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CONSTRAINED EGALITARIANISM IN A SIMPLE REDISTRIBUTIVE MODEL

ABSTRACT. The paper extends a result in Dutta and Ray's (1989) theory of constrained egalitarianism initiated by relying on the concept of proportionate rather than absolute equality. We apply this framework to redistributive systems in which what the individuals get depends on what they receive or pay *qua* members of generally overlapping groups. We solve the constrained equalization problem for this class of models. The paper ends up comparing our solution with the alternative solution based on the Shapley value, which has been recommended in some distributive applications.

KEY WORDS: Capacity, Egalitarianism, Inequality theory, Shapley value, Transferable utility cooperative game

1. INTRODUCTION

The body of work on distributive inequality called "inequality theory" is almost exclusively normative. It investigates ways of comparing distributions of achievements or resources, such as the Lorenz ordering and its variants, regardless of whether or not these distributions are actually available. This separation of normative issues and feasibility considerations is typical of standard microeconomics, as the basic model of the consumer illustrates. By assumption, the consumer's preference ordering is defined over all logically conceivable commodity baskets, whether they are feasible or not. But – of course – standard microeconomics does not stop at the stage of clarifying the agents' objectives; the next step is to discuss the agent's choices, given the constraints. Inequality theorists do not often take this further step. There does not yet exist *a theory of constrained egalitarianism* that can be compared with the familiar theories of constrained optimization in microeconomics.

Yet a pathbreaking paper has laid down the foundations of a theory of constrained egalitarianism. Dutta and Ray (1989) have defined a concept of an egalitarian distribution subject to particip-

Theory and Decision **54:** 33–56, 2003.

ation constraints. The resulting analysis is an admixture of normative and positive considerations, in which the latter stem from the fact that coalitions can defect and prevent the grand coalition from achieving its egalitarian aims. In particular, Dutta and Ray show how the Lorenz ordering can be maximized on the core of a transferable utility (TU) cooperative game. Using the essential assumption that the considered game is convex, they demonstrate that, despite being partial, the Lorenz ordering admits of a unique greatest element on the core, and they provide a simple algorithm to compute this solution.[1] The abstract terminology of TU games used by Dutta and Ray already suggests that their analysis should be widely applicable. The present paper will make this even clearer by extending their result in two directions.

First we propose a generalized version of Dutta and Ray's original method to the case where agents have different "weights", and thus some norm of *proportionate justice*, rather than straightforward equality, should be approximated given the constraints. This extension is in keeping with the work in inequality theory applying the Lorenz ordering to households having different sizes or different needs.[2] Our formulation of proportionate constrained egalitarianism formally applies to any convex TU cooperative game, and it will be explained as such.

Second, we single out a class of simple redistributive problems, to be called here *basic transfer problems*, in which the issue of constrained equalization naturally arises, and the Dutta–Ray method of resolution turns out to be often applicable. Essentially, these problems involve a "centre" which transfers money to, or receives money from, possibly many, and in general overlapping, groups of agents. What each agent (it may be either an individual or a decision-making entity of any sort) eventually receives depends on both its share in the various groups it belongs to and on what these groups get from, or pay to, the centre (the "basic transfers"). The question of constrained egalitarianism arises not because there are *participation* constraints, but because we assume that while striving towards equality, the centre regards the existing procedure of basic transfers as being unalterable.

Finally, we derive some informative consequences of the proportionate equalization principle for the basic transfer model. These properties put the constrained egalitarianism approach at large in clear contrast with the alternative approach of the Shapley value, which has sometimes been recommended to resolve fair division or cost-sharing problems.

The plan of the paper is as follows. Section 2 briefly explains the original Dutta–Ray solution for TU games, and then moves to our proportionate extension; Section 3 introduces the basic transfer model and shows how it can lead to a convex TU game representation; Section 4 assesses the normative properties of our solution in terms of basic transfers and compares it with the Shapley value. An appendix presents some complementary results and proofs.

2. PROPORTIONATE EGALITARIAN SOLUTIONS IN TRANSFERABLE UTILITY COOPERATIVE GAMES

2.1. *Transferable utility cooperative games*

A transferable utility (TU) cooperative game is a pair (N,v), where $N = \{1, \ldots, i, \ldots, n\}$, is a fixed population of agents (or players) and v – the *characteristic function* of the game – assigns to each nonempty subset S of N, called a *group*, or a *coalition*, a real number $v(S)$, called its *worth*. We denote by **S** the set of possible groups and put $v(\emptyset) = 0$. Then, v is a 2^n-dimensional vector.
A game (N, v) is *convex* if for all groups S, T,

$$v(S) + v(T) \leq v(S \cup T) + v(S \cap T);$$

it is *superadditive* if, for all disjoint groups S, T,

$$v(S) + v(T) \leq v(S \cup T);$$

and *monotonic* if for all $S \subset T$, $v(S) \leq v(T)$.

Since $v(\emptyset) = 0$, a convex game is also superadditive, and a superadditive game is monotonic if and only if $v(S) \geq 0$ for all $S \in \mathbf{S}$. For any $x \in R^n$ and $S \in \mathbf{S}$ we use the notation $x(S) = \sum_{i \in S} x_i$. An *allocation* for the coalition S is defined to be any vector x_S of $R^{|S|}$; it is *feasible* for S if $x(S) = v(S)$; a *feasible allocation* is an allocation which is feasible for the grand coalition N.

The usual interpretation of v is that it sets strategic constraints on possible allocations to individuals, or that a coalition S can block any feasible allocation x which would give S less than its worth. This suggests focusing attention on the *core* of (N,v), to be denoted by $C(N, v)$, i.e., the set of allocations (the *core allocations*) satisfying the constraints:

$$x(N) = v(N) \text{ and } x(S) \geq v(S) \text{ for all } S \in \mathbf{S} \tag{1}$$

It is well known that the core can be empty in general, but convex games have nonempty cores. Alternative interpretations of the characteristic function, which are more relevant for the games to be associated with the basic transfer model, are considered in the second part of the paper.

2.2. *The Lorenz criterion*

The notion of equality used throughout is the classic one of the Lorenz (partial) ordering. Given $x, y \in R^n$ for some $n \geq 2$, x is said to *Lorenz-dominate* (L-dominate) y, which is denoted by $x \geq^L y$, if for all $k = 1, \dots, n$,

$$\inf\{x(S) : |S| = k, S \in \mathbf{S}\} \geq \inf\{y(S) : |S| = k, S \in \mathbf{S}\},$$

or equivalently, if for all $k = 1, \dots, n$,

$$x_{(1)} + \cdots + x_{(k)} \geq y_{(1)} + \cdots + y_{(k)},$$

where $x_{(k)}$ ($y_{(k)}$) denotes the kth component of x (resp. y) in the increasing order. For any given $z \in R^n$ the mapping $L(z, .)$ obtained by putting $L(z, k) = z_{(1)} + \ldots + z_{(k)}$ for all k, and then making a linear interpolation, is called the *Lorenz curve* for z. In this terminology, "x L-dominates y" means that the Lorenz curve for x lies above the Lorenz curve for y. As early as 1929, Hardy, Littlewood and Pólya (henceforth HLP) proved the following result:

THEOREM 1. *For $x, y \in R^n$, the following conditions are equivalent:*

(i) There exists a bistochastic $(m \times m)$ matrix M such that $x = My$;
(ii) for all (continuous) concave functions $f : R \to R$,

$$f(x_1) + \ldots + f(x_n) \geq f(y_1) + \ldots + f(y_n);$$

(iii) $x \geq^L y$ *and* $x(N) = y(N)$.

(See Hardy, Littlewood and Pólya (1934), and, for easy reference, Berge (1966, pp. 193–194), or Marshall and Olkin (1979, pp. 107–108). It is now fairly well understood that most of the economists' formal discussions of inequality are either related to or even straightforwardly derived from the HLP theorem. This literature is huge and still on the move, so we will refrain from singling out specific references. The only technical result we need on the Lorenz ordering is the HLP theorem itself.

As was explained in the introduction, Dutta and Ray (1989) showed how to maximize the Lorenz ordering on the core of a convex game. In order to fully appreciate this contribution, the following reminder is to the point. Whatever the game (N,v), the core $C(N,v)$ is a compact subset of R^n. Using the HLP theorem, it is easy to see that if the core is nonempty, there exists at least one maximal element for $\geq^L$ on $C(N,v)$.[3] (We call an element *maximal* if it is not strictly dominated by any other element for the given partial ordering, and *greatest* if it weakly dominates any other for that ordering.) The existence question being readily solved, it remained to investigate the uniqueness or otherwise of maximal elements. Dutta and Ray (1989, pp. 625–626) establish a *uniqueness* property for convex games:

THEOREM 2. *If* (N, v) *is convex, there is a unique greatest element for* $\geq^L$ *in* $C(N, v)$.

Observe the way in which this uniqueness property is stated. It entails, but is stronger than, the property that there is a unique maximal element. We will refer to the unique greatest element of Theorem 2 as to the *egalitarian solution* for (N, v).[4] Importantly, the Dutta–Ray theorem is proved constructively, i.e., by defining an algorithm which delivers the desired solution.

The conclusion of Theorem 2 collapses when the convexity condition is weakened. Here is an example.[5] Let $N = \{1, 2, 3, 4\}$. Consider the TU game in which v is the smallest superadditive function on **S** compatible with the following data:

$$v(N) = 100,\ v(2, 4) = 60,\ v(3, 4) = 70.$$

Then, (15,15,25,45) and (10,20,30,40) are both maximal elements for the Lorenz ordering on the core, and any convex combination of these allocations shares this property.

We will present an extension of the theorem which is motivated by the following conceptual point. The grand theories of distributive justice rarely, if ever, recommend absolute equality between individuals. They typically select some individual characteristics – e.g., need, work, desert – according to which the society's worth should be apportioned among its members.[6] Similarly, in "micro-justice" problems, *proportionate* rather than *absolute* equalization will often emerge as the intuitively plausible notion to adopt. In inequality theory proportionate equalization is typically introduced in terms of equivalence scales between households (see Ebert, 1999, and the references in Section 2 of his paper).

In accord with these motivations, we introduce the notion of a constrained *proportionate* egalitarian solution.[7] The greatest element referred to in Theorem 2 is the best available approximation in $C(N, v)$ to the equal distribution vector $(k, k, \dots, k)$, with $k = v(N)/n$. What we will do is to approximate $(\alpha_1 k, \alpha_2 k, \dots, \alpha_n k)$ instead, where $\alpha = (\alpha_1, \dots, \alpha_n)$ is a vector of *positive* weights summing to 1, and for every i, α_i is i's normatively desirable share of the total worth $v(N)$. What is the technically precise sense in which distribution vectors in R^n can be said to "approximate" this proportionate egalitarian norm? Our strategy will be first to guess what a right approximation x° to $(\alpha_1 k, \dots, \alpha_n k)$ may be. It will then be shown that this particular allocation – let us call it the α-*proportionate* solution in order to contrast it with the egalitarian solution – formally satisfies an existing generalization of the Lorenz ordering to the proportionate case.

The guess is based on the egalitarian solution for an *auxiliary* game which replicates each player i of the initial game as many times as required by his proportionality coefficient α_i in $(\alpha_1, \dots, \alpha_n)$. (From now on, we assume that the α_i are rational numbers; to extend the analysis to real numbers would be a purely technical exercise.) Then, reverting to the initial game (N, v) we obtain a tentative solution x°. The analysis will be carried under the assumption that the initial game (N, v) is not only convex, but also monotonic, which – in view of an already made observation – amounts to assuming that $v \geq 0$. We have explained the sense in which the monotonicity assumption is insubstantial.[8]

Formally, we fix a given vector of positive integers $m = (m_1, \dots, m_n)$ such that $m_i/m_j = \alpha_i/\alpha_j$, and we associate with (N, v) the TU game (N^m, v^m) defined as follows. The player set is:

$$N^m = \{(1,1), \dots, (1, m_1), \dots, (i, 1), \dots, (i, m_i), \dots, (n, 1), \dots, (n, m_n)\}.$$

We call (i, j) a *replica* of i. In the notation already introduced for sums of vector components, the total number of players in the new game is $\Sigma_{i \in N} m_i = m(N)$. We will denote the set of coalitions of this game by $\mathbf{S}'$, the coalition $\{(i, 1), \dots, (i, m_i)\}$ by [i], and the coalition $\bigcup_{i \in S} [i]$ by $[S]$. The characteristic function v^m is defined as follows:
for all $S' \in \mathbf{S}'$, $v^m(S') = v(S)$, where S is the largest coalition in $\mathbf{S}$ such that $[S] \subset S'$, i.e., $v^m(S') = v(\{i \in N : [i] \subset S'\})$.

This completes the definition of the associated game. However we need also introduce the mapping $R^n \to R^{m(N)}$:

$$x \to x^m = (x_1/m_1, \dots, x_1/m_1, \dots, x_i/m_i, \dots, x_i/m_i, \dots, x_n/m_n, \dots, x_n/m_n),$$

and call a vector y of $R^{m(N)}$ *uniform* if $y = x^m$ for some x, i.e., if it gives the same amount to each replica (i, j) of any given i. More precisely we shall say that y is the *uniform image* of x.

To assess how far a vector of R^n is from the proportionate equality ideal we use its uniform image. Given $m = (m_1, \dots, m_n)$, we define the *m-Lorenz (partial) ordering* $\geq^{L_m}$ on R^n by the condition that:

$$x \geq^{L_m} y \quad \text{iff} \quad x^m \geq^L y^m.$$

Thus, x is declared to be closer to proportionate equality than y if and only if their respective uniform images in $R^{m(N)}$ are ranked in the appropriate way by the *ordinary* Lorenz criterion.

At this point it is necessary to check that the particular choice of m is irrelevant:

Fact 1. Ordering $\geq^{L_m}$ only depends on weights α.

Proof. Consider another vector m' such that $m'_i/m'_j = \alpha_i/\alpha_j = m_i/m_j$, for all i, j.

Since $x_i/m_i \geq x_j/m_j$ iff $x_i/m'_i \geq x_j/m'_j$, the Lorenz mappings $L(x^m, .)$ and $L(x^{m'}, .)$ satisfy $L(x^m, k) = L(x^{m'}, k')$ whenever $k = \Sigma_{i \leq^{i_0}} m_{(i)}$ and $k' = \Sigma_{i \leq^{i_0}} m'_{(i)}$ for some i_0. Thus, for any x, y and such k, k', $L(x^m, k) \geq L(y^m, k)$ iff $L(x^{m'}, k') \geq L(y^{m'}, k')$.

However, by linearity of $L(x^m,.)$ and $L(y^m,.)$, if $L(x^m, k) \geq L(y^m, k)$ is valid for $k = \Sigma_{i \leq^{i_0}} m_{(i)}$ and all i_0, it is in fact valid for all k; and similarly for $L(x^{m'}, k')$ and $L(y^{m'}, k')$. Therefore, $x^m \geq^L y^m$ iff $x^{m'} \geq^L y^{m'}$ hence $x \geq^L_m y$ iff $x \geq^{L_m{}'} y$.

Allocations in (N, v) give rise to allocations in (N^m, v^m) exactly as one could expect:

Fact 2: For all x, $x \in C(N, v)$, if and only if $x^m \in C(N^m, v^m)$.

Proof. For every $S' \in \mathbf{S}'$,
$x^m(S') = \Sigma_{(i,j)\in S'} x^m(i, j) = \Sigma_{i \in N} \mid [i] \cap S' \mid .x_i/m_i \geq \Sigma_{[i] \subset S'} x_i \geq v(\{i : [i] \subset S'\}) = v^m(S')$. Conversely, for every S in $\mathbf{S}$,
$x(S) = x^m([S]) \geq v^m([S]) = v(S)$.

Fact 3: (N^m, v^m) is convex.

Proof. Take two coalitions $S', T' \in \mathbf{S}'$, and consider the largest coalitions $S, T \in \mathbf{S}$ such that $[S] \subset S'$ and $[T] \subset T'$, so that $v^m(S') = v(S)$ and $v^m(T') = v(T)$. Then, $v^m(S' \cap T') = v(S \cap T)$ and (using monotonicity of v) $v^m(S' \cup T') \geq v(S \cup T)$. The convexity property for S, T then leads to the same property for S', T'.

From Theorem 2 for the convex game (N^m, v^m) there is a unique greatest element x^* for $\geq^L$ in $C(N^m, v^m)$. Define $x^\circ \in R^n$ from x^* as follows: for all $i \in N$,

$$x_i^\circ = \sum_{j=1}^{m_i} x^*_{(i,j)}$$

This vector is our notion of an α-proportional solution for the initial game (N, v). The next facts summarize the properties of x^* and x°, with a view of establishing that x° has indeed the properties required for a solution.

Fact 4: x^* is a uniform allocation.

Proof. Suppose that y^* is the vector obtained from x^* by permuting the components of (i, j) and (i, k), and keeping all other components the same. For any S' define T' to be S' if neither (i, j) nor (i, k) is in S'; otherwise, to be S' with (i, j) replaced by (i, k) if $(i, j) \in S'$, and (i, k) replaced by (i, j) if $(i, k) \in S'$. Thus $T' = S'$ when both, or neither one, belong to S'. Clearly, $v^m(S') = v^m(T')$, and $y^*(S') = x^*(T')$. Hence, $y^*(S') \geq v^*(S')$ for all S', i.e., $y^* \in C(N^m, v^m)$. But $y^* \equiv^L x^*$ (where $\equiv^L$ denotes the indifference relation of $\geq^L$) so that, by the uniqueness property of x^* (Theorem 2), $y^* = x^*$.

Fact 4 implies that $x^* = (x^\circ)^m$. In view of Fact 1, this means that x° is feasible:

Fact 5: $x^\circ \in C(N, v)$.

It remains to argue for the normative properties of x°:

PROPOSITION 1. *x° is the unique greatest element in $C(N, v)$ for the m-Lorenz ordering.*

Proof. Take any $y \in C(N, v)$. From the definition of x° and the optimality property of $x^* = (x^\circ)^m$, $x^* \geq^L y^m$, since $y^m \in C(N^m, v^m)$ from Fact 1. Thus, $x^\circ \geq^{L_m} y$.
For uniqueness: If y is a greatest element for $\geq^{L_m}$ in $C(N, v)$, then y^m is a greatest element for $\geq^L$ in $C(N^m, v^m)$, and $y^m = x^*$ from Theorem 2, so that $y = x^\circ$.

3. APPLICATION TO THE BASIC TRANSFER MODEL

3.1. *The basic transfer model*

In the three examples below, there is always a fixed population of agents $N = \{1, \ldots, i, \ldots, n\}$, and there is an entity, called *the centre*, which is in charge of allocating money between groups of agents. A *group* is simply some nonempty subset S of N; denote by $\mathbf{S}$ the set of possible groups. A *system of basic transfers* is any assignement of a real number $\varphi(S)$ to each S in $\mathbf{S}$. The subset of those S with $\varphi(S) \neq 0$ will be denoted by $\mathbf{S}^+$, and the budget of the centre by $M = \Sigma\varphi(S)$. Given a system of basic transfers, an *allocation* is any vector $x = (x_1, \ldots, x_n)$ that redistributes the net receipts of the groups (which may be a priori positive, zero, or negative) to their members, and *only* to their members, in a certain way. Specifically, an allocation for a basic transfer model (N,φ) should satisfy the set of inequalities:

$$\sum_{T \subset S} \varphi(T) \leq \sum_{i \in S} x_i \leq \sum_{T \cap S \neq \emptyset} \varphi(T) \quad \text{for all} S \in \mathbf{S}, \tag{2}$$

which the examples will serve to motivate. We already observe from this definition that an allocation x must satisfy the book-keeping condition that $M = \Sigma x_i$. Which allocation actually prevails will depend on both the net receipts of the groups and how the groups share them between their members. The centre always has the former information but may lack the latter, even *ex post*. Thus, in some applications the centre exactly knows the actual allocation, and in others it does not.

EXAMPLE 1. Here N is a population of farmers each of which produces one or several of p products. The farmers are subsidized in an indirect way. They form groups of producers, which will typically overlap, and some of these groups receive subsidies from the Department of Agriculture, which they have then to allocate between their members. If, say, the producers of potatoes are exactly the same as the producers of carrots, both will count as one and the same group. Any given farmer may belong to one, several, or none of the subsidized groups. The transfer to a subsidized group S is represented by $\varphi(S) > 0$. We define a basic transfer function on the whole of $\mathbf{S}$ by putting $\varphi(S) = 0$ if $S \notin \mathbf{S}^+$. In this context the inequalities (2) indicate that the farmers in S get at least all the money going to the subsidized subgroups of S, and at most all the money going to the subsidized groups with which S overlaps. Hence, these inequalities summarize all possible ways of allocating money to the individuals by way of the scheme φ. If the centre does not know how the groups S divide the net transfers $\varphi(S)$ between their members, the actual allocation is unknown and may be any x satisfying the basic inequalities (2). This is how we motivate the notion of an allocation adopted here.

EXAMPLE 2. Here the individuals in N are states in a federation, and the centre is the Federal Institution. The latter collects money from the states individually in order to fund various joint ventures between the states. If joint ventures can be identified with subsets of N, in the same way as groups of producers were in the last example, we get another instantiation of the basic transfer model. This time, however, the model has non-uniform sign restrictions. Singletons S will be assigned nonpositive $\varphi(S)$, while all other groups S receive nonnegative $\varphi(S)$. Supposing that the Federal Institution has no external resources, the total amount to be allocated is $M = \Sigma\varphi(S) = 0$. The set of possible allocations x is again bounded from below and above by inequalities (2). If the centre does not know how the states benefit from a joint project, it can only say that the actual allocation belongs to the set defined by these inequalities.

EXAMPLE 3. In a straightforward idealization of existing redistributive systems, households pay taxes individually, while they receive subsidies *qua* members of one or several categories (e.g., the category of households with children, that of households with some disabled member, and so on). If one is happy with this rough picture and willing to identify categories with subsets of the total household population N, the resulting basic transfer model will have the same sign restrictions as in Example 2. However, a slightly more realistic description would take into account the fact that households also pay money to the centre *qua* members of categories. For instance, there are taxes that only the employed part of the population has to pay, and similarly with landowners, foreign residents, etc. In an improved description of this sort the sign restrictions on φ can become indeterminate. As we explain in the next section, the Dutta–Ray techniques do not apply to basic transfer models with *any* sign restriction.

Whatever the basic transfer model (N, φ), the question arises, is the equal distribution vector $(x_i{=}M/n)$ compatible with the inequalities (2)? Since more often than not, equality is not feasible, this question becomes, how can equality be *approximated* within the set of constraints? This problem can be arrived at from rather different angles, depending on what is assumed on both the centre's information and policy goals. Specifically, assume that the centre does *not* know what actual allocation prevails. Just for evaluation purposes, it may wonder how far this unknown allocation is from equality. To find the most equal allocation(s) compatible with (2) is one way of answering this question; it amounts to conjecturing that the best possible case obtains. Alternatively, suppose that the centre exactly *knows* what allocation is prevailing. For evaluation purposes again, it makes sense to compare the actual allocation with the most equal among all allocations that are compatible with φ, and this leads to solve exactly the same constrained equalization problem.

Beyond and above evaluation, the centre's aim might be to prepare a more equal redistributive arrangement. If this is the case, a relevant question is whether or not the constraints (2) would still have to apply to the new arrangement. Specifically, suppose that the centre in Example 1 wants to replace the *group* distributive scheme φ by an *individual* distributive scheme x^*. Suppose further that for institutional or political reasons, the constraints (2) set lower bounds on what the producers must receive in x^*. For instance, the corn producers should receive no less than all the money going to the subgroups of corn producers, and so on. Under these assumptions, the solution of the constrained equalization problem becomes the centre's policy target.

In sum, for reasons which depend on each particular application, one is led to raise one and the same analytical question, *how can equality be approximated within the inequality set* (2)? We will discuss a natural answer to this question suggested by the possibility of turning basic transfer models into cooperative games.

3.2. *From basic transfer models to cooperative games*

In this section we transform the basic transfer model (N,φ) into a TU cooperative game (N, v), where v is the *characteristic function* of the game, and compare the resulting representation with the original one.[9] We say that (N, v) is the cooperative game associated with the basic transfer model (N,φ) if:

$$v(S) = \sum_{T\in\mathbf{S}} \varphi(T)v_T(S) \text{ for all } S \in \mathbf{S}, \tag{3}$$

where $v_T(S) = 1$ if $T \subset S$ and 0 otherwise.[10] This definition ensures that the notion of a core allocation for (N, v) coincides with that of an allocation for (N, φ). That is, for any $x \in R^n$, the set of inequalities (1) in Section 2 is equivalent to the present set (2) given the definition just provided for v. The implication from (2) to (1) is trivial. To check the reverse implication, notice that given the equality $x(N) = v(N)$, the set of upper inequalities becomes redundant with the set of lower inequalities in (2).

In fact, the set of all cooperative games (N, v) and the set of all basic transfer models (N,φ) can be mapped to each other one-to-one. That is, if one takes v to be the primitive term, it is possible to find φ such that (3) holds and the two transformations – i.e., from φ to v, and from v to φ – are inverse to each other. This observation was made early on in cooperative game theory (Shapley, 1953) but can also be found in the formally similar theory of nonadditive belief functions (Shafer, 1976). Here is the explicit formula to compute φ from v:

$$\varphi(S) = \sum_{T\subset S} (-1)^{|S\setminus T|} v(T) \text{ for all } S \in \mathbf{S}. \tag{4}$$

Given that v and φ are interdefinable, it is worth saying what interpretation φ could receive when v is taken to be the primitive term, as is the case in standard cooperative game theory. From the direct formula (3), we get $\varphi(\{i\}) = v(\{i\})$ for any individual i.
Hence, $\varphi(\{i, j\}) = v(\{i, j\}) - v(\{i\}) - v(\{j\})$ for any pair $\{i, j\}$, and generally:

$$\varphi(S) = v(S) - \Sigma_{T\subset S_{2\leq|T|<|S|}} \varphi(T) - \Sigma_{i\in S} v(\{i\}). \tag{5}$$

For instance, for any triple $\{i, j, k\}$, $\varphi(\{i, j, k\}) = v(\{i, j, k\}) - \varphi(\{i, j\}) - \varphi(\{i, k\}) - \varphi(\{j, k\}) - v(\{i\}) - v(\{j\}) - v(\{k\})$. In words, the φ function computes the *surplus brought about by a coalition to its members*. Since we have not yet imposed any sign restrictions, this surplus can be negative. It is obtained by subtracting from the worth of the coalition $v(S)$ not only the individual worths $v(\{i\})$, but also the surpluses that accrue from forming (nonsingleton) coalitions smaller than S – as Equation (5) makes clear. In TU game theory, a construction like this one exists in relation to the Shapley value.

We have not yet discussed the sign restrictions on φ that were illustrated in Examples 1, 2 and 3 of last section. As it turns out, sign restrictions on φ correspond to recognizable properties of v. The following equivalences are borrowed from Chateauneuf and Jaffray (1989, Propositions 2 and 3), who stated them in connection with the theory of nonadditive belief:

PROPOSITION 2. *(i)* $\Sigma_{\{i\}\subset T\subset S}\varphi(T) \geq 0$ *for all* $S \in \mathbf{S}$ *and* $i \in N$ *is equivalent to the condition that v is monotonic.*

(ii) $\Sigma_{\{i,j\}\subset T\subset S}\varphi(T) \geq 0$ *for all* $S \in \mathbf{S}$ *and distinct* $i, j \in S$ *is equivalent to the condition that v is convex.*

We conclude from these equivalences that convexity and monotonicity play a very different role vis-à-vis the basic transfer model. As (i) shows, it is possible to turn any basic transfer model (N,φ) into a monotonic one (N,φ'), by putting $\varphi' = \varphi$ on all nonsingleton sets, and $\varphi'(\{i\}) = \varphi(\{i\}) + c$ for all $i = 1, \ldots, n$, where c is some positive constant large enough to make all inequalities in (i) come true. Any allocation x for (N,φ') can trivially be turned into an allocation for the real model (N,φ) by subtracting c from each x_i. Arguably, the normative properties imposed on an allocation should not be affected by a common translation of only the $\varphi(\{i\})$, so that they can be studied as well on the translated model. No such reasoning is available for convexity. From (ii) the required translation would involve changing the value of φ for some nonsingleton sets. When an allocation is found for the translated model, there is no unique way of reverting to an allocation in the original model.

Example 1 involves setting $\varphi \geq 0$, and then corresponds to a game which is by itself both monotonic and convex.[11] In Example

2, $\varphi(\{i\}) \leq 0$ for all i, and $\varphi(A) \geq 0$, if $|A| \geq 2$, which corresponds to an initially nonmonotonic but convex game. However, taking advantage of the argument in the last paragraph, we can reduce this case to the previous one. Finally, Example 3 illustrates that not every basic transfer model can be turned into a convex game. If one is unhappy with the heroic assumption that households never pay money to the centre *qua* members of categories, the associated game may or may not be convex, depending on whether or not the restrictions in (ii) are met.

In standard TU game-theory, where v is the primitive term, the equivalences of Proposition 2 would have to be reinterpreted in terms of the surpluses brought about by various coalitions. Thus, property (i) states that the total sum of surpluses contributed by i to S is nonnegative, so that to add i to $S \setminus \{i\}$ is worthwhile. Similarly, property (ii) states that the total sum of surpluses contributed by $\{i, j\}$ to S is nonnegative, so that it pays to add $\{i, j\}$ to $S \setminus \{i, j\}$. Also, as the reader will check, (ii) restates the increasing marginal worth property of a convex v, that is to say:

$$[v(S) - v(S \setminus \{i\})] - [v(S \setminus \{j\}) - v(S \setminus \{i, j\})] \geq 0.$$

Equivalently:

$$v(S) - v(S \setminus \{i, j\}) \geq [v(S \setminus \{i\}) - v(S \setminus \{i, j\})] + [v(S \setminus \{j\}) - v(S \setminus \{i, j\})],$$

or:

$$v(S) - \max[v(S \setminus \{i\}), v(S \setminus \{j\})] \geq \min[v(S \setminus \{i\}), v(S \setminus \{j\})] - v(S \setminus \{i, j\}).$$

4. PROPORTIONATE EGALITARIAN SOLUTION FOR THE BASIC TRANSFER MODEL

4.1. *Proportionate egalitarian solution and share function*

It is convenient to frame the discussion in terms of the auxiliary notion of *share function*. A share function $\lambda(S, i)$ for a basic transfer model (N,φ) is defined on all $S \in \mathbf{S}$ and $i \in N$ by the conditions that:

- if $\varphi(S) \neq 0, \lambda(S,i) \geq 0, \lambda(S,i) = 0$ if $i \notin S$, and $\Sigma_{i \in S}\lambda(S,i) = 1$;
- if $\varphi(S) = 0, \lambda(S,i) = 0$ for all i.

A share function λ is meant to describe the way in which the basic transfers $\phi(S)$ are apportioned among the members of S. If the centre in Examples 1, 2 and 3 knows λ, it exactly knows the amount each individual ends up with, and conversely. Note that it is not true generally (i.e., regardless of φ) that every conceivable λ determines what we called an allocation for (N,φ), i.e., a vector x satisfying the basic inequalities:

$$\sum_{T \subset S} \varphi(T) \leq \sum_{i \in S} x_i \leq \sum_{T \cap S = \emptyset} \varphi(T) \text{ for all } S \in \mathbf{S}. \tag{6}$$

Clearly, sign restrictions on φ are relevant to whether or not this property holds. However, what we are interested in is the converse property. Is it the case that every allocation x for the basic transfer model (N,φ) can be obtained by dividing the basic transfers $\varphi(S)$ according to some (possibly nonunique) share function λ? Fortunately, this question has a positive answer for basic transfer models satisfying the monotonicity and convexity property.[12] Capitalizing on this fact, we will assess the α-proportionate solution x° in terms of its associated λ^*.

PROPOSITION 3. *Suppose that a basic transfer model (N,ϕ) satisfies the monotonicity and convexity conditions of Proposition 2. Then, there is a share function λ^* corresponding to the α-proportionate solution x° with the property that for all $S \subset N$,*

$$i, j \in S \text{ and } x_i^\circ/\alpha_i > x_j^\circ/\alpha_j \Rightarrow \lambda^*(S,i) = 0.$$

If a basic transfer model satisfies the stronger assumption that $\varphi(S) \geq 0$ for all $S \subset N$, the same conclusion holds for all λ corresponding to the α-proportionate solution.

Proof. The proof depends on the algorithm used to compute x°. See the appendix.

Normally, at the constrained optimum x°, some individuals i end up with more than other individuals j in terms of their desirable shares. In essence, Proposition 3 says that whenever this is the case, the i get nothing from participating in a group S in which some j also participates. In other words, the α-proportionate solution is *an "all-or-nothing" solution*. It makes sure that the final solution gives all of each $\varphi(S)$ to the relatively most deprived individual, and nothing to the relatively less deprived ones.

4.2. *The egalitarian solution vs the Shapley value*

In the particular case of egalitarian solutions, where the desirable shares α_i are all equal, this "all-or-nothing" property is even simpler to state. It puts the constrained egalitarianism approach at large in sharp contrast with another approach that game theorists and some economists have endowed with normative significance, namely that of the Shapley value. There are many equivalent definitions of the Shapley value of a TU game (N, v), among them a useful restatement in terms of the basic transfer model (N,ϕ) uniquely associated with (N, v). This restatement is all we need here. Specifically, we define the *Shapley value* of (N,ϕ) to be the vector $Sh = (Sh_1, \dots , Sh_n)$ that results from always sharing $\varphi(S)$ equally between the $i \in S$, i.e., by applying the share function: $\lambda(S, i) = 1/|S|$ for all S and i.[13] From well-known facts in TU game theory, if (N,φ) satisfies the convexity condition of Proposition 2, this defines an allocation, so that it becomes meaningful to compare the Shapley value with the egalitarian solution.

The comparison shows that the Shapley value and the egalitarian solution obey very different intuitive principles of equalization. The Shapley value can be said to be "egalitarian" in the formal sense that the sharing function is symmetric vis-à-vis each individual. This symmetry property also means that if the procedure to reach an allocation consists in sharing each $\varphi(S)$ in succession, then the individuals will be treated equally at *each* stage of the procedure (in contradistinction with the iterative procedure described in the Appendix). So we can find a sense, either formal or procedural, in which the Shapley value embodies "equality".[14] But in a distributive context like the present one, it does not seem to be a very plausible sense of the words "equality" and "egalitarian". Here, equality is a substantive aim to be achieved, i.e., what is at stake is equality of the distributed worth between the individuals rather than equal treatment of the individuals. Granting this, the Lorenz ordering appears to be the natural way of making comparisons, and the property stated in Proposition 3 logically follows. The next example straightforwardly illustrates that these two sharing methods can diverge widely from each other:

EXAMPLE 4. Take $N = \{1, 2, 3\}$, and $\varphi(1) = \varphi(2) = \varphi(1, 2) = \varphi(1, 3) = \varphi(2, 3) = 0$, $\varphi(3) = 1$, $\varphi(1, 2, 3) = 3$. (This corresponds to: $v(N) = 4$, $v(1, 3) = v(2, 3) = 1$, $v(1, 2) = v(1) = v(2) = 0$, $v(3) = 1$.) The convexity condition is satisfied. The Shapley value is (1, 1, 2). The vector (4/3,4/3,4/3) is feasible and is thus the egalitarian solution.

We can extend the comparison just made to the proportionate solution of this paper and the weighted Shapley value, respectively[15]. Given any vector β of proportionality coefficients (i.e., with $\beta_i > 0$ and $\Sigma\beta_i = 1$), we define the *β-weighted Shapley value* of (N,ϕ) to be the vector Sh^β resulting from always sharing the $\varphi(S)$ between the $i \in S$ according to the ratios β_i/β_j, i.e., by applying the share function: $\lambda(S, i) = \beta_i/\beta(S)$ for all S and i.[16] Again, if (N,φ) satisfies the convexity condition, a standard argument ensures that this defines an allocation, so that comparison with the α-proportionate solution makes sense. The important fact is that for typically many basic transfer models, an α-proportionate solution cannot be a Sh^β solution (for β possibly different from α), except in the degenerate case where the α-proportionate solution is the *unconstrained* α-proportionate solution. We single out a wide class of models (N,φ) in which this negative conclusion straightforwardly derives from the previous result:

COROLLARY 1. *Suppose that (N,φ) satisfies the condition that $\varphi(S) \geq 0$ for all $S \subset N$, with $\varphi(N) > 0$. Then, an α-proportionate solution x° cannot be a Sh^β allocation for any vector of proportionality coefficients β unless $x^\circ = (\alpha_1 k, \alpha_2 k, \ldots, \alpha_n k)$, where $k = \sum_{T\subset N} \varphi(T)$.*

Proof. See the Appendix.

This statement is related to the following geometric fact: whenever the constraints (2) are binding, the α-proportionate solution automatically belongs to the boundary of the set of allocations, whereas the α-weighted Shapley value is an interior point.[17] Here is an example to illustrate this typical situation.

EXAMPLE 5. Take $N = \{1, 2, 3\}$, and $\varphi(1) = 1$, $\varphi(2) = 2$, $\varphi(3) = 3$, $\varphi(1, 2) = 0$, $\varphi(1, 3) = 0.5$, $\varphi(2, 3) = 0$, $\varphi(1, 2, 3) = 0$. (This corresponds to: $v(1) = 1, v(2) = 2, v(3) = 3, v(1, 2) = 3, v(1, 3) = 4.5, v(2, 3) = 5, v(N) = 6.5$.) The sign conditions of the corollary are satisfied. The vector (6.5/3, 6.5/3, 6.5/3) is not feasible. The egalitarian solution is (1.5, 2, 3). It can be checked that the allocation set is the segment in R^3 from (1, 2, 3.5) to (1.5, 2, 3), so that the egalitarian solution is an extreme point of this set. The Shapley value is (5/4, 2, 13/4). Compared with the egalitarian solution, it is biased towards individual 3. The egalitarian solution cannot be recovered as a Sh^{β}, although it might be approached closely by a suitable choice of β.

Examples like this one are worth pondering about, given the well-documented use of the Shapley value as a sharing device in normative and public economics applications.[18] We think that especially when it is generalized in the way we proposed here to account for differences in needs or desert, the egalitarian solution makes more sense from the broad viewpoint of ethical intuition. At least, we have tried to identify a class of simple redistributive models in which the proposed solution has a good standing from this viewpoint.

5. APPENDIX

5.1. *An algorithm for the α-proportionate egalitarian solution*

Here is a procedure to compute the α-proportionate solution x° in terms of the initial data (N,ϕ). It is a straightforward adaptation of Dutta and Ray's (1989, p. 625) algorithm, so we have kept as close

as possible to their notation. We fix any $m = (m_1, \dots, m_n)$ as in Section 2.
Step 1. Find the greatest group S^1 maximizing:
$e(S, \varphi, m) =^{def} \sum_{T \subset S} \varphi(T)/m(S)$ over all $S \in \mathbf{S}$.
(There is a unique maximum because of convexity.)
Put $x_i^\circ = e(S^1, \varphi, m)$ for all $i \in S^1$.
Step k. Suppose that $S^1, \dots, S^{k-1}$ have been defined and that $A^{k-1} =^{def} S^1 \cup \dots \cup S^{k-1} \neq N$.
Find the greatest group S^k maximizing:
$e(S, \phi, m) =^{\mathrm{def}} \sum \phi(T))/m(S)$ where $S \subset N/A^{k-1}$ and the sum is taken over all distinct $T \subset A^{k-1} \cap S$, $T \cap S \neq \emptyset$.
Put $x_i^\circ = e(S^k, \varphi, m)$ for all $i \in S^k$.
(Again, there is a unique maximum because of convexity.)
By construction, the sets $S^1, \dots, S^k, \dots$ define a partition of N such that
$x_i^\circ/m_i = x_j^\circ/m_j$ if $i, j \in S^k$ for some k, and:
(+) $x_i^\circ/m_i > x_j^\circ/m_j$ if and only if $i \in S^k$, $j \in S^p$ for some $k < p$.
Also, for all $k : x^\circ(S^1 \cup \dots \cup S^k) = \Sigma\varphi(T)$, where the sum is over all $T \subset S^1 \cup \dots \cup S^k$.

That the algorithm just explained delivers the α-proportionate solution follows because it coincides with the original Dutta–Ray algorithm as applied to the auxiliary game (N^m,v^m). It is enough to apply Dutta and Ray's Theorem 2 (1989, p. 627) to this game.

5.2. *Proof of Proposition 3*

We prove the existential claim of the proposition by considering the above algorithm. We can associate a share function λ° with the algorithm in the following way. Consider first S^1. The basic transfer model (S^1, φ^1) with $\varphi^1(T) = \varphi(T)$ for all $T \subset S^1$ satisfies the same assumptions as (N,ϕ). Hence, by Dutta and Ray's Theorem 2 as applied to this game, the vector $x^1 \in R^{|S1|}$ such that $x_i^1 = x_i^\circ$ for all $i \in S^1$ is an allocation. From a result mentioned in the text, there is a share function λ^1 corresponding to x^1. Extending the argument inductively, we construct a sequence of share functions $\lambda^1, \dots, \lambda^p, \dots$ on S^p, $p \geq 1$, which amounts to defining a share function λ° corresponding to x°.

Suppose now that $x_i^\circ/m_i > x_j^\circ/m_j$. From property (+) above this can happen only if $i \in S^k$ and $j \in S^p$ for some $k < p$. Given the construction just made of λ° and the definition of x_i° in terms of $\Sigma\varphi(T)$, the condition that $\lambda^\circ(T, i) > 0$ would mean that $T \subset A^{k-1} \cup S^k = A^k$. But this is impossible since $T \cap S^p \neq \emptyset$, and $A^k \cap S^p = \emptyset$. Now, using the assumption that $\varphi \geq 0$, we set out to derive the same conclusion for any share function λ delivering the solution x°. Suppose first that p is the index of the last set in the sequence $A^1, A^2, \ldots$, i.e., that $A^p = S^1 \cup \ldots \cup S^p = N$. Then,

$$x^\circ(S^p) = m(S^p)e(S^p, \varphi, m) = \sum \varphi(T),$$

where the sum is taken over all distinct $T \subset N$, $T \cap S^p \neq \emptyset$, or equivalently:
where the sum is taken over all distinct T of the form $\{j\} \subset T \subset N$ for all $j \in S^p$.
Now, the given share function λ satisfies the condition that:

$$x^\circ(S^p) = \sum \lambda(T, j)\phi(T),$$

where, again, the sum is taken over all distinct T of the form $\{j\} \subset T \subset N$ and all $j \in S^p$. Given that $\phi(T) \geq 0$ for all such T, the comparison between the two formula for $x^\circ(S^p)$ shows that $\lambda(T, i) > 0$, $i \in S^k$, $k < p$, is impossible.

Assume now that the result has been proved for all indexes down from the last to some $p + 1 > 1$. We show that it holds for $j \in S^p$ as well. Now,

$$x^\circ(S^p) = \sum \phi(T),$$

where the sum is taken over all distinct T of the form $\{j\} \subset T \subset A^p$ for all $j \in S^p$. Also:

$$x^\circ(S^p) = \sum \lambda(T, j)\phi(T) + \sum \lambda(T', j)\phi(T'),$$

where the first sum is taken over all distinct T of the form $\{j\} \subset T \subset A^p$ and all $j \in S^p$, and the second sum is taken over all T' of the form $\{j\} \subset T'$, $T' \not\subset S^p$, and all $j \in S^p$. By the induction hypothesis, the second sum is zero, so that we can repeat the previous argument, and conclude that $\lambda(T, i) > 0$, $i \in S^k$, $k < p$, is impossible.

5.3. *Proof of Corollary 1*

As the inspection of the algorithm shows, x° is the nonconstrained α-proportionate solution if and only if it converges in one step, i.e., $A^1 = N$. Otherwise, there are at least two individuals i, j such that $x_i^\circ/\alpha_i > x_j^\circ/\alpha_j$. Applying Proposition 3 we conclude that $\lambda(N, i) = 0$ for all share functions leading to x°. This applies in particular to the share function of that Sh^β which coincides with x° if there is any. But there cannot be any such Sh^β, since its share function λ stipulates that

$$\lambda(N, i) = \beta i > 0.$$

6. ACKNOWLEDGEMENTS

The authors are grateful to Bhaskar Dutta, Udo Ebert, Michel Le Breton, Hervé Moulin, Alain Trannoy, and two anonymous referees for useful comments.

NOTES

1. There is more to Dutta and Ray's (1989) contribution to constrained egalitarism than this result; however, we shall focus on it exclusively.
2. The paper by Atkinson and Bourguignon (1987) is a major example. For further work and references along the same line, see Ebert (1999).
3. For some continuous, *strictly* concave f, there exists y^* which maximizes $\Sigma\, f(y_i)$ on the core. Now, suppose that there is x in the core such that $x \geq^L y^*$. Then, the HLP theorem implies that $\Sigma f(x_i) \geq \Sigma f(y_i^*)$, and by strict concavity of f, $x = y^*$, so that y^* is a maximal element.
4. Dutta and Ray's (1989, pp. 620–21) own definition of an egalitarian solution is actually more complex than this one. But in the case of convex games (N, v) their special definition coincides with that of a unique greatest element for the Lorenz ordering on $C(N, v)$. Since we restrict attention to convex games, the terminological slip is harmless.
5. Suggested to us by Bhaskar Dutta in private correspondence.
6. This is the famous problem, "Equality of What?", discussed in Sen (1992, ch. 1).
7. A similar idea is independently introduced and formalized by Hokari (2002).

8. That monotonicity is an insubstantial assumption in the present context is further confirmed by the following property of the egalitarian solution. Suppose that v' results from v by adding to it a uniform measure, i.e., $v'(S) = v(S) + c|S|$ for all S, where c is a fixed positive number. Then, the egalitarian solution for (N, v') is obtained from the egalitarian solution by adding c to each component (see the socalled weak covariance property in Dutta and Ray, (1989, p. 633), or in Dutta (1990).
9. For an exposition of cooperative game theory, see, e.g., Moulin (1988).
10. These v_T are characteristic functions of special games sometimes referred to as *unanimity games* .
11. Actually, $\varphi \geq 0$ gives to game (N, v) a stronger property called *infinite-order monotonicity*. Such games possess specific properties (Shafer, 1976), which however are not essential for the results derived below.
12. See Chateauneuf and Jaffray (1989, Proposition 5). The monotonicity condition is contained in the authors' definition of a "capacity".
13. That this definition is equivalent to the more traditional ones in terms of (N, v) is mentioned in Moulin (1988, exercise 5.10, p. 140).
14. Mas-Colell, Winston and Green (1995, p. 680) take a step in the direction. of arguing that the Shapley value embodies "egalitarianism". Other writers have emphasized the normative appeal of the Shapley value. In an application to cost-sharing, Champsaur (1975) claims that it is "equitable".
15. See Kalai and Samet (1988). Hokari (2002) also discusses the weighted Shapley value, and so do Sundberg and Wagner (1992) in the equivalent language of "p-smears".
16. This definition of the weighted Shapley is meant to extend the definition just given for the ordinary Shapley value. We leave it for the reader to check that it is equivalent to the better-known one in terms of (N, v).
17. In the *relative* interior sense, of course. If the dimension of the set of allocations is smaller than n, it may not have an interior in R^n. Example 5 precisely illustrates this case.
18. Dutta and Ray (1989) are critical of the Shapley value, as illustrated by their discussion of Champsaur (1975).

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