PHILIPPE MONGIN

A NOTE ON VERISIMILITUDE AND RELATIVIZATION TO PROBLEMS*

ABSTRACT. This note aims at critically assessing a little-noticed proposal made by Popper in the second edition of *Objective Knowledge* to the effect that verisimilitude of scientific theories should be made relative to the problems they deal with. Using a simple propositional calculus formalism, it is shown that the "relativized" definition fails for the very same reason why Popper's original concept of verisimilitude collapsed – only if one of two theories is true can they be compared in terms of the suggested definition of verisimilitude.

In the second edition of his *Objective Knowledge*, Popper offered the following, admittedly intuitive definition of verisimilitude:

> It appears intuitively that a statement b is nearer to the truth than a statement a, if, and only if, (1) the (relativized) truth content of b exceeds the truth content of a and (2) some of the consequences of a that are false (preferably all those accepted as being refuted, and even more preferably, some others beyond them) are no longer derivable from b, but replaced by their negations (1979, Appendix 2, p. 371)

If '(relativized) truth content' were replaced by 'truth content' in this sentence, Popper's proposal would just be a weakening of his original 'qualitative' definition, and it would fail for the very same reason as the latter did: suppose A and B are theories, B is false and $A_T \subseteq B_T$, then it must be the case that $A_F \subseteq B_F$ [1]. Thus, the novel point made in the Appendix 2 of *Objective Knowledge* can only relate to relativization, i.e. 'relativizing content to our relevant problems' (1979, p. 368). This is clear enough from the context of the passage just quoted [2]. Still, to the best of my knowledge, the 1979 suggestion has hardly attracted any attention at all – as if its treacherous proximity to the 'qualitative' definition had made a rebuttal redundant. This note aims at filling the lacuna by paying due attention to the relativization clause.

To do so involves one in the awkward task of connecting the propositional calculus formalism with a not altogether arbitrary definition of what "our relevant problems" are. I suggest the following shortcut: a problem is a question of the form 'is p true?', with p ranging over P, a predetermined set of statements subject to various restrictions. Accordingly, the *relativized content* of a theory θ will be defined as $\theta' =$

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$\theta \cap P$. Now, write $P = P_T \cup P_F$, where P_T (P_F) is the subset of all true (false) statements in P and define the *relativized truth* (*falsity*) *content* of any theory θ as the set $\theta'_T(\theta'_F)$ of those statements in θ' that are true (false), i.e. write $\theta'_T = \theta \cap P_T$ ($\theta'_F = \theta \cap P_F$). Note that θ' and θ'_T may or may not be theories, depending on P. Note also the grossly trivial fact that θ'_T and θ'_F are also $\theta_T \cap P$ and $\theta_F \cap P$ respectively. In the particular case where $B \vdash A$, Popper's comparison criterion collapses at once whatever P may be. For then, $A \subseteq B$, and clearly $A'_T \subseteq B'_T$ and $A'_F \subseteq B'_F$ hold at the same time, which means that the original argument against verisimilitude has been reinstated in terms of the newly defined contents. Disregarding the irrelevant case where $A \cap B = \emptyset$, we are left with one case only, i.e. where the consequence classes A and B overlap, neither of them being included in the other. This is, of course, the situation which is relevant to a verisimilitude theory. The remainder of this note is devoted to showing that it is hardly favourable to Popper's 1979 proposal *if some rather minimal restrictions are imposed on P.*

I shall indeed assume the following:

(A1) P is closed under negation;
(A2) P is closed under disjunction.

In effect, the former condition means that P_F consists of all negates of the statements in P_T, and those negates only. As $\neg$ and $\vee$ make a set of sufficient connectives, the two assumptions together imply that P is a propositional calculus. Now, Popper's suggested definition may be restated as follows:

(1′) $A'_T \subseteq B'_T$;
(2′) there exists $a \in A'_F$ such that $a \notin B'_F$ and $\neg a \in B'_T$.

The final subclause '$\neg a \in B'_T$' is formally needed to echo Popper's meaning, since B' may well not be complete, but it will play no role in the negative result of this note.

PROPOSITION 1. Assume (A1) and (A2). If (1′) and (2′) hold, B' is true.

Proof: It has to be shown that (1′) along with the assumption that B' is false leads to contradicting (2′). Suppose then that there is $f \in B'_F$ and $a \in A'_F$. Using the definitions as well as (A1) and (A2), we know that f, a and $a \vee \neg f$ are in P. Since $a \vee \neg f$ is a true consequence of A, it also belongs to A'_T. From (1′), $a \vee \neg f \in B'_T$, $a \vee \neg f \in B$, and, since B is a consequence class, $a \in B$; since a is false and is in P, $a \in B'_F$, which contradicts (2′). □

This proposition is intended to *trivialize* Popper's 1979 suggestion in the same way as Miller's and Tichy's results did his original "qualitative" definition. Assuming (A1) and (A2), the relevance of which will be discussed below, the former is seen to hold only in those uninteresting situations to which the latter had been shown to be restricted. The very straightforward proof used here parallels Tichy's in his 1976 article. It is easy to make the analogy even closer with the early negative results. There is a dual pair of conditions to which Popper may have resorted as an alternative to (1′) and (2′):

(3′) there exists $\tilde{a} \in B'_T$ such that $\tilde{a} \notin A'_T$ and $\neg\tilde{a} \in A'_F$;
(4′) $B'_F \subseteq A'_F$.

It is not obvious to me that (3′) and (4′) are any less faithful to the spirit of falsificationism than (1′) and (2′). Whatever may be the case, a corresponding trivialization result is easy to come by (the proof goes along the same line as that of Proposition 1 and is left to the reader):

PROPOSITION 2. Assume (A1) and (A2). If (3′) and (4′) hold, B' is true.

Collecting the two results together leads to the final analogy:

PROPOSITION 3. Assume (A1) and (A2). If A' and B' are false, $A'_T \subseteq B'_T$ if and only if $A'_F \subseteq B'_F$.

One may be willing to weaken conditions (1′) and (2′) while still retaining (A1) and (A2). A possibility which seems worth considering goes as follows:

(1′) $A'_T \subseteq B'_T$;
(2″) there exists $a \in A_F$ such that $a \notin B_F$.

That is, relativization makes it possible for B to retain at least as many true statements as were *relevant* in A; and there is a falsehood, *relative to whichever problem*, that A has and B does not have. This is at least literally compatible with Popper's quotation, since he does not explicitly mention relativization in connection with his own condition (2). The proof in support of Proposition 1 does not carry through anymore, but there are persuasive methodological arguments against conditions (1′)

and (2″). Beside using a non-homogeneous definition of content, they appear to be too lax. The two objections – non-homogeneity and laxness – are really two faces of the same coin. I take it that the philosophical point of relativizing content is to make clear what being nearer to truth *relative to some problem* means. Now, if (1′) and (2″) were substituted for (1′) and (2′), the removal of any *irrelevant* falsehood from A would lead to a progress towards the truth relative to the problem at hand. More precisely, suppose there is $a \in A_F$, $a \notin P$. Delete every statement in A involving a and take, if necessary, the deductive closure of the remainder. This will define a new theory B which trivially satisfies the requirements (1′) and (2″). The new theory will be described as nearer to the truth relative to P though it results from the old one through modifications which are immaterial to P.

The formalization of problem-relativization used throughout this paper calls for some comments. First of all, it is not quite satisfactory to define a problem as a question 'is p true?', where p belongs to a propositional calculus. For the latter is a decidable system whereas not every problem is decidable. The objection is strong in its own right, but it is a fact that the technical literature on verisimilitude relies on using propositional calculus. The point made in this note is precisely that Popper's suggestion to import concepts of problem and problem-relativization into such a logical framework does not serve his purpose. Second, defining relativized theories θ' by the simple set theoretical operation $\theta \cap P$ leads to seemingly paradoxical conclusions [3]. Take A = Kepler's laws, B = Newton's theory. The former makes it possible to predict the positions of any two planets from their earlier positions, whereas the latter requires also their masses to be known. Take P = the set of all physical statements which do *not* involve masses. There is a prima facie difficulty here since condition (1) cannot be met satisfactorily, i.e. it is not true that $A'_T \subseteq B'_T$. For Kepler's true statements about the planets' positions are unaffected by relativization, whereas they cannot be derived from the relativized Newtonian set of statements. My answer to this counterexample is that it involves no paradox at all. It is not surprising that *the more explicative theory cannot be shown to be nearer to the truth than the less explicative one when verisimilitude is made relative to the problem-situation of the latter*. That is, Kepler's laws fare no worse than Newton's where both are appraised on the background of what Kepler had to say on masses. The difficulty here, if there is any, is with the very notion of problem-relativization, and not with the formalism of this note. Relativizing content to problems should lead – at least prima facie – to as many verisimilitude measurements as are distinct theories. To limit proliferation and avoid some gross adhoceries, one may have to privilege the frame of reference of the latest theory, which is, of course, enriching the logical theory of verisimilitude with a historical, even possibly historicist component. Third, the axioms resorted to here may of course be called into question. They are exceedingly demanding from the viewpoint of any cognitive psychology. Presumably the concept of a problem should not be construed as a purely logical one. It should retain some connection with actual human capabilities, and it is, of course, beyond any human cognitive power consistently to apply (A2) – (A1) does not strike one as formidable. This objection may be less troublesome than it seems to be, since those axioms are obviously too strong compared with the actual work that they perform in the proof. The latter basically involve one-step formal operations on a small lot of predetermined statements; this suggests that trivialization results should be easy to come by in a formalism adapted to a more cognitively-oriented version of the 1979 proposal. I shall not engage in this line of discussion, since it would involve another leap from Popper's initial stand towards the empirical. In the context of a *logical* definition of verisimilitude and relativization, (A1) and (A2) are plausible closure axioms and there is even some evidence that they have been implicitly resorted to in previous discussion on Popper's attempt to relativize comparison of theories to problems [4].

NOTES

* This note was written when the author was visiting the Groupe de Recherche en Épistémologie Comparative (Université du Québec à Montréal).

[1] A theory θ is taken to mean a set of statements closed under deduction, i.e. a consequence class. $\theta_T(\theta_F)$ is the truth (falsity) content of θ as usually defined in the literature, e.g. in Miller (1974) and Tichy (1974) where Popper's 'qualitative' concept of verisimilitude was first disentangled.

[2] See also Popper, 1976, p. 155–159, from which his 1979 suggestion evolved.

[3] I have borrowed the counterexample below from a discussion between Popper and Watkins which is reported in an antecedent passage of *Objective Knowledge* (1979, pp. 369–370).

[4] In 1979, p. 368, Popper reports the following objection by David Miller. Suppose that there are two conflicting theories A and B, with $A \vdash c$ and $B \vdash \neg c$, Take any statement u which is not decidable within B. Statement $c \vee u$ cannot be decidable within B either, since $c \vee u$ is equivalent to u in the presence of B, but it is decidable within A. This

objection presumably relies on the assumption that u belongs to the set P of problematic statements and the latter satisfies some closure rule akin to (A2).

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DELTA
Ecole normale supérieure
48 bd Jourdan
75014 Paris
France