of the final victory, while the latter arranges the movements of a given battle in order to win it out.[39] This is a means-ends hierarchy, but it also taken to express differences in spatio-temporal locations - the movements of a campaign being more remote than those of a battle - and to reflect the hierarchical

[38] We add "in principle" because the uncertainty grows as the level of command decreases. For instance, there is no completely firm evidence of who exactly ordered the main cavalry charge on June 18, and some historians have questioned the received view that Ney did.
[39] This is essentially the distinction made by Clausewitz (*On War,* I, II).

organization - the general-in-command being the sole responsible for the strategy, whereas he shares or delegates responsibilities on the tactic.[40] The June 1815 sequence of events illustrates this multiple distinction neatly. On the basis of his plan of campaign, Napoleon entrusted Ney and Grouchy with the supervision of battles that were likely to be pitched in remote directions. Even on the battlefield of Waterloo, where he was present, he left much to Ney to decide. For the purpose of an "analytic narrative", it is very convenient to be able to investigate strategic decisions independently of tactical ones, especially if this division coincides with one in terms of individual actors.

Third, the overall goal is determined from without and once and for all; it is to win out the battle or the campaign, as the case may be. According to the older military definitions, the first occurs with the final occupation of the field, and the second with the conquest of a province or stronghold. The modern concept of a victorious campaign or battle is more abstract, holding it to be the destruction of the opposing forces or, failing that, their significant weakening, with their own admission of the fact if possible. This much is suggested by *On War*, although there has been some debate on Clausewitz's precise meaning.[41] The plural understanding of victory gives birth to some ambiguity in the assessment of success or failure as one goes back in time.[42] But this is by and large a second-order problem, which should be set against the broadly correct point that, in the present context, the teleological component of reasons is both fixed and simple.

Fourth, even a campaign decision of the highest degree of complexity is in principle assessed in terms of its final consequences on the field. An idealized general-in-command would reduce the content of his decisions to that of their successive effects, then pass back step by step from the evaluations of the latter to the comparative evaluation of the former. Seen from this *consequentialist* perspective, the choice of June 17 was relevant

[40] Before Clausewitz, Bülow emphasized the spatio-temporal and organizational aspects, i.e., the tactic has to do with military movements within the angle of vision or the possible reach of the general-in-command, and the strategy with what goes beyond.
[41] Arguing from the relationship of politics to military activity in *On War* (VIII), Aron (1976) concludes that Clausewitz promoted a novel conception of victory. But Paret (1992, p. 106) makes it clear that he did not altogether give up the notion that the military objective is to conquer terrain.
[42] Even some Napoleonic events are not easy to classify. With Borodino, Eylau is the classic example of a dubious victory, and Tolstoi's *War and Peace* goes as far as to claim the two battles for the Russian camp.

only in relation to the chances of victory or defeat which it established for June 18. A decision of that kind does exhibit further characteristics - for example it may or may not comply with the art of war, or it may require more or less courage than another. But, for consequentialism, these characteristics disappear from the evaluation insofar as they do not influence the outcome of the engagements (violation of the rules of war may catch the enemy off-guard; courage of the commander may fire up the troops). Rational choice theories endorse consequentialism mathematically, using combination rules like expected utility in order to evaluate actions by backward reasoning,[43] and that military campaigns by and large comply with this principle turns them into a promised land for the application of these theories.

But are the last three characteristics, which so conveniently excuse the simplicity of our work, not themselves simplifications of a military activity that is more varied than has been admitted? A historical prudence is necessary, given the changes undergone by the military activity and its reflection in strategic theorizing since Napoleon's time.[44] The concepts of the decisive battle and of the masterminded campaign that leads to it are marked with the stamp of an era. The indefinite battles of the First World War following that of the Marne, with the new concept of the total war they heralded, the colonial wars, the guerrilla combats and other dirty wars so common in the 20th Century - all shook the previous characteristics to a significant extent. In the other direction, scholars have pointed out that the wars of Ancien Regime embodied highly specific conceptions of the battle and the campaign, and even of the destructive nature of war. Taking these facts into account, strategic thinking today is more inclined to emphasize historical variability than it was at its Clausewitzian peak. Since our "analytical narrative" methodology aims at connecting the proposed models with existing accounts, we cannot ignore the intellectual shift. Thus, our game-theoretic application is open to the objection that it is *too* well chosen, revealing only limited potential for generalization beyond its time period. The same techniques as we used on Napoleon might still analyze Joffre at the Marne, but only

[43] Game theory which employs rules other than expected utility is regrettably not well developed.

[44] On these changes, see the Earle's collection (1943) and Aron's (1976) comments on Clausewitz's heritage.

with difficulty Falkenhayn at Verdun, and probably not at all Massu during the battle of Algiers, or the Israeli generals during their South Lebanon campaigns.

While we must emphasize relativity in some way, we are not committed to the *historical* relativism that the previous argument suggests. Military campaigns confront rational choice theories with a continuum of obstacles that exist in the abstract before taking shape at particular time locations. That is to say, these theories apply more closely as the distinction becomes clearer between war and peace, and combat and cessation; as the goals of the combatants in each camp turn out to be more closely aligned; as military decision-making adheres to a stricter hierarchy; and similarly with other properties. By exploring this kind of dependency further, we would eventually conclude that it is not history *per se* that decides whether or not an application is feasible. Indeed, casual reviews suggest promising examples in the mid-20th Century or in Ancien Regime or even in Ancient Rome, as well as discouraging ones at the height of 19th Century strategism.[45] A military historian adopting the methodology proposed here would not be in a very different position from an economist, whose success varies with the areas of social interaction to which he applies the maximizing and equilibrium assumptions of his theories.

A fifth significant consideration is that military actions are *already* seen as rational or irrational even before the observer applies these qualifiers. The actors themselves are usually the first to adopt them, either *ex ante* or *ex post*; then, polarized in the same way, come the judgments of witnesses, memoirs writers, military instructors, academic historians. It is out of the question to set aside any of these layers of commentary, because taken together, they make up almost all of the information history can collect, and also because they direct towards the topics in need of treatment. Napoleon produced the first systematic study of the Waterloo campaign, which made possible Clausewitz's and moved others, and on it went, right up to the current modest essay, which capitalizes much on its predecessors. Each step has brought out both new information and new

[45] Compare the campaigns investigated by Fuller (1954-56) on a very broad time range. Some are evidently more amenable to rational choice modeling, but there is no such obvious time-dependency as it seemed at first glance.

problems, sometimes unsuspected.[46] The existence of a reflective spiral is certainly not specific to military history among the various historical sub-disciplines, but its nature is; for more or less openly, all commentaries in this branch boil down to questioning the rationality of the main actors.

We close the list with a sixth and final point, whose weight will depend on the more or less conclusive instantiation of the third. Schematizing the *teleological* side of the reasons for action helps to underscore the explanatory power of the *cognitive* side. Rational choice theories – more generally speaking than just game theory – find their peak inferential capacity when they are specialized in this way.[47] For example, by assuming that firms maximize their profits, economists manage to connect the oligopolistic structure of a market with the conjectures that each firm makes about its competitors' strategic moves. Or again, by assuming that stock market traders maximize their expected utility, and that they are identically risk-averse, they can relate asset prices to the beliefs held by these traders. Both applications illustrate the logical power of models that, for one, postulate simple objectives, and for another, keep them fixed throughout. Military historians approximate this one-sided method when, proceeding from the assumption that victory is desirable, they focus their explanations on a few actors' beliefs and risk-attitudes. Military historians may be even more justified than economists in going this way, because their assumption that the general-in-command aims at victory is altogether more convincing than the peculiar teleological assumptions just mentioned on firms and traders.

Returning to the "analytic narrative" controversy, we see that the six-character list illustrates a possible handling of the objections of the lowest level. Instead of arguing about the studies on their individual merits, it would help if one related them to *a preexisting list of conditions of applicability*, and check the extent to which each study satisfies these conditions. Thus viewed, even that part of the controversy might find a

[46] Largeaud (2008) provides a thorough account of how interpretations of Waterloo have succeeded, and to an extent generated, each other on the French scene. One would welcome similar reviews for the British and German scenes.

[47] One can debate the explanatory dissymmetry that appears between desires and beliefs. Davidson (2004, p. 26) claims that it is structural, but others see only an accidental property of the available facts.

more balanced resolution than it did. No doubt, the characters selected above are fitted for the military application and not yet at the proper level of generality, but they do suggest directions in which more abstract criteria may eventually be found.

5. How to articulate rational choice modeling with historical narrative

Among other obstacles when they enter the study of history, rational choice theories confronts "the culture of the unique", to quote a recent discussant's felicitous expression.[48] This refers to a common tendency in the field to treat past events from the angle that maximizes their individuality. On the philosophical view that the historian seeks not just to evoke, reconstruct or describe that which has passed, but also to explain it, he shows a distinctive taste for singularity in the choice of both *explanandum* and *explanans*. A reproducible natural phenomenon such as the eruption of Vesuvius in 79 C.E. interests him only to the extent that it serves to establish surrounding events that are unique in character. The historian may ask, for instance, why Pompeii and its neighboring countryside developed so well under the threat of a destructive eruption. Although it would be conceivable to connect the farmers and merchants of Pompeii with those who, in another age, fell beneath the eruptions of Etna or the Java volcanoes, it is doubtful that the historian of ancient Rome would accept so to extend his *explanandum*. For this would again mean highlighting similarities, even if the regularity is now social and not geological in nature. The same predilection can be found in the tentative *explanans*. Rather than reduce the attitude in Pompeii to extemporal schemata of risk calculation, historians prefer invoking, say, the judicial system of Campanian agriculture in the first Century, or the complex subjective relationships that the pagans of antiquity had with natural forces. Although perhaps too didactic, this little example shows how the "culture of the unique" may permeate the whole of historical work, and if it does involve such an activity, the whole of *historical explanation*. Rational choice theories are excluded twice over - by the nature of the questions posed and that of the answers provided.

[48] Grenier (2001, p. 91).

But does not the taste for singularity run up against the obvious fact that history sometimes repeats itself? This objection calls for a straightforwardly semantic response; it is undone through a reinterpretative analysis that is in principle applicable to any possible case. By adducing the fact of repetition to those characteristics of the events whereby repetition occurs, one paradoxically reinstates the uniqueness of the events in question. If Louis-Napoleon Bonaparte's coup of 18 Brumaire is viewed as an purposeful reproduction of his uncle's 18 Brumaire, then, as Marx ironically underlines, the two acts differ for exactly that reason, the one being a parody of the other. A more abstract, but still comparable, semantic analysis would take account of nonintentional repetition. To take a scholastic example, if John Lackland passed twice by the same place, this return could well have been involuntary and even unknown to the principal, yet it holds a distinguishing connotation - the sad and rare destiny of a prince bereft of his dominion. It is enough for the historian to emphasize this secondary meaning for John's return to cease to seem as such; it has become a novel kind of event. Thus, it is always possible to embrace repetition from the point of view of historical singularity, and although this is but an option, not a necessity, historians generally follow it.[49]

The above insights on the "culture of unique" do not appear to clash with the view that it is an acquired disposition among historians – therefore a "culture" in the literal sense – rather than a structural constraint imposed by their discipline. Abstractly, the general can be contrasted with the *particular* as well as with the *singular*, and these are two different concepts. The singular is unique and inimitable, while the particular does not have to be so. Dupont, a French citizen, is listed under a certain number in the national registry upon his birth; this number is particular to him, but in no way does it singularize him, since every other French citizen also receives an equally particular number. Guided by this contrast, we raise the question, why must the historian favor the singular over the particular? Why must he refrain from considering the choice of location of the Pompeians from the perspective of a commonplace calculation of risks and benefits that also encompasses the peasants of Java and Sicily? The "culture of the unique" could simply

[49] Here we echo an elegant argument by Veyne (1971, ch. 1).

be *a historiographic trend among others*, related to the peculiar belief that anachronism is the gravest of professional errors. Since it may result in relativism, which not all historians accept, it is easy to conceive of an opposite trend that reverses the priority of evils, holding this consequence to be even worse than anachronism. To document the conflict between the two trends in historiographic research goes beyond this work, but perhaps enough has been said to conclude that the "culture of the unique" is not an objection; it would be one if it were more than a culture, which is precisely dubious.

We would end up with a stronger dismissal if we rallied Hempel's (1965) classic analysis of historical explanation. According to this analysis, a statement on the Pompeian judicial or religious system cannot explain why the inhabitants lived under the threat of Vesuvius if it is not made part of a larger formulation that includes general laws - and most typically, although Hempel was not dogmatic here, a rational choice theory. Then, the institutional facts just mentioned become "initial conditions" of the laws, or if the latter do belong to a rational choice theory, parameters describing the inhabitants' objectives and beliefs. It follows from Hempel's analysis that historians should either give up explanation as one of the discipline's tasks or overcome "the culture of the unique", which diverts their attention from generalities. Hempel himself took the view that historians do make, or at least sketch, explanations, and that they overemphasize the singularity of events only by a misperception of this process. Because they are not interested in generalities *per se*, they impoverish their picture of their own work, retaining only the "initial conditions" or the parameters of each historical situation. An alternative view is that they know exactly what they are doing, and indulge in other activities than explanation: for example, description, evocation, or comprehension, genres which Hempel carefully separates from explanation. By not practicing it, historians would diminish the scientific status of their discipline; this time, more than their self-image is at issue. Each Hempelian line delivers a direct argument against "the culture of the unique". But the initial analysis of explanation has been contested so often that we

cannot simply embark on it here.[50] We will be content with the less sweeping dialectical reply made above.

The worst obstacle impeding rational choice theories is yet to come. *Models*, which are their vectors for concrete application, do not get on well with *narratives*, the historians' canonical way of expression. This contrast has risen to the forefront of some recent methodological discussions.[51] What makes it serious is that the narrative genre is immensely flexible, hence apparently self-sufficient; to superimpose a modeling exercise seems to be not only cumbersome, but unnecessary. In order to evaluate the objection, let us briefly review the roles that narratives play outside the literary field of fiction. Certainly, they can evoke, describe, and help understand the events they report. It is an open question - actually connected with the Hempelian debate - to what extent they also perform explanations. A feature that has not yet been mentioned is that narrators pass judgments that are not always exclusively factual. Some are evaluative and some – more subtly – factual and evaluative at once; both this duality and its partial erasure are rooted in ordinary language, which is the narrator's medium.[52] We also insist, because it is so rarely mentioned, on the feature that narratives allow one to discuss human actions without deciding – or even addressing – the metaphysical question of determinism and liberty. "On June 18, the ground remained sodden until late morning, and the battle did not begin until that time." Such a sentence is, as it were, *metaphysically open*. At a closer examination, it may be either that the weather conditions made an early start unfeasible or that Napoleon preferred to wait, having a real choice. The narrator can carry on without committing himself to either view; his sentence is entirely comprehensible, although not very explanatory as it stands.

We are now to discuss two major causes of tension between models and narratives. The former are associated with *hypothetico-deductive reasoning*, which the latter cannot accommodate. Along with their special symbolism, the former carry with them a

[50] The most elaborate objections to Hempel are from Dray (1957), who simultaneously endorses the explanatory power of rationality and rejects the nomological component of explanation.
[51] *Le modèle et le récit* (2001) extends this duality to the social sciences generally.
[52] Such commonplace predicates as *progress* or *poverty* are, by nature and irreducibly, hybrids of the factual and the evaluative; see Mongin (2006).

semantic of their own that is inassimilable by the latter. It is worth stressing that conflicts occur despite the fact that the narratives of interest are nonfictional; we may assume that the same concern with truth is shared by the modeler and narrator.[53]

We must immediately qualify the first cause of tension. Obviously, a narrative is not a deduction, but because it deals with human action, it includes an inferential aspect of an everyday sort. "Having considered the sodden ground, Napoleon delayed the start of the battle." The temporal succession here covers an inference by the actor and, doubtless, by the narrator himself - nobody would send people and carts out if the mud would impede their progress. It is another virtue of the narrative that it need not make precise to what extent chronological order establishes logical order and, if it does, upon whom such logical order depends, the actor or the author. Beside inferences, narratives can leave room for hypotheses - the other ingredient of hypothetico-deductive reasoning. It would be a mistake to believe that nonfictional narratives rule them out because of the declarative nature of their statements. For these statement may be true or false even if there is no sure way to determine which in fact it is, and the narrator can in this case either confer on them an intermediate value, such as *probable*, or suspend judgment altogether. "Napoleon probably decided that the mud did not permit an attack." "One might ask whether the attack took place after 11:30 because of the earlier inclement weather, or for other reasons." Sentences like these are optional, because a narrator is under no obligation to make his epistemic states public, but they fall easily into place. They bring out hypothetical, even hypothetico-deductive, nuances that are acceptable within the limits of the genre.

The second cause of tension - the semantic duality - appears thornier. A rational choice model brings with it not only mathematical notions, but a customary interpretation of them, on which the effective work of this model is conditioned. In game theory, to mention but this major case, the various notions of equilibria are associated with scenarios that modelers use for reasoning at the same time as they write the algebra – and often in place of the algebra altogether. The problem is that *these semantics differ from*

[53] Grenier (2001, p. 82 and 89) identifies the same two points of tension, although without this proviso.

those of the historian. They have been elaborated within game theory's own cultural tradition. They combine vague generalities of common sense claimed unreservedly (e.g., that people prefer more to less), assumptions presented as convenient simplifications (e.g., numerical payoffs used to capture satisfaction), analogies elevated to schemes of reasoning (e.g., lotteries serving to describe all sorts of probabilistically uncertain outcomes). The historian may well be suspicious of such specialized meanings, which are likely not to match well with his own. A strategy, in the parlance of military historians, does not mean the same as for game theorists; a commonplace for the former, the distinction between a strategy and a tactic is unfamiliar to the latter. It would be easy to flesh out the list of mismatches between the two groups of researchers.

But this very real difficulty does not dash any hope of reconciling historical narratives and rational choice models. The last paragraph actually set a concretely identified rational choice theorist against an also concretely identified historian. The tension arises between two heterogeneous micro-cultures that are not forever fixed and may even coevolve with contact. Thus, the notions of tactic and strategy are not impossible to represent game-theoretically, and conversely, historians might absorb the technical distinction between strategies and actions. In fact, the semantics of the two groups both sink their roots into common sense, which needs only to be systematized, and certain natural connections will appear. Some existing campaign narratives exhibit these connections so well that it does not require too much extra-work to proceed to the modeling stage. We have illustrated this process when basing our game theory on Clausewitz's and Fuller's accounts. For sure, the process involved a semantic loss, but it appears neither that it destroyed all meaning, nor that it reassembled diverse meanings incoherently. In sum, the objection of incompatible semantics does not have the weight that was first afforded to it.

A generality now begins to make itself seen. The tension between historical narrative and rational choice modeling is relaxed as soon as we give up the attempt to put them on equal footing – as though the first were a well defined genre, after the fashion of the other. The historical narrative never discharges its functions entirely; only for that reason can he serve so many of them at a time. It is flexible all right, but not self-sufficient. At

the opposite, rational choice modeling lends its hand to a few functions thoroughly; it untangles ambiguity – that is its gain – but is too focused – that is its loss. Thus, when applied to a narrated sequence of events, it makes explicit the underlying inferences, assigning them to the actor, the narrator or both; it unearths epistemic values that the narrator had merely meant to suggest; it unifies to an extent the semantic values, favoring those with commonsense interpretations. Taking up other features of the narrative for comparison, we may add that rational choice modeling promotes explanation at the expense of evocation and description, stabilizes the distinction between the factual and the evaluative, and also clarifies that between determinism and liberty. On each score, the precision gained has a possible cost of irrelevance or even silliness. The modeler is chancy where the narrator was overcautious. The two would better work side by side, each sinning according to his own faults, each benefiting from the other's faults.

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APPENDIX

As stated in the text, the zero-sum game between Napoleon and Blücher has a unique equilibrium (S_2,B_2B_1) if some optional conditions hold, beyond those which are automatically ensured by the definitions of final payoffs, probabilities and the abatement coefficient θ. The present appendix gives some details on the computation of this equilibrium.

Let us label Napoleon's payoffs in the following way:

	B_1B_1	B_1B_2	B_2B_1	B_2B_2
S_1	V_{11}	V_{12}	V_{13}	V_{14}
S_2	V_{21}	V_{22}	V_{23}	V_{24}
S_3	V_{31}	V_{32}	V_{33}	V_{34}

Each V_{ij} is obtained by an expected utility calculation:

$$V_{11} = k\left[l'(a_1 + a_2) + (1 - l')(b_1 + b_2)\right] + (1 - k)\left[l(a_1 + a_2) + (1 - l)(b_1 + b_2)\right]$$
$$V_{12} = k\left[l'(a_1 + a_2) + (1 - l')(b_1 + b_2)\right] + (1 - k)\left[a_1 + \theta a_2)\right]$$
$$V_{13} = k(a_1 + \theta a_2) + (1 - k)\left[l(a_1 + a_2) + (1 - l)(b_1 + b_2)\right]$$
$$V_{14} = a_1 + \theta a_2$$
$$V_{21} = k\left[ma_1 + (1 - m)b_1 + a_2\right] + (1 - k)\left[ma_1 + (1 - m)b_1 + b_2\right]$$
$$V_{22} = k\left[ma_1 + (1 - m)b_1 + a_2\right] + (1 - k)\left[ma_1 + (1 - m)b_1 + \theta a_2\right]$$
$$V_{23} = k\left[ma_1 + (1 - m)b_1 + \theta a_2\right] + (1 - k)\left[ma_1 + (1 - m)b_1 + b_2\right]$$
$$V_{24} = ma_1 + (1 - m)b_1 + \theta a_2$$
$$V_{31} = k\left[l''(a_1 + a_2) + (1 - l'')(b_1 + b_2)\right] + (1 - k)(b_1 + b_2)$$
$$V_{32} = k\left[l''(a_1 + a_2) + (1 - l'')(b_1 + b_2)\right] + (1 - k)\left[ma_1 + (1 - m)b_1 + b_2\right]$$
$$V_{33} = k\left[ma_1 + (1 - m)b_1 + a_2)\right] + (1 - k)(b_1 + b_2)$$
$$V_{34} = k\left[ma_1 + (1 - m)b_1 + a_2)\right] + (1 - k)\left[ma_1 + (1 - m)b_1 + b_2)\right]$$

The definitions of k, m, l, l', l'', θ and the sign restrictions on a_1, b_1, a_2, b_2 imply a number of inequalities between the V_{ij}:

$V_{11} > V_{31},\ V_{21} > V_{31},\ V_{12} > V_{32},\ V_{22} > V_{32},\ V_{14} > V_{24},$
$V_{22} > V_{21},\ V_{21} > V_{23},\ V_{22} > V_{23},\ V_{24} > V_{23},$
$V_{32} > V_{31},\ V_{33} > V_{31},\ V_{34} > V_{33}.$

The strategic analysis on Napoleon's side proceeds as follows. It is the case that:

$V_{14} > V_{13}$ and $V_{12} > V_{11}$ iff (*) $\dfrac{a_1 + \theta a_2 - b_1 - b_2}{a_1 + a_2 - b_1 - b_2} > l,$

and:

$V_{13} > V_{11}$ iff (*') $\dfrac{a_1 + \theta a_2 - b_1 - b_2}{a_1 + a_2 - b_1 - b_2} > l'$.

We assume both (*) and (*') to hold, thus ensuring that $V_{11} = V_1$ is the security payoff of S_1 (cf. fn. 30).

By inspecting the definitional inequalities, we observe that $V_{23} = V_2$ is the security payoff of S_2 and that $V_{31} = V_3$ is the security payoff of S_3.

Now to compare the three values V_1, V_2, V_3. In view of (*'), the inequality $V_2 > V_1$ can be obtained from $V_2 > V_{13}$, which is equivalent to:

$(m - k)(a_1 - b_1) > (l - kl)(a_1 - b_1 + a_2 - b_2)$,

or:

(**) $m > k(1 - ld) + ld$, with $d = \dfrac{a_1 - b_1 + a_2 - b_2}{a_1 - b_1}$.

This inequality makes sense only if the right-hand side is between 0 and 1, i.e., only if $m > 0$, $k < 1$, and

(**') $l < 1/d$,

which implies that $l < 1$. We impose these conditions (cf. fn. 31). Notice that (**) implies that $m > k$, a condition to which we return below.

Since $V_{33} > V_3$ holds, the inequality $V_2 > V_3$ can be obtained from $V_2 > V_{33}$, or equivalently:

(***) $m > (\dfrac{k}{1-k})(1 - \theta)(\dfrac{a_2}{a_1 - b_1})$,

which requires that $k < 1$. The right-hand side is less than 1 under one of the two conditions:

(***') $k < \dfrac{1}{2}$, $a_2 < a_1 - b_1$,

or:

(***") $\theta > k$, $a_2 < a_1 - b_1$.

We assume (***) to hold, as well as either (***') or (***") (cf. fn. 32). Hence, V_2 is Napoleon's maxmin.

Here are the computations on Blücher's side. From what has just been shown in the last paragraphs, $-V_2$ is the security payoff of the conditional strategy B_2B_1. We will show that it is also the maxmin by checking that no other strategy can deliver a higher security payoff.

Concerning B_1B_1: from definitional inequalities, the security payoff is either $-V_{11}$ or $-V_{21}$. It cannot be $-V_{11}$ because $V_{11} > V_{21}$ would imply a cycle, given that $V_{21} > V_2 > V_{13} > V_{11}$; so it is $-V_{21}$, which cannot be the maxmin, given the first inequality in this sequence.

Concerning B_1B_2: definitional inequalities entail $-V_{12}$ or $-V_{22}$ being the security payoff, but neither can be the maxmin because $V_{22} > V_2$ holds (if $-V_{22}$ is the security payoff, it falls below $-V_2$, and the same if it is $-V_{12}$, since this implies $V_{12} > V_{22}$).

Concerning B_2B_2: again from definitional inequalities, either $-V_{14}$ or $-V_{24}$ is the security payoff, and by a similar argument, $V_{24} > V_2$ precludes either value from being the maxmin.
Thus, we conclude that the equilibrium of the game, in the von Neumann-Morgenstern sense, is (S_2, B_2B_1).

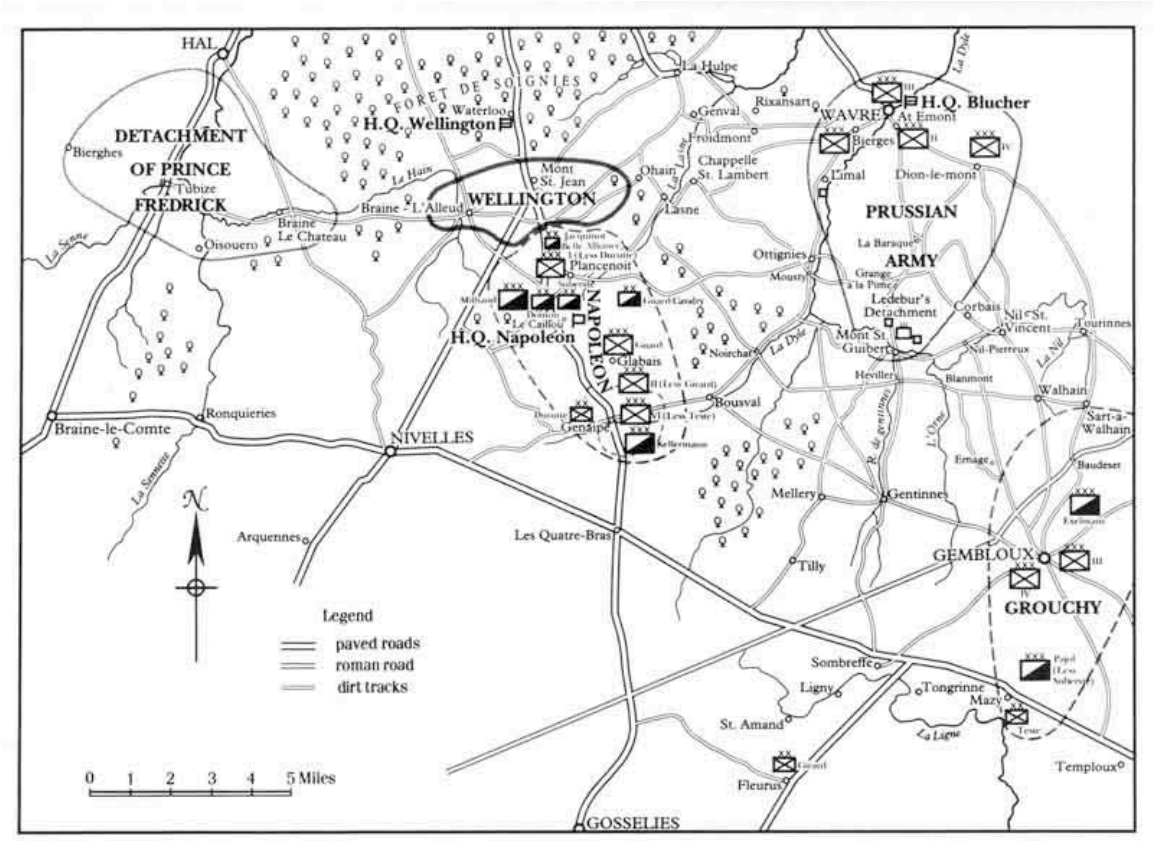
A Cournot-Nash equilibrium calculation would have reached the same conclusion somewhat differently and more quickly. It would have used the fact that Blücher's strategies B_2B_2 and B_1B_2 are dominated, respectively, by B_2B_1 and B_1B_1, once condition (*) is granted. So they are discarded from consideration for Napoleon too, and his strategy S_3 becomes dominated by S_2 from (***), (***') or (***"), and definitional inequalities. The game is now 2x2, and the remaining conditions, i.e., (*'), (**), (**'), ensure that (S_2, B_2B_1) is an equilibrium in the Cournot-Nash sense and that it is unique.

As mentioned in the text, the assumption that $l = 0$ simplifies the analysis. Then, (*), (*'), (**') are trivially satisfied. The binding conditions are (***), (***') or (***"), and (**), which reduces to the straightforward inequality $m > k$. Thus, in this limiting case, the necessary condition becomes sufficient.

Not all the probabilities can take extreme values. The conditions make it necessary that $m > 0$, $l < 1$ and $k < 1$, with k being further bounded from above and θ unrestrained between 0 and 1, or alternatively k and θ being mutually related. Notice that l" is the least constrained parameter, being only subjected to the definitional inequalities l', $m > l$".

It is trivial to find non-extreme values that satisfy all the conditions. For example, take as utility parameters:
$a_1 = 1, b_1 = -1, a_2 = 1/2, b_2 = -1/2, \theta = 1/2,$
and as probability parameters:
$l = 0.1, l' = 1/2, l'' = 1/3, k = 1/3, m = 2/3.$

MAP OF THE OPERATIONS AS OF JUNE 17-18, 1815



Positions of the Armies, 17 to 18 June