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ARE "ALL-AND-SOME" STATEMENTS FALSIFIABLE AFTER ALL?

The Example of Utility Theory

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Popper's well-known demarcation criterion has often been understood to distinguish statements of empirical science *according to their logical form*. Implicit in this interpretation of Popper's philosophy is the belief that when the universe of discourse of the empirical scientist is infinite, empirical universal sentences are falsifiable but not verifiable, whereas the converse holds for existential sentences. A remarkable elaboration of this belief is to be found in Watkins's early work (1957, 1958) on the statements he calls "all-and-some," such as: "For every metal there is a melting point." All-and-some statements (hereafter AS) are both universally and existentially quantified in that order.[1] Watkins argued that AS should be regarded as both nonfalsifiable and nonverifiable, for they partake in the logical fate of both universal and existential statements. This claim is subject to the proviso that the bound variables are

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1. As against those other mixed statements which can be called "some-and-all." Contrary to AS, the latter are rarely touched upon in the literature.

"uncircumscribed" (in Watkins's words); i.e., that the universe of discourse is infinite.[2]

Like many pieces of falsificationist philosophy, Watkins's analysis has eventually made its way into economic methodology. A recent paper by Boland (1981) aims at offering a new construal of the (by no means novel) claim that the fundamental assumptions of neoclassical economics are not falsifiable after all. Boland's substantial point is that "the neoclassical maximization hypothesis" (in the author's phrase) is irrefutable *by virtue of its logical form*. This is argued by (a) analyzing "the neoclassical maximization hypothesis" as an AS with an infinite domain, and (b) reviving Watkins's "demonstration" that such "uncircumscribed" AS are neither verifiable nor falsifiable.[3]

The primary aim of this paper is to show that move (b) is ineffective, for Watkins's early view and Boland's current belief[4] that uncircumscribed AS are *always* irrefutable is simply false. This will be argued in section 1 by means of a counterexample borrowed from elementary utility theory. The latter was very easy to come by. Ironically enough, it sufficed properly to work out Boland's step (a) in order to turn "the neoclassical maximization hypothesis" into an unambiguous counterexample to the alleged irrefutability of "uncircumscribed" AS!

After shaking confidence in the view that existential or all-and-some statements are irrefutable, the paper aims at suggesting that it could be salvaged if properly restricted. All the counterexamples thus far known involve second-order quantification. It will be argued that if an appropriate translation rule is applied to them, the counterexamples turn out to be innocuous first-order statements of the usual, refutable type. As a result, the conventional wisdom on falsifiability is left untouched provided it is not extended to second-order scientific statements.

1. THE "MAXIMIZATION HYPOTHESIS" AS A FALSIFIABLE "UNCIRCUMSCRIBED" AS

Let us consider the static neoclassical theory of consumer choice. Among the axiom set of the latter there are the well-known requirements that preferences are complete, reflexive, and transitive. It is

2. Another standard assessment of the empirical undecidability of AS is to be found in Hempel (1950), where it is related to the issue of cognitive significance rather than the science/metaphysics demarcation criterion.
3. See Boland, 1981, p. 1034. Surprisingly enough, none of the standard pieces on existentials and AS are quoted on this page.
4. Watkins's more recent work on metaphysics (e.g., 1975) suggests that he would no longer analyze refutability solely in terms of the logical form of scientific statements. It does not appear, however, to imply a clear-cut rejection of the thesis that "uncircumscribed" AS are empirically undecidable.

difficult to make empirical sense of the reflexivity assumption. The completeness and transitivity axioms, however, can be given the form of *universal empirical laws* in a straightforward way. In order to achieve this, it is enough to take the primitive term $\gtrsim$ ("is at least as good as") as a directly observable one; for instance, we shall define as a direct empirical counterpart of $\gtrsim$ any actual choice made by any observable consumer between any given two observable bundles of goods. Now, if x, y, and z are variables that are to be interpreted as referring to observable bundles of goods, the completeness axiom:

$$(1) \qquad (\forall x)\,(\forall y)\,((x \gtrsim y) \vee (y \gtrsim x) \vee (x \gtrsim y \;\&\; y \gtrsim x))$$

and the transitivity axiom:

$$(2) \qquad (\forall x)\,(\forall y)\,(\forall z)\,((x \gtrsim y \;\&\; y \gtrsim z) \rightarrow x \gtrsim z)$$

are obviously of the universal empirical type. Note that the above interpretation completely dispenses with the variable "consumer." This means that a test of consumer theory would have to be understood as a sequence of orderings involving either two or three bundles of goods no matter whether chosen orderings refer to the same or different individuals. This is of course not satisfactory. Neoclassical consumer theorists are interested in *individual* preference sets and utility functions. We should therefore substitute a three-place (observable) predicate P(c, x, y)—"c regards x as at least as good as y"—for the two-place predicate $\gtrsim$ and rephrase (1) and (2) accordingly, quantifying on the x, y, z, *and* c, as in the following:

$$(1') \quad (\forall c)\,(\forall x)\,(\forall y)\,[P(c,x,y) \vee P(c,y,x) \vee (P(c,x,y) \;\&\; P(c,y,x))]$$

$$(2') \quad (\forall c)\,(\forall x)\,(\forall y)\,(\forall z)\,[(P(c,x,y) \;\&\; P(c,y,z)) \rightarrow P(c,x,z)].$$

We take it that if they are to qualify as an interesting scientific theory, the axioms of neoclassical economics should be understood as referring to an *infinite domain* of consumers and baskets of goods. This assumption is in agreement with Popper's discussion of "strict universality" (1934, § 15) and more generally with standard philosophical practice.

Along with the three axioms above, textbooks usually mention the assumption that preferences should be taken as continuous (in some well-defined sense). As is always the case with continuity requirements, such an axiom should be analyzed as an unfalsifiable AS.[5] But it

5. Continuity assumptions are AS *par excellence* as is visible from the definition of a continuous function: "For every open set in F (the range) there is an open set in E (the domain) such that . . . ." Also, leaving aside degenerate cases, continuity assump-

has to be recalled that continuity is *not* the reason why Boland claims to have found an AS among the axioms of neoclassical theory. He makes the very different claim that "the neoclassical maximization hypothesis" *as a whole* can in some way be constructed as an AS:

> "The neo-classical premise is not a strictly universal statement. Properly stated, the neo-classical premise is 'For all decision makers there is something they maximize' " (1981, p. 1034).

How is it possible to make sense of this hint within the framework of the above four-axiom theory?

"Something" in Boland's phrase should of course be construed as referring to the agent's objective function. Now, there is a sense in which the four-axiom theory can be shown to collapse into an "uncircumscribed" AS of the type mentioned by Boland. The axioms taken in conjunction with the well-known mathematical theorem that a well-behaved preference ordering can be represented by a continuous function logically imply the following statement (S):

> "For every consumer there exists a continuous function V that represents his preferences."

If the preferences-representing (more concisely, utility) function is understood as being rather a (precisely defined) *family* of functions, (S) is not only a consequence of, but also an equivalent statement to, the four axiom-theory (Debreu, 1959, pp. 55–59).

Now, it is a classic (though not universally accepted) requirement of philosophy of science that the falsifiability of a scientific theory is invariant with changes in its logical form, i.e., two distinct, but logically equivalent statements of the same theory should have the same potential falsifiers. By virtue of this "equivalence requirement" (Hempel, 1945), the unrestricted all-and-some statement (S) in the last paragraph should be regarded as a falsifiable one. It is indeed equivalent to a conjunction of axioms two of which, the completeness and the transitivity axioms, have the form of universal empirical—i.e., on the commonly received view, falsifiable—laws. There is, of course, nothing novel in that. It is well known to psychologists and empirical decision theorists that the standard neoclassical theory of rational choice can be

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tions are AS with infinite domain, thus qualifying for Watkins's irrefutability claim. They are indeed not falsifiable, as it is well known that any finite number of observations cannot disconfirm a continuity assumption. The same would apply to differentiability assumptions as well as any statements involving limits. A useful discussion of the way in which the empirical sciences deal with such statements is to be found in Simon, 1985.

tested on some of its axioms. Some would even claim that it *has* already been falsified along those lines—the completeness and transitivity axioms being notoriously difficult to meet in some empirical cases.

Worse than that, testing statement (S) on its equivalent axiomatic form is a convenient, but by no means necessary procedure if one is to attempt to falsify it. As a potential falsifier of transitivity, take any "cyclic" sequence of three choices in which at least one of the choices involves the strong, rather than weak, preference-ordering relation. If α, β, γ are constant symbols that refer to given observable bundles of goods, the statement:

$$P(c, \alpha, \beta) \,\&\, \sim P(c, \beta, \alpha) \,\&\, P(c, \beta, \gamma) \,\&\, P(c, \gamma, \alpha) \tag{3}$$

is a potential falsifier of the transitivity axiom. It is easily seen as a *direct* potential falsifier of (S) as well. For statement (3) taken in conjunction with (S) would imply the following self-contradictory system of inequalities:

$$\begin{cases} V(\alpha) > V(\beta) \\ V(\beta) \geqq V(\gamma) \\ V(\gamma) \geqq V(\alpha). \end{cases}$$

It was worth emphasizing that there are direct falsifiers of the "uncircumscribed" AS (S): the counterexample just set out can be made completely independent of Hempel's "equivalence requirement."

The refutable AS that has just been discussed is by no means a *curiosum*. Economic theory involves further counterexamples, at least one of which should be mentioned here—the celebrated "expected-utility theorem." It is usually given the simple existential form: "There exists a von Neuman–Morgenstern utility function" (cf. Savage, 1972, theorem 4, p. 75), but is more accurately rephrased as the following AS:

> (EUT) "For every individual c, there exists a continuous utility function W that represents his preferences over the set of lotteries and has the von Neuman-Morgenstern property."

Falsifiability of the expected-utility theorem is very well documented; many writers would even claim that it is actually falsified. Thus, Watkins's thesis is again in trouble.

Note that (EUT) is *uncircumscribed* in the sense adhered to in this paper and therefore qualifies as a valid counterexample. Still, there is a way out in the case of (EUT) that is not available in the case of (S). Contrary to the latter, the former involves a refutable restriction (i.e., the so-called von Neuman-Morgenstern property) which makes it too easy a game to find out potential or actual falsifiers. If Watkins's irrefutability claim is to have at least some *prima facie* plausibility, it should be understood as excluding such functional restrictions as are exhibited in (EUT). This requires that the thesis under attack be rephrased: *unrestricted* (rather than only uncircumscribed) AS are irrefutable. From his 1957 and 1958 papers, it is clear that Watkins was mainly concerned with excluding AS the existential consequences of which are restricted either spatio-temporally or numerically (for metal c, there is a solvent to be found in the next room, a melting point between 80° C and 80.5° C, etc.).[6] He did not consider, although he presumably meant to dispose of, the more complex case of functional restrictions.

Even in this generously enlarged version, the thesis is at a loss, for the static consumer theory (which is also implied by (EUT)) does not involve anything such as a testable functional restriction (continuity does not qualify for it). In the simple, but illuminating case of (S), it becomes obvious that *even generally unrestricted* existential predictions could sometimes be empirically defeated.[7]

2. THE MORAL OF THE COUNTEREXAMPLES

Two attempts will be made here to explain what was faulty in Watkins's thesis. The first proposal to be discussed is due to H. Simon (1985). In view of various scientific examples Simon has suggested that a distinction should be drawn between existential sentences that have their existential quantifier running on a *theoretical* term (henceforth QTT) and sentences where the existentially quantified variable is an *observable* term. In the absence of any restriction of a spatio-temporal or other sort, the latter are definitely not falsifiable, as is visible from the example: "There is a unicorn." The former may be falsifiable, however, as is apparent from Simon's following example:

6. Popper apparently had the same class of restrictions in mind when he wrote (in a rather doubtful way) on Watkins's thesis: see Popper, 1934–1972, p. 193, note 2, and also Popper, 1974, II, pp. 993, 1039.
7. Before attempting to rationalize the above counterexamples, there is a final point to be stressed regarding Boland's phrase "the neoclassical maximization hypothesis." There are clearly a variety of ways in which the latter sentence can be related to the formalism of neoclassical theory, (S) and (EUT) being only prominent examples. It might perhaps be asked whether alternative translations would not lead to "well behaved," i.e., nonfalsifiable AS. There are strong intuitive reasons for believing that this cannot happen. Maximization is deeply connected with transitivity (maximizing being meaningless in the case of cyclic preferences), and transitivity surely has an empirical import on *any* reasonable interpretation of neoclassical economics as an empirical science.

$$(4) \qquad (\exists\, k)\, (\forall\, i)\, (y_i = k\, x_i).$$

Here, the "theory" described by (4) simply asserts that there is a linear relationship to be found between two *observable* variables, y and x; i is the index number of observations; k is taken as a *theoretical* term. Statement (4) is obviously falsifiable.

Note carefully that Simon's distinction does not lead to a sufficient condition for falsifiability of unrestricted existentials or AS. That it is only a *necessary* one can be seen from the following, trivial example. Let us turn statement (4) into an AS by interchanging quantifiers:

$$(5) \qquad (\forall\, i)\, (\exists\, k)\, (y_i = k\, x_i).$$

As an example of a scientific theory, (5) would be rather absurd; however, it is enough to show that allowing existential quantifications on theoretical variables only does not insure falsifiability per se.

Simon's demarcation line is plainly relevant to the issue discussed in this paper. On the one hand, all the standard examples of irrefutable AS either are of the empirical type (e.g., "For every metal there exists some acid that will dissolve it") or can at least be construed as such (e.g., atomism and other "haunted universe doctrines" in Watkins's 1957 and 1958 articles). Thus, Simon helps us understand the kind of inductive fallacy that lies beyond the claim that *all* "uncircumscribed" (or even possibly all "unrestricted") AS are irrefutable. On the other hand, the counterexamples just set out in this article neatly fall on the right side of the demarcation line. (S) and (EUT) have their existential quantifier running on variables V and W which are clearly theoretical. Simon's necessary condition rationalizes falsifiability of (S) and (EUT) in the somewhat weak, but nonetheless informative, sense of being logically compatible with it.

As an alternative rationalization sketch, the difference between first- and second-order calculi should be stressed. It has not escaped the careful reader of section 1 that (S) and (EUT) have the syntactical form of AS only if quantification over predicates and functions is allowed. This is really the clue to the translation of the axiomatic form of utility theory into the more compact form of (S) or (EUT). Recognition of this fact does not diminish the value of utility theory as a counterexample to Watkins's irrefutability claim, for, again, it was an unguarded one. However, it now remains to be seen *whether or not a suitable restriction of the irrefutability claim to first-order calculi could salvage it after all.*

To answer this question fully would involve developing a model theoretic framework, which is beyond the scope of this article. Still, some progress can be made at an intermediate level of analysis by carefully discussing examples. To start with, take Simon's "some-and-all" statement (4). The appearances notwithstanding, this is a formula in the *second*-order calculus. A first-order calculus involves individual variable symbols as well as constant symbols, but only the former could have a quantifier running on them. Thus, if k is to refer to a constant in (4), the latter cannot belong to the first-order calculus. Once it is realized that (4) is really second-order, a translation into first order might be looked for. In this particular case, it is enough to drop the second-order quantification and introduce a constant symbol K in the first-order vocabulary in order to get the desired translation:

$$(\forall i)\ y_i = K\,x_i. \tag{4'}$$

We are left with a first-order universal empirical sentence. Now, if we stick to the usual methodological rule of regarding such sentences as refutable, the paradox of a "refutable existential" in (4) has simply vanished. To resolve it, there was no need to use the fact that k is a theoretical term; only simple-minded translation into first-order logic was required.

It may be asked what logical concept of "translation" is being used here. Suppose we are given a formula F involving second-order existential quantification:

$$(\exists\, \Pi_1)\,(\exists\, \Pi_2)\ldots(\exists\, \Pi_k)\ T\,(\Pi_1, \Pi_2, \ldots, \Pi_k)$$

where $\Pi_1, \Pi_2, \ldots, \Pi_k$ are variables referring to predicates and functions. Call L the language of F and λ the set of predicate and function symbols belonging to L. A first-order translation of F will be defined to be the formula F′:

$$T(P_1, P_2, \ldots, P_k)$$

obtained from F by canceling second-order quantification and replacing each bound occurrence Π_i, $i = 1 \ldots k$, in F by a new predicate or symbol function P_i. F′ is a formula of a new language L′ which is first order, and has therefore fewer variables (but more predicate and function symbols) than L. Let us call $\mu = \{ P_1, \ldots, P_k \}$ the added set of predicate and function symbols. To show the correctness of the translation concept just defined, it is enough to mention a simple result proved in 1967 by Bohnert in a different context (cf. Tuomela, 1973, pp. 57–58).

Proposition (Bohnert). If φ is a formula of L, $F \vdash \varphi$ if and only if $F' \vdash \varphi$. ("$\vdash$" stands for provability in second-order logic.)

Bohnert's proposition insures that applying the above-defined translation concept to a second-order existential F will not alter its consequence set. Thus, if we manage to show that F′ has refutable consequences, we can safely conclude that F is also refutable, as we did on intuitive grounds in the case of (4) and (4′).

There is no difficulty in extending this translation concept to the more complex case of AS of the form:

$$(\forall\, c)\ (\exists\, \Pi_1)\ (\exists\, \Pi_2) \ldots (\exists\, \Pi_k)\ T\ (\Pi_1, \Pi_2, \ldots, \Pi_k)$$

where c is an individual variable and $\Pi_1, \ldots, \Pi_k$ are as before. For instance, statement (S) in section 1 implies the second-order AS:

$$(6) \qquad (\forall\, c)\ (\exists\, V)\ (\forall\, x)\ (\forall\, y)\ [V(x) \geqq V(y) \leftrightarrow P(c, x, y)]$$

which has the following translation:

$$(6') \qquad (\forall\, c)\ (\forall\, x)\ (\forall\, y)\ [U(c, x) \geqq U(c, y) \leftrightarrow P(c, x, y)].$$

Here the added vocabulary U consists of the symbol U(•, •), which is to be interpreted as a two-place function varying over consumers and baskets of goods. Again, checking the correctness of the extended translation concept would make it possible safely to conclude that if (6′) is falsifiable, (6) is as well. Note that to claim falsifiability for (6′) requires a definitely stronger methodological decision than that required for (4′).[8]

Generally speaking, the availability of a well-defined and safe translation for certain classes of second-order existentials and AS means that the methodological decisions relative to first-order sentences will automatically reflect in the former. Hence, examples such as those discussed in this paper act as *test cases* for the standard wisdom on falsifiability. For instance, the tenet that universal empirical sentences are falsifiable would have to be dramatically revised if it should lead to the false prediction that some of the AS discussed here are not refutable. Fortunately, such conflict is not in view. All throughout this paper, consistency can be preserved between the recognition of (S), (EUT), or (4) being falsifiable and adherence to the usual tenets of falsifiability as applied to first-order scientific theories.

The construal above makes no use of the theoretical/observable distinction, but it is very easy now to take it into account. In standard

8. We need more than the usual rule that universal empirical sentences should be regarded as falsifiable, for (6′) is only *partly* empirical (it involves a theoretical predicate U[•, •] as well as the observable predicate P[•, •, •]). (6′) is an example of what is sometimes called a "correspondence rule" in the literature.

philosophical practice, a theoretical term is analyzed as being either a predicate or a function.[9] Therefore, quantification over theoretical terms is *ipso facto* second order, and the translation process just defined applies to Simon's QTT-sentences as a particular case. An even more specific case is when not only are all the Π_i theoretical predicate or function variables, but there is no theoretical predicate or function symbol apart from the Π_i in $T(\Pi_1, \Pi_2, \ldots, \Pi_k)$. Then, translation into first-order language amounts to reversing the well-known procedure of Ramsey elimination.[10] As far as observable terms are concerned, they are either individual or predicate/function symbols. When quantifiers run on the former only, as in "There exists a unicorn," it is enough to apply standard methodological rules. Quantification over *observable* predicate or function symbols is perhaps an abstract possibility only; at least, no simple example is forthcoming. Still, should such a case arise in the empirical science, the above translation procedure would again be applicable.

CONCLUSION

Boland's claim is to count as one among numerous failed attempts to demonstrate that neoclassical economics has no empirical import—a conclusion that many economists will no doubt welcome. More importantly, the above investigation has led to suggest a way of accommodating the counterexample of utility theory within a refined logical framework for falsificationism. Such a happy ending will probably strengthen the confidence of some (and the present writer among them) in the feasibility of implementing Popper's demarcation criterion into properly defined logical distinctions.

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9. In other words, there is no such thing as a *theoretical individual*. This no doubt debatable assumption is implicit in most of the model theoretic work made on theoretical terms (see for instance Tuomela, 1973, chap. I, or Groen and Simon, 1973).

10. Note that Bohnert's proposition was proved in view of giving a logical foundation to Ramsey's procedure. In this context, it amounts to the familiar fact that, given a (first-order) theory T and its Ramsey sentence T^R, T and T^R have the same first-order empirical consequences.

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