Order Choice and Information in Limit Order Markets

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Abstract

This article surveys recent developments in the study of limit order markets. The focus is on the decisions of traders under symmetric or asymmetric information, and in particular on their choice between limit orders and market orders. We discuss also the price impact of different types of orders, and their information content.

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1 Introduction

In recent years, trading via limit orders and market orders has become the dominant form of trading in most exchanges around the world, whether these are pure electronic limit order markets, or hybrid markets in which limit order traders are in competition with floor traders, specialists, or dealers.

A pure limit order market is defined as a market in which there are essentially only limit orders and market orders. A market order demands immediate execution, irrespective of the price. A limit order is an instruction to buy or sell only at a pre-specified price, and is placed in a queue based on price/time priority. A limit order is usually executed only after a market order clears it from the queue. A limit order that gets immediate execution because the price is already met is called a marketable order, and is not differentiated from a market order. A sell limit order is also called an ask (or offer), while a buy limit order is also called a bid. The *limit order book*, or simply the *book*, is the collection of all outstanding limit orders. The lowest ask in the book is called the ask price, or simply ask, and the highest bid is called the bid price, or simply bid.[1] This article does not discuss *hidden* limit orders, which are limit order for which some of the quantity is not visible to the market.

Given the importance of limit order markets, there have been relatively few models that describe price formation and order choice in these markets. The main reason for this scarcity is the difficulty of the problem. In dealer or specialist markets, liquidity provision is restricted to one or several individuals, who are easier to model, especially if they are assumed to be uninformed.[2] In contrast, in limit order markets, liquidity provision is open to everyone via limit orders. Therefore, in order to achieve the tractability of previous models, one may assume, as in Glosten (1994), Rock (1996), Seppi (1997), or Biais, Martimort, and Rochet (2000), that limit order traders are uninformed. If instead informed traders are allowed to choose how to trade, the problem becomes significantly more difficult, especially in dynamic models. This paper focuses on the order choice, therefore we do not discuss the literature in

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[1]One can think of limit orders as supplying liquidity (i.e., immediacy) to the market, and market orders as demanding liquidity. In reality, the distinction between the supply and demand of liquidity is not as clear cut. For example, an aggressive limit order, e.g., a limit order submitted close to the ask or the bid, is likely to be executed, and can be thought as demanding liquidity, at least relative to the limit orders that are submitted further away from the market.

[2]Dealer and specialist markets are modeled, e.g., in Amihud and Mendelson (1980), Kyle (1985), Glosten and Milgrom (1985). See also the equivalence result of Back and Baruch (2007).

which traders cannot choose between market orders and limit orders.

The author has benefited from some excellent surveys on the market microstructure literature, e.g., O'Hara (1995), Madhavan (2000), Biais, Glosten, and Spatt (2005), and especially Parlour and Seppi (2008), which focuses on limit order markets.

This survey begins by an analysis of order choice in the presence of symmetric information (Section 2), and then discuss the same choice under asymmetric information (Section 3). The next section discusses the empirical literature on order choice, as well as on the information content and price impact of various types of orders (Section 4). The survey ends with some questions for future research (Section 5).

2 Order Choice with Symmetric Information

In the absence of asymmetric information, the choice between market orders and limit orders is decided by comparing the certain execution of market orders at possibly disadvantageous prices with the uncertain execution of limit orders at more advantageous prices. If, moreover, limit orders cannot be freely canceled or modified, one additional cost of limit orders is that they can be picked off by the new traders.

The intuition for the tradeoff between price and time goes back to Demsetz (1968), but it was first modeled explicitly in Cohen, Maier, Schwartz, and Whitcomb (1981). This paper assumes an exogenous price process which determines whether a limit order gets executed by the next period or not, in which case the limit order gets canceled. The model produces a non-zero bid-ask spread. Moreover, if the market has a smaller trading intensity (it is thinner), the equilibrium bid-ask spreads is larger. To understand these results, consider an investor who contemplates placing a limit buy order below the ask. Due to the frictions in the model, the probability of execution is always significantly less than one, while the price improvement converges to zero as the limit buy order approaches the ask. This "gravitational pull" makes it certain that above a certain price below the ask it is always more advantageous to buy with a market order than with a limit order. Thus, there is a minimum distance between the bid and the ask, i.e., a minimum bid-ask spread. One can see also that this minimum spread must be larger when the probability of execution of the limit order gets smaller, which happens when the market is thinner.

Parlour (1998) presents a model of a limit order book in which there are only two prices: the bid, B, and the ask, A. The buy limit orders queue at B, and the limit orders queue at A. The trading day is divided into $T+1$ subperiods, $t = 0, 1, \ldots, T$. On each trading subperiod, one trader arrives, that is either a seller (with probability π_S), a buyer (with probability π_B), or neither. A buyer can make one of three choices: submit a buy market order, submit a buy limit order, or stay out of the market. Each trader is characterized by a parameter $\beta \in [\underline{\beta}, \overline{\beta}]$ (with $0 \leq \underline{\beta} \leq 1 \leq \overline{\beta}$), which determines the tradeoff between consumption on the trading day and on the next day, when the asset value is realized. The parameter β can be interpreted as an impatience coefficient, or alternatively as a private valuation for the asset. Traders with extreme values of β prefer to trade immediately, and use market orders. Traders with intermediate values of β prefer to wait, and use limit orders. A low β indicates that the trader prefers consumption today to consumption tomorrow, which means that, given the choice, she prefers to sell the asset to get cash today. Similarly, high β characterizes a propensity to buy. A trader arriving at t has only one opportunity to submit an order. Once submitted, orders cannot be modified or canceled.

The model leads to a stochastic sequential game, and has a cutoff equilibrium. For example, buyers with high β submit a buy market order; with intermediate β submit a limit buy order; and with low β stay out of the market. The model produces interesting liquidity dynamics, and in particular it provides an explanation for the diagonal effect of Biais, Hillion, and Spatt (1995), who find that, e.g., buy market orders (BMO) are more likely after BMO than after sell market orders (SMO). To see this, consider the arrival of a BMO at t. This reduces the liquidity available at the ask by one unit, and, if the next trader is a seller, the trader is more likely to submit a sell limit order (SLO) than a sell market order (SMO). This in turn makes future buyers prefer BMO to BLO. Note that this type of dynamics also involve a correlation between BMO and SLO, which provides an additional empirical prediction.

Foucault, Kadan, and Kandel (2005) explicitly model waiting costs as linear in the expected waiting time. Limit orders cannot be canceled and must be submitted inside the existing bid-ask spread. Prices are constrained in a fundamental band $[B, A]$: a competitive fringe of traders stands ready to buy at B and sell at A an unlimited number of shares. Traders arrive according to a Poisson process with parameter λ, and can trade only *one* unit. Traders can be either patient, by incurring a waiting cost of δ_P per unit of time, or impatient,

with a waiting cost of $\delta_I \geq \delta_P$. The fraction of impatient traders out of the total population is θ_P.

The assumption that limit orders must always improve the spread implies that the only relevant state variable is the bid-ask spread, which is an integer multiple of the minimum tick size, Δ. This in turn means that a limit order can be characterized simply by the bid-ask spread that it creates after being submitted: a j-limit order is a limit order that results in a bid-ask spread of size $j\Delta$. The resulting Markov perfect equilibrium depends on the current spread: if the spread is below a cutoff, both patient and impatient traders submit market orders; if the spread is above a cutoff, both types of traders submit limit orders; and if the spread is of intermediate size, the patient traders submit limit orders, and impatient traders submit market orders.

Foucault, Kadan, and Kandel (2005) focus on the case when it is never optimal for impatient traders to submit limit orders. Not all spreads can exist in equilibrium, but only a subset: $n_1 < n_2 < \cdots < n_q$. Then a patient trader facing a spread between $n_h + 1$ and n_{h+1} is to submit an n_h-limit order. The expected time to execution of such a limit order has a closed-form expression: $T(n_h) = \frac{1}{\lambda}\left(1 + 2\sum_{k=1}^{h-1}\rho^k\right)$, where $\rho = \frac{\theta_P}{\theta_I}$ is the ratio of the proportions of patient and impatient traders. Relatively more patient traders (higher ρ) induce more competition for providing liquidity, hence higher expected waiting time for limit orders. Aware of this, each patient trader submits more aggressive orders, and consequently reduces the spread more quickly.

The paper discusses the notion of resiliency: e.g., the propensity for the bid-ask spread to revert to its lower level after, a market order has consumed liquidity in the book. Note that in the context of the model there is no *pressure* to revert to the former level, which would be due for example to the arrival of more liquidity providers when the bid-ask spread gets wider. Here resilience is due to the ergodicity of the Markov equilibrium: any possible equilibrium spread is attainable after a while. Intuitively, more patient traders make the book more resilient. Surprisingly, a higher arrival rate λ *decreases* the resiliency of the book, i.e., fast markets are less resilient than slow markets. The intuition is that, in a faster market, limit traders become less aggressive in improving the spread, and the spread reverts more slowly to smaller levels.

Goettler, Parlour, and Rajan (2005) propose a model in which traders have private val-

uations about the value of an asset. Each period, the common value v_t stays the same; or moves up by one tick, with probability $\frac{\lambda}{2}$; or moves down by one tick with probability $\frac{\lambda}{2}$. Each trader can trade up to $\bar{z}$ units of the asset. Limit orders can be submitted on a discrete price grid, $p^{-N}, \ldots, p^{-1}, p^1, \ldots, p^N$. The common value of the asset is at the midpoint between p^{-1} and p^1. A competitive crowd of traders provides an infinite depth of buy orders at p^{-N} and sell orders at p^N. The limit order book is described by a vector $L_t = \{l_t^i\}_{i=-(N-1)}^{N-1}$, where l_t^i represents the number of units of limit orders on level i at time t. Limit orders cannot be modified, but can be canceled according to an exogenously specified cancellation function, which may depend on whether the common value moves in an adverse direction. Indeed, the payoff of a trader with private valuation β_t who submits at t an order of one share with price p^i is: $(p^i + v_t) - (\beta_t + v_\tau)$ if the share will be sold at $\tau \geq t$; $(\beta_t + v_\tau) - (p^i + v_t)$ if the share will be bought at $\tau \geq t$; or 0 if the share is canceled before it is executed.

The state space is too large to be able to find the Markov perfect equilibrium in closed form, so the authors find a numerical solution, using an algorithm by Pakes and McGuire (2001). The initial parameter choices are: $N = 4$ (8 ticks); maximum trading volume per trader $\bar{z} = 2$; and the trading day has $M = 250$ transactions. Since the common value moves over time and traders cannot modify their limit orders, there are picking-off risks, and this creates patterns in the book. For example, BMO is more likely after BMO than after SLO: following an increase in v_t, sell orders in the book become stale and the incoming traders are more likely to submit BMO until the book fully adjusts.

Interestingly, the common value is often shown in simulations to be outside of the bid-ask spread, and this is not due only to stale orders. Indeed, consider the case when a trader with a very high private valuation encounters a book with a deep buy side, but a thin sell side. Then it is often optimal for this trader to submit a BLO above the common value v_t (which places the common value outside the spread), in order to attract more future SMO, hence potentially execute at a better price than from a BMO.

Large (2009) analyzes the choice between market orders and limit orders based on indirect waiting costs. The model assumes that limit orders are always submitted inside the spread. The slippage of an order is defined as $sign \times (p - m_t)$, where $sign$ is $+1$ or -1 if the order is to buy or sell, respectively; p is the trade price; and m_t is the bid-ask midpoint. A market order has positive slippage, a limit order has negative slippage. Negative slippage is preferable,

because it means trading at a more advantageous price. The payoff from trading is given by $sign \times \beta - slippage$, where β is the trader type. Given a time discount $\rho < 1$, an extreme β implies a bigger loss from waiting, i.e., extreme β is essentially equivalent to high waiting costs. Therefore, extreme β types tend to place market orders, while moderate β types tend to place limit orders. This is a cutoff strategy similar to that in Parlour (1998), or to the strategy of the informed trader in Roşu (2011), except that in Large (2009) there is no asymmetric information. The rest of the model relies on the idea that limit orders and the market orders have to be in equilibrium: the volume of market orders and uncanceled limit orders must be mechanically equal.

Roşu (2009) presents the first dynamic model in which agents are allowed to freely modify or cancel their limit orders. The model is similar to that of Foucault, Kadan, and Kandel (2005), except that prices are continuous rather than discrete, and there are no restrictions on order submissions. Surprisingly, allowing traders to be fully strategic turns out to simplify the problem. This is because, with waiting costs, all traders on the same side of the book have the same expected utility—even if they are on different levels. E.g., a seller with a higher limit price gets a better expected execution price, but also waits longer. The seller with the lower limit price cannot move above the price at which the two expected utilities are equal: otherwise, he will instantly be undercut by the other trader.

Since all traders on the same side of the book have the same expected utility, there exists a Markov perfect equilibrium in which the state variable is (m, n), with m the number of sellers, and n the number of buyers in the book. Their expected utility follows a recursive system of difference equations. The recursive system can be solved numerically, in many cases in closed form. In equilibrium, impatient agents submit market orders, while patient agents submit limit orders and wait, except for the states in which the limit order book is full. When the book is full, some patient agent either places a market order or submits a quick (fleeting) limit order, which some trader from the other side of the book immediately accepts.[3] This result provides one explanation for the very short-lived limit orders documented by Hasbrouck and Saar (2009). In states in which the book is not full, new limit orders are always placed inside the bid-ask spread, thus endogenizing one assumption of Foucault, Kadan, and Kandel (2005).

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[3]This comes theoretically as a result of a game of attrition among the buyers and sellers. The game is set in continuous time, and there is instant undercutting.

In Roşu (2009) orders have a price impact, even if the information is symmetric. To study price impact, one has to go beyond one-unit orders, and allow impatient traders to use multi-unit market orders. There is a *temporary* (or instantaneous) price impact function, which is the actual price impact suffered by the market order trader, and also a *permanent* (or subsequent) price impact, which reflects the fact that traders modify their orders in the book to account for the new reality. This permanent price impact is the difference between the new ask price and the ask price before the market order was submitted. In this setup, the temporary price impact is larger than the permanent price impact, which is equivalent to price overshooting. The intuition is that, before a multi-unit market order comes, the traders who expect their limit orders to be executed do not know the exact order size, so they stay higher in the book. Once the size becomes known, the sellers regroup lower in the book.

Also, if multi-unit market orders arrive with probabilities that do not decrease too fast with order size, then the price impact function is typically first concave and then convex. This is the same as saying that the limit orders cluster away from the bid and the ask, or that the book exhibits a "hump" shape. Empirically, this is documented by Bouchaud, Mezard, and Potters (2002) and or Biais, Hillion, and Spatt (1995). In the model, the hump shape arises because patient traders cluster away from the bid-ask spread when they expect to take advantage of large market orders that are not too unlikely.

The general, two-sided case is more difficult, and the solution is found numerically. An empirical implication is the *comovement* effect between bid and ask prices, documented by Biais, Hillion, and Spatt (1995). For example, a sell market order not only decreases the bid price—due to the mechanical execution of limit orders on the buy side—but also subsequently decreases the ask price. Moreover, the decrease in the bid price is larger than the subsequent decrease in the ask price, which leads to a wider bid-ask spread. The comovement effect is stronger when there are more limit traders on the side of the subsequent price move, and the competition among them is stronger.

3   Order Choice with Asymmetric Information

Models of the limit order book with asymmetric information provide further insights into the choice between market and limit orders. Handa and Schwartz (1996) note two risks of limit

orders: (1) an adverse information event can trigger an undesirable execution, that is limit orders can be "picked off" either by traders with superior information, or simply by traders who react more quickly to the public news (this is also called a *winner's curse* problem); and (2) favorable news can result in desirable execution not being obtained (this is also called as the *execution risk* problem).[4] Furthermore, there is a balance between liquidity demand and supply: a relative scarcity of limit orders makes prices more volatile, and thus increases the benefits of a limit order.

Chakravarty and Holden (1995) presents a static model of order choice with asymmetric information. In this model, limit order traders compete with market makers in a hybrid market (e.g., NYSE). The market makers set quotes first, and then market orders and limit orders are set simultaneously by one informed trader, and two or more uninformed trader. Since this is a one-period model, limit orders must be quote improving in order to be executed. The presence of uninformed limit orders inside the spread creates an additional reason for informed market orders, even if the true asset value is within the spread. (Normally, informed market orders should not be submitted when the true value is within the spread.) Furthermore, Chakravarty and Holden (1995) show that a combination of *BMO* and *SLO* is sometimes better for the informed trader than a *BMO* alone. This is because *SLO* provides insurance against the uncertainty about whether uninformed traders will supply liquidity on the sell side; a *BMO*-only might execute at a disadvantageous price.

Foucault (1999) studies the mix between market and limit orders in a model with moving fundamental value. For tractability, limit orders have a one-period life. They are subject to both the winner's curse problem and the execution risk problem. Similar to Goettler, Parlour, and Rajan (2005), Foucault (1999) is technically not an asymmetric information model, but the presence of trading frictions makes it resemble such a model. The fundamental value is assumed to move by $\pm\sigma$ according to a binomial tree with the same up and down probabilities. Each trader's private valuation is either L or $-L$, and this determines whether the trader is a buyer or a seller, and whether it uses a market order or a limit order. This decision also depends on whether the limit order book is empty or has already one limit order (it cannot have more than one).

The mix between market and limit orders depends essentially on the volatility of the

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[4]E.g., if a trader has a *BLO* in the book, good news about the asset raises the price and makes it less likely that the *BLO* will be executed, i.e., its execution costs increase.

asset, σ. When σ is higher, the picking off risks are larger, and the limit order therefore incorporates a larger compensation. This increases the cost of market orders, and makes them less frequent. In turn, this decreases the execution probability of a limit order. This analysis leads to two empirical implications: (1) the fraction of limit orders in the order flow increases with volatility; and (2) the fill rate (ratio of filled limit orders to the number of submitted limit orders) is negatively related to volatility.

Handa, Schwartz, and Tiwari (2003) extend Foucault (1999) by introducing privately informed traders, whose information becomes public after one trade. Additionally, the proportion of traders with high and low valuation is allowed to be unbalanced. One implication of their model is that spreads should be higher in the balanced case, compared with the unbalanced case. This is because in unbalanced markets, the relatively scarce category of traders can extract better terms of trade from the other side, and this translates into tighter spreads.

Goettler, Parlour, and Rajan (2009) numerically solve the first dynamic model of limit order markets with asymmetric information. The model is similar to that of Goettler, Parlour, and Rajan (2005), with the main difference that traders can acquire information about the fundamental value. Traders arrive according to a Poisson process with intensity λ, and upon entry decide whether to acquire information, for a fixed cost c. Traders who remain uninformed observe the fundamental value with a fixed lag. The fundamental value changes according to a Poisson process with intensity μ, in which case with equal probability can go up or down by k ticks. Each agent can trade at most one unit, and have a type $\theta = (\rho, \alpha)$, where ρ is the discount rate, and α is a private valuation for the asset. Traders with $\alpha = 0$ are called *speculators,* while those with high α are usually buyers, and those with low α are usually sellers. There are cancellation costs: a limit trader can cancel or modify his order only at a time which is randomly drawn.

The equilibrium concept used is stationary Markov perfect Bayesian equilibrium, and it is solved numerically using the Pakes and McGuire (2001) algorithm. Because there are cancellation costs, there are stale limit orders and picking-off risks. Speculators have a high demand for information, but a low desire to trade, so they usually become informed and use limit orders. However, in very volatile times more orders become stale, so in the equilibrium when only speculators are informed, they demand more liquidity via market orders, and submit limit orders at more conservative prices. Thus, limit order markets act as a volatility

multiplier: a small increase in fundamental volatility can lead to a large increase in price volatility.

Harris (1998) considers the dynamic order submission decisions of some stylized traders: liquidity traders, informed traders, and value traders. Liquidity traders are informed, and their choice of market order versus limit order depends on their deadline. If the deadline by which they must fill their order is distant, liquidity traders use limit orders. Gradually, if their limit orders do not fill, they replace them with more aggressively priced limit orders, and eventually with market orders. Informed traders have transitory private information about the fundamental value, and in general submit market orders to use their "hot information" unless the bid-ask spreads are wide and trading deadlines are distant, in which case they use limit orders to minimize transaction costs. Value-motivated traders have a flow of information, and have a cutoff strategy: when they believe the price is far from the fundamental value (and likely to revert quickly), they use market orders. Otherwise, they use limit orders.

Kaniel and Liu (2006) extend the partial equilibrium model of Harris (1998) to a general equilibrium model, similar to Glosten and Milgrom (1985), but with only three trading dates. In their model, informed traders submit limit orders when information is long-lived, to the extent that the limit orders can be even more informative than the market orders.

Roşu (2011) proposes a dynamic model of limit order markets, in which traders can acquire information about a moving fundamental value. As in Roşu (2009), there are no order cancellation costs or monitoring costs, and the same technique is used to construct a stationary Markov perfect equilibrium. The fundamental value v_t moves according to a diffusion process with normal increments. Traders arrive to the market according to a Poisson process with intensity λ, and upon arrival can acquire information: they pay a fixed cost c and observe v_t at the moment of entry. After that, they do not observe v anymore, which means that the precision of their private information decays over time. The resulting equilibrium is pooling: informed traders disguise their information while waiting in the book, and mimic the behavior of the uninformed traders. The informed traders could cancel at any time their limit order and modify it to a market order, but it is shown that they do not have an incentive to do so.

Traders are risk-neutral and have waiting costs proportional to the expected waiting time until execution. The waiting costs of the patient traders are very small, so that the model focuses on a set of states which appears with probability very close to one. In this "average

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limit order book" one can forget about the shape of the limit order book, and focus only on the information content of orders. The model assumes that only patient traders can be informed. If impatient traders can also be informed, some of the quantitative results change, but the qualitative results remain. In equilibrium, impatient traders always submit market orders, while patient traders are either uninformed, in which case they use limit orders; or informed, in which case they submit either a limit order or a market order.

The first set of results concerns the nature of the equilibrium and the strategy of the informed traders. We note that, alongside the fundamental value process there is also the *efficient price*, which is the expectation of the fundamental value conditional on all public information. In the model, since traders can cancel or revise their orders instantaneously, there are no stale limit orders or picking-off risks. The limit order book always moves up and down along with the efficient price: after each order, all traders modify their limit orders up or down to take into account the update of the efficient price due to the information contained in that order.

The strategy of the informed trader depends on how far the fundamental value, v, is from the efficient price, v^e. The optimal order of the informed trader is: a buy market order (BMO) if v is above v^e plus a cutoff value; a buy limit order (BLO) if v is between v^e and v^e plus the cutoff; a sell limit order (BLO) if v is between v^e minus the cutoff and v^e; and a sell market order (SMO) if v is below v^e minus the cutoff. The cutoff value is proportional to the *efficient volatility*, which is the conditional volatility of the fundamental value given all public information.

Put differently, an informed trader who observes an extreme fundamental value submits a market order, while one who observes a moderate fundamental value submits a limit order. This makes rigorous an intuition present, e.g., in Harris (1998), Bloomfield, O'Hara, and Saar (2005), Hollifield, Miller, Sandås, and Slive (2006). To understand why, we note that in the absence of private information a patient trader would use a limit order in order to take advantage of the bid-ask spread. But an informed trader who observes, e.g., a fundamental value well above the efficient price realizes that the order book will drift upwards due to the action of future informed traders. This reduces the expected profit from a buy limit order, thus making the buy market order more attractive. If the fundamental value is above a cutoff, cashing in on the information advantage with a market order is better than waiting to

be compensated for the limit order.

The next set of results regards the price impact of a trade and the efficient price process. Since both limit orders and market orders carry information about the fundamental value, the efficient price adjusts after each type of order. A key result is that all types of orders (BMO, BLO, SLO, SMO) are equally likely on average. First, if market orders were more likely than limit orders, the bid-ask spread would increase as limit orders were consumed by market orders. As the bid-ask spread increased, limit orders would become more likely, to the point where market orders and limit orders are equally likely. Second, if buy orders were more likely than sell orders, price impact would increase on average the efficient price to the point at which buy and sell orders are equally likely. Therefore, all order types are equally likely. This argument also shows that the midpoint between the bid and the ask is very close to the efficient price, so it provides a good empirical proxy for it.

The presence of informed traders ensures that the efficient price approximates the fundamental value. The speed of convergence is increasing in the information ratio: more informed traders make the efficient price converge more quickly to the fundamental value. Moreover, in a stationary equilibrium, the volatility of the efficient price is approximately equal to the fundamental volatility. This result shows that the volatility of the bid-ask spread midpoint is a good proxy for the fundamental volatility.

The average price impact of each type of order has a particularly simple form: Δ for BMO; $u\Delta$ for BLO; $-u\Delta$ for SLO; and $-\Delta$ for SMO; where $u \approx 0.2554$ is a constant, and the price impact parameter Δ is proportional to the fundamental volatility and inversely proportional to the square root of total trading activity (the sum of arrival rates of informed and uninformed traders). This implies that the price impact of a limit order is on average about four times smaller than the price impact of a market order, and that this ratio ($\frac{1}{u} \approx 3.912$) is independent of all the variables in the model. This result can be tested empirically.

A surprising result is that the price impact parameter Δ is independent of the ratio of the arrival rates of informed and uninformed traders (the *information ratio*). E.g., suppose the information ratio is low, and that the market sees a buy market order. A low information ratio means that it is unlikely that the market order comes from an informed trader. This should decrease Δ. At the same time, if the market order does come from an informed trader, this trader knows there is not much competition from other informed traders, so the only

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reason to submit a market order is if the observed fundamental value v is very far from the efficient price v^e. For this reason Δ should be larger.

The fact that these two effects exactly cancel each other is due to the stationarity of the equilibrium. A stationary equilibrium implies that the volatility between two trades of the fundamental value v and of the efficient price v^e must be equal. But the volatility of v between two trades only depends on the fundamental volatility and on the average time between two trader arrivals, neither of which depends on the information ratio. On the other hand, since v^e changes between two trades either by $\pm\Delta$ or by $\pm u\Delta$, with equal probability, it follows that the volatility of v^e is proportional to Δ. So the price impact parameter Δ is independent of the information ratio.

The third set of results in Roşu (2011) concerns the shape of the limit order book, as measured by the bid-ask spread or price impact, in relation to trading activity, volatility, and information asymmetry. A surprising result is that the average bid-ask spread is decreasing in the information ratio. In a static model such as Glosten (1994), in which the fundamental value is constant, a larger fraction of informed traders increases adverse selection and makes spreads larger. But with a moving fundamental value, the bid-ask spread has two components: one due to adverse selection costs (proportional to the price impact parameter Δ), and one due to the public uncertainty about the fundamental value (proportional to the efficient volatility σ^e). As explained above, Δ is independent of the information ratio; while σ^e is decreasing in the information ratio: more informed traders, smaller public uncertainty.

This suggests another way of estimating the information ratio. The ratio of the intra-day volatility of the spread midpoint to the average bid-ask spread depends only on the information ratio, and not on the fundamental volatility or trading activity. This ratio therefore can be used as a measure of the probability of informed trading, in the spirit of Easley, Hvidkjaer, and O'Hara (2002).

Note that the definition of asymmetric information in Roşu (2011) is different from the typical adverse selection models, such as Kyle (1985) or Glosten (1994). Here, informed traders can more accurately be described as "smart" liquidity traders. This feature gives a realistic description of order driven markets in which traders are relatively small and information is decentralized, as opposed to a market dominated by large insiders who know the fundamental value and are present at all times. In fact, as one can see from the results, competition among

more "smart" traders leads to *less* adverse selection. Indeed, when the information ratio is higher, efficient prices are usually closer to the fundamental value, and the bid-ask spreads are smaller.

4   The Information Content of Orders

If informed agents can trade with both limit orders and market orders, both types of orders should have an information content. One way to test that empirically is to look at the price impact after each type of order. Biais, Hillion, and Spatt (1995) find that limit orders do attract future price moves in the same direction, indicating that limit orders contain some information. Kaniel and Liu (2006) study the price informativeness of limit orders and market orders by comparing the conditional probabilities of the bid-ask midpoint moving in the direction of the order right after the order was submitted. With this measure of price informativeness, they find that limit orders are more informative than market orders.

Cao, Hansch, and Wang (2008) analyze the information content present in limit orders behind the best bid and ask, and find that the order book is indeed moderately informative, in the sense that its contribution to price discovery is about 22%. The other 78% comes from the best bid and ask, as well as the last transaction price. Also, imbalances between the bid and ask side in the limit order book are informative, even after controlling for return autocorrelation, inside spread, and trade imbalance. Contrary to the evidence above, Griffiths, Smith, Turnbull, and White (2000) find that non-marketable limit orders have a significant price impact in the opposite direction.

Kavajecz and Odders-White (2004) study the connection between technical analysis and liquidity provision. They argue that the limit orders do not reveal just information related to the fundamental value, but also discover pockets of depth already in place in the limit order book. Whether these pockets themselves are actually informed is not clear.

Eisler, Bouchaud, and Kockelkoren (2010) presents a VAR framework for the description of the impact of market orders, limit orders and cancellations. Assuming an additive model of impact, they use an empirical databases consisting only of trades and quotes information. They find that the impact of limit orders is similar, but smaller than that of market orders, in agreement with Roşu (2011). Limit order cancellations also impact prices, beyond their

effect of attracting limit orders. Latza and Payne (2011) find evidence that both limit orders and market orders have forecasting power for stock returns at very high frequencies. The predictive power of limit order flows is greater and more persistent than that of market order flows. The forecasting power of limit order flows relative to market order flows is larger when both bid-ask spreads and return volatility are high.

Some empirical studies have considered at the order submission strategies of various types of institutional or individual traders. Keim and Madhavan (1995) find that indexers and technical traders are more likely to submit market orders, while value traders are more likely to submit limit orders. Ellul, Holden, Jain, and Jennings (2005) provide evidence that orders routed to the automatic execution system of NYSE (as opposed to those routed to the floor auction process) display extreme impatience, yet they appear to be the less information sensitive ones – which indicates that the informed traders need not be impatient in taking advantage of their superior information;

Toth et al. (2011) present an empirical study based on a database where the broker that initiates an order book event can be identified. They find that brokers are very heterogeneous in liquidity provision: some are consistently liquidity providers while others are consistently liquidity takers. The behavior of brokers is strongly conditioned on the actions of other brokers, while they are only weakly influenced by the impact of their own previous orders.

Ranaldo (2004) finds evidence for the waiting costs as the explanation of the choice between limit orders and market orders: limit traders become more aggressive when (1) the own side of the book is thicker; (2) the opposite side of the book is thinner; (3) the bid-ask spread is wider; and (4) the temporary volatility increases. Ranaldo (2004) also finds experimental evidence for Foucault (1999): the bid-ask spread increases in price volatility. Ahn, Bae, and Chan (2001) show that market depth (volume of limit order submissions) increases with transitory volatility, and transitory volatility declines after an increase in market depth. If transitory volatility arises from, e.g., the ask side, investors submit more sell limit orders than sell market orders. They argue that this is simply evidence for limit order traders who are ready to supply liquidity when this is needed.

Menkhoff, Osler, and Schmeling (2010) identify informed traders by high trading activity and by central trading location. The paper provides evidence from an foreign exchange market that informed traders take advantage of market conditions such as widening spreads

or higher volatility, in which case they shift strongly towards limit orders, while uninformed traders respond modestly or not at all. The same is true for changes in market momentum, or changes in depth, but less so for changes in the expected time to execution. Anand, Chakravarty, and Martell (2005) identify informed traders by institutional investors, and find that informed traders use market orders at the beginning of the trading day, and limit orders later in the day. Furthermore, limit orders submitted by informed traders perform better than limit orders submitted by uninformed (i.e., individual) traders.

Bae, Jang, and Park (2003) find that traders submit more limit orders relative to market orders when the spread is large (also found by Chung, Van Ness, and Van Ness (1999)); when the order size is large; and when traders expect high transitory price volatility. Goldstein and Kavajecz (2000) find that during circuit breakers and extreme market movements in the NYSE, limit order traders are significantly less willing to supply liquidity and migrate to the floor.

Harris and Hasbrouck (1996) provide evidence from the NYSE SuperDOT limit order book that limit orders perform better than market orders, even after introducing a penalty for unexecuted orders. A similar result is obtained by Wald and Horrigan (2005), who also consider the investor type (information, risk aversion).

Hollifield, Miller, and Sandås (2004) test empirically whether optimal order submission is monotone in the trader's valuation for the asset. The idea is that, due to the tradeoff between price and execution costs, the trader's strategy should depend on the trader's private valuation: if the valuation is above a threshold the trader submits a *BMO*; if the valuation is not that high, the trader submits a *BLO*; and so on. The paper does not reject the monotonicity restriction for buys and sells considered separately, but it does reject when buys and sells are considered jointly. The expected payoffs from submitting limit orders with low execution probabilities are too low relative to the expected payoffs from submitting limit orders with high execution probabilities. This may be due to not being able to properly model monitoring costs for limit orders with low execution probabilities.

Hollifield, Miller, Sandås, and Slive (2006) use data from the Vancouver exchange to estimate the gains from trade. They find that traders with more extreme private values usually submit orders with low execution risk and low picking-off risk, while traders with moderate private values submit limit orders with higher execution risk and higher picking-off

risk.

Lo and Sapp (2005) analyze in an ordered probit model the choice of both price aggressiveness and quantity. They find a tradeoff between the two dimensions: more aggressive orders are smaller in size. The competition from increased depth on the same side of the market leads to less aggressive orders in smaller size.

5   Questions for Future Research

As Parlour and Seppi (2008) point out, we know relatively little about limit order markets. In particular, the question about how informed agents choose to trade in such markets is crucial. The extant models are stylized, and it is important to know which findings remain robust to a more realistic specification.

For tractability, most models of limit order markets assume one-unit trading, with risk-neutral traders. If instead risk-neutral agents were allowed to trade more than one unit, they would clear the whole limit order book up to the perceived fundamental value. In fact, under risk-neutrality even a noisy signal would determine informed traders to use very aggressive order submission strategies. A solution is to allow multi-unit trading, and at the same time to introduce risk aversion in order to make agents trade less. Risk aversion however complicates the problem, because one has to explicitly model inventories of risk-averse traders.

Modeling inventories is also related to the possibility of market makers arising endogenously in limit order markets. In an experimental study, Bloomfield, O'Hara, and Saar (2005) show that informed traders tend to assume both the role of speculators on their private information, and of market makers. Thus, market making arises endogenously in markets, and cannot be ignored. But allowing traders to perform the role of market makers runs into the difficult problem of modeling inventories.

Understanding the reasons to trade is crucial if we want to understand order choice. Do people trade to hedge non-tradable income, or because the investment opportunity sets have shifted? Do they trade because of liquidity shocks? What is the role of delegated asset management? Without an answer to these more general questions, it is difficult to give a definitive answer to the order choice problem.

On the empirical side, endogeneity is a key issue. For example, several studies, e.g.,

Ranaldo (2004), find that the bid-ask spread increases in volatility. But it is not clear which way the causality goes. The theoretical story, e.g., in Foucault (1999), is that volatility determines the bid-ask spread. However, the converse might also hold for mechanical reasons: prices are more volatile when the bid-ask spread is wider.

Another important topic is to find a way to separate waiting costs from information, both theoretically and empirically. Limit orders suffer from both execution costs (waiting costs, monitoring costs) and adverse selection (asymmetric information). Agents can be patient or impatient, and also informed and uninformed. Therefore, being able to identify the different types of costs and the strategies of the various types of traders in limit order markets is a key topic for future research.

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