Mathematical Social Sciences 79 (2016) 6–10

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/econbase

A theorem on aggregating classifications[☆]

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ARTICLE INFO

Article history:
Received 23 October 2014
Received in revised form
8 October 2015
Accepted 8 October 2015
Available online 19 October 2015

ABSTRACT

Suppose that a group of individuals must classify objects into three or more categories, and does so by aggregating the individual classifications. We show that if the classifications, both individual and collective, are required to put at least one object in each category, then no aggregation rule can satisfy a unanimity and an independence condition without being dictatorial. This impossibility theorem extends a result that Kasher and Rubinstein (1997) proved for two categories and complements another that Dokow and Holzman (2010) obtained for three or more categories under the condition that classifications put at most one object in each category. The paper discusses an interpretation of its result both in terms of Kasher and Rubinstein's group identification problem and in terms of Dokow and Holzman's task assignment problem.

1. Introduction

While preference aggregation still looms large in the agenda of social choice theory, there is a small, but growing body of literature on the aggregation of *classifications*. The general scheme is that the members of a group each propose dividing a given set of objects into categories, and that a collective division of the set results from these individual proposals by respecting various conditions of association, which are partly reminiscent of those usually defined for preference aggregation. In one version of this scheme, which appears to date back to Mirkin (1975), the individuals and the collective can partition the set in any possible ways. (See Chambers and Miller, 2011 and Dimitrov et al., 2012 for recent developments; the latter paper also surveys the field.) In another version, which can be traced to Kasher and Rubinstein (1997), there is a given list of designated categories in which the objects must be fitted. This version has been explored, both by Kasher and Rubinstein and followers, in the particular case where the objects to be classified are the very individuals who propose the classifications. As a typical application, some countries legally divide their citizens according to racial, ethnic or religious criteria. Since the citizens themselves have opinions on how this should be done, one may investigate how the legal division should reflect these opinions. Put in axiomatic form, this has come to be called the *group identification problem*. (See, among others, Samet and Schmeidler, 2003; Dimitrov et al., 2007 and Miller, 2008.)

The present paper investigates the aggregation of classifications with designated categories, and hence belongs to the second branch of analysis, but does not pursue the group identification problem specifically. Rather, it proves an impossibility theorem for this second branch at large. In a nutshell, if there are $p \geq 3$ categories in the list and $m \geq p$ objects to be classified in these categories, and if moreover both individual and collective classifications satisfy the *surjectivity* (ontoness) restriction that each category is filled with at least one object, then the collective classifications are dictatorial if they satisfy a unanimity and an independence condition. The unanimity condition says that if the individuals in a profile agree on how to classify an object, the aggregate for this profile endorses the agreed on classification. The independence condition says that if there are two profiles and each individual classifies an object identically in both of them, the corresponding two aggregates also classify the object identically. These two conditions are reminiscent of familiar ones in preference aggregation, but have a *unary* form, which leads to a distinctive analytical treatment.

Kasher and Rubinstein (1997, Theorem 2), have stated this theorem for the special case $m \geq p = 2$. They relate it to the group identification problem, but their proof is in fact independent

☆ The authors thank Franz Dietrich, Ron Holzman, Marcus Pivato, the referees and the associate editor for useful comments on previous versions.

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http://dx.doi.org/10.1016/j.mathsocsci.2015.10.001

of this context.[1] Thus, our work can be seen as an extension of theirs. To free the impossibility result from the limitation to two categories is a non-trivial step, as will appear from the proof and technical comments below. Although Kasher and Rubinstein have not emphasized this point, it is essential to the impossibility that the categories, whatever their number $p \geq 2$, are never left empty by either the individual or the collective classifications. This is the surjectivity restriction mentioned above.

If one is ultimately interested in the group identification problem, this is a natural restriction to consider. One may expect lawmakers to confer legal status on a social category only if they believe it to be applicable at least to some citizens, and in a country where democratic principles hold, one may further expect that the categories have been agreed on between the lawmakers and the citizens prior to being used in practice. Accordingly, citizens would no more than lawmakers leave any category unfilled, even though they would no doubt disagree on its precise extension. The group identification literature alludes to political examples that seem to warrant this analysis. Kasher and Rubinstein (1997) implicitly draw inspiration from the legal religious denominations in Israel, and Miller (2008) explicitly mentions the racial divisions recognized by the US Census. If Israeli or US citizens were asked to classify a significant sample of their respective populations, they would be very unlikely to leave any of the available categories vacuous, except perhaps for strategic purposes that we will not consider in this paper. Concerning the group identification problem, our view is that the most troublesome idealizing assumption is not surjectivity, but the very form of the poll, which requires each citizen to classify any other, whereas most political examples only involve self-designation.

If one is not particularly interested in this problem, one may turn to more direct cases of aggregating classifications for which surjectivity appears to be appropriate. Consider a panel of astronomers who meet to classify distant celestial bodies into stars, exoplanets, brown dwarfs and other less identifiable objects. Each astronomer proposes his own classification, and the chair tries to turn these individual data into an authoritative classification. The classification is well-established on prior grounds, so if the set of celestial bodies under consideration is large enough, neither the individual astronomers nor the chair will leave any of the four categories empty.[2] This is of course a theoretical example, but it is worth noting that the status of celestial bodies is currently discussed at a collective level, with aggregative steps – typically votes – being sometimes taken (for an intriguing account of the discussions surrounding Pluto, see Marschall and Maran, 2009).

We will provide further motivation for surjectivity while interpreting our framework in terms of a *collective task assignment problem*, as in Dokow and Holzman (2010). Having this interpretation in view, these authors investigate the same problem of aggregating classifications as ours, but make the opposite assumption that there are $m \geq 3$ objects and $p \geq m$ positions. They show that if individual and collective classifications satisfy the *injectivity* restriction that each category is filled with at most one object, then the collective classifications are dictatorial if they satisfy unanimity (in a reinforced version) and independence. The two impossibility results complement each other very naturally. Dokow and Holzman's actually belongs to an abstract theory of nonbinary evaluations, which they develop for its own sake, and we had borrowed this powerful apparatus to carry out our first proof (Maniquet and Mongin, 2014). For ease of exposition, we have shifted here to the language and ultrafilter proof technique of standard social choice theory, but the interested reader may consult this earlier version, which also discusses the connections between social choice theory and the recently developed judgment aggregation theory.

2. The formal setup and the theorem

There are a set $N = \{1, \ldots, n\}$ of individuals, a set $X = \{1, \ldots, m\}$ of objects, and a set $P = \{1, \ldots, p\}$ of positions (or categories), with $p \geq 3$. The individuals classify the objects by putting each of them in a position. Formally, classifications are mappings $X \to P$. By assumption, there are at least as many objects as positions, and each classification assigns at least one object to each position. Formally, $m \geq p$, and the set of classifications is the *surjectivity* (ontoness) *domain*:

$$\mathcal{C} = \{k : X \to P \mid \forall r \in P, \exists x \in X : k(x) = r\}.$$

An *aggregation function* associates a social classification with any profile of individual classifications:

$$F : \mathcal{C}^n \to \mathcal{C}, (c_1, \ldots, c_n) \mapsto F(c_1, \ldots, c_n).$$

We abridge $F(c_1, \ldots, c_n)$, $F(c'_1, \ldots, c'_n)$,..., as c, c'.... The definition of F encapsulates a universal domain condition. We introduce three more conditions axiomatically. *Independence* requires that if an object occupies the same positions in two profiles of individual classifications, x occupies the same position in the associated social classifications.

Condition 1. Independence*: For all $(c_1, \ldots, c_n)$, $(c'_1, \ldots, c'_n) \in \mathcal{C}^n$ and all $x \in X$, if for all $i \in N$, $c_i(x) = c'_i(x)$, then $c(x) = c'(x)$.*

Unanimity requires that if all individual classifications in a profile give an object the same position, the social classification give it that position.

Condition 2. Unanimity*: For all $(c_1, \ldots, c_n) \in \mathcal{C}^n$, all $x \in X$, all $r \in P$, if for all $i \in N$, $c_i(x) = r$, then $c(x) = r$.*

The last condition states that one individual imposes his classification to society.

Condition 3. Dictatorship*: There is $j \in N$ such that for all $(c_1, \ldots, c_n) \in \mathcal{C}^n$, $c = c_j$.*

Independence and *Unanimity* are reminiscent of Independence of Irrelevant Alternatives and the Pareto conditions in Arrovian social choice theory. They can be defended normatively by roughly parallel arguments—*Independence* being connected with computational ease and nonmanipulability, and *Unanimity* with the individuals' sovereignty. Dictatorship is meant to be as undesirable here as it is there. Notice however that the present conditions are *unary*, i.e., bear on one object at the time, as suits a classification aggregation problem, whereas the Arrovian conditions are *binary*, as suits a preference aggregation problem.

Theorem 1. *If an aggregation function F satisfies* Independence *and* Unanimity, *it satisfies* Dictatorship.

The proof of the theorem consists in showing that the set of decisive subsets of N is an ultrafilter. In the present context, a subset of N is *decisive* if for every profile, every object and every position, when the profile is such that all individuals in this subset agree to put the given object in the given position, then society endorses this agreement. This notion of a decisive subset appears only as Definition 4 in the course of the proof. We first introduce weaker variant notions of decisiveness that are graded in logical strength, i.e., Definitions 1–3, exploring their properties in

[1] This two-category case is a corollary to an impossibility theorem proved by Rubinstein and Fishburn (1986, Theorem 3).

[2] Notice that surjectivity here follows as a fact of the situation, and not on normative grounds. A referee alerted us to this distinction.

a corresponding sequence of lemmas, i.e., Lemmas 2–4. This serves as a groundwork for the pivotal Lemma 5, which states that the set of decisive subsets, in the sense of Definition 4, is an *ultrafilter*. It then follows by a classic argument that this set identifies a dictator, i.e., an individual $j \in N$ as in the statement of *Dictatorship*. Both the use of graded decisiveness notions and the final ultrafilter argument are very familiar from social choice theory, but we had to adapt these tools to our unary framework of classifications.[3]

We will often represent profiles by tables like the following:

	N_1	N_2	$\cdots$	$N \diagdown N_1 \cup N_2 \cup \ldots$
r				
r'		x		
r''				

The lines are determined by the positions in P, and the columns, by the individuals, who will be grouped according to some partition of N. The objects in X appear in the entries; e.g., if x appears in the entry (r', N_2), this means that $c_i(x) = r'$ for all $i \in N_2$. We do not necessarily describe the full content of an entry and never list more than three lines at a time. All lemmas below require *Unanimity* and *Independence*, and by the latter axiom, the missing content of a table can be completed consistently with the chosen data; as this argument occurs repeatedly, we will sometimes omit it from the proofs.

We begin with an important preliminary lemma, to the effect that *Unanimity* can be logically reinforced, given the surjectivity domain adopted here. In effect, the reinforcement says that an object can occupy a position in the social classification only if it occupies this position in at least one of the individual classifications.[4]

Lemma 1. *For all $(c_1, \ldots, c_n) \in \mathcal{C}^n$, all $x \in X$ and all $r \in P$, if $c(x) = r$, then $c_i(x) = r$ for at least one $i \in N$.*

Proof. If this is not the case, there exists an object x and a profile $(c_1, \ldots, c_n)$ s.t. x appears in $1 \le p_x < p$ lines, but x is in a social position different from any of these lines. By *Unanimity*, we must have $p_x \ge 2$. Consider any profile $(c'_1, \ldots, c'_n)$ s.t. (i) x appears in the same entries as in $(c_1, \ldots, c_n)$, (ii) the p_x lines where x appears are filled with $p_x - 1$ other objects than x, and (iii) the remaining $m - p_x$ objects are distributed so that the remaining $p - p_x$ lines are filled and each of these objects always appears on a single line. Then, by *Unanimity*, the $m - p_x$ objects are in the $p - p_x$ social positions, and since by *Independence* c' and c give x the same social position, c' has only $p_x - 1$ objects to fill the p_x positions. This contradicts the fact that $c' \in \mathcal{C}$.[5] ■

In the next proofs, we will use Lemma 1 as follows. We will devise profiles in table form such that the entries of a line r contain only two objects x and y, and then conclude from Lemma 1 that society can fill r only with either x or y. If our assumptions also ensure that x's social position cannot be r, we fully determine what y's social position is—it must then be r. Variants of this reasoning serve to prove Lemmas 2–5.

[3] As a brief reminder, an ultrafilter U on a set N is a set of subsets of N that does not contain the empty set, is closed under inclusions and intersections, and is such that for every subset of N, either this subset or its complement belongs to U. If N is finite, U has the form of the set of all subsets of N that contain a given element j of N.

[4] In Dokow and Holzman's (2014) more abstract framework, this property – labelled *Supportiveness* – plays a critical role; see below.

[5] To illustrate the last step in the proof with $p = 3$ and $m = 4$, here is a profile $(c'_1, \ldots, c'_n)$ leading to the contradiction:

	N_1	$N \setminus N_1$	*Society*
r	x	y	?
r'	y	x	?
r''	wz	wz	xwz

Definition 1. Let $x \in X$, $N_1 \subseteq N$ and $r, r' \in P$, $r \neq r'$. The subset N_1 is *semi-decisive for r against r' over x* if, for all $(c_1, \ldots, c_n) \in \mathcal{C}^n$, the two conditions that

- for all $i \in N_1$, $c_i(x) = r$, and
- for all $i \in N \setminus N_1$, $c_i(x) = r'$,

entail that $c(x) = r$.

Lemma 2. *If the subset N_1 is semi-decisive for r against r' over x, then for all $s, s' \in P$ and all $y \in X$, it is semi-decisive for s against s' over y.*

Proof. If $N_1 = N$, *Unanimity* secures the conclusion. If $N_1 \subsetneq N$, take any $(c_1, \ldots, c_n)$ as follows:

	N_1	$N \setminus N_1$
r	x	
r'		x

Take $y \neq x$, and s, s' s.t. $s \neq s'$ and $s, s' \neq r, r'$. By *Independence*, we can fix $c_i(y) = r'$ for all $i \in N_1$, and $c_i(y) = s$ for all $i \in N \setminus N_1$, while completing the profile in such a way that only x and y appear in line r':

	N_1	$N \setminus N_1$
r	x	
r'	y	x
s		y

(From now on, we skip the details of completion of our auxiliary profiles; they are easily adapted from those of the last proof.) Since $c(x) = r$ by the semi-decisiveness assumption, it follows from Lemma 1 (in the way explained above) that $c(y) = r'$, and by *Independence* again, we conclude that N_1 is semi-decisive for r' against s over y. The same proof for r', s, s' instead of r, r', s shows that N_1 is semi-decisive for s against s' over y, as desired. Variant arguments take care of the cases where $\{r, r'\} \cap \{s, s'\} \neq \varnothing$. ■

Definition 2. Let $x \in X$, $N_1 \subseteq N$, $r \in P$. The subset N_1 is *semi-decisive for r over x* if, for all $(c_1, \ldots, c_n) \in \mathcal{C}^n$, the two conditions that

- for all $i \in N_1$, $c_i(x) = r$, and
- for all $i \in N \setminus N_1$, $c_i(x) \neq r$,

entail that $c(x) = r$.

Lemma 3. *If the subset N_1 is semi-decisive for r against r' over x, then for all $s \in P$ and all $y \in X$, it is semi-decisive for s over y.*

Proof. If $N_1 \subsetneq N$, take any $(c_1, \ldots, c_n)$ with $c_i(y) = s$ for all $i \in N_1$ and $c_i(y) \neq s$ for all $i \in N \diagdown N_1$. If $c_i(y)$ is the same for all $i \in N \diagdown N_1$, Lemma 2 already delivers the result, so we assume that $c_i(y)$ takes $k \ge 2$ values when i ranges over $N \diagdown N_1$. We only deal with $k = 2$, as the general case follows from adapting the proof. Assume then that y appears on line s and two other lines s', s''. If $x \neq y$, we can apply *Independence* and ensure that $(c_1, \ldots, c_n)$ satisfies the following (with no more objects on line s'):

	N_1	N_2	$N \setminus N_1 \cup N_2$
s	y	x	x
s'	x	y	
s''			y

By Lemma 2, N_1 is semi-decisive for s' against s over x, whence $c(x) = s'$, so that by Lemma 1, $c(y) = s$, as desired. If $x = y$, the argument uses $z \neq x$ in the role of x. ■

Definition 3. Let $x \in X$, $N_1 \subseteq N$. The subset N_1 is *decisive for r over x* if, for all $(c_1, \ldots, c_n) \in \mathcal{C}^n$, the condition that

- for all $i \in N_1$, $c_i(x) = r$,

entails that $c(x) = r$.

Lemma 4. *If, for all $r \in P$ and all $x \in X$, the subset N_1 is semi-decisive for r over x, then for all $s \in P$ and all $y \in X$, it is decisive for s over y.*

Proof. Take $N_1 \subsetneq N_1' \subsetneq N$, and $(c_1, \ldots, c_n)$ in which $c_i(y) = s$ for all $i \in N_1'$, and $c_i(y) \neq s$ for all $i \in N \setminus N_1'$, with $c_i(y)$ taking $k \geq 1$ values when i ranges over $N \setminus N_1'$. Assume that $k = 2$. Then, there are three positions and four groups to consider, i.e., $c_i(y) = s$ for all $i \in N_1$, $c_i(y) = s'$ for all $i \in N_2$, $c_i(y) = s''$ for all $i \in N_3$, and again $c_i(y) = s$ for all $i \in N_4 = N \setminus N_1 \cup N_2 \cup N_3$. If $x \neq y$, we take $z \neq x, y$, and invoking *Independence*, ensure that $(c_1, \ldots, c_n)$ satisfies the following (with no more objects on line s):

	N_1	N_2	N_3	N_4
s	y	z	x	y
s'	x	y	z	z
s''	z	x	y	x

By assumption, N_1 is semi-decisive for s' over x, and for s'' over z, so that $c(x) = s'$ and $c(z) = s''$. Then, Lemma 1 entails that $c(y) = s$, as desired. The cases $k = 1$ and $k \geq 3$ result from adapting this argument. ■

Definition 4. Let $x \in X$, $N_1 \subseteq N$. The subset N_1 is *decisive* if, for all $x \in X$ and $r \in P$, N_1 is decisive for r over x.

Lemma 5. *The set of decisive subsets of N is an ultrafilter.*

Proof. If N_1 is decisive, then any superset of N_1 is decisive; this simply follows from the definition. We proceed to show that if N_1 and N_2 are decisive, then so is $N_1 \cap N_2$. Fix $s, s' \in P$ and $y \in X$. We will show in two steps that if N_1 and N_2 are decisive, then $N_1 \cap N_2$ is semi-decisive for s against s' over y. When this conclusion is reached in the second step, the stronger conclusion that $N_1 \cap N_2$ is decisive will follow from applying the previous lemmas in succession.

Step 1. Here we fix $r, r', r'' \in P$ and $x \in X$ and consider a profile $(c_1, \ldots, c_n)$ in which $c_i(x) = r$ for all $i \in N_1 \cap N_2$, $c_i(x) = r'$ for all $i \in N_1 \setminus N_2$, $c_i(x) = r''$ for all $i \in N_2 \setminus N_1$, and $c_i(x) = r'$ for all $i \in N \setminus N_1 \cup N_2$. By *Independence*, we make sure that the three lines r, r', r'' satisfy the following (with no more objects appearing on line r).

	$N_1 \setminus N_2$	$N_1 \cap N_2$	$N_2 \setminus N_1$	$N \setminus N_1 \cup N_2$
r'	x	z	z	x
r	z	x	y	z
r''	y	y	x	y

Since N_1 is decisive for r'' over y, and N_2 for r' over z, $c(y) = r''$ and $c(z) = r'$; hence $c(x) = r$ follows from Lemma 1.

Step 2. Consider $(c_1, \ldots, c_n)$ in which $c_i(y) = s$ for all $i \in N_1 \cap N_2$, and $c_i(y) = s'$ for all $i \in N \setminus N_1 \cap N_2$. Taking some other line s'', we make sure by *Independence* that s, s', s'' satisfy the following (with no more objects on line s):

	$N_1 \setminus N_2$	$N_1 \cap N_2$	$N_2 \setminus N_1$	$N \setminus N_1 \cup N_2$
s	x	y	z	x
s'	y	x	y	y
s''	z	z	x	z

Because N_1 is decisive, $c(z) = s''$. Now x is in the same pattern as in Step 1, with s, s', s'' instead of the (arbitrarily fixed) r, r', r'', whence $c(x) = s'$. It then follows from Lemma 1 that $c(y) = s$, hence that $N_1 \cap N_2$ is semi-decisive for s against s' over y, as was to be proved.

It remains to be shown that for all $N_1 \in N$, either N_1 or $N \setminus N_1$ is decisive. Consider $(c_1, \ldots, c_n)$ satisfying the following:

	N_1	$N \setminus N_1$
r	x	
r'		x

By Lemma 1, either $c(x) = r$ or $c(x) = r'$, meaning that either N_1 is semi-decisive for r against r' over x, or $N \setminus N_1$ is semi-decisive for r' against r over x. The conclusion then follows from the earlier lemmas. ■

3. More comments

The formalism of this paper can be interpreted in terms of a collective task assignment problem. Suppose that there is a set P of p tasks and a set X of m workers, and that exactly one task must be assigned to each worker; then an assignment of tasks to workers is a well-defined mapping $X \rightarrow P$. If several workers can be occupied on the same task, the condition $m \geq p$ makes sense, and if furthermore all tasks must be attended to, surjectivity applies. As a pictorial example of all these conditions being met, think of a conservation camp, in which each enrollee has one and only one activity, like planting a tree or opening a trail, such activities typically require enrollees to work side by side, and the camp wants each of them to be carried out. Suppose now that there are several supervisors, and each proposes an assignment $X \rightarrow P$. Our theorem challenges the way in which one may wish to define a collective task assignment from these individual proposals.[6]

Now suppose that the problem is still to assign p tasks in P to m workers in X, under the earlier constraint that exactly one task is assigned to each worker, and the new constraint that some tasks can be left unattended to, so that the reverse inequality $p \geq m$ makes sense. If one adds that no two workers can be occupied on the same task, the assignments $X \rightarrow P$ will be *injective*. As an example of all conditions being met, think of tasks as being jobs that are advertised by an employment agency, and of workers as being applicants to this agency, with potentially more jobs than applicants because the jobs are somehow unattractive. As before, we may suppose that several proposals are made for the task assignment and that these need to be aggregated. Dokow and Holzman (2010, Example A and Corollary 1) have analyzed a related example in their abstract formalism of nonbinary evaluations, and remarkably, they have come up with an impossibility theorem.[7]

Formally, Dokow and Holzman introduce the *injectivity domain*:

$$\mathcal{C}^{inj} = \left\{ k : X \rightarrow P \mid \forall x, x' \in X : k(x) = k(x') \Longrightarrow x = x' \right\}$$

and define an *aggregation function* to be:

$$F : (\mathcal{C}^{inj})^n \rightarrow \mathcal{C}^{inj}, (c_1, \ldots, c_n) \mapsto F(c_1, \ldots, c_n).$$

They also introduce the following variant of the *Unanimity* condition:

Condition 4. Supportiveness*: For all $(c_1, \ldots, c_n) \in (\mathcal{C}^{inj})^n$, all $x \in X$, all $r \in P$, if $c(x) = r$, then there is $i \in N$ such that $c_i(x) = r$.*

Supportiveness trivially entails *Unanimity*, but the converse does not hold in general. Using a general theorem of theirs (2010, Theorem 1), Dokow and Holzman show that if $F : (\mathcal{C}^{inj})^n \rightarrow \mathcal{C}^{inj}$ satisfies *Independence* and *Supportiveness*, it satisfies *Dictatorship*. If we manage to prove *Dictatorship* from *Independence* and *Unanimity*, a more standard derivation, this is because our surjectivity domain supports the problematic converse; indeed, Lemma 1 states that *Unanimity* and *Independence* entail *Supportiveness* on $\mathcal{C}$.

6 In an alternative, but perhaps less natural, interpretation of the formalism, X is a set of m *tasks*, P is a set of p *workers*, and the latter are assigned to the former rather than the converse. Now the interpretational constraints for $X \rightarrow P$ to be a surjective mapping, with $m \geq p$, are that exactly one worker is assigned to each task, each worker can be occupied on more than one task, and no worker is left unoccupied.

7 Dokow and Holzman literally consider the alternative interpretation in which p *candidates* in P are assigned to m *jobs* in X. Now the interpretational constraints for $X \rightarrow P$ to be an injective mapping, with $p \geq m$, are that exactly one candidate is assigned to each job, candidates can end up without a job, and no candidate can end up with more than one job.

While the present paper uses the language and ultrafilter proof technique of social choice theory, an earlier version derived the same result using Dokow and Holzman's apparatus of nonbinary judgments. This involved a significant technical detour but permitted including our result into the large set of applications covered by these authors. The interested reader is referred to Maniquet and Mongin (2014). On the occasion of this proof, the earlier version touches on the broader question of how the recent body of work on judgment aggregation contributes to the progress of social choice theory (see also the discussion in Dietrich and Mongin, 2010, or Mongin, 2012).

A word may be added in connection with the motivating examples of this paper. In our view, the conflict between *Unanimity*, *Independence* and either the *surjectivity* or *injectivity* domain restriction is not easy to resolve once it is recognized. The two axioms have some normative standing, as was said, and the domain restriction cannot be removed if it is part of the institutional context (as we argued surjectivity was in the group identification problem) or if it reflects a feasibility constraints (variants of the collective assignment problem illustrated this for either surjectivity or injectivity). Then, one is left with the thorny choice of sacrificing one of two *prima facie* relevant axioms.

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