Internet Appendix for "Quoting Activity and the Cost of Capital"

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This document includes supplementary material to the paper. Section 1 provides tables on the robustness of some of our empirical results. Section 2 shows that under certain assumptions, the traders' order flow is approximately of the linear form described in equation (5) in the paper. Section 3 analyzes a version of the baseline model in which there is only one trader, selected at random from the population described in Section 2 of this Internet Appendix. Section 4 analyzes an extension of the baseline model in the paper to multiple dealers. Section 5 analyzes an extension of the baseline model in the paper to multiple trading rounds. Section 6 provides micro-foundations for the dealer's precision function in the multi-trade model of Section 5 in this Internet Appendix.

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Contents

1 Additional Tables and Figures

In this section, we provide additional tables related to the robustness of some of our empirical results.

FIGURE IA.1

Portfolio Alphas for Different Holding Horizons and Formation Periods

The figure shows the long-short alpha for the difference between risk-adjusted returns for low-quote-to-trade ratio (QT1) and high-quote-to-trade ratio (QT25) portfolios for 25 QT-sorted portfolios across different holding and formation periods. The alphas are estimated using the FF4+PS model (market, size, value, momentum and the Pástor and Stambaugh (2003) traded liquidity factor). Stocks are assigned into portfolios based on their quote-to-trade ratio level over the past 1, 3, 6, and 12 months (formation period), and holding horizons range from 1 to 12 months.

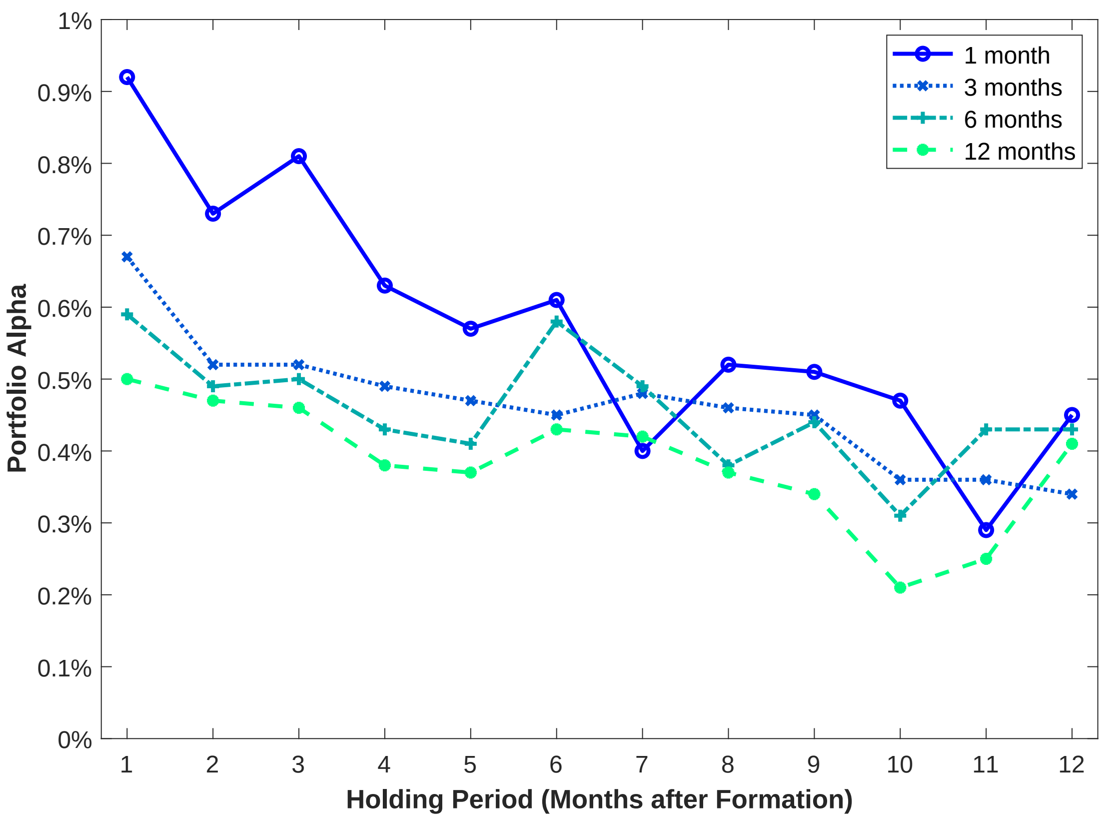

TABLE IA.1
Variable Description

$QUOTE_{i,t}$	Total number of best quote updates in stock i over period t. (Source: TAQ)
$TRADE_{i,t}$	Total number of trade executions in stock i over period t. (Source: TAQ)
$QT_{i,t} = \frac{QUOTE_{i,t}}{TRADE_{i,t}}$	Quote to trade ratio for stock i over period t. (Source: TAQ)
$R_{f,t}$	Risk free rate, one month Treasury bill rate. (Source: WRDS/Kenneth French Webpage)
$R_{m,t}$	Value weighted return on the market portfolio. (Source: WRDS/Kenneth French Webpage)
$R_{i,t}$, $R_{p,t}$	Return on stock i or portfolio p. (Source: WRDS/CRSP)
$r_{p,t} = R_{p,t} - R_{f,t}$	Excess return on portfolio p. (Source: WRDS/TAQ)
$r^a_{i,t}$	Risk-adjusted return on stock (or portfolio) i. (Source: WRDS/TAQ)
$r_{HML,t}$	Value factor constructed by Kenneth French. (Source: WRDS/Kenneth French Webpage)
$r_{SMB,t}$	Size factor constructed by Kenneth French. (Source: WRDS/Kenneth French Webpage)
$r_{UMD,t}$	Momentum factor (up-minus-down) constructed by Kenneth French. (Source: WRDS/Kenneth French Webpage)
$r_{Liq,t}$	Liquidity factor constructed by Pástor and Stambaugh (2003). (Source: WRDS)
$r_{PIN,t}$	PIN factor constructed by Easley et al. (2002). (Source: Sören Hvidkjaer Webpage)
$QSPREAD_{i,t}$	Quoted spread. Difference between best ask quote and best bid quote (measured in USD). (Source: TAQ)
$SPREAD_{i,t}$	Relative spread. The quoted spread divided by the mid-quote price (measured in %). (Source: TAQ)
$PRC_{i,t}$	Price in USD. (Source: WRDS/TAQ)
$USDVOL_{i,t}$	Trading volume (measured in mill. USD). (Source: WRDS/TAQ)
$VOLUME_{i,t}$	Share volume turnover (measured in thousand shares). (Source: WRDS/TAQ)
$ILR_{i,t}$	Amihud (2002) illiquidity ratio for stock i over period t calculated as $ILR_{i,t} = [\sum(USDVOL_{i,t})/\lvert r_{i,t}\rvert] \cdot 10^6$. (Source: WRDS/TAQ)
$R1$	Previous month return (Source: WRDS)
$R212$	Cumulative return from month $t-2$ to $t-12$. (Source: WRDS)
$VOLAT_{i,t}$	Return volatility for stock i calculated as absolute return over period t. (Source: WRDS/TAQ)
$IVOLAT_{i,t}$	Idiosyncratic volatility for stock i measured as the standard deviation of the residual from a three-factor Fama/French model on daily data as in Ang et al. (2009). (Source: WRDS/TAQ)
$MM_{i,t}$	Number of registered NASDAQ market makers in stock i
$MCAP_{i,t}$	Market Capitalization of a stock, calculated as the number of outstanding shares multiplied by price (measured in mill. USD).
$BM_{i,t}$	Book-to-Market value for stock i calculated as the log of the book value of equity divided by the market value of equity measured for the previous fiscal year.
$ANF_{i,t}$	The number of analysts following firm i in month t. (Source: IBES)
$INST_{i,t}$	Equity holdings of institutions in firm i at the end of year t, constructed from 13F files. (Source: WRDS)

TABLE IA.2
Sample Stock Characteristics

The table presents the monthly time-series averages of the cross-sectional 25th percentiles, means, medians, 75th percentiles, and standard deviations of the variables for the sample stocks. The sample period is June 1994 through December 2017, and only NYSE/AMEX and NASDAQ listed stocks are included in the sample. Stocks with a price less than USD 2, above USD 1000, or with less than 100 trades in month $t-1$ are removed. Stocks that change listings exchange, CUSIP or ticker symbol are removed. The description of variables is in Table IA.1.

	p25	Mean	Median	p75	Std.dev
Number of sample stocks (whole sample=10,670)	2,549	2,914	2,910	3,237	476
MCAP (in mill. USD)	96	3,819	367	1,568	18,305
PRC (in USD)	9	26	18	33	32
USDVOL (in mill. USD)	4	594	37	261	2,880
VOLUME (in 1,000 shares)	392	16,850	2,321	10,195	78,797
QUOTE (in 1,000)	1	281	31	243	687
TRADE (in 1,000)	0	38	4	25	126
QT	1.17	27.59	4.58	16.24	200.75
SPREAD (%)	0.13	1.65	0.66	2.20	2.59
QSPREAD	0.02	0.22	0.10	0.31	0.46
ILR (%)	0.02	4.40	0.16	1.41	26.09
VOLAT	0.00	0.03	0.01	0.03	0.07
BM (log)	0.30	0.70	0.53	0.86	0.97
r_i (indiv. stock midpoint excess returns, delist adj.)	-0.056	0.011	0.004	0.068	0.144
INST	0.228	0.489	0.498	0.732	0.298
$R1$ (lagged 1 month return in month $t-1$)	-0.055	0.015	0.005	0.070	0.152
$R212$ (cumulative returns month $t-12$ through $t-2$)	-0.106	0.140	0.112	0.338	0.480

TABLE IA.3

Characteristics of Quote-to-Trade Ratio Portfolios by Subsample

The table presents the monthly average characteristics for 10 quote-to-trade ratio (QT) portfolios constructed in month t in two subsamples. Portfolio 1 consists of stocks with the lowest QT and portfolio 10 consists of stocks with the highest QT in month t. Each portfolio contains on average 290 stocks. Stocks priced below \$2 or above \$1000 at the end of month t are removed. For each QT decile, we compute the cross-sectional mean characteristic for month t. The reported characteristics are computed as the time-series mean of the mean cross-sectional characteristic. Column (2) is the QT level, column (3) shows market capitalization (in million USD), columns (4) and (5) show the share volume (in million shares) and USD volume traded (in million USD), columns (6) and (7) show the quoted spread and relative spread (in % of the mid-quote), column (8) shows the Amihud illiquidity ratio (ILR) in %, column (9) shows volatility (calculated as the absolute monthly return in %) (VOLAT), column (10) shows price, column (11) shows the average Book-to-Market value measured at the end of the previous calendar year (BM), column (12) shows the average number of analysts following the stock (ANF), and column (13) shows the average institutional ownership (INST). Panel A presents results for the first half of the sample June 1994 to December 2005, and Panel B for the second half of the sample January 2006 to December 2017.

QT portf	QT	MCAP (mill.)	VOLUME (mill.) Shares	USD	SPREAD Quoted	Rel.(%)	ILR (%)	VOLAT (%)	PRC	BM	ANF	INST
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Panel A. Period 1994-2005												
1	0.6	4,801	34	975	0.199	2.02	3.40	4.99	15.4	0.59	11.60	0.38
2	0.9	1,746	9	267	0.227	2.27	4.13	3.90	16.2	0.62	7.87	0.39
3	1.2	1,575	6	212	0.250	2.42	4.79	3.68	17.8	0.63	7.13	0.39
4	1.4	1,582	5	200	0.283	2.57	5.38	3.56	19.4	0.63	6.76	0.39
5	1.7	1,573	5	182	0.330	2.80	6.74	3.41	21.1	0.65	6.25	0.38
6	2.3	2,251	5	213	0.392	3.04	9.32	3.23	22.3	0.71	6.34	0.37
7	3.6	4,178	8	338	0.362	2.73	7.74	2.38	23.1	0.80	10.19	0.42
8	6.1	2,648	5	219	0.373	2.41	4.76	1.72	25.6	0.74	10.16	0.45
9	12.9	1,904	3	144	0.416	2.36	4.90	1.45	27.4	0.73	9.00	0.45
10	54.7	1,095	1	70	0.523	2.63	7.07	1.17	29.1	0.84	6.43	0.41
Panel B. Period 2006-2017												
1	2.5	17,947	144	3,171	0.021	0.23	0.52	2.84	16.9	0.69	21	0.67
2	5.8	10,142	43	1,815	0.039	0.37	1.41	2.55	23.8	0.63	17	0.65
3	8.5	8,181	28	1,418	0.047	0.40	1.76	2.23	29.5	0.62	16	0.66
4	11.3	6,498	20	1,114	0.051	0.41	1.81	1.97	33.3	0.61	15	0.66
5	14.4	4,818	14	829	0.057	0.44	1.98	1.91	35.8	0.60	13	0.66
6	18.5	3,518	10	604	0.067	0.49	2.08	1.80	36.9	0.63	12	0.64
7	24.5	2,261	6	381	0.078	0.57	2.37	1.80	36.0	0.66	10	0.60
8	35.1	1,372	4	213	0.093	0.72	3.11	1.78	33.1	0.72	7	0.55
9	60.2	848	2	109	0.130	0.95	4.08	1.77	30.2	0.80	5	0.46
10	325.0	502	1	40	0.261	1.50	7.66	1.74	31.7	1.17	3	0.36

TABLE IA.4
Additional Determinants of the Quote-to-Trade Ratio

The table shows panel regressions of the quote-to-trade ratio (QT) on different characteristics. The dependent variable is the monthly QT. The independent variables are: annual number of analysts following the stock (*ANF*), quarterly institutional ownership (*INST*), log-book-to-market as of the previous year end (*BM*), previous month return (*R*1), as well as contemporaneous (monthly) variables: log-market capitalization (*MCAP*), price (*PRC*), trading volume in mill. U.S. dollars (*USDVOL*), Amihud illiquidity ratio (*ILR*), relative bid-ask spread (*SPREAD*), idiosyncratic volatility (*IVOLAT*) measured as the standard deviation of the residuals from a FF3 regression of daily raw returns within each month as in Ang et al. (2009), number of NASDAQ market makers (*MM*); *AQ* is a dummy variable that takes the value one after the staggered introduction of Autoquote; *SSBAN* is a dummy variable that takes the value one during the 2008 U.S. short-selling ban; *TICK*1, *TICK*2 and *REG-NMS* are dummy variables that take the value one, respectively, after June 1997, May 2001, and December 2006; *TICKPILOT-x* is a dummy for stocks in the 2016 tick pilot experiment ("*t*" stands for "treated" and "*c*" for "control"). Time fixed-effects are at the month-year level. Standard errors are double-clustered at the stock and month-year level.

	(1)	(2)	(3)	(4)	(5)	(6)
ANF	0.14	-0.25**	-0.59***	-0.59***	-0.62***	-0.61***
	(1.09)	(-2.48)	(-5.30)	(-5.33)	(-5.39)	(-5.38)
INST	-53.44***	9.58**	-16.57**	-18.43**	-12.51**	-13.24**
	(-9.56)	(2.01)	(-2.40)	(-2.74)	(-2.21)	(-2.39)
BM	-3.06	2.11	-1.92	-1.93	-3.96	-3.96
	(-0.38)	(0.35)	(-0.33)	(-0.33)	(-0.54)	(-0.53)
*R*1	-2.90	-7.16***	-4.73*	-4.75*	-5.88**	-5.92**
	(-1.18)	(-2.65)	(-1.84)	(-1.86)	(-2.07)	(-2.09)
MCAP	1.00	2.98	-2.78	-2.49	-3.97	-3.80
	(0.34)	(1.42)	(-1.26)	(-1.13)	(-1.44)	(-1.39)
PRC	0.23***	0.14***	0.19***	0.19***	0.19**	0.19**
	(3.32)	(3.03)	(3.68)	(3.68)	(2.59)	(2.59)
USDVOL	-0.85***	-0.52***	-1.01***	-1.04**	-0.54***	-0.55***
	(-4.38)	(-3.04)	(-2.65)	(-2.66)	(-2.77)	(-2.82)
ILR	3.72	2.06	-1.10	-1.12	1.16	1.17
	(1.21)	(0.81)	(-0.45)	(-0.45)	(0.38)	(0.38)
SPREAD	-162.81***	-321.30***	-162.98***	-161.98***	-203.41***	-202.89***
	(-2.68)	(-6.30)	(-3.51)	(-3.49)	(-3.55)	(-3.54)
IVOLAT	-46.49	-163.56***	-146.74***	-146.48***	-134.22***	-134.05***
	(-1.53)	(-5.87)	(-4.87)	(-4.85)	(-4.07)	(-4.07)
MM	-1.40***				-0.33**	-0.33**
	(-10.16)				(-2.14)	(-2.18)
AQ		5.04**	-20.76***	-20.61***	-30.88***	-30.72***
		(2.00)	(-6.10)	(-6.03)	(-6.22)	(-6.20)
SSBAN		4.38	-12.62**	-12.85**	-13.84**	-13.95**
		(0.86)	(-2.31)	(-2.35)	(-2.00)	(-2.02)
*TICK*1			0.76	0.76	-0.37	-0.37
			(0.81)	(0.80)	(-0.26)	(-0.27)
*TICK*2			23.29***	23.42***	33.23***	33.32***
			(7.28)	(7.32)	(7.28)	(7.29)
REG-NMS			28.79***	29.28***	29.19***	29.44***
			(5.80)	(5.87)	(4.33)	(4.36)
TICKPILOT-t				-31.71***		-32.35***
				(-7.75)		(-6.57)
TICKPILOT-c				-27.36***		-23.10***
				(-7.19)		(-4.36)
Stock FE	YES	YES	YES	YES	YES	YES
Time FE	YES	NO	NO	NO	NO	NO
N	470,082	805,655	805,655	805,655	547,255	547,255
Adj. R^2	0.411	0.292	0.296	0.297	0.319	0.320

TABLE IA.5

Risk-Adjusted Returns for Quote-to-Trade Ratio Portfolios

The table shows risk-adjusted monthly returns for various portfolios sorted on the quote-to-trade ratio (QT). The alphas reported in the table are the intercepts (risk-adjusted returns) of regressions of portfolio returns on risk factors. The monthly returns of the QT portfolios are risk-adjusted using several asset pricing models: CAPM, Fama and French (1993) model (FF3), a model that adds the Pástor and Stambaugh (2003) traded liquidity factor (FF3+PS), a five factor model that adds a momentum factor (FF3+PS+MOM), the Fama and French (2015) five factor model (FF5), and a model that adds the PIN factor for the period June 1994 to December 2002 (FF3+PS+MOM+PIN). We show the alpha for the lowest and highest QT portfolios and the alpha for the difference in returns between the low and high portfolios. In Panel A, stocks are assigned to 25 portfolios based on their QT level in month t. Then returns are calculated for each portfolio for month $t+1$. Panel B shows stocks assigned to 50 portfolios. ***, **, and * indicate rejection of the null hypothesis that the risk-adjusted portfolio returns are significantly different from zero at the 1%, 5%, and 10% level, respectively.

	CAPM	FF3	FF3+PS	FF3+PS +MOM	FF5	FF3+PS +MOM+PIN
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: 25 QT portfolios						
α_1	0.87	1.10**	1.11**	1.91***	1.02***	1.94***
α_{25}	0.25	-0.21	-0.20	-0.02	-0.11	(0.04)
α(QT1-QT25)	0.62	1.31***	1.31**	1.92***	1.13***	1.98***
Panel B: 50 QT portfolios						
α_1	0.60	0.84**	0.84**	1.59***	1.01***	1.61***
α_{50}	0.15	-0.28	-0.29	-0.10	-0.10	(0.12)
α(QT1-QT50)	0.45	1.12**	1.13**	1.70***	1.11***	1.73***

TABLE IA.6

Fama–MacBeth Regressions Using $t-2$ Information

The table reports the Fama and MacBeth (1973) coefficients from a regression of risk-adjusted returns using the two-period lagged quote-to-trade ratio (QT). The firm characteristics are measured in month $t-2$, except $R1$ and $R212$. The variables included are: quote-to-trade ratio (QT), relative bid/ask spread ($SPREAD$), Amihud illiquidity ratio (ILR), market value of equity ($MCAP$), book to market ratio (BM) calculated as the log of the book value of equity divided by the market value of equity measured for the previous fiscal year, previous month return ($R1$), and the cumulative return from month $t-2$ to $t-12$ ($R212$), idiosyncratic volatility ($IVOLAT$) measured as the standard deviation of the residuals from a Fama and French (1993) three factor model regressed on daily raw returns within each month as in Ang et al. (2009), trading volume in mill. U.S. dollars ($USDVOL$), and price (PRC). All characteristics apart from returns are logged and all coefficients are multiplied by 100. The standard errors are corrected by using the Newey-West method with 12 lags. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

	(1)	(2)	(3)	(4)	(5)
Const.	0.004***	0.013***	0.010***	0.038***	0.027***
$QT_{i,t-2}$	-0.163***	-0.212***	-0.219***	-0.229***	-0.159***
$SPREAD_{i,t-2}$	0.168***		0.086**		0.028
$ILR_{i,t-2}$		0.098***	0.069***		-0.035
$MCAP_{i,t-2}$				-0.079	0.006
$BM_{i,t-2}$				0.018	-0.003
$R1_{i,t-2}$				-3.787***	-3.880***
$R212_{i,t-2}$				0.052	0.165
$IVOLAT_{i,t-2}$				-2.561	-8.167***
$USDVOL_{i,t-2}$				-0.082	-0.078
$PRC_{i,t-2}$					-0.349***
R^2	0.01	0.01	0.01	0.03	0.03
Time series (months)	278	278	278	278	278

TABLE IA.7

Stock Risk-Adjusted Returns and Quote-to-Trade Ratio Robustness

The table reports the Fama and MacBeth (1973) coefficients from several robustness regressions of risk-adjusted returns for single stocks, given by $r^a_{i,t} = r_{i,t} - \sum_{j=1}^{J} \beta_{i,j,t-1} F_{j,t}$. Excess return uses excess returns instead of the risk adjusted returns as the dependent variable. The firm characteristics are measured in month $t-1$. The variables included are: quote-to-trade ratio (QT), relative bid/ask spread ($SPREAD$), Amihud illiquidity ratio (ILR), market value of equity ($MCAP$), book to market ratio (BM) calculated as the log of the book value of equity divided by the market value of equity measured for the previous fiscal year, previous month return ($R1$), and the cumulative return from month $t-2$ to $t-12$ ($R212$), idiosyncratic volatility ($IVOLAT$) measured as the standard deviation of the residuals from a Fama and French (1993) three factor model regressed on daily raw returns within each month as in Ang et al. (2009), trading volume in mill. U.S. dollars ($USDVOL$), price (PRC), short interest (SI), quarterly institutional ownership ($INST$), and the annual number of analysts following the stock (ANF). All characteristics apart from returns are logged and all coefficients are multiplied by 100. The standard errors are corrected by using the Newey-West method with 12 lags. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

	(1)	(2)	(3)	(4)	Excess Return
Const.	0.033***	0.032***	0.029***	0.043***	0.033***
$QT_{i,t-1}$	-0.128**	-0.143***	-0.112**	-0.149***	-0.130***
$SPREAD_{i,t-1}$	-0.017	0.009	-0.026	0.011	-0.015
$ILR_{i,t-1}$	-0.006	-0.007	-0.005	-0.007	0.016
$MCAP_{i,t-1}$	-0.126*	-0.162**	-0.145*	-0.170**	-0.119
$BM_{i,t-1}$	0.017	0.044	0.028	0.025	0.101
$R1_{i,t-1}$	-3.373***	-3.443***	-3.465***	-3.307***	-2.047***
$R212_{i,t-1}$	0.055	0.064	0.036	0.106	0.433*
$IVOLAT_{i,t-1}$	-9.627***	-10.058***	-10.033***	-9.364***	-9.143**
$USDVOL_{i,t-1}$	0.143*	0.096	0.181**	0.038	0.112
$PRC_{i,t-1}$	-0.451***	-0.347***	-0.455***	-0.327***	-0.466***
$SI_{i,t-1}$	-0.096***		-0.098***		
$INST_{i,t-1}$		-0.115**	-0.097*		
$ANF_{i,t-1}$				0.009**	
R^2	0.04	0.04	0.04	0.04	0.06
Time series (months)	278	278	278	278	278

TABLE IA.8

Stock Risk-Adjusted Returns, Quote-to-Trade Ratio and Market Makers

The table reports the Fama and MacBeth (1973) coefficients from regressions of risk-adjusted monthly returns on firm characteristics for the NASDAQ and market makers subsample. The dependent variable is the risk-adjusted return $r^a_{i,t} = r_{i,t} - \sum_{j=1}^{J} \beta_{i,j,t-1} F_{j,t}$, where the risk factors $F_{j,t}$ come from the FF3+PS+MOM model (market, size, value, momentum and the Pástor and Stambaugh (2003) traded liquidity factor). Columns (1) and (2) report the results for the addition of the NASDAQ market makers as a conditioning variable, and Column (3) reports the results for the NASDAQ subsample. The firm characteristics are measured in month $t-1$. The characteristics included are: quote-to-trade ratio (QT), relative bid/ask spread ($SPREAD$), Amihud illiquidity ratio (ILR), log-market capitalization ($MCAP$), book-to-market ratio (BM) calculated as the natural logarithm of the book value of equity divided by the market value of equity from the previous fiscal year, previous month return ($R1$), cumulative return from month $t-2$ to $t-12$ ($R212$), idiosyncratic volatility ($IVOLAT$) measured as the standard deviation of the residuals from a FF3 regression of daily raw returns within each month as in Ang et al. (2009), trading volume in mill. U.S. dollars ($USDVOL$), price (PRC), and the number of NASDAQ market makers (MM). All characteristics apart from returns are logged and all coefficients are multiplied by 100. The standard errors are corrected by using the Newey-West method with 12 lags. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

	(1)	(2)	(3)
Const.	0.033***	0.030***	0.038***
$QT_{i,t-1}$	-0.128*		-0.113*
$SPREAD_{i,t-1}$	0.048	0.068	0.036
$ILR_{i,t-1}$	0.038	0.030	0.039
$MCAP_{i,t-1}$	-0.226**	-0.218**	-0.250**
$BM_{i,t-1}$	0.053	0.054	0.057
$R1_{i,t-1}$	-3.763***	-3.724***	-3.739***
$R212_{i,t-1}$	0.102	0.109	0.108
$IVOLAT_{i,t-1}$	-10.283**	-10.648**	-10.361**
$USDVOL_{i,t-1}$	0.212*	0.213*	0.203*
$PRC_{i,t-1}$	-0.496***	-0.556***	-0.473***
$MM_{i,t-1}$	-0.004	-0.001	
R^2	0.04	0.04	0.04
Time series (months)	278	278	278

TABLE IA.9

Stock Risk-Adjusted Returns and Quote-to-Trade Ratio Subsample

The table reports the Fama and MacBeth (1973) coefficients from regressions of risk-adjusted returns for single stocks, given by $r^a_{i,t} = r_{i,t} - \sum_{j=1}^{J} \beta_{i,j,t-1} F_{j,t}$ for different subsample period. The first period is the period before and after the introduction of Autoquote in 2002. "Pre-Autoquote" refer to the period from June 1994 to December 2002, and "Post-Autoquote" refers to the period from January 2003 to December 2017. The second period is the period before and after the introduction of Regulation NMS. "Pre-NMS" refer to the period from June 1994 to December 2006, and "Post-NMS" refers to the period from January 2007 to December 2017. The firm characteristics are measured in month $t-1$. The variables included are: quote-to-trade ratio (QT), relative bid/ask spread ($SPREAD$), Amihud illiquidity ratio (ILR), market value of equity ($MCAP$), book to market ratio (BM) calculated as the log of the book value of equity divided by the market value of equity measured for the previous fiscal year, previous month return ($R1$), and the cumulative return from month $t-2$ to $t-12$ ($R212$), idiosyncratic volatility ($IVOLAT$) measured as the standard deviation of the residuals from a Fama and French (1993) three factor model regressed on daily raw returns within each month as in Ang et al. (2009), trading volume in mill. U.S. dollars ($USDVOL$), and price (PRC). All characteristics apart from returns are logged and all coefficients are multiplied by 100. The standard errors are corrected by using the Newey-West method with 12 lags. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

	Autoquote		Reg NMS	
	Pre	Post	Pre	Post
Const.	0.031***	0.038***	0.029***	0.043***
$QT_{i,t-1}$	-0.181***	-0.143**	-0.125**	-0.191***
$SPREAD_{i,t-1}$	0.073	-0.008	0.085**	-0.051
$ILR_{i,t-1}$	0.045	-0.038	0.037	-0.058*
$MCAP_{i,t-1}$	-0.383***	0.001	-0.254***	-0.006
$BM_{i,t-1}$	0.308**	-0.122*	0.200*	-0.155*
$R1_{i,t-1}$	-4.987***	-2.436*	-4.001***	-2.626
$R212_{i,t-1}$	0.629**	-0.221	0.507**	-0.388
$IVOLAT_{i,t-1}$	-17.191***	-5.242	-14.197***	-4.323
$USDVOL_{i,t-1}$	0.428***	-0.158**	0.270**	-0.190**
$PRC_{i,t-1}$	-0.535**	-0.239**	-0.502***	-0.170
R^2	0.04	0.03	0.04	0.03
Time series (months)	99	178	146	131

2 Micro-Foundations of Order Flow

In this section, we provide assumptions under which the traders' order flow is approximately of the linear form described in equation (5) in the paper:

$$\text{(IA.1)} \qquad \begin{aligned} Q^b &= \frac{k}{2}(v-a) + \ell - m + \varepsilon^b, \quad \text{with} \quad \varepsilon^b \overset{IID}{\sim} \mathcal{N}(0, \Sigma_L/2), \\ Q^s &= \frac{k}{2}(b-v) + \ell + m + \varepsilon^s, \quad \text{with} \quad \varepsilon^s \overset{IID}{\sim} \mathcal{N}(0, \Sigma_L/2), \end{aligned}$$

where Q^b is the buy demand and Q^s is the sell demand. The proofs are in Section 2.3.

2.1 Environment

The risky asset is in positive net supply $M > 0$. There are two types of traders: investors and liquidity (or noise) traders. Liquidity traders are either buyers or sellers. When trading occurs, liquidity buyers submit to the exchange an aggregate buy order for L^b shares, and liquidity sellers submit an aggregate buy order for L^s shares. Both L^b and L^s have IID normal distribution $\mathcal{N}(\ell_L, \Sigma_L/2)$, therefore by subtracting the mean we decompose them as follows:

$$\text{(IA.2)} \qquad L^b = \ell_L + \varepsilon^b, \quad L^s = \ell_L + \varepsilon^s, \quad \text{with} \quad \varepsilon^b, \varepsilon^s \sim \mathcal{N}(0, \Sigma_L/2).$$

Investors have CARA utility with coefficient $A > 0$. A mass one of investors starts with an initial endowment in the risky asset that is normally distributed as $\mathcal{N}(M, \sigma_M^2)$.[1] Investors observe the asset value v before trading, and then trade on the exchange at the quotes set by the dealer: the ask quote a and the bid quote b. The asset liquidates at $v + u$, where u has a normal distribution $\mathcal{N}(0, \sigma_u^2)$.[2]

[1]In addition, suppose the liquidity traders' initial average endowment is $x_L = 0$. As the investors' average endowment is the asset supply M, market clearing implies that the dealer's initial inventory must be zero. If instead we allow other values of x_L, this section applies to any initial dealer inventory.

[2]The notation here is slightly different from the notation used in the paper, where v denotes the liquidation value. Here the liquidation value is $v + u$ and v is the forecast of the informed investors. This notation, however, is compatible with the rest of the paper as long as the dealer learns about v rather than $v + u$.

2.2 Equilibrium

Before we analyze the equilibrium, we describe the behavior of a CARA investor in the presence of ask and bid quotes. Define the lower target $\underline{X}$ and the upper target $\overline{X}$ by:

$$\text{(IA.3)} \qquad \underline{X} = \frac{v-a}{A\sigma_u^2}, \qquad \overline{X} = \frac{v-b}{A\sigma_u^2}.$$

Lemma IA.1 shows that a CARA investor trades only when his initial endowment in the risky asset is outside of the target interval $[\underline{X}, \overline{X}]$. In that case, he trades exactly so that his final inventory is equal to the closest target.

Lemma IA.1. *Consider a risky asset with liquidation value $v + u$, with $u \sim \mathcal{N}(0, \sigma_u^2)$, and a CARA investor with coefficient A who observes the value v and has endowment x_0 in the risky asset. The investor can buy any positive quantity at the ask quote a, or sell any positive quantity at the price b, where $a > b$. Suppose the risk-free rate is zero. Let $\underline{X}$ and $\overline{X}$ be defined as in (IA.3). Then, the investor's optimal trade makes his final inventory equal to either (i) $\underline{X}$, if $x_0 < \underline{X}$, (ii) x_0, if $x_0 \in [\underline{X}, \overline{X}]$, or (iii) $\overline{X}$, if $x_0 > \overline{X}$.*

Next, define the following numeric constants:

$$\text{(IA.4)} \qquad \rho_0 = \frac{1}{\sqrt{8\pi}} \approx 0.1995, \qquad \rho_1 = \frac{1}{2\pi} + \frac{1}{4} \approx 0.4092.$$

Proposition IA.1 provides an approximate expression for the aggregate order flow.

Proposition IA.1. *Investors submit aggregate orders Q^b and Q^s of the form:*

$$\text{(IA.5)} \qquad \begin{aligned} &Q^b \approx \frac{k}{2}(v-a) + \ell - m + \varepsilon^b, \qquad Q^s \approx \frac{k}{2}(b-v) + \ell + m + \varepsilon^s, \\ &\textit{with} \quad k = \frac{2\rho_1}{A\sigma_u^2}, \qquad \ell = \ell_L + \rho_0\sigma_M, \qquad m = \rho_1 M, \end{aligned}$$

and the error terms ε^b and ε^s are IID with normal distribution $\mathcal{N}(0, \Sigma_L/2)$. Both approximations in (IA.5) represent equality up to terms of the order of $1/\sigma_M$.

We thus provide micro-foundations for the order flow equations (IA.1). E.g., the imbalance parameter m arises from the fact that investors are risk averse and therefore are more likely to be sellers than buyers when the asset is in positive net supply (i.e., when $M > 0$).

The informed investors' precision $1/\sigma_u^2$ is a key determinant of the investor elasticity k. Intuitively, if the informed investors have more precise signals, they trade more aggressively and therefore their demands are more sensitive in the mispricing (k is larger). The informed investors' risk aversion A is also a determinant k: if informed investors are more risk averse (A is larger), they trade less aggressively and therefore their demands are less sensitive in the mispricing (k is smaller).

2.3 Proofs of Results

Proof of Lemma IA.1. This is a standard result in asset pricing, and therefore we only provide the intuition. First, suppose there is only one trading price p (the buy and sell prices are equal). Then, an investor with constant absolute risk aversion has an optimal target inventory of the form $X = \frac{v-p}{A\sigma_u^2}$. Therefore, regardless of his initial endowment x_0, the investor submits a market order such that his final inventory equals X. When the buy and sell prices are different, there are two targets corresponding to each price: $\underline{X} < \overline{X}$. A key fact is that the investor optimally must either buy at the ask, or sell at the bid, but not both.[3] In the first case, when the investor only buys, he behaves like a CARA agent that faces the ask quote a, hence optimally trades up to the lower target $\underline{X}$. For this trade to be a buy, however, his initial endowment x_0 must be below $\underline{X}$. Similarly, when x_0 is above the upper target $\overline{X}$, he sells down to $\overline{X}$. Finally, when x_0 is in between the two targets, there is no incentive to trade and the CARA agent's target inventory in this case remains equal to x_0. □

Proof of Proposition IA.1. We first introduce some notation. Define:

$$\begin{aligned} \phi(x) &= \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{x^2}{2}\right), \quad \Phi(x) = \int_{-\infty}^{x} \phi(t)\mathrm{d}t, \\ \psi(x) &= \Phi(-x)\Big(\phi(x) - x\Phi(-x)\Big), \end{aligned} \tag{IA.6}$$

where $\phi(x)$ is the standard normal density, and $\Phi(x)$ is the standard cumulative den-

[3] Because of the positive bid-ask spread, any quantity simultaneously bought and sold represents a deadweight loss.

sity. One can check that the function $\psi(x)$ defined in equation (IA.6) is positive and decreasing.

By assumption, there is a mass one of investors whose endowments are independent and distributed according to the normal distribution $\mathcal{N}(M, \sigma_M^2)$, with density function:

$$\text{(IA.7)} \qquad \phi_M(x) \;=\; \frac{1}{\sigma_M}\,\phi\Big(\frac{x-M}{\sigma_M}\Big).$$

Then, investors' endowments integrate to $\int_{-\infty}^{\infty} x\phi_M(x)\mathrm{d}x = M$, which, since the dealer has zero endowment, is indeed equal to the net supply of the risky asset.

To compute investor i's optimal demand, note that by assumption his liquidation value is $v+u$, where v is known by the investor, and $u \sim \mathcal{N}(0, \sigma_u^2)$ is unknown. Thus, investor i computes $\mathsf{E}(v+u|v) = v$ and $\mathsf{Var}(v+u|v) = \sigma_u^2$. Thus, the targets $\underline{X} = \frac{v-a}{A\sigma_u^2}$ and $\overline{X} = \frac{v-b}{A\sigma_u^2}$ are common to all investors.

According to Lemma IA.1, the optimal demand of an investor depends on his initial endowment. By assumption, traders' endowments are IID with density $\phi_M(x)$ as in (IA.7). Therefore, investors' aggregate buy market order is equal to $I^b = \underline{P}\int_{-\infty}^{\underline{X}}(\underline{X} - x)\phi_M(x)\mathrm{d}x$, where $\underline{P} = \int_{-\infty}^{\underline{X}}\phi_M(x)\mathrm{d}x$ is the mass of investors with endowments below $\underline{X}$. Similarly, investors' aggregate sell market order is equal to $I^s = \overline{P}\int_{\overline{X}}^{\infty}(x-\overline{X})\phi_M(x)\mathrm{d}x$, where $\overline{P} = \int_{\overline{X}}^{\infty}\phi_M(x)\mathrm{d}x$ is the mass of investors with endowments above $\overline{X}$. Finally, investors with endowments between $\underline{X}$ and $\overline{X}$ do not submit any order. We compute:

$$\text{(IA.8)} \qquad I^b \;=\; \psi\left(\frac{M-\underline{X}}{\sigma_M}\right), \qquad I^s \;=\; \psi\left(\frac{\overline{X}-M}{\sigma_M}\right),$$

where ψ is as in (IA.6. Consider the linear approximation of ψ near $x=0$:

$$\text{(IA.9)} \quad \psi(x) \;=\; \rho_0 - \rho_1 x + O(x^2), \quad \rho_0 \;=\; \psi(0) \;=\; \tfrac{1}{\sqrt{8\pi}}, \quad \rho_1 \;=\; -\psi'(0) \;=\; \tfrac{1}{2\pi} + \tfrac{1}{4},$$

where $O(x^2)$ represents the standard "big O" notation.[4] The investors' aggregate buy order is thus $I^b = \rho_0\sigma_M + \rho_1(\underline{X} - M) + O(1/\sigma_M) = \frac{\rho_1}{A\sigma_u^2}(v-a) + \rho_0\sigma_M - \rho_1 M + O(1/\sigma_M)$. Also, from (IA.2), the liquidity buyers' aggregate order is $L^b = \ell_L + \varepsilon^b$, with $\varepsilon^b \sim \mathcal{N}(0, \Sigma_L/2)$. By adding I^b and L^b, we obtain that the aggregate traders' buy order,

[4] This means that there is a number $B > 0$ such that $\big|\psi(x) - (\rho_0 - \rho_1 x)\big| < Bx^2$.

$Q^b = I^b + L^b$, satisfies:

$$Q^b \;=\; \frac{\rho_1}{A\sigma_u^2}(v-a) + \big(\ell_L + \rho_0\sigma_M\big) - \rho_1 M + \varepsilon^b + O(1/\sigma_M). \tag{IA.10}$$

Let $k = \frac{2\rho_1}{A\sigma_u^2}$, $\ell = \ell_L + \rho_0\sigma_M$, $m = \rho_1 M$. Thus, we have $Q^b = \frac{k}{2}(v-a) + \ell - m + \varepsilon^b + O(1/\sigma_M)$ and similarly $Q^s = \frac{k}{2}(b-v) + \ell + m + \varepsilon^s + O(1/\sigma_M)$. This proves (IA.5). □

3 Random Trader Arrivals

In this section we consider the same baseline setup as in the paper, except that the aggregate demands are no longer assumed to be of the form:

$$\text{(IA.11)} \qquad \begin{aligned} Q^b &= \frac{k}{2}(v-a) + \ell - m + \varepsilon^b, \quad \text{with} \quad \varepsilon^b \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right), \\ Q^s &= \frac{k}{2}(b-v) + \ell + m + \varepsilon^s, \quad \text{with} \quad \varepsilon^s \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right). \end{aligned}$$

Instead, in the spirit of Glosten and Milgrom (1985), we allow traders to be selected at random from the population. Specifically, at the trading time τ a trader is randomly selected from the population described in Section 2 of this Internet Appendix. The main result of this section is that the equilibrium in this new setup remains qualitatively the same. The proofs are in Section 3.3 of this Internet Appendix.

3.1 Environment

At trading time τ a trader is selected at random and with equal probability he is an investor or a liquidity trader. If a liquidity trader is selected, he submits either a buy order or a sell order with equal probability, and the quantity is randomly chosen from the normal distribution $\mathcal{N}(\ell_L, \Sigma_L/2)$. If an investor is selected, his initial endowment is randomly chosen from the normal distribution $\mathcal{N}(M, \sigma_M^2)$, where M is the supply of the risky asset. Investors have CARA utility with coefficient A. Investors observe the asset value v before trading, and then trade on the exchange at the quotes set by the dealer: the ask quote a and the bid quote b. The asset liquidates at $v + u$, where u has a normal distribution $\mathcal{N}(0, \sigma_u^2)$. Finally, the dealer maximizes:[5]

$$\mathsf{E}_\tau \Big(x_0 \, v \; + \; \big((v-b) Q^s + (a-v) Q^b \big) \; - \; \gamma \, x_\infty^2 \Big), \tag{IA.12}$$

where Q^s is the random quantity sold by the trader, Q^b is the random quantity bought by the trader, and x_∞ is the final inventory of the dealer:

$$x_\infty \; = \; x_0 - Q^b + Q^s. \tag{IA.13}$$

3.2 Equilibrium

The investor's equilibrium behavior is the same as in Section 2 of this Internet Appendix: his optimal trade depends on how his initial endowment is positioned relative to the "lower target" $\underline{X}$ and the "upper target" $\overline{X}$ defined by:

$$\underline{X} \; = \; \frac{v-a}{A\sigma_u^2}, \qquad \overline{X} \; = \; \frac{v-b}{A\sigma_u^2}. \tag{IA.14}$$

The investor trades only when his initial endowment in the risky asset is outside of the target interval $[\underline{X}, \overline{X}]$. In that case, he trades exactly so that his final inventory is equal to the closest target. E.g., if the investor's initial endowment x is below $\underline{X}$, then the investor submits a buy order for $\underline{X} - x$. Ex ante, an initial endowment x occurs with probability density:

$$\phi_M(x) \; = \; \frac{1}{\sigma_M} \, \phi\Big(\frac{x-M}{\sigma_M} \Big), \tag{IA.15}$$

where ϕ is the standard normal density.

Proposition IA.16 describes approximately the dealer's equilibrium behavior when the volatility σ_M of the investors' initial endowment is large. For simplicity, we consider

[5] We ignore the dealer's monitoring costs in this analysis. Also, recall that for simplicity in Section 2 of this Internet Appendix we have assumed that the asset liquidates at $v+u$, while the investors observe the signal v. Since, however, u appears linearly in the objective function, it can be ignored.

only the case when the dealer's initial inventory is $x_0 = 0$. Define:

$$k \;=\; \frac{1}{A\sigma_u^2}, \qquad \ell \;=\; \frac{\ell_L + 2\phi(0)\gamma k \sigma_M}{6(1+\gamma k)}, \qquad m \;=\; \frac{M}{2}, \tag{IA.16}$$

where $\phi(0) = 1/\sqrt{2\pi}$.

Proposition IA.2. *Suppose the dealer has initial inventory $x_0 = 0$ and forecast w. Then the dealer's optimal quotes are:*

$$a \;=\; w + h - \delta + O\Big(\frac{1}{\sigma_M}\Big), \qquad b \;=\; w - h - \delta + O\Big(\frac{1}{\sigma_M}\Big), \tag{IA.17}$$

where h and δ are given by:

$$h \;=\; \frac{\ell}{k}, \qquad \delta \;=\; \frac{m}{k}\,\frac{1+2\gamma k}{1+\gamma k}, \tag{IA.18}$$

and k, ℓ, m are as in (IA.16).

Proposition IA.2 shows that the behavior of the dealer is the same as in the baseline model (see Proposition 1 in the paper). The only difference lies in the formulas for the coefficients k, ℓ, m. Recall that in Section 2 of this Internet Appendix, we provide micro-foundations for the aggregate demand equations (IA.11). In that context, the coefficients are:

$$\begin{aligned} k \;&=\; \frac{2\rho_1}{A\sigma_u^2}, \qquad \ell \;=\; \ell_L + \rho_0 \sigma_M, \qquad m \;=\; \rho_1 M, \\ \rho_0 \;&=\; \frac{1}{\sqrt{8\pi}} \approx 0.1995, \qquad \rho_1 = \frac{1}{2\pi} + \frac{1}{4} \approx 0.4092 \end{aligned} \tag{IA.19}$$

(see equation (IA.5) in the paper). Note that the formulas in (IA.19) are similar to the formulas in (IA.16), indicating that the random arrival model in this section produces results that are qualitatively the same as in the baseline model in the paper.

3.3 Proofs of Results

Proof of Proposition IA.2. The dealer's choice variables are the quotes a and b, or equivalently the half spread $h = (a-b)/2$ and pricing discount $\delta = w - (a+b)/2$. The dealer's forecast error is $e = v - w$, which has a normal distribution $e \sim \mathcal{N}(0, G)$, where $G = 1/F > 0$ is the dealer's inverse precision function (given by monitoring). As in the baseline model, one can show that G does not influence the optimal choice of h and δ, and hence we can set from the beginning $G = 1$. Thus, we assume that the dealer's forecast error has a standard normal distribution:

$$e \;=\; v - w \;\sim\; \mathcal{N}(0,1). \tag{IA.20}$$

At time τ a trader arrives, which can be an investor with probability 1/2, or a liquidity trader with probability 1/2. Let x be the investor's initial endowment. Then, Lemma IA.1 in Section 2 of this Internet Appendix shows that the trader submits the following quantities Q^b and Q^s:

$$\begin{aligned}
Q^b &= \ell_L + \varepsilon^b, \quad Q^s \;=\; 0, \quad \text{with probability } 1/2,\\
Q^b &= 0, \quad Q^s \;=\; \ell_L + \varepsilon^s, \quad \text{with probability } 1/2,\\
Q^b &= \underline{X} - x, \quad Q^s \;=\; 0, \quad \text{with probability } \tfrac{1}{2}\textstyle\int_{-\infty}^{\underline{X}} \phi_M(x)\mathrm{d}x,\\
Q^b &= 0, \quad Q^s \;=\; 0, \quad \text{with probability } \tfrac{1}{2}\textstyle\int_{\underline{X}}^{\overline{X}} \phi_M(x)\mathrm{d}x,\\
Q^b &= 0, \quad Q^s \;=\; x - \overline{X}, \quad \text{with probability } \tfrac{1}{2}\textstyle\int_{\overline{X}}^{\infty} \phi_M(x)\mathrm{d}x,
\end{aligned} \tag{IA.21}$$

where ε^b and ε^s are IID with normal distribution $\mathcal{N}(0, \Sigma_L/2)$, and $\phi_M(x)$ is the density function in (IA.15). Substituting the formulas for e, h and δ in (IA.12) and setting $x_0 = 0$, it follows that the dealer maximizes:

$$\mathsf{E}_{e,x}\Big(\big((h+\delta+e)Q^s + (h-\delta-e)Q^b\big) \;-\; \gamma\,(Q^s - Q^b)^2\Big), \tag{IA.22}$$

where $e \sim \mathcal{N}(0, G)$ and $x \sim \mathcal{N}(M, \sigma_M^2)$. Using the formulas in (IA.21), we recompute the dealer's objective function after multiplying by 2 and removing the terms that do not involve h or δ. Hence, the dealer maximizes:

$$\begin{aligned}
V \;=\; h\ell_L &+ \int_{-\infty}^{+\infty}\int_{-\infty}^{\underline{X}} \Big((h-\delta-e)(\underline{X}-x) - \gamma(x-\underline{X})^2\Big)\,\phi_M(x)\mathrm{d}x\,\phi(e)\mathrm{d}e\\
&+ \int_{-\infty}^{+\infty}\int_{\overline{X}}^{+\infty} \Big((h+\delta+e)(x-\overline{X}) - \gamma(x-\overline{X})^2\Big)\,\phi_M(x)\mathrm{d}x\,\phi(e)\mathrm{d}e
\end{aligned} \tag{IA.23}$$

where

$$\text{(IA.24)} \qquad \underline{X} = k(\delta - h + e), \qquad \overline{X} = k(\delta + h + e), \qquad k = \frac{1}{A\sigma_u^2}.$$

By computing first the inner integral (with respect to x), we obtain a linear combination of terms of the form $\phi\big(\frac{\underline{X}-M}{\sigma_M}\big)$, $\Phi\big(\frac{\underline{X}-M}{\sigma_M}\big)$, $\phi\big(\frac{M-\overline{X}}{\sigma_M}\big)$ and $\Phi\big(\frac{M-\overline{X}}{\sigma_M}\big)$, with coefficients which are polynomial in e and the choice variables h and δ. Write:

$$\text{(IA.25)} \qquad \begin{aligned} &\frac{\underline{X} - M}{\sigma_M} = \alpha_1 e + \beta_1, \qquad \frac{M - \overline{X}}{\sigma_M} = \alpha_2 e + \beta_2, \\ &\alpha_1 = \frac{k}{\sigma_M}, \quad \beta_1 = \frac{k(\delta - h) - M}{\sigma_M}, \quad \alpha_2 = -\frac{k}{\sigma_M}, \quad \beta_2 = \frac{M - k(\delta + h)}{\sigma_M}. \end{aligned}$$

Thus, to comput the outer integral (with respect to e), we need to be able to compute:

$$\text{(IA.26)} \qquad I_n = \int_{-\infty}^{+\infty} u^n \phi(\alpha u + \beta)\phi(u)\mathrm{d}u, \qquad J_n = \int_{-\infty}^{+\infty} u^n \Phi(\alpha u + \beta)\phi(u)\mathrm{d}u.$$

Note that $\phi'(u) = -u\phi(u)$. We now perform (i) direct computation for $n = 1$, and (ii) integration by parts to obtain recursive formulas for I_n and J_n.[6] We get:

$$\text{(IA.27)} \qquad \begin{aligned} I_0 &= \frac{1}{\sqrt{\alpha^2+1}}\,\phi\left(\frac{\beta}{\sqrt{\alpha^2+1}}\right), \qquad I_n = \frac{(n-1)I_{n-2} - \alpha\beta I_{n-1}}{\alpha^2+1}, \\ J_0 &= \Phi\left(\frac{\beta}{\sqrt{\alpha^2+1}}\right), \qquad J_n = (n-1)J_{n-2} + \alpha I_{n-1}, \end{aligned}$$

where $I_{-1} = J_{-1} = 0$. We obtain:

$$\text{(IA.28)} \qquad I_1 = -\frac{\alpha\beta}{\alpha^2+1} I_0, \qquad J_1 = \alpha I_0, \qquad J_2 = J_0 - \frac{\alpha^2\beta}{\alpha^2+1} I_0.$$

Using these formulas, we compute the dealer's objective function. Up to terms that do not depend on h and δ, the objective function is equal to:

$$\text{(IA.29)} \quad V_1 = \big(2\phi(0)\gamma k\sigma_M + \ell_L\big)h - 3k(1+\gamma k)\big(h^2+\delta^2\big) + 3M(1+2\gamma k)\delta + O(1/\sigma_M).$$

[6]The formula for I_0 is computed by noticing that $\phi(u)$ and $\phi(\alpha u + \beta)$ are log-quadratic in u. The formula for J_0 is obtained by noticing that I_0 is the differential of J_0 with respect to β.

This is a linear-quadratic problem in h and δ, therefore up to terms of the order of $1/\sigma_M$, the unique solution is:

$$\text{(IA.30)} \qquad h \;=\; \frac{\ell_L + 2\phi(0)\gamma k \sigma_M}{6k(1+\gamma k)}, \qquad \delta \;=\; \frac{M(1+2\gamma k)}{2k(1+\gamma k)}.$$

Using the notations in (IA.16), we obtain the formulas in (IA.18), which finishes the proof. □

4 Model with Multiple Dealers

In this section we provide an extension to multiple dealers of our baseline model in Section III.A of the paper. The main result of this section is that the equilibrium in this new setup remains qualitatively the same. The proofs are in Section 4.5 of this Internet Appendix.

4.1 Monitoring with Unique Signal

Corollary IA.1 shows that the equilibrium described in the paper (with one trading round and monitoring at a constant rate q), the equilibrium is essentially the same if we replace the monitoring process by a unique signal with the appropriate precision.

Corollary IA.1. *Suppose instead of monitoring at the rate q and receiving signals with precision $F(q)$ the dealer receives a unique signal with precision*

$$\text{(IA.31)} \qquad \tilde{F}(q) \;=\; \frac{qF(q)}{\ln(q+1)}.$$

Then, in the new equilibrium the dealer chooses the same half spread h, pricing discount δ, and monitoring rate q.

From the previous section it is clear that the equilibrium half spread and pricing discount do not depend on the dealer's signal structure. Thus, the main statement of Corollary IA.1 is the equivalence of monitoring rates under the two different signal structures. In particular, if we choose the monitoring precision $F(q) = f\ln(q+1)$ as in (13), the equivalent signal precision becomes linear: $\tilde{F}(q) = fq$. In the Internet

Appendix we use this equivalent formulation to simplify the presentation of the various extensions of our model.

4.2 Environment

The market is composed of one risk-free asset and one risky asset. Trading in the risky asset takes place in a market exchange based on the mechanism described below. There are two types of market participants: (a) $N \geq 1$ market makers called dealers (individually referred to as "she") who monitor the market and set ask and bid quotes at which others trade, and (b) traders, who submit market orders.

Assets. The risk-free asset is used as numeraire and has a return of zero. After trading takes place, the risky asset liquidates at a value of v per share called the fundamental value or asset value. The random variable v has a normal distribution $v \sim \mathcal{N}(v_0, \sigma_v^2)$, where σ_v is the fundamental volatility.

Trading. Trading occurs at at the first arrival τ in a Poisson process with frequency parameter normalized to one. Dealers continuously submit ask and bid quotes to the exchange. If at time t dealer $i = 1, \ldots, N$ submits an ask quote a_i and a bid quote b_i, the exchange computes the average ask and bid quotes:

$$a = \frac{1}{N} \sum_{i=1}^{N} a_i, \qquad b = \frac{1}{N} \sum_{i=1}^{N} b_i. \tag{IA.32}$$

Suppose a is the ask quote, and b is the bid quote at the trading time τ. Then, traders submit an aggregate buy market order Q^b and an aggregate sell market order Q^s according to:

$$\begin{aligned} Q^b &= \frac{k}{2}(v - a) + \ell - m + \varepsilon^b, \quad \text{with} \quad \varepsilon^b \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right), \\ Q^s &= \frac{k}{2}(b - v) + \ell + m + \varepsilon^s, \quad \text{with} \quad \varepsilon^s \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right), \end{aligned} \tag{IA.33}$$

where k, ℓ, m and Σ_L are exogenous constants. The parameter k is the investor elasticity, ℓ is the inelasticity parameter, and m is the imbalance parameter.[7] The quantity Q^b is

[7] Micro-foundations for the formulas in (IA.33) are provided in Section 2 in this Internet Appendix.

the buy demand and Q^s the sell demand. Together, Q^b and Q^s are called the traders' order flow.

Allocation Mechanism. After the exchange receives the market orders in (IA.33), dealer $i = 1, \dots, N$ receives a buy order Q_i^b and a sell order Q_i^s, where:

$$\text{(IA.34)} \quad \begin{aligned} Q_i^b &= \frac{1}{N}\left(\frac{k}{2}\Big((v-a)+\mu(a-a_i)\Big)+\ell-m+\varepsilon^b\right) = \frac{1}{N}\,Q^b + \frac{k\mu}{2N}(a-a_i), \\ Q_i^s &= \frac{1}{N}\left(\frac{k}{2}\Big((b-v)+\mu(b_i-b)\Big)+\ell+m+\varepsilon^s\right) = \frac{1}{N}\,Q^s + \frac{k\mu}{2N}(b_i-b), \end{aligned}$$

which executes, respectively, at the effective quotes $a_{e,i}$ and $b_{e,i}$:[8]

$$\text{(IA.35)} \qquad a_{e,i} = a + \nu(a_i - a), \qquad b_{e,i} = b + \nu(b_i - b).$$

The constants $\mu, \nu \in [0, 1]$ are fixed by the exchange from the beginning. In order to allow for many possible allocation mechanisms, we regard both μ and ν as free parameters. Intuitively, μ measures how strongly a dealer's quotes affect the order flow she receives; at one end the case $\mu = 1$ can be regarded as representing decentralized quoting (each dealer's quotes attract individual order flow), while $\mu = 0$ can be regarded as centralized quoting (the order flow that results from the average quotes simply gets divided equally among dealers). The parameter ν measures how sensitive the actual trading prices ($a_{e,i}$ and $b_{e,i}$) are to the initial dealer quotes (a_i and b_i); at one end the case $\nu = 1$ can be regarded as representing decentralized trading (each dealer trades at the quotes posted by her), while $\nu = 0$ can be regarded as centralized trading (dealers trade at identical prices, equal to the average quotes).

Dealer Monitoring. Dealer $i = 1, \dots, N$ monitors the fundamental value according to mutually independent Poisson processes with frequency $q_i > 0$ called the monitoring frequency (or monitoring rate). We assume that each dealer reveals her signal to the other dealers.[9] Thus, each dealer receives signals according to a Poisson process with

[8] Note that the quantities Q_i^b and Q_i^s indeed sum up, respectively, to Q^b and Q^s, and the effective quotes $a_{e,i}$ and $b_{e,i}$ average out, respectively, to a and b. Nevertheless, the dollar amounts $Q_i^b a_{e,i}$ and $Q_i^s b_{e,i}$ do not add up, respectively, to $Q^b a$ and $Q^s b$ unless dealers submit identical quotes—which is true in a symmetric equilibrium.

[9] We do not specify the mechanism by which these signals become known by the other dealers. One possibility is that in equilibrium each dealer's quotes are in one-to-one correspondence to her last signal,

frequency:

$$q = \sum_{i=1}^{N} q_i. \tag{IA.36}$$

Using Corollary IA.1 in the paper, we take a reduced form approach and replace the signals obtained from monitoring at the frequency q with a unique signal with precision:

$$F(q) = \frac{1}{\mathsf{Var}(v - w)}, \tag{IA.37}$$

where w is the dealer's forecast after observing the signal. We assume that $F(q)$ is increasing in q. Intuitively, an increase in the aggregate monitoring rate produces more precise forecasts for the dealers. The cost for dealer i of monitoring at the rate q_i is $C(q_i)$, and is paid only once at $t = 0$, before monitoring begins.

To simplify the equilibrium formulas, we assume that the precision function F and the monitoring cost C are linear increasing functions:

$$F(q) = f\,q, \qquad C(q_i) = c\,q_i, \tag{IA.38}$$

where f and c are positive constants.

Dealers' Quotes and Objective. Each time dealer i monitors, she sets the ask and bid quotes. Therefore, we interpret the monitoring rate q_i as dealer i's quote rate. Because monitoring is considered here in reduced form, we are interested only in the quotes (a_i, b_i) that are prevalent when trading occurs at τ. Thus, a quoting strategy for dealer i is a pair (a_i, b_i) where a_i is the ask quote and b_i is the bid quote. Each dealer starts with an initial inventory in the risky asset equal to x_0.[10] Let Q_i^b and Q_i^s be the buy and sell quantities, respectively, that dealer i trades according to the previously described allocation mechanism. Then dealer i's inventory after trading is:

$$x_{i,\text{end}} = x_0 - Q_i^b + Q_i^s. \tag{IA.39}$$

which can therefore be inferred by the others. However, deviations from equilibrium could make the other dealers infer a different signal. We avoid this type of complication by simply assuming it away.

[10]As in the paper, we first let the initial inventory x_0 as a free parameter, and later we set it equal to a particular value called the neutral (or preferred) inventory.

Then, given a quoting strategy (a_i, b_i) and a monitoring rate q_i, dealer' i's expected utility before trading at τ is equal to the expected profit minus the quadratic penalty in the inventory and minus the monitoring costs:

$$\mathsf{E}_\tau \left(x_0\, v \;+\; \big((v - b_{e,i}) Q^s + (a_{e,i} - v) Q^b \big) \;-\; \gamma\, x_{i,\text{end}}^2 \;-\; C(q_i) \right), \tag{IA.40}$$

where the parameter $\gamma > 0$ is dealer i's "inventory aversion," and $a_{e,i}$, $b_{e,i}$ are the effective quotes at which dealer i trades, as in (IA.35).

Equilibrium Concept. The structure of the game is as follow: First, each dealer $i = 1, \ldots, N$ chooses a constant monitoring rate q_i. Second, in the trading game dealer i chooses the ask quote a_i and the bid quote b_i such that objective function (IA.40) is maximized. After observing the dealers' quotes, the traders submit their order flow, which is then allocated to the dealers according to the allocagtion mechanism.

4.3 Equilibrium

Proposition IA.3 describes the equilibrium behavior of the N dealers. As in the baseline model, the description of the equilibrium depends on the parameters of the order flow in (IA.33), and the dealers' forecast w of the fundamental value (which is the same for all dealers).

Proposition IA.3. *In the model with N dealers, suppose all dealers have the same initial inventory x_0. Then there exists a symmetric equilibrium in which dealer i's optimal quotes are:*

$$a_i \;=\; w + h - \delta, \qquad b_i \;=\; w - h - \delta, \tag{IA.41}$$

and the half spread h, pricing discount δ, and total monitoring rate q satisfy:

$$
\begin{aligned}
h &= \frac{\ell}{k}\,\frac{2 + 2(N-1)\nu}{2 + (N-1)(\mu+\nu)}, \\
\delta &= \frac{2}{k}\,\frac{m\Big(1 + (N-1)\nu + 2\gamma k\big(1 + (N-1)\mu\big)/N\Big) + \gamma k\big(1 + (N-1)\mu\big)x_0}{2 + (N-1)(\mu+\nu) + 2\gamma k(1 + (N-1)\mu)/N}, \\
q &= \sqrt{\frac{k}{N}\left(1 + \frac{k\gamma}{N}\right)\frac{1}{fc}}.
\end{aligned}
\tag{IA.42}
$$

Corollary IA.2 describes the equilibrium corresponding to the "neutral inventory" value $x_{0,\text{neutral}}$ at which the dealers balance the order flow, i.e., their expected buy and sell demands are equal. Define the "neutral discount" to be the equilibrium discount δ_{neutral} corresponding to the case in which all dealers start with the neutral inventory.

Corollary IA.2. *In the model with N dealers, the neutral inventory, neutral pricing discount and neutral mid-quote price are given, respectively, by:*

$$
x_{0,\text{neutral}} = \frac{m}{\gamma k}, \qquad \delta_{\text{neutral}} = \frac{2m}{k}, \qquad p_{\text{neutral}} = w - \frac{2m}{k}.
\tag{IA.43}
$$

Corollary IA.3 provides comparative statics for the neutral discount δ, which is in one-to-one correspondence with the cost of capital r:

$$
r = \frac{\mathsf{E}_\tau(v) - p_{\text{neutral}}}{p_{\text{neutral}}} = \frac{2m/k}{w - 2m/k}.
\tag{IA.44}
$$

Corollary IA.3. *In the model with N dealers, the cost of capital is increasing in the elasticity parameter k, and is not affected by the inventory aversion γ.*

When the elasticity parameter k is higher, investors trade more aggressively on their information. Then, the dealers have an incentive to set a small risk premium δ (the difference between their forecast w and the mid-quote price), to reduce their expected inventory.

The fact that the neutral discount does not depend on inventory aversion has the same intuition as in the baseline model: this discount depends only on the properties of the order flow, and not on the dealers' inventory aversion.

4.4 Representative Dealer

We now compare the equilibrium for $N \geq 1$ dealers, each having inventory aversion γ, with the equilibrium for one dealer with inventory aversion $\gamma^{(1)}$ given by:

$$\gamma^{(1)} = \frac{\gamma}{N}. \tag{IA.45}$$

Intuitively, we verify whether we can replace N dealers with a representative dealer with proportionally smaller inventory aversion.

Corollary IA.4 describes the equilibrium behavior in the case of N dealers.

Corollary IA.4. *Let the superscript "(N)" describe the variables in the equilibrium with N dealers. Then, the equilibrium half spread, monitoring rate, neutral inventory and neutral discount respectively satisfy:*

$$\begin{aligned} h^{(N)} &= \frac{2+2(N-1)\nu}{2+(N-1)(\mu+\nu)}\, h^{(1)}, \qquad & q^{(N)} &= \frac{q^{(1)}}{\sqrt{N}} \\ x^{(N)}_{0,\text{neutral}} &= \frac{x^{(1)}_{0,\text{neutral}}}{N}, \qquad & \delta^{(N)}_{\text{neutral}} &= \delta^{(1)}_{\text{neutral}}. \end{aligned} \tag{IA.46}$$

One implication of Corollary IA.4 is that the neutral inventory of a representative dealer is N times larger than the neutral inventory of each dealer in the N-dealer equilibrium. This implies that the aggregate neutral inventory is the same in the two models, which justifies our choice of the representative dealer inventory aversion in (IA.45).

The first equation in (IA.46) implies that the equilibrium bid-ask spread in the two models depends on the market allocation policy, that is, on the allocation coefficients μ and ν. If the allocation coefficients are equal, then the equilibrium spread is the same in the two models.

From (IA.46) we also see that the neutral discount, and therefore the cost of capital, does not depend on the number of dealers, but only on the properties of the order flow, and in particular on the ratio between the imbalance parameter m and the investor elasticity k. This is in line with Prediction 1 in Section III.E of the paper: the number of market makers in a stock does not affect its cost of capital.

The only important difference between the two model arises for the total monitoring rate. The monitoring rate in the N-dealer model is lower than in the representative dealer model by the square root of N. Intuitively, this is because each dealer exerts a positive externality on the other dealers: when she monitors, she reveals her signal via her quotes and thus improves every dealer's estimate of the fundamental value. Because of this externality, each dealer under-monitors the market in equilibrium.

This last result justifies Prediction 1 in Section III.E of the paper: a large number of market makers in a stock is associated to a low quote-to-trade ratio. In that section, a large number of market makers is interpreted as a low value of the parameter γ (the inventory aversion of the representative market maker), which implies that the representative dealer can afford to monitor less often and thus sets a lower quote rate. In this Internet Appendix, we have an additional interpretation for Prediction 1: each market maker's public quotes exert a positive externality on the other market makers and therefore reduce everyone's incentive to monitor the market.

4.5 Proofs of Results

Proof of Corollary IA.1. The proof of Proposition 1 in the paper implies that the dealer's equilibrium choice of h and δ does not depend on the forecast variance. It remains to show under what conditions the dealer chooses the same monitoring rate. Equation (A-4) in the proof of Proposition 1 shows that it is enough to have the same ex ante variance $G(q)$. Equation (A-5) in the proof of Proposition 2 implies that the ex ante forecast variance $G(q)$ satisfies $G(q) = \frac{\ln(q+1)}{qF(q)}$. With a unique signal, the ex ante forecast variance is $\tilde{G}(q) = \frac{1}{\tilde{F}(q)}$. If we want the two variances to be equal, we need $\frac{\ln(q+1)}{qF(q)} = \frac{1}{\tilde{F}(q)}$, which is equivalent to (IA.31). □

Proof of Proposition IA.3. Fix the monitoring rates q_i, $i = 1, \ldots, N$. Let $\mathcal{I}_\tau$ be the dealers' information set just before trading at τ, and by E_τ the expectation operator conditional on $\mathcal{I}_\tau$. Let $w = \mathsf{E}_\tau(v)$ be the dealers' forecast of the fundamental value, and G the variance of the forecast error:

$$G = \mathsf{Var}(v - w). \tag{IA.47}$$

We now compute dealer i's expected utility coming from a quoting strategy (a_i, b_i). Recall that if a and b are the average quotes, the total order flow is (Q^b, Q^s), where:

$$\text{(IA.48)} \qquad Q^b = \frac{k}{2}(v - a) + \ell - m + \varepsilon^b, \qquad Q^s = \frac{k}{2}(b - v) + \ell + m + \varepsilon^s,$$

ε^b and ε^s are independent and normally distributed by $\mathcal{N}(0, \Sigma_L/2)$. Recall that the allocation mechanism requires that dealer i trades the quantities:

$$\text{(IA.49)} \qquad \begin{aligned} Q_i^b &= \frac{1}{N}\left(\frac{k}{2}\Big((v - a) + \mu(a - a_i)\Big) + \ell - m + \varepsilon^b\right), \\ Q_i^s &= \frac{1}{N}\left(\frac{k}{2}\Big((b - v) + \mu(b_i - b)\Big) + \ell + m + \varepsilon^s\right), \end{aligned}$$

at the prices:

$$\text{(IA.50)} \qquad a_{e,i} = a + \nu(a_i - a), \qquad b_{e,i} = b + \nu(b_i - b),$$

respectively, where $\mu, \nu \in [0, 1]$. Denote the sum of the other traders' quotes as:

$$\text{(IA.51)} \qquad a_{-i} = \sum_{j \neq i} a_j, \qquad b_{-i} = \sum_{j \neq i} b_j.$$

To further simplify notation, define:

$$\text{(IA.52)} \qquad \begin{aligned} &\tilde{a}_i = \frac{a_i}{N}, \qquad \tilde{a}_{-i} = \frac{a_{-i}}{N}, \qquad \tilde{b}_i = \frac{a_i}{N}, \qquad \tilde{b}_{-i} = \frac{a_{-i}}{N}, \\ &\tilde{k} = \frac{k}{N}, \qquad \tilde{\ell} = \frac{\ell}{N}, \qquad \tilde{m} = \frac{m}{N}, \qquad \tilde{\Sigma}_L = \frac{\Sigma_L}{N^2}, \\ &h_i = \frac{a_i - b_i}{2}, \qquad \delta_i = w - \frac{a_i + b_i}{2}, \qquad e = v - w. \end{aligned}$$

Note that $a = \tilde{a}_i + \tilde{a}_{-i}$ and $b = \tilde{b}_i + \tilde{b}_{-i}$. The order flow satisfies:

$$\text{(IA.53)} \qquad \begin{aligned} Q_i^b &= \frac{\tilde{k}}{2}\Big((v - a) + \mu(a - a_i)\Big) + \tilde{\ell} - \tilde{m} + \frac{\varepsilon^b}{N}, \\ Q_i^s &= \frac{\tilde{k}}{2}\Big((b - v) + \mu(b_i - b)\Big) + \tilde{\ell} + \tilde{m} + \frac{\varepsilon^s}{N}. \end{aligned}$$

and dealer i's inventory at liquidation is $x_{i,\text{end}} = x_0 - Q_i^b + Q_i^s$. Dealer i solves:

$$\text{(IA.54)} \qquad \max_{a_i, b_i} \mathsf{E}_\tau\Big(x_0 v + \big(v - b_{e,i}\big) Q_i^s + \big(a_{e,i} - v\big) Q_i^b - \gamma x_{i,\text{end}}^2 \Big),$$

where $a_{e,i}$ and $b_{e,i}$ are the effective quotes at which dealer i trades (see equation (IA.35)).

We also require that the equilibrium is symmetric, i.e., we impose that the equilibrium quotes satisfy $a_{-i} = (N-1)a_i$ and $b_{-i} = (N-1)b_i$. Note that the quoting strategy (a_i, b_i) is equivalent to choosing (h_i, δ_i). It is straightforward (although computationally tedious) to obtain the optimal strategy of dealer i in the symmetric equilibrium:

$$\text{(IA.55)} \qquad \begin{aligned} h_i &= \frac{\ell}{k} \frac{2 + 2(N-1)\nu}{2 + (N-1)(\mu + \nu)}, \\ \delta_i &= \frac{2}{k} \frac{m\Big(1 + (N-1)\nu + 2\gamma k\big(1 + (N-1)\mu\big)/N\Big) + \gamma k\big(1 + (N-1)\mu\big) x_0}{2 + (N-1)(\mu+\nu) + 2\gamma k(1 + (N-1)\mu)/N}. \end{aligned}$$

This proves the first two formulas in (IA.42).

Regardless of the initial inventory, dealer i's maximum expected utility (ignoring monitoring costs) is of the form:

$$\text{(IA.56)} \qquad U_{i,c=0} \;=\; D - \frac{k}{N}\Big(1 + \frac{k\gamma}{N}\Big) G,$$

where D is a constant that does not depend on the dealer i's monitoring rate q_i. With the usual notation, we assume the other dealers choose the monitoring rates q_j which sum to q_{-i}. Then we have:

$$\text{(IA.57)} \qquad G \;=\; \mathsf{Var}(v - w) \;=\; \frac{1}{f(q_i + q_{-i})}.$$

If we include the monitoring costs $C(q_i) = cq_i$, equation (IA.56) implies that dealer i's maximum expected utility is of the form:

$$\text{(IA.58)} \qquad U_i \;=\; D - \frac{k}{N}\Big(1 + \frac{k\gamma}{N}\Big) \frac{1}{f(q_i + q_{-i})} - cq_i.$$

The first order condition in q_i implies that the optimum q_i satisfies $(q_i + q_{-i})^2 = \frac{k}{N}\big(1 + \frac{k\gamma}{N}\big)\frac{1}{fc}$. Because the total monitoring rate is $q = q_i + q_{-i}$, we obtain $q^2 = \frac{k}{N}\big(1 + \frac{k\gamma}{N}\big)\frac{1}{fc}$, which proves the last formula in (IA.42). □

Proof of Corollary IA.2. We use the notation from the proof of Proposition IA.3. Note that the symmetry of the equilibrium implies $a = a_i$ and $b = b_i$. From (IA.53) it follows that in equilibrium $Q_i^b - Q_i^s = \tilde{k}\big(v - \frac{a_i+b_i}{2}\big) - 2\tilde{m} + \frac{\varepsilon^b-\varepsilon^s}{N}$. Since $\mathsf{E}_\tau(v) = w$ and $w - \frac{a_i+b_i}{2} = \delta_i$, we obtain:

$$\mathsf{E}_\tau(Q_i^b - Q_i^s) \;=\; \tilde{k}\delta_i - 2\tilde{m}. \tag{IA.59}$$

We now compute the neutral inventory $x_{0,\text{neutral}}$ at which $\mathsf{E}_\tau(Q_i^b) = \mathsf{E}_\tau(Q_i^s)$. Equation (IA.59) implies that $\delta_i = \delta_{\text{neutral}}$ satisfies:

$$\delta_{\text{neutral}} \;=\; \frac{2m}{k}, \tag{IA.60}$$

which also proves the formula $p_{\text{neutral}} = w - \delta_{\text{neutral}} = w - \frac{2m}{k}$. Substituting the neutral discount in (IA.55), we compute:

$$x_{0,\text{neutral}} \;=\; \frac{m}{\gamma k}. \tag{IA.61}$$

□

Proof of Corollary IA.4. The proof follows from equations (IA.42) and (IA.43). □

5 Model with Multiple Trading Rounds

This section builds an extension with multiple trading rounds of the baseline model in Section III.A of the paper. One notable difference from the baseline model is that in the multi-trade extension we assume that trading takes place at deterministic times (in event time) rather than at random times.[11] This extension is closely related to the price pressures model of Hendershott and Menkveld (2014, henceforth HM2014). The main

[11]Corollary IA.1 in the paper shows that the baseline model in which the dealer receives multiple signals arriving at Poisson times produces essentially the same equilibrium as a static model in which the dealer receives only one signal (with appropriate precision) at a deterministic time. For more discussion regarding this modelling choice, see Section 6 of this Internet Appendix.

result of this section is that the equilibrium in this new setup remains qualitatively the same. In addition, we show that the dealer's neutral inventory has a natural interpretation in the multi-trade model as the average inventory of the dealer, regarding of the starting inventory. The proofs of the results are given in Section 5.6 of this Internet Appendix.

5.1 Environment

The market is composed of one risk-free asset and one risky asset. Trading in the risky asset takes place in a market exchange, at discrete dates $t = 0, 1, 2, \ldots$ such that the trading frequency is normalized to one. There are two types of market participants: (a) one monopolistic market maker called the dealer ("she") who monitors the market and sets the quotes at which others trade, and (b) traders, who submit market orders.

Note that here the trading times are deterministic rather than follow a Poisson process with frequency equal to one (as in the model with a single trading round in the paper). To justify this choice, note that Corollary IA.1 in the paper shows that the baseline model with Poisson monitoring produces essentially the same equilibrium as in a static model in which the dealer receives only one signal (with appropriate precision) at a deterministic time. Section 6 of this Internet Appendix provides additional justification by considering an actual monitoring process at fractional deterministic times.

Assets. The risk-free asset is used as numeraire and has a return of zero. The risky asset pays a dividend $D > 0$ before each trading date.[12] The ex-dividend fundamental value v_t follows a continuous random walk process for which the increments have variance per unit of time equal to $\Sigma_v = \sigma_v^2$, where σ_v is the fundamental volatility. One possible interpretation for v_t is that it is the cash value that shareholders receive at liquidation, an event which can occur in each trading round with a fixed probability.[13]

[12]The dividend D is introduced so that the model generates a positive expected price appreciation each period and thus a positive expected return (or cost of capital). This modelling choice is made to avoid the complications that arise when mixing microstructure and asset pricing models (see the discussion in the second part of Footnote 22) in this Internet Appendix.

[13]Suppose there exists $\pi \in (0, 1)$ such that the asset liquidates in each period with probability π, in which case the shareholders receive v_t per share. Then it can be showed that the expected profits of a trader with quantities bought and sold at t equal to $-Q_t^b$ and $-Q_t^s$, respectively, has the form described in equation (IA.66) with $\beta = 1 - \pi$, and $\gamma = C(q) = 0$.

Trading. At trading date $t = 1, 2, \ldots$, after observing the ask quote a_t and the bid quote b_t, traders submit the following aggregate market orders:

$$\text{(IA.62)} \qquad \begin{aligned} Q_t^b &= \frac{k}{2}(v_t - a_t) + \ell - m + \varepsilon_t^b, \quad \text{with} \quad \varepsilon_t^b \overset{IID}{\sim} \mathcal{N}(0, \Sigma_L/2), \\ Q_t^s &= \frac{k}{2}(b_t - v_t) + \ell + m + \varepsilon_t^s, \quad \text{with} \quad \varepsilon_t^s \overset{IID}{\sim} \mathcal{N}(0, \Sigma_L/2), \end{aligned}$$

where Q_t^b is the buy demand and Q_t^s is the sell demand. The numbers k, ℓ, m and Σ_L are exogenous constants. Together, Q_t^b and Q_t^s are called the traders' order flow. The parameter k is the investor elasticity, ℓ is the inelasticity parameter, and m is the imbalance parameter. Micro-foundations for the order flow are provided in Section 2 in this Internet Appendix.

Dealer Monitoring. The dealer monitors the market according to an independent Poisson process with frequency parameter $q > 0$ called the monitoring frequency (or monitoring rate). In the spirit of Corollary IA.1 in the paper, we take a reduced form approach and replace the signals obtained from monitoring at the frequency q with signals that summarize the dealer's information just before trading at each t. Denote by w_t the dealer's forecast of the fundamental value v_t just before trading occurs at t. The forecast is the expected fundamental value of the asset conditional on all the information available until t. We define the "precision function" F_t as the inverse variance of the forecast error $v_t - w_t$. We assume that the precision function does not depend on t, and is an increasing function of the monitoring rate q:[14]

$$\text{(IA.63)} \qquad F(q) = \frac{1}{\mathsf{Var}(v_t - w_t)}.$$

The intuition is that an increase in the monitoring rate produces more precise forecasts for the dealer. Per unit of time, the cost of monitoring at the rate q is $C(q)$, which is an increasing function of q.

To simplify the equilibrium formulas, we assume that the precision function $F(q)$

[14] In Section 6 of this Internet Appendix we show how to generate $F(q)$ using a specific signal structure obtained by monitoring at fractional times $1/q$.

and the monitoring cost $C(q)$ are linear increasing functions:

$$F(q) \;=\; f\,q, \qquad C(q) \;=\; c\,q, \tag{IA.64}$$

where f and c are positive constants.[15]

Dealer's Quotes and Objective. As in the baseline model in the paper, we interpret the monitoring rate q as the dealer's quote rate. Because monitoring is considered here in reduced form, we are interested only in the quotes (a_t, b_t) that are prevalent when trading occurs at integer times t.

Thus, a quoting strategy for the dealer is a set of processes a_t (the ask quote) and b_t (the bid quote) that are measurable with respect to the dealer's information set. Let x_t be the dealer's inventory in the risky asset just before trading at t.[16] If Q_t^b is the aggregate buy market order at t, and Q^s is the aggregate sell market order at t, the dealer's inventory evolves according to:

$$x_{t+1} \;=\; x_t - Q_t^b + Q_t^s. \tag{IA.65}$$

Then, for a given quoting strategy, the dealer's expected utility at τ is equal to the expected profit from date τ onwards, minus the quadratic penalty in the inventory, and minus the monitoring costs:

$$\mathsf{E}_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \Big(x_t\, D \;+\; \big((v_t - b_t) Q_t^s + (a_t - v_t) Q_t^b \big) \;-\; \gamma\, x_t^2 \;-\; C(q) \Big), \tag{IA.66}$$

where $\beta \in (0,1)$ and $\gamma > 0$. Thus, the dealer maximizes expected profit, but at each t faces a utility loss that is quadratic in the inventory. Note that except for the dividend payment this utility function is essentially the same as the one specified in HM2014.[17]

[15] In the proof of Proposition IA.5, we describe equilibrium conditions for more general F and C.

[16] We let the initial inventory x_0 as a free parameter, although later (in Section 5.5) we set it equal to the parameter $\bar{x}$ from equation (IA.72), which is the long-term mean of the dealer's equilibrium inventory.

[17] This penalty can be justified either by the dealer facing external funding constraints, or by her being risk averse. The latter explanation is present in HM2014 (Section 3). There, the dealer maximizes quadratic utility over non-storable consumption. To solve the dynamic optimization problem, HM2014 consider an approximation of the resulting objective function (see their equation (16)). This approximation coincides with our dealer's expected utility in (IA.66) when $C(q) = 0$.

Equilibrium Concept. The structure of the game is as follows: First, before trading begins (before $t = 0$), the dealer chooses a constant monitoring rate q. Second, in the trading game the dealer continuously chooses the quotes (the ask quote a_t and the bid quote b_t) such that objective function (IA.66) is maximized.

We solve for the equilibrium in two steps. In the first step (Section 5.2), we take the dealer's monitoring rate q as given and describe the optimal quoting behavior. In the second step (Section 5.3), we determine the optimal monitoring rate q as the rate which maximizes the dealer's expected utility.

5.2 Optimal Quotes

We fix the monitoring rate q. Consider the game described in Section 5.1, with positive parameters $D, k, \ell, m, \Sigma_L, \gamma$. Define the following constants:

$$\text{(IA.67)} \qquad \begin{aligned} h &= \frac{\ell}{k}, \quad \omega = \frac{1-\beta}{\beta k}, \quad \alpha = \beta\,\frac{(\gamma-\omega)+\sqrt{(\gamma-\omega)^2+\frac{4\gamma}{\beta k}}}{2}, \\ \lambda &= \frac{\alpha}{1+k\alpha} = \frac{-(\gamma+\omega)+\sqrt{(\gamma-\omega)^2+\frac{4\gamma}{\beta k}}}{2}, \\ \delta &= \frac{1-\beta+2k\alpha}{k(1-\beta+k\alpha)}\,m - \frac{\beta}{2(1-\beta+k\alpha)}\,D. \end{aligned}$$

Proposition IA.4 describes the dealer's optimal behavior.

Proposition IA.4. *The dealer's optimal quotes at* $t = 0, 1, \dots$ *are:*

$$\text{(IA.68)} \qquad a_t = w_t - \lambda x_t + h - \delta, \qquad b_t = w_t - \lambda x_t - h - \delta,$$

where w_t *is the dealer's value forecast, and* x_t *is her inventory. The mid-quote price* $p_t = (a_t + b_t)/2$ *satisfies:*

$$\text{(IA.69)} \qquad p_t = w_t - \lambda x_t - \delta = w_t - \lambda x_t - \frac{1-\beta+2k\alpha}{k(1-\beta+k\alpha)}\,m + \frac{\beta}{2(1-\beta+k\alpha)}\,D.$$

To get intuition for this result, suppose the imbalance parameter m and the dividend D are both zero (hence $\delta = 0$). Consider first the particular case when the dealer is risk-neutral: $\gamma = 0$. In that case, both α and λ are equal to zero, and the dealer's inventory x_t does not affect her strategy. Equation (IA.68) implies that the dealer sets her quotes at equal distance around her forecast w_t. Hence, the ask quote at t is $a_t = w_t + h$, and the bid quote is $b_t = w_t - h$, where h is the constant half spread. The equilibrium value $h = \ell/k$ reflects two opposite concerns for the dealer: If she sets too large a half spread, then investors (whose price sensitivity is increasing in k) submit a smaller expected quantity at the quotes.[18] If she sets too small a half spread, this decreases the part of the profit that comes from the inelastic part ℓ of traders' order flow.

When the dealer has inventory concerns ($\gamma > 0$), her inventory affects the optimal quotes: according to equation (IA.68), the quotes are equally spaced around an inventory-adjusted forecast ($w_t - \lambda x_t$). The effect of the dealer's inventory on the mid-quote price is in fact the "price pressure" mechanism identified by HM2014. To understand this phenomenon, suppose that before trading at t the dealer has zero inventory, and at t traders submit a net demand Q. The dealer's inventory then becomes negative ($-Q$). To avoid the inventory penalty, the dealer must bring back the inventory to zero. For that, the dealer must raise the quotes to convince more sellers to arrive. Quantitatively, according to (IA.68) the dealer must increase both quotes by λQ, with the coefficient λ as in equation (IA.67). This makes the corresponding slope coefficient λ essentially a price impact coefficient, in the spirit of Kyle (1985).[19]

According to (IA.69), the mid-quote price is decreasing in the imbalance parameter m, and increasing in the dividend D. To understand why, suppose the imbalance parameter m is large, yet the dealer sets the mid-quote price equal to her forecast (that is, $p_t = w_t$). The dealer then expects the sell demand to be much larger than the buy demand. Thus, in order to avoid inventory buildup and to attract more buyers, she must

[18] E.g., equation (IA.62) implies that the expected quantity traded at the ask is $\mathsf{E}_t(Q_t^b) = \frac{k}{2}(w_t - a_t) + \ell$, which is decreasing in a_t.

[19] We stress that in our model price impact is caused by inventory considerations and not by adverse selection between the dealer and the traders. Nevertheless, adverse selection occurs as long as the dealer's signal precision f is not infinite. The interested reader can separate the effect of inventory and information by analyzing more carefully the dealer's signal structure described in Section 6.3 of this Internet Appendix. There we see that the informativeness of trading depends on the noise parameter Σ_L. There, however, the signal structure is chosen to justify the reduced-form assumption regarding the formula (IA.63) for $F(q)$. Under that structure, the dealer is only concerned about her forecast just before trading, and not on what effect trading has on this forecast. But under a different signal structure this fact is no longer true, e.g., if we set $\tilde{V}_\eta = V_\eta$ and $\tilde{V}_\psi = V_\psi$ (see the discussion before equation (IA.104)).

lower her price well below her forecast. A similar intuition works when the dividend D is large, but the argument reverses: because investors prefer getting a large dividend, to attract more sellers the dealer must set a price higher than the forecast.

5.3 Optimal Monitoring and the Quote Rate

In this section, we determine the dealer's optimal monitoring rate q. As the trading rate is normalized to one, we interpret the monitoring rate q as the quote-to-trade ratio, or simply the quote rate:

$$q = \text{Quote Rate.} \tag{IA.70}$$

Thus far, the description of the equilibrium does not depend on a particular specification for the precision function $F(q)$ or the monitoring function $C(q)$. To provide explicit formulas, however, we now assume that both functions are linear: $F(q) = fq$ and $C(q) = cq$. In the proof of Proposition IA.5, we describe the equilibrium conditions for more general F and C. Proposition IA.5 shows how to compute the dealer's optimal monitoring rate, which is the equilibrium quote rate.

Proposition IA.5. *The dealer's optimal monitoring rate q satisfies:*

$$q^2 = \frac{k(1+k\alpha)}{fc} = \frac{k\beta}{fc} \frac{(\gamma-\omega) + \sqrt{(\gamma-\omega)^2 + \frac{4\gamma}{\beta k}}}{-(\gamma+\omega) + \sqrt{(\gamma-\omega)^2 + \frac{4\gamma}{\beta k}}}. \tag{IA.71}$$

Corollary IA.5 provides some comparative statics for the quote rate.

Corollary IA.5. *The quote rate q is increasing in investor elasticity k and inventory aversion γ, and is decreasing in signal precision f and in monitoring cost c.*

If the investor elasticity k is larger, investors are more sensitive to the quotes, and the dealer increases her monitoring rate to prevent both adverse selection and large fluctuations in inventory. Indeed, there are two reasons for this increase, which can be understood by writing equation (IA.71) as a sum: $q^2 = \frac{k}{fc} + \frac{k^2\alpha}{fc}$. The first term (which does not depend on the dealer's inventory aversion γ) simply reflects that by increasing her monitoring rate, the dealer reduces the adverse selection that comes from trading with investors with superior information. The second term depends on the parameter α, which is increasing in the inventory aversion γ (see the proof of the Corollary). If γ is larger, the dealer is relatively more concerned about her inventory than about her profit. She then increases her monitoring rate to stay closer to the fundamental value, such that her inventory is not expected to vary too much.

If the signal precision parameter f is smaller, the dealer gets noisier signals each time she monitors, hence she must monitor the market more often in order to avoid getting a large inventory. As a result, in neglected stocks where we expect the dealer's signals to be noisier, the quote rate q should be larger. Similarly, if the monitoring cost parameter c is smaller, the dealer can afford to monitor more often in order to maintain the same precision, which increases the quote rate.

5.4 Neutral Inventory and Pricing Discount

In this section, we study the equilibrium evolution of the dealer's inventory. As we see in Proposition IA.4, the dealer's inventory is an important state variable. Corollary IA.6 computes its long-term mean and describes the equilibrium quotes by considering deviations of the dealer's inventory from its long-term mean.

Corollary IA.6. *The dealer's inventory is an* $AR(1)$ *process:*

$$\text{(IA.72)} \qquad x_{t+1} - \bar{x} \;=\; \frac{1}{1+k\alpha}\big(x_t - \bar{x}\big) + \varepsilon_t, \qquad \bar{x} \;=\; \frac{1+k\alpha}{k\alpha}\,\frac{(1-\beta)m + \beta k D/2}{1-\beta+k\alpha}.$$

where ε_t *is IID with mean zero and variance* $\frac{k^2}{fq} + \Sigma_L$. *The mid-quote price satisfies:*

$$\text{(IA.73)} \qquad p_t \;=\; w_t - \lambda\big(x_t - \bar{x}\big) - \bar{\delta}, \qquad \bar{\delta} \;=\; \frac{2m}{k}.$$

The mean inventory $\bar{x}$ represents the dealer's bias in holding the risky asset. In HM2014 both m and D are zero, and therefore the mean inventory $\bar{x}$ is also zero. In our case both m and D are positive, hence $\bar{x}$ is also positive. Intuitively, the case when m is positive corresponds to the case when investors are risk averse and the risky asset is in positive net supply (see the order flow micro-foundations in Section III.A of the paper). But the dealer also behaves approximately as a risk averse investor because of the quadratic penalty in inventory (see Footnote 17). Therefore, our model becomes essentially a risk sharing problem, in which the dealer holds a positive inventory on average.[20]

If we write the mid-quote equation (IA.73) at both t and $t+1$, we compute:

$$p_{t+1} - p_t \;=\; w_{t+1} - w_t + \psi\left(x_t - \bar{x}\right) - \lambda\varepsilon_{t+1}, \qquad \psi \;=\; \frac{\lambda k \alpha}{1 + k\alpha}. \tag{IA.74}$$

We say that the system is the "neutral state" if the dealer's inventory is equal to its long-term mean, i.e., $x_t = \bar{x}$. In this state, equation (IA.74) implies that the expected change in price is zero, which in the language of HM2014 means that there is no price pressure.

In general, we define the "pricing discount" as the difference between the dealer's forecast and the mid-quote price:

$$\delta_t \;=\; w_t - p_t. \tag{IA.75}$$

Equation (IA.73) implies that the pricing discount in the neutral state is the same as its long-term average, and is equal to $\bar{\delta} = 2m/k$. Note that this value is independent on the characteristics of the dealer, that is, on the inventory aversion γ, the signal precision f, or the monitoring cost c. We have thus proved Corollary IA.7.

Corollary IA.7. *The average pricing discount is $\bar{\delta} = 2m/k$, and does not depend on dealer characteristics.*

In particular, the average pricing discount does not depend on the dealer's inventory aversion γ. This is because in the neutral state there is no price pressure and the dealer just needs to balance the order flow such that the inventory does not accumulate in either direction. This result is surprising, because one may expect the discount to be larger if the dealer has a larger inventory aversion γ. But while a larger coefficient γ

[20]Even if $m = 0$, the dealer tends to hold inventory when the dividend D is positive. Indeed, in that case the dealer must increase her quotes to attract sellers (see equation (IA.69)), which tends to raise her inventory and thus increase the dividend collected.

just increases the speed of convergence of the pricing discount to its mean, it does not change the mean itself, which depends only on the properties of the order flow.[21]

The average pricing discount $\bar{\delta}$ does depend on the properties of the order flow: the imbalance parameter m and the investor elasticity k. If the imbalance parameter m is larger, the dealer expects the difference between the sell and buy demands to be larger. To compensate, the dealer must lower price to encourage demand, and therefore increase the discount. If the investor elasticity k is larger, investors are more sensitive to mispricing and therefore trade more intensely when the price is different from the fundamental value. To prevent an expected accumulation of inventory, the dealer must then set the price closer to her forecast, which implies a lower discount.

5.5 Cost of Capital

In this section, we define and analyze the cost of capital in the context of our model. We consider the point of view of an econometrician that has access to the quote and trade information, but not necessarily to the dealer's inventory and forecast (in practice, dealers' inventories and forecasts are not public information). The expected return (including dividends) at date t is then:

$$r_t = \frac{\mathsf{E}_t(p_{t+1}) + D - p_t}{p_t}, \tag{IA.76}$$

where E_t be the expectation operator conditional on the past information, p_t is the mid-quote price, and D is the dividend per share.

To simplify the presentation, we assume that the dealer's inventory starts at its long-term mean, that is, we set $x_0 = \bar{x}$. In this neutral state the price does not change in expectation (see Section 5.4). We define the "cost of capital" to be the expected return in the initial state.[22] Denote the initial dealer forecast by $w_0 = \overline{w}$. Then, the cost of

[21] According to (IA.73), the equilibrium discount satisfies $\delta_t - \bar{\delta} = \lambda(x_t - \bar{x})$, and thus δ_t and x_t are both $AR(1)$ processes with the same autoregressive coefficient: $1/(1 + k\alpha)$. From (IA.67), α is increasing in γ, therefore the speed of mean reversion of both processes is also increasing in γ.

[22] We define the cost of capital only in the initial (neutral) state, because we want to avoid price pressures that appear later in other states (which are the focus of HM2014). Another reason is that in general it is difficult to analyze risk premia in dynamic microstructure models. Indeed, if the expected return is constant, return compounding implies that the price process grows exponentially on average, and to keep up the fundamental value should also follow a geometric Brownian motion. But to maintain

capital is:

$$r = \frac{D}{\overline{w} - \bar{\delta}} = \frac{D}{\overline{w} - \frac{2m}{k}}. \tag{IA.77}$$

Note that the cost of capital is in one-to-one correspondence with the pricing discount $\bar{\delta} = 2m/k$. Thus, the cost of capital does not depend on the dealer characteristics either. The cost of capital does depend on the characteristics of the order flow: the imbalance parameter m and the investor elasticity k. The intuition for this dependence is the same as in the discussion after Corollary IA.7.

Corollary IA.8 connects the cost of capital to the equilibrium quote rate.

Corollary IA.8 (Quote Effect)**.** *Holding all parameters constant except for the investor elasticity k, there is an inverse relation between the cost of capital and the quote rate.*

Thus, the key driver of the quote effect in our model is investor elasticity. When k is larger, Corollary IA.5 shows that the quote rate is also larger: because traders are more sensitive to the quotes, in order to prevent large fluctuations in inventory the dealer must monitor more often. At the same time, when k is larger, the cost of capital is smaller: because investors trade more intensely when the price differs from the fundamental value, in order to prevent an expected accumulation of inventory the dealer must set the price closer to her forecast, which implies a lower discount and hence a lower cost of capital.

If we consider also the micro-foundations for the order flow (see Section III.A), the investor elasticity k is larger when the investors have more precise information or when they are less risk averse. Therefore, at a more fundamental level the quote effect is driven by traders' information precision: more precise investors cause both a larger quote rate and a smaller cost of capital.

5.6 Proofs of Results

Proof of Proposition IA.4. Fix the monitoring rate $q > 0$. Let $\mathcal{I}_t$ be the dealer's information set just before trading at t, and by E_t the expectation operator conditional

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a tractable model we need the fundamental value to follow an arithmetic Brownian motion.

on $\mathcal{I}_t$. Let $w_t = \mathsf{E}_t(v_t)$ be the dealer's forecast of the fundamental value, and G the variance of the forecast error. From (IA.63), we have:

$$G = \mathsf{Var}(v_t - w_t) = \frac{1}{fq}. \tag{IA.78}$$

We now compute the dealer's expected utility coming from a quoting strategy (a_t, b_t). If we define:

$$h_t = \frac{a_t - b_t}{2}, \qquad \delta_t = w_t - \frac{a_t + b_t}{2}, \qquad e_t = v_t - w_t, \tag{IA.79}$$

then the quoting strategy is equivalent to choosing (h_t, δ_t). Equation (IA.62) implies that traders' buy and sell demands at t are given, respectively, by $Q_t^b = \frac{k}{2}(v_t - a_t) + \ell - m + \varepsilon_t^b$ and $Q_t^s = \frac{k}{2}(b_t - v_t) + \ell + m + \varepsilon_t^s$, with $\varepsilon_t^b, \varepsilon_t^s \sim \mathcal{N}(0, \Sigma_L/2)$. Let $\varepsilon_t = -ke_t + \varepsilon_t^s - \varepsilon_t^b$. This is uncorrelated with the past information and has a normal distribution $\mathcal{N}(0, k^2G + \Sigma_L)$. If x_t is the dealer's inventory before trading at t, equation (IA.65) shows that x_t describes the recursive equation $x_{t+1} = x_t - Q_t^b + Q_t^s$, which translates into:

$$x_{t+1} = x_t - k\delta_t + 2m + \varepsilon_t \quad \text{with} \quad \varepsilon_t \overset{IID}{\sim} \mathcal{N}\big(0, k^2G + \Sigma_L\big). \tag{IA.80}$$

Substituting Q_t^b and Q_t^s in the dealer's objective function from (IA.66), and ignoring the monitoring costs $C(q)$, we get $\mathsf{E}_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau}\, \mathsf{E}_t\Big(Dx_t - \frac{k}{2}(a_t - v_t)^2 - \frac{k}{2}(v_t - b_t)^2 + (\ell - m)(a_t - v_t) + (\ell + m)(v_t - b_t) - \gamma x_t^2\Big)$. We decompose $\mathsf{E}_t(v_t - b_t)^2 = \mathsf{E}_t(v_t - w_t + w_t - b_t)^2 = G + (w_t - b_t)^2$, and similarly $\mathsf{E}_t(a_t - v_t)^2 = G + (a_t - w_t)^2$. Using the notation in (IA.79), it follows that the dealer's maximization problem at τ is:

$$\max_{(h_t,\delta_t)_{t\geq\tau}} \mathsf{E}_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau}\Big(Dx_t - kG - k\delta_t^2 - kh_t^2 + 2\ell h_t + 2m\delta_t - \gamma x_t^2\Big), \tag{IA.81}$$

where x_t evolves according to (IA.80). Using the Bellman principle of optimization, we reduce the dynamic optimization in (IA.81) to the following static optimization problem:

$$V(x_t) = \max_{h_t,\delta_t}\Big(2dx_t - kG - k\delta_t^2 - kh_t^2 + 2\ell h_t + 2m\delta_t - \gamma x_t^2 + \beta\, \mathsf{E}_t\, V(x_{t+1})\Big), \tag{IA.82}$$

where $d = \frac{D}{2}$. We guess that $V(x)$ is a quadratic function of the form:

$$V(x) \;=\; W_0 - 2W_1 x - W_2 x^2 \tag{IA.83}$$

for some constants W_0, W_1, W_2. Substituting x_{t+1} from (IA.80), the problem becomes:

$$\begin{aligned} V(x_t) \;=\; & \max_{h_t,\delta_t} \Big(2dx_t - kG - k\delta_t^2 - kh_t^2 + 2\ell h_t + 2m\delta_t - \gamma x_t^2 \\ & + \beta W_0 - 2\beta W_1(x_t - k\delta_t + 2m) - \beta W_2(x_t - k\delta_t + 2m)^2 - \beta W_2(k^2 G + \Sigma_L) \Big). \end{aligned} \tag{IA.84}$$

The first order condition in (IA.84) with respect to h_t implies $h_t = \frac{\ell}{k}$, which shows that the optimal $h_t = h$, the constant defined in (IA.67). The first order condition in (IA.84) with respect to δ_t implies $\delta_t = \frac{\beta W_2}{1+k\beta W_2}\, x_t + \frac{m+k\beta W_1+2km\beta W_2}{k(1+k\beta W_2)}$, which shows that the optimal $\delta_t = \lambda x_t + \Delta$, where:

$$\lambda \;=\; \frac{\alpha}{1+k\alpha}, \qquad \Delta \;=\; \frac{m + k\alpha_1 + 2km\alpha}{k(1+k\alpha)}, \qquad \alpha_1 \;=\; \beta W_1, \qquad \alpha \;=\; \beta W_2. \tag{IA.85}$$

Because $V(x_t) = W_0 - 2W_1 x_t - W_2 x_t^2$, we solve for W_0, W_1, W_2:

$$\begin{aligned} W_0 \;&=\; \frac{1}{1-\beta}\Big(\frac{\ell^2}{k} - k(1+k\alpha)G - \alpha\Sigma_L + \frac{(1+k\alpha)\big((1-\beta)m + \beta k d\big)^2}{k(1-\beta+k\alpha)^2} \Big), \\ W_1 \;&=\; \frac{\alpha}{1-\beta+k\alpha}\, m - \frac{1+k\alpha}{1-\beta+k\alpha}\, d, \qquad W_2 \;=\; \frac{\beta W_2}{1+k\beta W_2} + \gamma. \end{aligned} \tag{IA.86}$$

For a maximum, we need to have $W_2 > 0$. The quadratic equation for W_2 in (IA.86) has a unique positive solution:

$$W_2 \;=\; \frac{\gamma - \omega + \sqrt{(\gamma-\omega)^2 + 4\frac{\gamma}{\beta k}}}{2}, \quad \text{with} \quad \omega \;=\; \frac{1-\beta}{\beta k}. \tag{IA.87}$$

This implies that $\alpha = \beta W_2$ indeed satisfies (IA.67).

If the dealer has an inventory of $x_t = x$, from equation (IA.83) it follows that the maximum expected utility she can achieve at t is $V(x) = W_0 - 2W_1 x - W_2 x^2 = \frac{1}{1-\beta}\big(\frac{\ell^2}{k} - \alpha\Sigma_L - k(1+k\alpha)G + \frac{(1+k\alpha)((1-\beta)m+\beta k d)^2}{k(1-\beta+k\alpha)^2} \big) - 2W_1 x - W_2 x^2$. Since $G = \frac{1}{fq}$, we get:

$$\text{(IA.88)} \qquad U(q) \;=\; \frac{1}{1-\beta}\left(\widetilde{W}_0 - \frac{k(1+k\alpha)}{fq}\right) - 2W_1x - W_2x^2,$$

where $\widetilde{W}_0$, W_1 and W_2 do not depend on q. Also, using $\alpha_1 = \beta W_1$, we compute $\Delta = \frac{1-\beta+2k\alpha}{k(1-\beta+k\alpha)}\, m - \frac{\beta}{1-\beta+k\alpha}\, d$. Since $d = \frac{D}{2}$, this proves that the formula for Δ in (IA.67). □

Proof of Proposition IA.5. Consider a more general function $F(q) = 1/\mathsf{Var}(v_t - w_t)$ that is increasing in the monitoring rate q. If $G(q) = 1/F(q)$, we have showed in the proof of Proposition IA.4 that the dealer's maximum expected utility is of the form $V(x_t) = W_0 - 2W_1x_t - W_2x_t^2$, where W_0, W_1 and W_2 are as in (IA.86). This formula, however, ignores the monitoring costs per unit of time, $C(q)$. If we include these costs, the dealer's maximum utility is $W_0 - 2W_1x_t - W_2x_t^2 - \frac{C(q)}{1-\beta}$. But up to a constant that does not depend on q, this utility is equal to $\frac{-k(1+k\alpha)G(q)-C(q)}{1-\beta}$. The first order condition with respect to q is equivalent to $-k(1+k\alpha)G'(q) - C'(q) = 0$. Thus, the optimal monitoring rate satisfies:

$$\text{(IA.89)} \qquad -\frac{C'(q)}{G'(q)} \;=\; \frac{C'(q)F^2(q)}{F'(q)} \;=\; k(k\alpha+1).$$

The second order condition for a maximum is $k(k\alpha+1)G''(q) + C''(q) > 0$, which is satisfied if the functions G and C are convex, with at least one of them strictly convex.

We now use the linear specification $C(q) = cq$ and $F(q) = fq$, and compute the optimal monitoring rate q. Since $G(q) = \frac{1}{fq}$, from (IA.89) it follows that q satisfies $fcq^2 = k(k\alpha+1)$, which proves the first part of equation (IA.71). Because the function G is strictly convex, note that the second order condition is satisfied.

The second part of (IA.71) follows by using the expression for α in (IA.67). □

Proof of Corollary IA.5. We first prove that α is decreasing in k and increasing in γ. Equation (IA.86) implies that $\alpha = \beta W_2$ satisfies the equation $\frac{\alpha}{\beta} - \gamma = \frac{\alpha}{1+k\alpha}$. Differentiating this equation with respect to k, we get $\frac{\partial\alpha}{\partial k} = -\frac{\beta\alpha^2}{(1+k\alpha)^2-\beta} < 0$. Similarly, differentiation with respect to γ implies $\frac{\partial\alpha}{\partial\gamma} = \frac{\beta(1+k\alpha)^2}{(1+k\alpha)^2-\beta} > 0$.

Equation (IA.71) implies that q and the term $Q = k(1+k\alpha)$ have the same dependence on the parameters k and γ. Using the formula for $\frac{\partial\alpha}{\partial k}$, we compute $\frac{\partial Q}{\partial k} =$

$\frac{(1+k\alpha)^2(1-\beta+2k\alpha)}{(1+k\alpha)^2-\beta} > 0$. Finally, Q is increasing in α, which (as previously shown) is increasing in γ, hence Q is also increasing in γ.

By visual inspection of equation (IA.71), it is clear that the quote rate q is decreasing in f and increasing in σ_v. $\square$

Proof of Corollary IA.6. Using equation (IA.80) and the fact that in equilibrium $\delta_t = \lambda x_t + \Delta$, it follows that the dealer's inventory evolves according to $x_{t+1} = (1-k\lambda)x_t - k\Delta + 2m + \varepsilon_t$, with $\varepsilon_t \sim \mathcal{N}(0, k^2 G + \Sigma_L)$ and $G = \frac{1}{F} = \frac{1}{fq}$. From (IA.67), the coefficient $\phi = 1-k\lambda = \frac{1}{1+k\alpha} \in (0,1)$, hence $x_{t+1} = \frac{1}{1+k\alpha}x_t - k\Delta + 2m + \varepsilon_t$. Thus, x_t follows an $AR(1)$ process with auto-regressive coefficient ϕ, mean $\bar{x} = (2m - k\Delta)/(1-\phi)$, and variance $\Sigma_x = \left(\frac{k^2}{fq} + \Sigma_L\right)/(1-\phi^2)$. Using the formula for Δ in (IA.67), it is straightforward to prove the formula for $\bar{x}$ in (IA.72). One can also show that $\Sigma_x = \frac{k\alpha(2+k\alpha)}{(1+k\alpha)^2}\left(\frac{k^2}{fq} + \Sigma_L\right)$. $\square$

Proof of Corollary IA.7. This has already been proved in the discussion that precedes the statement of the Corollary. Alternatively, Proposition IA.4 implies that the pricing discount at t is equal to $w_t - p_t = \lambda x_t + \Delta$, whose average equals $\lambda\bar{x} + \Delta$. Using (IA.67), we compute the average discount to be $2m/k$, which is the same as $\bar{\delta}$. $\square$

Proof of Corollary IA.8. First, we prove rigorously equation (IA.77). Since the system is initially in the neutral state ($x_0 = \bar{x}$), according to (IA.74) the expected price change $\mathsf{E}_0\, p_1 - p_0$ is zero. But, if $\overline{w}$ is the initial forecast, by definition $\overline{w} - p_0$ is the pricing discount. Since in the neutral state the pricing discount is $\bar{\delta} = 2m/k$, it follows that $p_0 = \overline{w} - \bar{\delta}$, which proves (IA.77). Suppose now we hold all parameters constant except for k. Clearly, the cost of capital is decreasing in k, as the pricing discount $\bar{\delta}$ is decreasing in k. At the same time, Corollary IA.5 implies that the quote rate q is increasing in k. This proves the inverse relation between r and q. $\square$

6 Monitoring and Signals

6.1 Preliminaries

The purpose of this section is to provide micro-foundations for the dealer's precision function $F(q)$ in the multi-trade model of Section 5 in this Internet Appendix. For simplicity, we assume that both trading and monitoring occur at deterministic times, equal to the averages of the corresponding random times.[23] This is equivalent to timing trades in event time (at equally spaced intervals of length one), and timing the *average* number of monitoring times that occur between subsequent trading rounds. The downside of this approach is that in principle the monitoring rate q must be an integer. Nevertheless, the description of the equilibrium remains valid also for non-integer q, and thus we can think of the equilibrium being valid for all values of the monitoring rate.

Recall that in that model trading takes place at integer times $t = 0, 1, 2, \ldots$, and the monitoring rate is $q > 0$. We now assume that monitoring takes place at fractional times $\frac{0}{q}, \frac{1}{q}, \frac{2}{q}, \ldots$, where q is a positive integer. To simplify notation, we index monitoring times by $\tau = 0, 1, 2, \ldots$ rather than by the corresponding fractional times. With this notation, trading takes place at $\tau = 0, q, 2q, \ldots$, which are integer multiples of the monitoring rate. By convention, we assume that when a trading date coincides with a monitoring date, monitoring occurs before trading. Equations (IA.62) imply that traders' order flow satisfies:

$$\text{(IA.90)} \qquad \begin{aligned} Q^b_\tau &= \frac{k}{2}(v_\tau - a_\tau) + \ell - m + \varepsilon^b_\tau, \quad \text{with} \quad \varepsilon^b_\tau \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right), \\ Q^s_\tau &= \frac{k}{2}(b_\tau - v_\tau) + \ell + m + \varepsilon^s_\tau, \quad \text{with} \quad \varepsilon^s_\tau \overset{IID}{\sim} \mathcal{N}\left(0, \Sigma_L/2\right), \end{aligned}$$

6.2 Uninformative Trading

We first analyze the simpler case when the trading process is uninformative to the dealer. Formally, this occurs when the trading noise measured by Σ_L is sufficiently large (see

[23] We already know that in the baseline (single-trade) model in the paper we can replace the signals obtained from monitoring at the frequency q with signals that summarize the dealer's information just before trading at each t. But to extend this result to a multi-trade extension would be very difficult, because one would have to keep track of the different numbers of monitoring rounds between all the trading rounds. To avoid this difficulty, we essentially consider only the average outcome of the trading and monitoring processes.

equation (IA.106)). In this case, we ignore the trading process altogether and focus instead on the monitoring process. Denote by $\mathcal{I}_\tau$ the dealer's information set after monitoring at τ, and by $w_\tau = \mathsf{E}(v_\tau | \mathcal{I}_\tau)$ the dealer's forecast at τ.

We now show that any positive function $F(q)$, not necessarily linear, can arise as the dealer's precision function for a certain set of signals. Define:

$$\text{(IA.91)} \qquad G \;=\; G(q) \;=\; \frac{1}{F(q)}.$$

Fix $q > 0$, and define $V_\eta = V_\eta(q) > 0$ as follows: if $F(q) \leqq /\Sigma_v$, choose any $V_\eta > 0$; and if $F(q) > q/\Sigma_v$, choose any $V_\eta \in \big(0, \frac{1}{F(q) - q/\Sigma_v}\big)$. Also, define $V_v = V_v(q)$ and $V_\psi = V_\psi(q)$ by

$$\text{(IA.92)} \qquad V_v \;=\; \frac{\Sigma_v}{q}, \qquad V_\psi \;=\; G^2 \, \frac{V_\eta + V_v}{V_\eta V_v} - G.$$

Clearly, $V_v > 0$. We show that $V_\psi > 0$ as well. Indeed, from the definition of V_η, we see that $\big(F(q) - q/\Sigma_v\big) V_\eta < 1$ for all $q > 0$. Using previuos notation, this is the same as $\big(\frac{1}{G} - \frac{1}{V_v}\big) V_\eta < 1$, which is equivalent to $\frac{1}{G} < \frac{1}{V_v} + \frac{1}{V_\eta}$. Thus, $G \frac{V_\eta + V_v}{V_\eta V_v} > 1$ or equivalently $V_\psi = G\big(G \frac{V_\eta + V_v}{V_\eta V_v} - 1\big) > 0$. Note that equation (IA.92) implies:

$$\text{(IA.93)} \qquad \frac{G^2}{G + V_\psi} \;=\; \frac{V_v V_\eta}{V_v + V_\eta}.$$

We define the signal observed by the dealer at $\tau = 0$. Since we can choose freely the initial variance $\mathsf{Var}(v_0) = \Sigma_{v_0}$, consider $\Sigma_{v_0} > G$, and suppose that at $\tau = 0$ the dealer observes $s_0 = v_0 + \nu$, with $\nu \sim \mathcal{N}\big(0, \frac{G\Sigma_{v_0}}{\Sigma_{v_0} - G}\big)$. Then, the dealer's forecast is $w_0 = \mathsf{E}(v_0 | s_0) = \beta_0 s_0$, where $\beta_0 = G/\Sigma_{v_0}$. A direct computation shows that indeed $\mathsf{Var}(v_0 - w_0) = G$. Thus, if we define:

$$\text{(IA.94)} \qquad G_\tau \;=\; \mathsf{Var}(v_\tau - w_\tau), \quad \tau \;\geq\; 0,$$

we have $G_0 = G$.

At each $\tau = 1, 2, \ldots$, the dealer observes two signals:

$$\text{(IA.95)} \qquad \begin{cases} r_\tau = (v_{\tau-1} - w_{\tau-1}) + \psi_\tau, \text{ with } \psi_\tau \overset{IID}{\sim} \mathcal{N}(0, V_\psi), \text{ and} \\ s_\tau = (v_\tau - v_{\tau-1}) + \eta_\tau \text{ with } \eta_\tau \overset{IID}{\sim} \mathcal{N}(0, V_\eta). \end{cases}$$

Since the forecast is $w_\tau = \mathsf{E}(v_\tau | r_\tau, s_\tau, r_{\tau-1}, s_{\tau-1}, \ldots)$, its increment is $\Delta w_\tau = w_\tau - w_{\tau-1} = \mathsf{E}(v_\tau - w_{\tau-1} | r_\tau, s_\tau) = \mathsf{E}(v_\tau - v_{\tau-1} | s_\tau) + \mathsf{E}(v_{\tau-1} - w_{\tau-1} | r_\tau)$. Then:

$$\text{(IA.96)} \qquad \Delta w_\tau \;=\; \frac{V_v}{V_v + V_\eta}\, s_\tau + \frac{G_{\tau-1}}{G_{\tau-1} + V_\psi}\, r_\tau.$$

We compute $v_\tau - w_\tau = v_{\tau-1} - w_{\tau-1} + \Delta v_\tau - \Delta w_\tau = \frac{V_\psi}{G_{\tau-1}+V_\psi}(v_{\tau-1} - w_{\tau-1}) - \frac{G_{\tau-1}}{G_{\tau-1}+V_\psi}\psi_\tau + \frac{V_\eta}{V_v+V_\eta}\Delta v_\tau - \frac{V_v}{V_v+V_\eta}\eta_\tau$. Taking variance on both sides, we obtain the recursive equation:

$$\text{(IA.97)} \qquad G_\tau \;=\; \frac{G_{\tau-1} V_\psi}{G_{\tau-1} + V_\psi} + \frac{V_v V_\eta}{V_v + V_\eta}.$$

From (IA.93), we substitute $\frac{V_v V_\eta}{V_v+V_\eta}$ by $\frac{G^2}{G+V_\psi}$, and the recursive equation (IA.97) becomes

$$\text{(IA.98)} \qquad G_\tau - G_{\tau-1} \;=\; \left(1 - \frac{V_\psi^2}{(G + V_\psi)(G_{\tau-1} + V_\psi)}\right)(G - G_{\tau-1}).$$

Because $G_0 = G$, equation (IA.98) implies that G_τ is constant and equal to G for all τ.[24] Since $G = \frac{1}{F(q)}$, this finishes the proof.

For future reference, we use equation (IA.96) to compute $\mathsf{Var}(\Delta w_\tau) = \frac{V_v^2}{V_v+V_\eta} + \frac{G^2}{G+V_\psi}$. Equation (IA.93) then implies that $\mathsf{Var}(\Delta w_\tau) = \frac{V_v^2}{V_v+V_\eta} + \frac{V_v V_\eta}{V_v+V_\eta} = V_v$. Thus, we have proved that:

$$\text{(IA.99)} \qquad \mathsf{Var}(\Delta w_\tau) \;=\; \mathsf{Var}(\Delta v_\tau) \;=\; V_v \;=\; \frac{\Sigma_v}{q}.$$

6.3 Informative Trading

We now analyze the general case when the trading process is informative, meaning that the noise parameter Σ_L can be any positive real number. Thus, beside the monitoring

[24]Note that the coefficient in front of $G - G_{\tau-1}$ in equation (IA.98) is a number in the interval $(0, 1)$. It is then straightforward to show that G_τ converges monotonically to the constant G regardless of the initial value G_0.

times, we also need to analyze the dealer's inference at the trading times $\tau = 0, q, 2q, \ldots$, where q is the monitoring rate and is a positive integer. (Recall that on these dates monitoring occurs before trading.)

We now show that any linear function $F(q) = fq$ that satisfies a mild condition (see equation (IA.103)) can arise as the dealer's precision function for a set of signals. As before, given $F(q)$ we define $G = G(q) = \frac{1}{F(q)} = \frac{1}{fq}$. Denote by $\mathcal{I}_\tau$ the dealer's information set after monitoring at τ, by $w_\tau = \mathsf{E}(v_\tau | \mathcal{I}_\tau)$ the dealer's forecast at τ, and by $e_\tau = v_\tau - w_\tau$ her forecast error. Then, equations (IA.90) become:

$$\text{(IA.100)} \qquad \begin{aligned} Q_\tau^b &= \frac{k}{2} e_\tau - (a_\tau - w_\tau) + \ell + \varepsilon_\tau^b, \quad \text{with} \quad \varepsilon_\tau^b \overset{IID}{\sim} \mathcal{N}(0, \Sigma_L/2), \\ Q_\tau^s &= -\frac{k}{2} e_\tau - (w_\tau - b_\tau) + \ell + \varepsilon_\tau^s, \quad \text{with} \quad \varepsilon_\tau^s \overset{IID}{\sim} \mathcal{N}(0, \Sigma_L/2), \end{aligned}$$

At trading time $t = 0, q, 2q, \ldots$, define also:

$$\text{(IA.101)} \qquad w_{\tau+} = \mathsf{E}(v_\tau \mid \mathcal{I}_\tau, Q_\tau^b, Q_\tau^s), \qquad G_{\tau+} = \mathsf{Var}(v_\tau - w_{\tau+}).$$

As in the informative case, we look for a stationary equilibrium, which here means that we want the dealer to have a periodic signal precision with periodicity equal to the monitoring rate q. Thus, the signal precision follows a periodic sequence of the form:

$$\text{(IA.102)} \qquad G_0 \,,\; G_{0+} \,,\; G_1 \,,\; \cdots \,,\; G_q = G_0 \,,\; G_{q+} \,,\; G_{q+1} \,,\; \cdots$$

We show that there is a simple solution for which G_τ are equal to $G = \frac{1}{fq}$, as long as the following condition is satisfied:

$$\text{(IA.103)} \qquad f > \frac{1}{\Sigma_v} \qquad \text{or} \qquad \frac{\Sigma_L f^2}{k^2} > \frac{1}{\Sigma_v} - f.$$

To understand intuitively the role played by this condition, suppose (IA.103) fails to hold. This means that the noise component of trading, measured by Σ_L, is small. Then, the increase in precision $(1/G_0 - 1/G_{0+})$ that comes from the information content of trading is also small. By contrast, the decrease in precision $(1/G_{0+} - 1/G_1)$ that comes from the diffusion in fundamental value during the interval $[0, 1]$ is large, and thus the equation $G_0 = G_1$ cannot hold when (IA.103) fails. Note that the condition (IA.103) also translates into the requirement that the dealer's monitoring precision f is sufficiently high.

Suppose now condition (IA.103) is satisfied. We then assume that the dealer receives the same signals r_τ and s_τ as in the uninformative case, except for the monitoring times that come just after trading: $\tau = 1, q+1, 2q+1, \ldots$. At those times, we modify the variance of r_τ and s_τ, by defining new values for V_ψ and V_η. To see how this is done, consider the following cases:

- If $f > 1/\Sigma_v$, we multiply by q to obtain $fq = F = 1/G > 1/V_v$, where $V_v = \Sigma_v/q$. In this case, we choose $\frac{1}{\tilde{V}_\eta}$ in the positive interval $\big(\frac{1}{G} - \frac{1}{V_v}, \frac{\Sigma_L}{k^2G^2} + \frac{1}{G} - \frac{1}{V_v}\big)$.
- If $f \leq 1/\Sigma_v$, we have $1/G \leq 1/V_v$. Because q is a positive integer, condition (IA.103) implies $\frac{\Sigma_L f^2}{k^2} q^2 > \big(\frac{1}{\Sigma_v} - f\big)q$, which is equivalent to $\frac{\Sigma_L}{k^2G^2} > \frac{1}{V_v} - \frac{1}{G}$. In this case, we choose $\frac{1}{\tilde{V}_\eta}$ in the interval $\big(0, \frac{\Sigma_L}{k^2G^2} + \frac{1}{G} - \frac{1}{V_v}\big)$. Since $1/G - 1/V_v \leq 0$, it follows that $\frac{1}{\tilde{V}_\eta}$ also belongs to the larger interval $\big(\frac{1}{G} - \frac{1}{V_v}, \frac{\Sigma_L}{k^2G^2} + \frac{1}{G} - \frac{1}{V_v}\big)$.

Thus, in both cases $\frac{1}{\tilde{V}_\eta}$ lies in the interval $\big(\frac{1}{G} - \frac{1}{V_v}, \frac{\Sigma_L}{k^2G^2} + \frac{1}{G} - \frac{1}{V_v}\big)$, or equivalently $\frac{1}{\tilde{V}_\eta} + \frac{1}{V_v} - \frac{1}{G}$ lies in the interval $\big(0, \frac{\Sigma_L}{k^2G^2}\big)$. Next, define:

$$\text{(IA.104)} \qquad \tilde{V}_\psi = \frac{\Sigma_L}{k^2} \frac{\big(\frac{\Sigma_L}{k^2G^2} + \frac{1}{G} - \frac{1}{V_v}\big) - \frac{1}{\tilde{V}_\eta}}{\tilde{V}_\eta - \big(\frac{1}{G} - \frac{1}{V_v}\big)}.$$

It follows that both $\tilde{V}_\eta$ and $\tilde{V}_\psi$ are positive, and hence when $\tau = 1, q+1, 2q+1, \ldots$, the modified signals r_τ and s_τ are well defined.

We show $G_\tau = G$ for all $\tau \geq 0$. Because the only difference between the informative and the uninformative case occurs at $\tau = 1, q+1, 2q+1, \ldots$, without loss of generality we only need to prove that $G_1 = G$. Since trading at $\tau = 0$ is informative for the dealer, her forecast after trading is $w_{0_+} = \mathsf{E}(v_0 | \mathcal{I}_0, Q_0^b, Q_0^s) = w_0 + \mathsf{E}(e_0 | Q_0^b, Q_0^s)$, where $e_0 = v_0 - w_0$ and:

$$\text{(IA.105)} \qquad \mathsf{E}(e_0 | Q_0^b, Q_0^s) = \frac{kG}{k^2G + \Sigma_L}\big(Q_0^b - Q_0^s\big), \qquad \mathsf{Var}(e_0 | Q_0^b, Q_0^s) = \frac{G\Sigma_L}{k^2G + \Sigma_L}.$$

We apply the recursive formula (IA.97) for $\tau = 1$, by replacing (i) V_η with $\tilde{V}_\eta$, (ii) V_ψ

with $\tilde{V}_\psi$, and (iii) G_0 with $G_{0_+} = \frac{G\Sigma_L}{k^2G+\Sigma_L}$. Then, a direct computation shows that $G_1 = G$. Since all G_τ are equal to G, it follows that $F(q) = fq$.

We can now determine when trading is uninformative for the dealer, i.e., the update $w_{0_+} - w_0$ is much smaller than a generic increment $w_\tau - w_{\tau-1}$ (for τ not of the form $1, q+1, 2q+1, \ldots$). This translates into the condition that the variance Σ_L is sufficiently large:

$$\Sigma_L \gg \frac{k^2}{f^2\Sigma_v}. \tag{IA.106}$$

Indeed, using equations (IA.99) and (IA.105), the condition $\mathsf{Var}(w_{0_+} - w_0) \ll \mathsf{Var}(\Delta w_\tau)$ becomes $\frac{k^2G^2}{k^2G+\Sigma_L} \ll \frac{\Sigma_v}{q}$, which translates to $\frac{\Sigma_L}{k^2G^2} \gg \frac{q}{\Sigma_v}$, or since $G = \frac{1}{fq}$, to $\Sigma_L \gg \frac{k^2\Sigma_v}{qf^2\Sigma_v}$. But the monitoring rate q is a positive integer, hence the condition is equivalent to $\Sigma_L \gg \frac{k^2}{f^2\Sigma_v}$.

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