Testing the Monotonicity Property of Option Prices

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Many option pricing models imply that the price of a call option is a monotonically increasing function of the value of its underlying asset, and the price of a put option is a monotonically decreasing function of the value of its underlying asset. This property is known as the monotonicity property. In this article, the author tests the empirical validity of the monotonicity property using all transaction prices in 2002 for five option contracts written on the European, British, French, German, and Swiss stock indices. The author finds that sampled intra-day option prices violate the monotonicity property 6–35% of the time. The author uncovers evidence that the frequent violations of the monotonicity property are attributable, in large part, to microstructure effects and arise from rational trading tactics.

> Financial science too can boast about very high explanatory power if the phenomenon is artfully selected. For instance, the daily change in an option's price can be accurately "predicted" by the concurrent change in the associated stock price.[1]

Empirical tests of option pricing models usually consist of contrasting theoretical and market prices of options. An alternative line of attack is to test properties that should hold for all models based on a given stochastic process for the underlying asset (see Aït-Sahalia [2002], Carr and Wu [2003]). Following this approach, we test the empirical validity of the monotonicity property, or homogeneity property, for option prices. This property states that the price of a call option is a monotonically increasing function of the value of its underlying asset, and that the price of a put option is a monotonically decreasing function of the value of its underlying asset.[2] It is shared by all option pricing models assuming that the underlying asset price follows a one-dimensional diffusion process, such as the models of Black and Scholes [1973], Merton [1973], Cox and Ross [1976], Derman and Kani [1994], and Rubinstein [1994], as well as in certain restricted stochastic volatility settings (see Bergman, Grundy, and Wiener [1996]).

Using a six-month sample of S&P 500 options quotes, Bakshi, Cao, and Chen [2000] show that the sign of option price changes differs quite often from that implied by the monotonicity property. That is, call option prices move in the opposite direction of the underlying asset 7–16% of the time, and put option prices and index value move in the same direction 5–16% of the time, depending on the intra-day sampling interval.[3] They interpret these results as evidence that option prices are not generated by a univariate diffusion model. Thus, they argue that option pricing models should allow more than one state variable to evolve stochastically. On the contrary, Dennis and Mayhew [2005] claim that, when option prices are observed with noise, a significant portion of the reported violations may be caused by microstructural biases, which are due, for instance, to bid-ask spreads or tick sizes.

In this article, we endeavor to test empirically the validity of the two main arguments (i.e., concurrent change in volatility and microstructural bias) explaining the frequent deviations from the monotonicity property. Furthermore, we investigate whether the violations of the monotonicity property arise from rational trading tactics followed by option traders. To provide a comprehensive analysis of this issue, we study all transaction prices in 2002 for five option contracts written on the European, French, German, Swiss, and British stock indices. These options are traded on different exchanges, under different trading mechanisms, and vary greatly in terms of trading activity and liquidity.

Since the monotonicity property is defined as a partial derivative, it cannot directly be tested using a time series of option prices and underlying asset values. For this reason, we define an empirically testable version of the monotonicity property stating that the price of a call option and the value of its underlying asset move in the same direction, and that the price of a put option and the value of its underlying asset move in opposite directions.

Our testing procedure of the monotonicity property differs from the one in Bakshi, Cao, and Chen [2000] in several respects. We conduct our empirical analysis using observed transaction prices instead of average bid-ask quotes. While more cumbersome to deal with, transaction prices allow us to control explicitly for the impact of microstructure effects on our conclusions. Following Bakshi, Cao, and Chen [2000], Fahlenbrach and Sandas [2005] study co-movements in the index options and futures quotes on the British derivatives market. They show that the violations of the monotonicity property are much more frequent around trades than during periods without any trades. Furthermore, our international dataset allows us to test the empirical validity of the monotonicity property across derivatives markets.

Our empirical analysis leads to the following conclusions. We show that, depending on the sampling interval (from tick-by-tick price changes to daily price changes) and the option contract considered, call option prices move in the opposite direction of the underlying asset 7–32% of the time. Similarly, the associated violation rates for put option prices range from 6–35%. Overall, the occurrence rates generally decrease with the length of the time interval considered and with the trading activity of the option contract. These empirical results may be helpful in determining the optimal rebalancing frequency of hedging strategies in option markets.

We then investigate the causes of the frequent violations of the monotonicity property. Our findings suggest that part of the violations are due to concurrent changes in volatility. We also consider the microstructure aspects of the markets that are generating the prices. Controlling explicitly for the direction of option trades (i.e., seller-initiated or buyer-initiated), we uncover evidence that a substantial portion of the reported violations of the monotonicity property are attributable to the bid-ask bounce. Furthermore, we show that violations of the monotonicity property can result from rational trading tactics followed by traders in a market with relatively limited liquidity.

The remainder of the article proceeds as follows. The next section describes the testing procedure, the following section presents the dataset and empirical results, and the last section offers a summary and concluding comments.

EMPIRICAL MONOTONICITY PROPERTY: DEFINITIONS AND VIOLATIONS

The monotonicity property asserts that the price of a call option C is a monotonically increasing function of the value of its underlying asset S and that the price of a put option P is a monotonically decreasing function of the value of its underlying asset. That is, the partial derivative of the call option price with respect to the value of the underlying asset is non-negative, $C_S \geq 0$, and the partial derivative of the put option price with respect to the value of the underlying asset is non-positive, $P_S \leq 0$. Since this property is defined as a partial derivative, it cannot directly be tested using a time series of option prices and underlying asset values. Therefore, we define the following empirically testable version of the monotonicity property for call and put options:

Definition 1 *For a given call option and its underlying asset, the empirical monotonicity property is satisfied over a given time interval if the change in the call option price, ΔC, and the change in the underlying asset value, ΔS, have the same sign.*

Definition 2 *For a given put option and its underlying asset, the empirical monotonicity property is satisfied over a given* *time interval if the change in the put option price, ΔP, and the change in the underlying asset value, ΔS, have opposite signs.*

An equivalent formulation is that, over a given time interval, the price of a call option and the value of its underlying asset move in the same direction, and that the price of a put option and the value of its underlying asset move in opposite directions. Thus, if the empirical monotonicity property (EMP) is satisfied between any two points in time, the empirical delta of a call option, $\Delta C/\Delta S$, is always non-negative, and the empirical delta of a put option, $\Delta P/\Delta S$, is always non-positive.

Intuitively, the EMP makes a lot of sense. Indeed, as option and underlying asset prices are affected by the same news at the same time, they should conform to this property. Note that this property should also hold when option returns lead cash returns as reported by Manaster and Rendleman [1982] and Bhattacharya [1987], or when cash returns lead option returns as claimed by Stephan and Whaley [1990]. This is true as long as the sampling interval used to measure the price changes is longer than the estimated lead.

Formally, if the option is assumed to be a function of the underlying asset price and time, we have by Ito's lemma:

$$dC = C_t dt + C_S dS + \frac{1}{2} C_{SS}(dS)^2 \qquad (1)$$

$$dP = P_t dt + P_S dS + \frac{1}{2} P_{SS}(dS)^2 \qquad (2)$$

When, in addition, S is assumed to depend on a single standard Brownian motion $W(t)$, so that

$$\frac{dS(t)}{S(t)} = \alpha(S_t, t)dt + \sigma(S_t, t)dW(t) \qquad (3)$$

then $(dS)^2 = \sigma^2 S^2 dt$ in equations (1) and (2). Since under rather general conditions $C_S \geq 0$ (see Bergman, Grundy, and Wiener [1996]), the EMP is respected if $(C_t + \frac{1}{2} C_{SS} \sigma^2 S^2)dt$ is negligible. Analogously, the property is valid for a put option as long as $(P_t + \frac{1}{2} P_{SS} \sigma^2 S^2)dt$ is negligible. Although the theory does not claim that the EMP should be satisfied for any arbitrary time interval, this property is likely to hold over small time intervals.

Using a dataset of option and underlying asset prices, it is relatively simple to assess the validity of the EMP by counting the number of occurrences when ΔC and ΔS do not have the same sign or when ΔP and ΔS do have the same sign. Specifically, two types of violations can be derived for call options and two others can be derived for put options.

Violations for Call Options
Type I : $\Delta S < 0, \Delta C > 0$
Type II : $\Delta S > 0, \Delta C < 0$

Violations for Put Options
Type I : $\Delta S > 0, \Delta P > 0$
Type II : $\Delta S < 0, \Delta P < 0$

Both violations I and II imply that the empirical delta of a given option does not display the expected sign.[4] We study the signs of contemporaneous option price and index value changes instead of the sign of their product. This separate analysis permits us to compare the magnitude and the frequency of the deviations from the EMP in an up or a down movement in the underlying asset value. This partition turns out to be central to studying the implications of our results for hedging practices. Indeed, a delta-hedged position that combines a long position in one call option with an appropriate short position in the underlying asset, generates a gain on *both* the derivatives position and underlying position when a type-I violation occurs ($\Delta S < 0$, $\Delta C > 0$). In contrast, this hedged position faces a double loss when a type-II violation arises ($\Delta S > 0$, $\Delta C < 0$). Analogously, a short-call delta-hedged position suffers from a type-I violation, but benefits from a type-II violation.

EMPIRICAL ANALYSIS

Data

We analyze the price dynamics of five index options written on the European (DJ EURO STOXX-50), French (CAC 40), German (DAX), Swiss (SMI), and British (FTSE 100) stock indices.[5] These contracts are European-style options and are traded on three leading derivatives exchanges, namely, EUREX, EURONEXT, and LIFFE.[6] For each of the five contracts, we get all transaction prices in 2002, which represents a total of almost 1.4 million observations of call and put option prices and more than 173 million contracts traded. The sources for the option data are the Deutsche Börse Group for the options on the

EXHIBIT 1

Option Contracts and Underlying Assets

This Exhibit presents, for each option contract, the underlying stock index, the option code, the number of different option series (calls and puts), the number of trades on options, the number of contracts traded, the minimum option-price change (in points and in value), and the number of observations on the stock index. For all contracts, the sample covers the period 01/01/02–12/31/02. Option data are tick data. Cash data are available with a one-minute frequency, except for the DAX index which is available with a 15-second frequency. Sources for option data: Deutsche Börse Group for ODAX, OSMI, OESX; EURONEXT for PXL; and LIFFE for ESX. Sources for cash data: Olsen Data for DJ EURO STOXX 50, CAC 40, SMI, FTSE 100; and Karlsruhe University for DAX.

	Europe	France	Germany	Switzerland	U.K.
Underlying	DJ EURO STOXX 50	CAC 40	DAX	SMI	FTSE 100
Option Ticker	OESX	PXL	ODAX	OSMI	ESX
Option Exchange	EUREX	EURONEXT	EUREX	EUREX	LIFFE
# Series	1,355	1,181	1,784	1,335	1,317
# Transactions	221,759	130,226	878,744	103,278	63,131
# Contracts Traded	39,560,190	83,708,170	44,048,879	4,238,734	1,897,735
Tick Size (Value)	0.1 (EUR 1)	0.1 (EUR 0.1)	0.1 (EUR 0.5)	0.1 (CHF 1)	0.5 (£ 5)
# Index Observations	165,980	130,259	681,883	126,091	127,587

European, German and Swiss indices; EURONEXT for the options on the French index; and LIFFE for options on the British index. Additional information on the index options, such as tick size, number of series, and ticker symbols, is presented in Exhibit 1. Our dataset also includes the intra-day value of the underlying stock indices observed every 15 seconds for the DAX and every 60 seconds for other indices, with a total number of observations exceeding 1.2 million. Intra-day cash indices have been obtained from Olsen data, except for the DAX data that have been provided by the Karlsruhe University, Germany.

EUREX, EURONEXT, and LIFFE all offer fully electronic trading platforms. There are several differences in the market models used on these three exchanges that are worthwhile mentioning. On EUREX, there are market makers obliged to promptly supply bid and ask quotes for any options. Furthermore, all orders entered into the EUREX system are assigned a time stamp that is used to prioritize orders with the same price. Market orders have the highest priority for matching. In the case of limit orders, orders with the best possible prices (i.e., highest price limit for buy orders and lowest price limit for sell orders) take precedence in the matching process. If the limit orders have the same price limit, the extra criterion used for establishing matching priority is the order time stamp. When a given order is not completely executed, the unexecuted part of the order remains in the order book until it is totally executed.[7] On EURONEXT, there are designated market makers for actively traded derivatives, including the CAC 40 index options. Consistent with the matching rule on EUREX, the central order book applies a price-time trading algorithm. In contrast, there are no designated market makers on LIFFE with special quoting obligations in the FTSE 100 index option market.[8] Traders submit orders electronically to a central order book in which orders are prioritized for execution on the basis of price. Orders at the same price are filled in a pro rata fashion according to order size.

A notable feature of our dataset is the use of transaction prices instead of bid-ask midpoint prices. Transaction prices are less subject to bid-ask spread manipulations than bid-ask midpoint prices. Indeed, spread manipulations by market makers can potentially create average-price-based violations even in absence of any trading activity. Moreover, transaction prices can be used to measure actual violations, while arbitrarily sampled bid-ask midpoints only allow the measurement of potential violations. In contrast, the bid-ask bounce can critically affect our conclusions and, thus, will be properly accounted for as will be shown later in the article.

We apply four exclusion filters to the original option data. First, options with less than six days to expiration are omitted to alleviate expiration-related bias. Second, we eliminate options with a Black-Scholes implied volatility greater than 100%.[9] Third, options with a transaction price strictly below two tick sizes are not used in our analysis in order to mitigate the impact of price-discreteness-related bias. Fourth, option series with less than ten trades during a given day are dropped from the day's sample. For all option contracts, transaction times are in year-month-day-hour-minute-second format. Once each transaction time has been converted into seconds, we match every option price with the index value observed at the closest point in time. Since the DAX index is observed every 15 seconds, the present matching process limits the time discrepancy between option prices and index values to 7 seconds. For the same reason, the maximum time discrepancy cannot exceed 30 seconds for the other four contracts, which limits the impact of imperfect synchronization between option prices and index values.

We divide the option data into several categories according to either moneyness or time-to-expiration. A call option is said to be *in-the-money* (ITM) if $S/X \geq 1.03$; *at-the-money* (ATM) if $S/X \in (0.97, 1.03)$; and *out-of-the-money* (OTM) if $S/X \leq 0.97$. A similar partition is obtained for put options by replacing S/X with X/S. Moreover, an option contract is classified as *short-term* if its time-to-expiration $\tau \leq 30$ calendar days to maturity; *medium-term* if $\tau \in (30, 60)$; and *long-term* if $\tau \geq 60$. Exhibit 2 describes certain sample properties of the option data used in this study. Summary statistics are reported for the average transaction price and the number of observations for each moneyness–maturity category. Not surprisingly, we note that short-term options tend to be more actively traded than medium-term and long-term options, and for each contract, OTM options have the highest trading volume.

Price Change Measurement

Our testing procedure requires the calculation of contemporaneous price changes for the options and the underlying assets. Since we use transaction prices, which are unequally spaced, we compute for each option contract and its underlying asset both 1) tick-by-tick price changes and 2) price changes over a fixed sampling interval. For tick-by-tick price changes, the time interval between two trades, $\widetilde{\Delta t}$, is random. For fixed sampling interval price changes, the time between two trades, Δt, is enforced to be 30 minutes, 1 hour, 2 hours, 3 hours, and 1 day.[10] The procedure yielding price changes for both random and fixed Δt is illustrated in Exhibit 3 and proceeds as follows. For ease of presentation, we consider five trades, denoted trades A, B, C, D, and E, that take place at times t_A, t_B, t_C, t_D, and t_E, respectively. For this sample, four tick-by-tick price changes can straightforwardly be computed (between t_A and t_B, t_B and t_C, t_C and t_D, and t_D and t_E) but only two Δt -price changes can be obtained (between t_A and t_C, and t_B and t_D). A Δt-price change is obtained by starting from a given trade, say trade A, adding Δt to t_A, and considering a $\Delta t/2$ window around $t_A + \Delta t$. The fixed-interval price change is computed between the price at time t_A and the price at the closest trade to $t_A + \Delta t$, with the constraint to remain within the $\Delta t/2$ window.[11] This procedure guarantees that Δt-price changes are computed from trades spaced about Δt apart. For instance, it guarantees that 30-minute price changes are only computed from prices spaced by at least 22.5 minutes but not more than 37.5 minutes. The total number of option price changes resulting from this procedure is 3,187,059 for the five option contracts, among which 752,360 are tick-by-tick price changes.

Violation Rates

Exhibit 4 reports the occurrence frequency of violations I and II for each of the five option contracts. Our main empirical findings are as follows. First, the violation rates for tick-by-tick price changes are substantial, call option prices go up and index value goes down between 6.1% for DAX options and 13.9% for CAC options of the time (Type I: $\Delta S < 0$, $\Delta C > 0$). Similarly, call option prices go down and index value goes up between 6.4% for DAX options and 14.2% from CAC options of the time (Type II: $\Delta S > 0$, $\Delta C < 0$). Thus, at the tick-by-tick frequency, call option prices move in the opposite direction of the underlying asset 12.5–28.1% of the time. Second, type-I violations are more frequent than type-II violations, which means that EMP-violating option price drops are more frequent than EMP-violating option price rises. Third, while for fixed time intervals both occurrence rates are globally decreasing with the length of the time interval considered, the drop is much more pronounced for type-I violations. In particular, for one-day price changes, type-I violation rates are typically below 3%, but type-II violation rates remain in the 6–10% range. Fourth, the most actively traded contract in our sample—DAX options exhibits lower violation rates. Finally, when put option prices are used in place of call option prices, a similar picture emerges for all five contracts.

EXHIBIT 2

Summary Statistics for the Five Option Contracts

Each panel reports, for a given option contract, average transaction prices and number of observations for the following categories of call options: short-term options (time to expiration less than 30 days), medium-term options (time to expiration between 30 and 60 days), long-term options (time to expiration less than 60 days), out-of-the-money options (moneyness less than 0.97), at-the-money options (moneyness between 0.97 and 1.03), and in-the-money options (moneyness greater than 0.97). Figures for puts are indicated in parentheses. We omit options with less than six days to expiration, options with an implied volatility greater than 100%, and options with quoted price strictly below two tick sizes. Option contracts with less than ten trades during a given day are dropped from the day's sample.

Panel A: Options on the European Stock Index (DJ EURO STOXX 50)

	Time to Maturity			
Moneyness	**SHORT**	**MEDIUM**	**LONG**	**All Maturities**
OTM	26.5; 13,061	42.8; 9.194	76.4; 16,503	51.6; 38,758
	(28.0; 15,756)	(49.3; 10,975)	(93.2; 17,147)	(58.8; 43,878)
ATM	71.5; 8,750	112.7; 2,949	206.9; 3,801	112.5; 15,500
	(75.5; 9,482)	(124.3; 3,955)	(199.1; 4,304)	(116.4; 17,741)
ITM	230.4; 547	282.7; 217	423.7; 400	306.6; 1,164
	(217.0; 1,143)	(277.1; 742)	(1,148.7; 1,948)	(702.2; 3,833)
All Moneyness	49.1; 22,358	63.7; 12,360	107.1; 20,704	74.0; 55,422
	(53.3; 26,381)	(79.0; 15,672)	(200.6; 23,399)	(112.1; 65,452)

Panel B: Options on the French Stock Index (CAC 40)

	Time to Maturity			
Moneyness	**SHORT**	**MEDIUM**	**LONG**	**All Maturities**
OTM	32.5; 11,855	55.6; 3,218	84.9; 2,148	43.4; 17,221
	(34.8; 12,184)	(52.1; 3,375)	(106.7; 2,520)	(48.1; 18,079)
ATM	80.9; 4,475	139.8; 761	232.8; 283	96.8; 5,519
	(90.2; 4,829)	(142.6; 685)	(249.2; 402)	(107.0; 5,916)
ITM	238.6; 141	278.4; 17	0.0; 0	242.9; 158
	(222.7; 322)	(668.9; 25)	(371.4; 178)	(294.4; 525)
All Moneyness	47.4; 16,471	72.6; 3,996	102.2; 2,431	57.6; 22,898
	(53.7; 17,335)	(71.1; 4,085)	(140.4; 3,100)	(67.6; 24,520)

Panel C: Options on the German Stock Index (DAX)

	Time to Maturity			
Moneyness	**SHORT**	**MEDIUM**	**LONG**	**All Maturities**
OTM	38.1; 122,254	61.2; 65,088	100.7; 55,208	58.6; 242,550
	(43.5; 107,638)	(67.6; 60,636)	(118.9; 38,628)	(64.6; 206,902)
ATM	112.5; 44,618	176.0; 13,723	284.3; 5,258	140.5; 63,609
	(115.2; 53,253)	(172.6; 17,282)	(258.8; 9,996)	(145.4; 80,531)
ITM	426.4; 6,820	856.5; 2,639	940.7; 1,849	610.9; 11,308
	(401.9; 18,521)	(441.6; 6,593)	(767.5; 6,374)	(484.2; 31,488)
All Moneyness	72.5; 173,692	106.3; 81,450	141.2; 62,325	94.6; 317,467
	(101.8; 179,412)	(118.3; 84,511)	(219.5; 54,998)	(126.4; 318,921)

EXHIBIT 2 *(Continued...)*

Panel D: Options on the Swiss Stock Index (SMI)

	Time to Maturity			
Moneyness	**SHORT**	**MEDIUM**	**LONG**	**All Maturities**
OTM	46.4; 3,692	53.9; 2,084	98.2; 2,483	63.9; 8,259
	(50.8; 4,511)	(82.7; 1,778)	(141.9; 2,362)	(82.2; 8,651)
ATM	84.8; 5,989	139.6; 1,783	282.1; 716	112.9; 8,488
	(112.6; 6,870)	(164.1; 2,028)	(327.3; 1,394)	(151.8; 10,292)
ITM	272.6; 100	351.2; 120	754.6; 81	433.6; 301
	(501.8; 690)	(415.8; 156)	(746.8; 544)	(588.0; 1,390)
All Moneyness	72.5; 9,781	101.1; 3,987	154.5; 3,280	94.8; 17,048
	(111.7; 12,071)	(137.5; 3,962)	(278.5; 4,300)	(152.0; 20,333)

Panel E: Options on the British Stock Index (FTSE 100)

	Time to Maturity			
Moneyness	**SHORT**	**MEDIUM**	**LONG**	**All Maturities**
OTM	50.9; 4,506	75.7; 1,642	145.5; 768	67.3; 6,916
	(57.7; 3,378)	(76.9; 1,571)	(161.7; 676)	(75.6; 5,625)
ATM	146.4; 4,361	217.5; 977	382.1; 80	162.7; 5,418
	(166.0; 3,788)	(235.0; 935)	(452.5; 294)	(195.6; 5,017)
ITM	545.1; 67	808.8; 23	918.3; 52	724.5; 142
	(716.9; 272)	(916.4; 192)	(954.4; 88)	(824.1; 552)
All Moneyness	101.2; 8,934	134.5; 2,642	211.2; 900	116.2; 12,476
	(137.0; 7,438)	(191.4; 2,698)	(308.4; 1,058)	(166.3; 11,194)

The sum of type-I and type-II violations gives the frequency with which the empirical delta of a given option ($\Delta C/\Delta S$ or $\Delta P/\Delta S$) does not display the expected sign. This simple transformation allows us to compare our results for fixed sampling intervals with the ones obtained by Bakshi, Cao, and Chen [2000] using S&P 500 options (see their Table 3, Column "Type I"). We see in Exhibit 4 that the violation rates obtained for DAX and FTSE options are smaller than the ones reported for S&P 500 options. However, the EMP is more strongly violated by options written on the European, French, and Swiss stock indices.

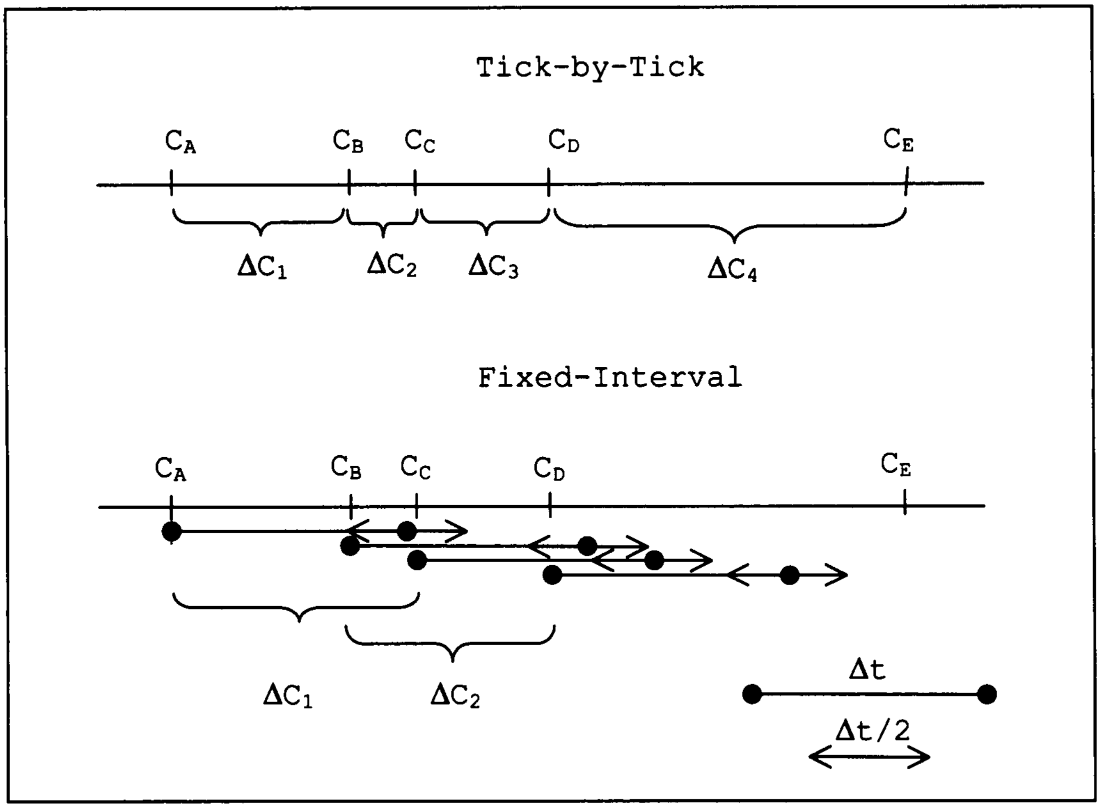
Exhibit 5 allows us to take a more detailed look at the violation rates across moneyness and maturities. The exhibit successively reports occurrence frequencies for OTM, ATM, ITM, short-term, medium-term, and long-term options. In the interest of brevity, we only present the results for tick-by-tick price changes. We find comparable occurrence rates across option categories, which indicates that deviations from the EMP are not limited to a special class of options, but seem to be a market-wide phenomenon.

To get an idea of the magnitude of a typical price change in a violation, we report in Exhibit 6 the average absolute option price change (expressed as a percentage of option prices) when a violation occurs. Not surprisingly, the average price change increases with the sampling interval, ranging, for instance, between 2–7% with a tick-by-tick frequency and between 4–14% with a daily frequency. We cannot exclude that the average price change remains within the typical bid-ask spread for all five contracts, and especially for short time intervals. Indeed, using a sample that overlaps with ours (FTSE 100 options, August 2001–July 2002), Fahlenbrach and Sandas [2005] find that the average bid-ask spread is 5.0–8.9% for ATM options, 11.8–28.2% for OTM options, 29.7% for short-term options, 11.2% for medium-term options, and 8.0% for long-term options. On EURONEXT, Sahut [1998] shows that the bid-ask spread ranges from 7.1–19.5%. Finally, on EUREX,

EXHIBIT 3

Tick-by-Tick and Fixed-Interval Price Changes

This Exhibit details the construction of the 1) tick-by-tick price changes and 2) price changes over a fixed sampling interval (Δt). In this exhibit, we consider five trades, denoted trades A, B, C, D, and E. Tick-by-tick price changes are computed between two consecutive transaction prices irrespective of the elapsed time between the two trades. Fixed-Δt price changes are computed between a price at time t and the price at the closest trade to $\Delta t + t$, with the constraint to remain within a $\Delta t/2$ window.



average spreads are in the 4–6% range (see Bartram and Fehle [2006] for details). These figures suggest that the frequent violations of the EMP reported above are due, in large part, to the bid-ask bounce.

Causes of the Violations

Are violations caused by changes in other underlying variables? According to option pricing theory, violations of the EMP can be due to simultaneous changes in the value of other variables affecting option prices. For instance, it is well known that the value of an option increases with the volatility of the underlying asset or that the time value of a given option gradually decreases through time. We know that the value of an option V depends on several underlying variables, namely, the current value of the underlying asset S, the volatility of the underlying asset σ, time t, the strike price K, the dividend rate q, and the risk-free interest rate r such that:

$$V = V(S, \sigma, t, K, q, r) \tag{4}$$

Using the denomination of each rate of change of the option price with respect to the underlying variables (i.e., the Greeks), the change in the value of the option can be rewritten as

$$\Delta V = Delta \cdot \Delta S + Vega \cdot \Delta\sigma + Theta \cdot \Delta t + Diva. \Delta q + Rho \cdot \Delta r + \tfrac{1}{2} Gamma \cdot (\Delta S)^2 + \ldots \tag{5}$$

We clearly see in equation (5) that when one analyzes the empirical relationship between ΔV and ΔS, the influence

EXHIBIT 4
Violation Rates by Sampling Interval

This Exhibit presents type-I and type-II violation occurrences for each of the five contracts, as a percentage of total observations at a given sampling interval defined as: Type I: $\Delta S < 0$, $\Delta C > 0$ for calls and $\Delta S > 0$, $\Delta P > 0$ for puts; and Type II: $\Delta S < 0$, $\Delta C > 0$ for calls and $\Delta S < 0$, $\Delta P < 0$ for puts. Figures for puts are indicated in parentheses.

Sampling	Europe		France		Germany		Switzerland		U.K.	
Interval	I	II	I	II	I	II	I	II	I	II
Tick-by-tick	9.9	10.3	13.9	14.2	6.1	6.4	9.4	9.9	6.2	6.5
	(9.4)	(9.2)	(14.4)	(14.5)	(6.2)	(6.2)	(10.5)	(9.7)	(5.1)	(5.8)
30 minutes	14.9	16.5	15.4	16.1	3.8	4.5	15.6	16.0	6.3	6.1
	(16.3)	(14.8)	(17.2)	(16.5)	(3.9)	(4.2)	(20.1)	(14.8)	(5.9)	(5.5)
1 hour	15.3	16.7	14.4	15.4	3.1	3.9	15.1	15.7	5.1	5.5
	(16.4)	(14.8)	(15.7)	(15.8)	(3.2)	(3.9)	(17.2)	(16.0)	(4.2)	(4.9)
2 hours	11.4	12.1	10.1	10.8	2.9	4.0	11.3	10.7	4.7	5.3
	(11.2)	(11.5)	(10.9)	(12.1)	(2.7)	(3.9)	(12.4)	(12.0)	(4.4)	(4.3)
3 hours	8.0	12.3	7.8	10.3	2.6	3.9	9.8	11.2	4.5	5.1
	(10.7)	(10.1)	(9.6)	(11.3)	(2.3)	(3.8)	(11.8)	(9.7)	(2.8)	(5.1)
1 day	3.0	10.1	1.8	9.8	0.6	6.6	2.0	9.5	1.0	7.9
	(3.3)	(8.0)	(2.9)	(7.9)	(0.6)	(5.7)	(4.1)	(7.6)	(0.9)	(8.6)

EXHIBIT 5
Violation Rates Across Maturities and Moneyness

This Exhibit presents type-I and type-II violation occurrences for out-(OTM), at-(ATM), and in-the-money (ITM) options, as well as for short-term, medium-term, and long-term options with violation occurrences defined as: Type I: $\Delta S < 0$, $\Delta C > 0$ for calls and $\Delta S < 0$, $\Delta P > 0$ for puts; and Type II: $\Delta S > 0$, $\Delta C < 0$ for calls and $\Delta S < 0$, $\Delta P < 0$ for puts. The results are based on tick-by-tick price changes. Figures for puts are indicated in parentheses.

	Europe		France		Germany		Switzerland		U.K.	
Breakdown	I	II	I	II	I	II	I	II	I	II
OTM	10.1	10.5	14.1	14.0	6.2	6.5	8.7	9.1	6.0	6.3
	(9.1)	(8.9)	(14.2)	(14.2)	(6.4)	(6.4)	(9.3)	(8.7)	(5.0)	(5.0)
ATM	9.5	10.1	13.4	14.7	5.9	6.4	9.9	10.6	6.3	6.9
	(10.0)	(9.9)	(14.7)	(15.1)	(6.3)	(6.1)	(11.5)	(10.6)	(5.2)	(6.8)
ITM	8.1	7.9	17.6	15.7	4.3	4.3	12.2	9.2	5.8	3.8
	(10.4)	(10.3)	(16.0)	(17.0)	(4.6)	(4.7)	(9.8)	(9.1)	(5.4)	(4.5)
SHORT	9.4	9.8	13.8	14.2	6.3	6.7	9.7	10.5	6.1	6.7
	(9.3)	(9.1)	(14.2)	(14.5)	(6.4)	(6.3)	(10.7)	(10.1)	(5.5)	(6.2)
MEDIUM	9.3	10.0	14.8	14.1	5.9	6.3	9.5	9.4	6.7	6.4
	(9.0)	(8.6)	(15.7)	(13.6)	(6.1)	(6.3)	(9.9)	(8.4)	(4.0)	(5.2)
LONG	10.9	11.1	13.7	13.7	5.7	5.8	8.2	8.6	4.9	4.8
	(9.9)	(9.8)	(13.4)	(15.6)	(5.7)	(5.7)	(10.5)	(9.6)	(5.1)	(4.3)

Exhibit 6

Percent Price Changes When a Violation Occurs

This Exhibit presents the average absolute option price changes expressed as a percentage of option prices when a type-I or type-II violation occurs which are defined as: Type I: $\Delta S < 0$, $\Delta C > 0$ for calls and $\Delta S > 0$, $\Delta P > 0$ for puts; and Type II: $\Delta S > 0$, $\Delta C < 0$ for calls and $\Delta S < 0$, $\Delta P < 0$ for puts. Figures for puts are indicated in parentheses.

Sampling	Europe		France		Germany		Switzerland		U.K.	
Interval	I	II	I	II	I	II	I	II	I	II
Tick-by-tick	7.0	3.7	5.0	5.1	2.0	1.9	5.3	4.7	3.6	3.2
	(3.2)	(3.3)	(4.8)	(4.2)	(1.7)	(1.6)	(5.8)	(3.3)	(2.9)	(2.5)
30 minutes	6.7	5.4	7.4	6.8	2.3	2.4	7.2	6.6	4.0	3.6
	(2.7)	(12.2)	(7.1)	(6.0)	(2.2)	(2.1)	(5.3)	(4.1)	(3.4)	(2.7)
1 hour	8.0	6.8	9.7	8.3	2.6	2.6	9.6	8.2	4.7	4.0
	(4.1)	(4.5)	(9.2)	(7.6)	(2.4)	(2.2)	(8.8)	(5.2)	(3.0)	(3.3)
2 hours	9.1	5.9	9.5	8.2	3.0	2.8	7.6	8.3	5.2	4.3
	(5.8)	(4.2)	(9.3)	(7.6)	(2.7)	(2.5)	(9.1)	(5.8)	(3.8)	(4.0)
3 hours	6.5	7.7	9.0	8.0	2.8	2.8	8.8	7.7	3.2	3.4
	(4.2)	(3.9)	(9.0)	(6.4)	(2.8)	(2.5)	(9.8)	(5.7)	(3.9)	(3.1)
1 day	8.7	8.5	14.3	9.8	4.2	5.1	14.3	9.8	4.8	6.4
	(7.1)	(17.1)	(12.7)	(10.3)	(3.0)	(4.3)	(12.0)	(9.7)	(3.0)	(5.9)

of the other underlying variables is neglected. In equation (6), we focus on volatility shocks and restrict equation (5) as follows:[12]

$$\Delta V = Delta \cdot \Delta S + Vega \cdot \Delta \sigma \qquad (6)$$

Controlling for shocks in the volatility of the underlying asset is of primary importance. Indeed, a violation I may be caused by a sudden spike in the level of the expected annualized volatility over the remaining life of the option. Moreover, in a simulation experiment, Bakshi, Cao, and Chen [2000] show that option prices generated according to a stochastic volatility model violate the EMP as frequently as actual option prices.

When attempting to account for intra-day changes in volatility, we invariably face the problem that volatility cannot be readily observable. To overcome this problem, we simply claim that if a violation I ($\Delta S < 0$, $\Delta C > 0$) is triggered by a sudden rise in volatility ($\Delta\sigma > 0$), this volatility shock should also affect other options written on the same underlying asset. Specifically, the likelihood of having a violation on call option i ($\Delta S < 0$, $\Delta C_i > 0$) should be positively affected by the fact that there is a concurrent violation on call option j ($\Delta S < 0$, $\Delta C_j > 0$), where calls i and j have the same underlying asset, the same expiration date, but different strike prices.

To test the latter hypothesis, we apply the following matching procedure. For each call option price change ΔC_i measured over Δt_i, we identify another call option price change ΔC_j measured over Δt_j that maximizes the overlap between the two time intervals Δt_i and Δt_j. If the overlap of the best pair of price changes does not exceed 90%, we drop this particular ΔC_i from the sample. By applying this matching rule to all 30-minute, 1-, 2-, and 3-hour call option price changes, we end up with 673,100 valid matchings, which corresponds to approximately one-third of the original sample.

We compute the total number of type-I and type-II violations for all matching observations ΔC_j knowing that ΔC_i has 1) a type-I violation, 2) a type-II violation, or 3) no violations. Empirical results are presented in Exhibit 7.[13] The first column of the exhibit shows that this sample yields the same violation rates as shown in Exhibit 4. For instance, of the 17,128 price changes for options on the European index, we find 1,732 type-I violations, or 10.1%. Turning to the matching sample, we record a very large number of type-I violations (i.e., 1,086 out of 1,732). This result suggests that violations tend to occur in bunches; they concurrently affect options written on the same underlying asset. A similar pattern arises with type-II violations. Furthermore, similar clustering effects can be seen for the other four contracts. This empirical

EXHIBIT 7
Violation Clustering for Call Options

This Exhibit presents, in the column headed "Original Sample," the number and the percentage of type-I and type-II violations for each of the five contracts defined as: Type I: $\Delta S < 0$, $\Delta C > 0$ and Type II: $\Delta S > 0$, $\Delta C < 0$. The original sample contains all 30-minute, 1, 2, and 3 hour call option price changes ΔC_i that have been successively matched with a concurrent option price change ΔC_j, where calls i and j have the same underlying asset and expiration date, but different strike prices. The column headed "Matching Sample" presents the number and the percentage of type-I and type-II violations for the matching observation ΔC_j knowing that ΔC_i has a type-I violation, a type-II violation, or no violation, respectively.

	Original Sample Violation	Matching Sample I	Matching Sample II	Matching Sample Not I, not II
		Europe		
I	1,732 (10.1)	1,086	25	621
II	2,463 (14.4)	21	1,705	737
Not I, not II	12,933 (75.5)	721	824	11,388
Total	17,128 (100.0)	1,828 (10.7)	2,554 (14.9)	12,746 (74.4)
		France		
I	846 (9.3)	567	10	269
II	933 (10.2)	10	591	332
Not I, not II	7,334 (80.5)	297	295	6,742
Total	9,113 (100.0)	874 (9.6)	896 (9.8)	7,343 (80.6)
		Germany		
I	17,614 (2.8)	3,882	633	13,099
II	24,457 (3.8)	598	6,829	17,030
Not I, not II	597,108 (93.4)	11,985	16,074	569,049
Total	639,179 (100.0)	16,465 (2.6)	23,536 (3.7)	599,178 (93.7)
		Switzerland		
I	489 (12.3)	339	3	147
II	496 (12.4)	5	395	96
Not I, not II	3,004 (75.3)	103	103	2,789
Total	3,989 (100.0)	447 (11.2)	501 (12.6)	3,041 (76.2)
		U.K.		
I	175 (4.7)	57	4	114
II	194 (5.3)	5	66	123
Not I, not II	3,322 (90.0)	92	134	3,096
Total	3,691 (100.0)	154 (4.2)	204 (5.5)	3,333 (90.3)

Exhibit 8

Violation Rates When Controlling for the Bid-Ask Bounce

This Exhibit reports the type-I and type-II violation occurrences for index options written on the FTSE 100 stock index at a given sampling interval with violation occurrences defined as: Type I: $\Delta S < 0$, $\Delta C > 0$ for calls and $\Delta S > 0$, $\Delta P > 0$ for puts; and Type II: $\Delta S > 0$, $\Delta C < 0$ for calls and $\Delta S < 0$, $\Delta P < 0$ for puts. For each transaction, we know whether the trade occurred at the bid price or at the ask price. For type-I (type-II) violations, we break down the sample between option price changes computed between a bid (ask) price and an ask (bid) price and other option price sequences. Figures for puts are indicated in parentheses.

Sampling Interval	All	Violations I Bid to Ask	Others	All	Violations II Ask to Bid	Others
Tick-by-tick	6.1	8.7	5.4	6.5	8.4	5.9
	(5.1)	(7.0)	(4.6)	(5.8)	(7.9)	(5.0)
30 minutes	6.3	12.9	4.7	6.1	12.7	4.3
	(5.9)	(10.0)	(4.9)	(5.5)	(13.1)	(3.5)
1 hour	5.1	11.3	3.4	5.5	11.3	3.9
	(4.1)	(7.9)	(3.2)	(5.0)	(9.9)	(3.5)
2 hours	4.7	9.4	3.4	5.3	11.5	3.5
	(4.5)	(8.7)	(3.1)	(4.3)	(7.0)	(3.6)
3 hours	4.5	8.6	3.3	5.1	9.2	3.9
	(2.8)	(5.5)	(2.0)	(5.1)	(7.5)	(4.4)
1 day	1.0	1.8	0.7	7.9	7.4	8.1
	(0.9)	(1.5)	(0.7)	(8.6)	(9.4)	(8.3)

evidence supports the claim in Bakshi, Cao, and Chen [2000] that violations of the EMP are mainly due to volatility shocks. Although these clusters of violations of the EMP are consistent with the volatility story, they may also reflect the microstructure of the options market. In the next two subsections, we will take different angles to study the role of market microstructure.

Are violations caused by the bid-ask bounce? We question whether deviations from the EMP are caused by the bid-ask bounce. Indeed, even when both bid and ask prices remain constant, a type-I violation ($\Delta S < 0$, $\Delta C > 0$ and $\Delta S > 0$, $\Delta P > 0$) would mechanically arise between a seller-initiated trade (bid price) and a buyer-initiated trade (ask price). In this case, the option price change would be equal to the bid-ask spread. For similar reasons, a type-II violation ($\Delta S > 0$, $\Delta C < 0$ and $\Delta S < 0$, $\Delta P < 0$) would arise between a buyer-initiated trade and a seller-initiated trade.

To assess the impact of microstructure effects on the EMP violations, we explicitly account for the direction of option trades. We test if the occurrence frequency of type-I and type-II violations are critically affected by whether option price changes are computed between a bid and an ask price, an ask and a bid price, two bid prices, or two ask prices. Our intuition suggests that type-I violations should be more frequent when option price changes are computed between a bid and an ask price. In the same way, type-II violations should be more frequent when option price changes are computed between an ask and a bid price. We recompute the type-I and type-II violation rates using an additional dataset provided by the LIFFE, which contains all quotes on FTSE 100 options in 2002. For each transaction price, we check in this additional file if the trade occurred at the bid price or at the ask price.

We compute occurrence frequency of type-I violations when option price changes are computed between a bid price and an ask price, and other combinations (i.e., ask-bid, bid-bid, or ask-ask). For type-II violations, we break down the sample between option price changes computed between an ask price and a bid price, and other combinations (i.e., bid-ask, bid-bid, or ask-ask). The results in Exhibit 8 lend a great deal of support to a microstructural explanation for the frequent violations of the EMP. Indeed, it turns out that type-I violations are two or three times more frequent between a bid and an ask price than for other price sequences. Analogously, type-II violations are two or three times more frequent

Exhibit 9

Violations and Rational Trading Tactics

This Exhibit presents the coefficient estimates and the associated p-values of the following PROBIT model:

$$\text{Violation}_i = \alpha + \beta_1 \left|\frac{\Delta S_i}{S_i}\right| + \beta_2 \cdot \text{Trading\_Activity}_i + \beta_3 \cdot \text{Late\_Trading}_i + \beta_4 \cdot \text{Friday}_i + e_i$$

where Violation is a discrete variable equal to one when a given option price change computed over a time interval Δt is characterized by a type-I violation or a type-II violation, and zero, otherwise; $|\Delta S_i/S_i|$ is the absolute value of the relative change in the level of the stock index over Δt; Trading_Activity is the daily number of trades on a given series (a call or a put with a given strike price and a given maturity); Late_Trading is a discrete variable equal to one if the transaction takes place during the last three hours of the trading day, and zero, otherwise; and Friday is a discrete variable equal to one if the transaction takes place on a Friday, and zero, otherwise. The last column reports the number of observations used in each regression. For each subsample (Europe, France, Germany, Switzerland, U.K., and all contracts combined), we report in the first line the values for call options and, in the second line (in parentheses), the values for put options.

	β_1	P-value	β_2	P-value	β_3	P-value	β_4	P-value	Obs.
Europe	-30.25	0.000	-0.002	0.000	0.028	0.053	0.084	0.000	79,675
	(-35.38)	(0.000)	(0.001)	(0.000)	(0.103)	(0.000)	(0.011)	(0.000)	(90,531)
France	-48.83	0.000	-0.003	0.006	0.223	0.000	0.006	0.768	32,518
	(-36.41)	(0.000)	(0.001)	(0.142)	(0.193)	(0.000)	(0.100)	(0.000)	(32,621)
Germ.	-87.52	0.000	-0.001	0.000	-0.017	0.019	0.034	0.000	839,388
	(81.88)	(0.000)	(-0.001)	(0.000)	(0.002)	(0.739)	(0.066)	(0.316)	(809,779)
Switz.	-39.43	0.000	0.001	0.747	0.021	0.442	-0.030	0.141	23,344
	(-35.64)	(0.000)	(0.003)	(0.000)	(0.042)	(0.060)	(-0.017)	(0.344)	(28,847)
U.K.	-171.35	0.000	-0.011	0.000	0.045	0.290	0.093	0.003	17,391
	(-229.76)	(0.000)	(-0.006)	(0.003)	(-0.054)	(0.304)	(0.013)	(0.737)	(12,494)
All	-89.57	0.000	-0.003	0.000	0.028	0.000	0.069	0.000	992,316
	(-88.39)	(0.000)	(-0.003)	(0.000)	(0.088)	(0.000)	(0.086)	(0.000)	(974,272)

between an ask and a bid price. Once the effect of the bid-ask bounce has been isolated, violation rates of the EMP for sampled intra-day option prices remain in the 3–6% range for call options and 2–5% range for put options.

Do violations arise from rational trading tactics? Another potential reason for having violations of the EMP may simply be tactical trading in a market characterized by 1) price/time priority and 2) moderate liquidity.[14] To illustrate our point, let us consider the case of a given trader who aims to sell 50 call contracts. Currently, the bid price for the call option is $99, the ask price is $100, and the most recent trade was at $100. She might be willing to sell at $99.50 if she knew she could execute right away, especially if her order is not first in line. She does not want to lower her offer price because she would still have uncertainty about when and whether she could trade, but she would give up the opportunity to sell at $100. Over time, the underlying stock index would jiggle around, maybe rising a small amount from where it was at the last option trade, without causing any changes in the options market. Now a new bid price comes into the options market at $99.50, so she decides to hit the bid and sell. The data would show that the index level went up, but the option price went down. In this case, the violation of the EMP would arise from rational trading tactics.

Such tactical trading behavior might be more likely when the underlying asset has not changed very much since the last option trade, when there have been relatively few trades on the option contract, late in the trading day, or on Fridays, when option traders may feel increasing urgency to get their trades done before the market closes. To investigate the validity of the trading-tactics hypothesis, we estimate the following PROBIT model using all tick-by-tick and fixed-interval option price changes (up to three hours):

$Violation_i =$

$$\alpha + \beta_1 \cdot \left|\frac{\Delta S_i}{S_i}\right| + \beta_2 \cdot Trading\_Activity_i + \beta_3 \cdot Late\_Trading_i + \beta_4 \cdot Friday_i + e_i \quad (7)$$

where *Violation* is a discrete variable equal to one when a given option price change computed over a time interval Δt is characterized by a type-I violation or a type-II violation, and zero, otherwise; $|\Delta S_i / S_i|$ is the absolute value of the size of the relative change in the stock index over Δt; *Trading_Activity* is the daily number of trades on a given series (a call or a put with a given strike price and a given maturity); *Late_Trading* is a discrete variable equal to one if the transaction takes place during the last three hours of the trading day, and zero, otherwise; and *Friday* is a discrete variable equal to one if the transaction takes place on a Friday, and zero, otherwise.

We present the regression results in Exhibit 9. For each contract, we report in the first line the coefficient estimates and the p-values for call options and, in the second line, the coefficient estimates and the p-values for put options. Consistent with the trading-tactics hypothesis, violations of the EMP are negatively related to the relative changes in the underlying asset and to the level of activity of the option contract. Furthermore, our regression results indicate that violations of the EMP are more likely to happen right before the options market close, and on Fridays. These findings are valid for individual contracts as well as for the combined sample containing all five contracts. Moreover, we get consistent results with call and put options.

CONCLUSION

In this article, we test the empirical validity of the monotonicity property of option prices using all transaction prices in 2002 for five option contracts written on the European, French, German, Swiss, and British stock indices. We show that, depending on the sampling interval (from tick-by-tick price changes to daily price changes) and the option contract considered, call option prices move in the opposite direction of the underlying asset 7–32% of the time. Similarly, the associated violation rates for put option prices range from 6–35%. Furthermore, the occurrence rates generally decrease with the length of the time interval considered and with the liquidity of the contract.

Our findings contribute to the debate on the optimal hedging frequency. Recently, Bossaerts and Hillion [2003] used information on concurrent index option and futures price changes to value and hedge DAX index options with intra-day data. They claim that the poor performance of their method on real data is caused by the frequent negative empirical deltas that arise using tick-by-tick price changes. They conclude their study by stating that the "issue of whether such anomalous co-movements disappear over longer time intervals deserves further investigation." Our empirical results shed light on this important issue.

Furthermore, we investigate the causes of the frequent violations of the monotonicity property. Our findings suggest that part of the violations are due to concurrent changes in volatility. We also consider the microstructure aspects of options markets. Controlling explicitly for the direction of option trades (i.e., seller-initiated or buyer-initiated), we uncover evidence that a substantial portion of the reported violations of the monotonicity property are attributable to the bid-ask bounce. Finally, we show that violations of the monotonicity property can result from rational trading tactics followed by traders in a market with relatively limited liquidity.

ENDNOTES

I thank Lei Han for excellent research assistance and Avi Bick, Thomas Gilbert, Dusan Isakov, Peter Klein, Andrey Pavlov, Daniel Smith, Chris Veld, seminar participants at Catholic University of Louvain (CORE) and Simon Fraser University, and participants at the Northern Finance Association 2004 conference for helpful comments. Special thanks go to Stephen Figlewski (the Editor) for his thorough review of the previous draft of the article. Research support from Simon Fraser University is gratefully acknowledged. I am responsible for all errors and omissions.

[1]Richard Roll "R2," *Journal of Finance,* Vol. 43, No. 2 (July 1988), pp. 541.

[2]The monotonicity property is not one of the model-free that have to be respected in arbitrage-free option markets, such as upper or lower bounds for option prices (see Smith [1976] and Hull [2006]).

[3]Bakshi, Cao, and Chen [2000] show that their conclusions remain valid after controlling for moneyness, maturity, time decay, bid-ask spread, trading volume, number of quotes revisions, and time of day.

[4]Note that a situation where either the option price or the stock index remains constant over the sampling interval is not considered as a violation. This alleviates the bias introduced by the minimum tick size. In particular, we do not record a spurious

violation when the "true" option price changes by less than one tick size, and then the observed option price remains constant.

[5]The DJ EURO STOXX 50 is an index of 50 European blue chip stocks compiled by Dow Jones. The CAC 40 Index comprises 40 of the largest-cap companies listed in France. The DAX Index is based on 30 large stocks trading on the German exchange. The SMI Index is made up of the 30 largest companies trading in Switzerland. The FTSE 100 Index is a portfolio of 100 large companies listed on the London Stock Exchange. Besides the DAX Index, which is a total return index, all indices are price indices (i.e., dividends are not assumed to be reinvested).

[6]EUREX, the world's leading derivatives exchange (in terms of number of contracts traded), is jointly operated by Deutsche Börse AG and SWX Swiss Exchange. It closed out 2002 with over 800 million contracts traded, and broke the one billion contract barrier in 2003 (source: EUREX). EURONEXT was created in 2000 after the merger of the Amsterdam, Brussels, and Paris stock and derivatives exchanges. EURONEXT took control of the London International Financial Futures and Options Exchange (LIFFE) in 2001 and migrated part of its operations during the first half of 2003 onto the LIFFE, making EURONEXT.LIFFE the world's second largest derivatives exchange (again, in terms of number of contracts traded).

[7]An exception is a limit order with a "fill-or-kill" feature.

[8]Beginning in 2005, a newly designated market-maker scheme is operating in the European-style FTSE 100 index option contract.

[9]To compute the implied volatility, we interpolate, on a daily basis, a risk-free rate with a maturity matching that of the option from the term structure of the interbank offered rates for Euros, British Pounds, and Swiss Francs, obtained from Datastream. We assume that the dividends are known over the life of the option and replace q by the annualized average realized dividend rate. We obtain daily dividend rates on the stock indices from Datastream. Because the DAX index is a total return index, we do not account for the dividend yield.

[10]The use of small time intervals mitigates the impact of time decay on option prices.

[11]For the 1-day sampling interval, we use a $\Delta t/6$ window (4 hours).

[12]We reject the idea that time decay can have any measurable impact over the short time intervals used in this study. We do not account for changes in the risk-free interest rate r and in the dividend rate q because the effect on option prices of changes in r or q has been shown to be relatively small (see, among others, Bakshi, Cao, and Chen [1997] and Jarrow and Turnbull [2000, p. 278]), and that Δr and Δq cannot be measured with a sufficient precision over very small time intervals. We also neglect the Gamma effect as it is often treated as a second-order effect in the literature.

[13]We obtain similar results with put options, but do not report the results to save space.

[14]We thank Stephen Figlewski (the editor of this journal) for suggesting this potential cause of violation of the EMP.

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