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Asset Pricing with Systematic Skewness: Two Decades Later

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ABSTRACT

We reexamine the asset pricing performance of systematic skewness ("coskewness"), a risk factor in the three-moment CAPM model of Kraus and Litzenberger (1976). In an influential paper, Harvey and Siddique (2000) test a coskewness factor constructed by sorting stocks on past coskewness. We replicate and extend their paper. Overall, coskewness appears to be priced in the cross section of stocks, especially when using an alternative coskewness proxy like (i) the predicted systematic skewness (*PSS*) of Langlois (2020), where coskewness is predicted by various firm characteristics, or (ii) a modified *PSS* factor (*mPSS*) that uses only return-based characteristics.

Keywords: Skewness, coskewness, three-moment CAPM, persistent factors, expected return.

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1 Introduction

An important goal of asset pricing is to determine a security's expected return, or cost of capital. The traditional model of expected returns, the CAPM, expresses a relation between an asset's expected return and its market beta, or normalized covariance with the aggregate market return. The CAPM is usually derived by relying on economic foundations involving either restrictions on investor utility or on the distribution of asset returns.

Confronted with the empirical shortcomings of the CAPM (see Fama and French, 1993), researchers have relied on a multitude of factors to explain the cross-section of asset returns. In recent years, there has been increased interest in the study of the "zoo" of factors (see Cochrane, 2011; Harvey *et al.*, 2016; McLean and Pontiff, 2016; Linnainmaa and Roberts, 2018; Feng *et al.*, 2020; Hou *et al.*, 2020).

In this context, we pursue a "back to the basics" approach and consider a three-moment CAPM model, as in Kraus and Litzenberger (1976), that works with more general utility functions and return distributions. This extension of the CAPM has an appealing intuition that involves skewness, the third moment of returns: In the traditional CAPM, investors have preferences over a portfolio's mean and variance, and the systematic risk of an asset (beta) is measured as its contribution to the variance of the market portfolio. Beside beta, the three-moment CAPM involves an additional characteristic (gamma) related to skewness, which is the normalized covariance with the squared market return. If investors are prudent (i.e., have positive third derivative of utility) and in particular if they have decreasing absolute risk aversion, then they prefer more positively-skewed portfolios, all else being equal.[1] Thus, assets that decrease the investor portfolio's skewness (assets with low gamma) are less desirable and should command higher expected returns.

In the first part of the paper, we follow Harvey and Siddique (2000) and test a conditional version of the three-moment CAPM by constructing a long-short portfolio sorted by systematic skewness, from low to high. We

[1] Intuitively, investors with decreasing absolute risk aversion are more risk averse in low wealth states than in high wealth states, thus they avoid negatively skewed assets and prefer positively skewed assets. In Section 2, it is shown that investors with decreasing absolute risk aversion are prudent. Kimball (1990) shows that precautionary saving occurs if and only if investors are prudent.

call this portfolio the *HS* factor and we test its performance in the cross-section of U.S. stock returns. Initially, our empirical tests follow closely Harvey and Siddique (2000), except that their sample period July 1963 to December 1993 is extended to December 2019. Beyond the three factors of Fama and French (1993) (market, size, and value), we also consider the investment and profitability factors from Fama and French (2015), and the momentum factor from Carhart (1997).

The first thing to notice when we replicate the results in Harvey and Siddique (2000) is that their estimates are fairly different from ours, even though many of the qualitative results stay the same. One possible reason is the discrepancy between the number of stocks in their sample (9,268) and ours (14,988) over the replication sample period (July 1963 to December 1993). This discrepancy may be caused by updates in CRSP over the years, especially for NASDAQ stocks.[2]

We compare the results with the intuition of the three-moment CAPM: If an asset has negative coskewness with the investor's portfolio, then investing in the asset makes the overall portfolio more negatively skewed, hence makes the asset less desirable and increases its expected return. Therefore, in the data we should observe a negative relation between an asset's average return and (i) its coskewness and (ii) its beta on the coskewness factor *HS*, which is the return of a long-short portfolio containing stocks with low coskewness minus stocks with high coskewness. The results (in Table 1) are mixed: For the replication sample period (1963 to 1993) and the extended period (July 1963 to December 2019), the replicated results are qualitatively similar to the original ones: Result (i) is true but Result (ii) is not. The converse is true (i.e., Result (ii) is true but Result (i) is not) over the pre-sample period (July 1926 to June 1963), and none of the results is true for the post-sample period (January 1994 to December 2019). Replication for other test assets (32 industry portfolios, 10 portfolios sorted on size, and 27 portfolios sorted on book-to-market, size, and momentum) leads to similarly mixed results over the various sample periods.

Harvey and Siddique (2000) propose several formal tests to check whether the addition of a coskewness factor to a standard model, such as the CAPM or the three-factor Fama and French (1993) model, improves the

[2] This explanation was suggested by Campbell Harvey in private communication to the authors.

pricing results. They employ four types of tests: First, the GRS test from Gibbons *et al.* (1989) measures whether the introduction of a coskewness factor significantly decreases the GRS F-statistic (of portfolio alphas being jointly zero). The second test measures whether the alphas from the standard model are correlated with the coskewness factor. The third test measures whether the introduction of a coskewness factor significantly increases the R^2 in the asset pricing regressions. Fourth, a version of the Fama and MacBeth (1973) test measures whether the risk premium associated to the coskewness factor is positive and significant.[3] Note that the second and third tests are less strong, as they do not measure directly whether coskewness is priced or whether there is an improvement in pricing due to the addition of coskewness.

The results for the GRS test (in Table 2) are inconclusive. Harvey and Siddique (2000) report significant decreases in the GRS F-statistics for all portfolio groups once the coskewness factor is introduced, while our replication shows little change in the GRS F-statistics. These changes are small for all portfolio groups, both in the replication sample period and in the other sample periods.

For the correlation test (in Table 2), our replicated results coincide with those of Harvey and Siddique (2000), as we generally find a positive correlation between the coskewness factor and intercepts from regressions on the three Fama and French (1993) factors, for all portfolio groups and all sample periods. For the R^2 test (in Table 3), we confirm qualitatively the results of Harvey and Siddique (2000) that the adjusted R^2 increases when a coskewness factor is added to the CAPM or FF3 regressions, although the replicated changes in R^2 are generally smaller than the original changes. The results hold in the other sample periods, but the changes in R^2 are generally even smaller.

The results for the Fama-MacBeth test at the individual stock level (in Table 4) are mixed. For the replication sample period (1963 to 1993), the replicated risk premium of the coskewness factor *HS* is positive and significant for two of the five individual stock groups, in line with the original estimates. For the post-sample period (1994 to 2019), the coskewness risk premium is positive and significant for three of the five stock groups, while for the pre-sample period (1926 to 1963), the coskewness risk premium

[3] Harvey and Siddique (2000) perform the Fama-MacBeth test at the individual stock level, using a weighting scheme to mitigate the large idiosyncratic variation.

is not significant in any of the five groups.

Harvey and Siddique (2000) unveil interesting relations between momentum strategies and portfolio skewness (in Table 5). They find that buying winner portfolios (based on past return performance) and selling loser portfolios requires acceptance of significant negative skewness. This is in line with winner portfolios having a larger average return than loser portfolios, which they also document. Our replicated results are in line with the original results in Harvey and Siddique (2000), except that we do not find significant positive average return differences in the momentum strategies for which performance is estimated over the past 24 months. This is, however, not a problem as momentum strategies are usually constructed using performance over the past 6 or 12 months.

An additional test of the performance of the *HS* factor is to calculate its average return over the sample period, which is the spread in average return between low- and high-coskewness portfolios. If coskewness is priced in the data, then the portfolio *HS* should have a positive and significant premium. Harvey and Siddique (2000) report an average return of *HS* of 3.60% per year over the replication period (1963 to 1993) that is significant at a 95% level. This figure is lower in our replication (2.58% per year) and not statistically significant even at a 90% level (see Table IA.12 in the Internet Appendix). When we extend our empirical analysis to other periods, the average annual return of *HS* is −3.15% over the pre-sample period (1926 to 1963) and 1.64% over the post-sample period (1994 to 2019), with neither of these figures being statistically significant at a 90% level.

Overall, based on the replicated results in the first part of the paper, the evidence of the coskewness factor *HS* being priced is inconclusive. The favorable results for the correlation and R^2 tests suggest that the coskewness factor contains pricing information in addition to the CAPM or FF3 factors. However, the more formal GRS test does not show a clear improvement in pricing performance once coskewness is introduced, which may be caused by the fact that the *HS* coskewness proxy is very noisy.

In the second part of the paper, we compare *HS* with two alternative coskewness factors: (i) *PSS*, the predicted systematic skewness of Langlois (2020), which uses many stock characteristics as conditioning information in predicting coskewness, and uses the normalized cross-sectional ranks of variables rather than their actual values, and (ii) *mPSS*, a modified *PSS* factor that we construct by using only return-based characteristics and is thus available for a longer time period (starting in 1926 rather than in 1963).

We evaluate coskewness factors along two dimensions: First, the factor should be priced, i.e, it should have a significant and positive risk premium. Second, it should be persistent, by which we mean it should have a negative realized coskewness. Indeed, the factors we analyze are built by sorting stocks on predicted coskewness using past characteristics. If the factor is not persistent, it means the prediction is not reliable and the factor is less likely to capture true coskewness.[4]

To analyze whether a factor is priced, we perform Fama-MacBeth tests on several portfolio groups, with independent sorts on (i) size and book-to-market, (ii) size and momentum, and (iii) size and coskewness. The coskewness factor is added to the benchmark Fama-French five factor model, while in the Internet Appendix we also consider as benchmarks the CAPM model and the Carhart-Fama-French four factor model (market, size, value, and momentum). The *HS* factor has a mixed pricing performance and is not persistent (it has in fact a positive realized coskewness). The *mPSS* and *PSS* factors have better pricing performance and are generally persistent. The *mPSS* factor performs usually less well than *PSS*, suggesting that conditioning only on return-based characteristics does not capture all the information on coskewness that matters to investors.

Overall, we confirm the intuition in Harvey and Siddique (2000) that coskewness is priced in the cross section of stocks. The empirical evidence is significantly stronger if, instead of *HS*, we use as coskewness proxy *PSS* or *mPSS*, which are persistent factors and therefore likely to be closer to true coskewness. This is especially relevant since, compared to many other factors in the "zoo," coskewness is motivated by a compelling theoretical model such as the three-moment CAPM.

Related Literature

This paper is part of a larger literature on asset pricing with skewness (see, e.g., Bali *et al.*, 2016). Early theoretical papers, e.g., Rubinstein (1973) and Kraus and Litzenberger (1976), show that only systematic skewness

[4]An alternative method to build persistent coskewness factors is to use option prices to measure skewness (see Schneider *et al.*, 2020). However, this method involves short sample periods and requires further assumptions to extract the systematic part of option-implied skewness.

(coskewness) carries a risk premium. Kraus and Litzenberger (1976) derive a three-factor CAPM by approximating the marginal utility function with a Taylor series expansion.[5] As we show in Section 2, the method in Kraus and Litzenberger (1976) is equivalent to assuming that the investor's utility is specified directly as a function of the mean, standard deviation, and skewness of the investors' portfolio wealth. Harvey and Siddique (2000) derive a conditional three-factor CAPM from the simpler assumption that the pricing kernel has a quadratic expression in the market portfolio's return. Dittmar (2002) shows that the pricing kernel can be obtained endogenously as a function of the aggregate wealth by imposing some assumptions on preferences. Dittmar (2002) shows that the pricing kernel is nonlinear, and in addition to coskewness, further considers cokurtosis and human capital.

An alternative method to obtain a three-factor CAPM is to impose an assumption on the distribution of shocks to returns. Simaan (1993) provides a model with spherical shocks and one common nonspherical shock to create systematic skewness. Dahlquist *et al.* (2017) use Gaussian shocks and one common exponential shock to create systematic skewness. Both papers derive the asset pricing implications of coskewness in stock returns, the first in an expected utility framework and the second with generalized disappointment aversion preferences. Dahlquist *et al.* (2017) show that using generalized disappointment aversion instead of expected utility leads to a larger importance for return coskewness.

Other authors argue that both systematic and idiosyncratic skewness are important in pricing assets. Behavioral studies, e.g., Brunnermeier *et al.* (2007) and Barberis and Huang (2008), use biased beliefs that optimize investor well-being and prospect theory, respectively, to justify investor preferences for assets' total skewness. Mitton and Vorkink (2007) introduce heterogenous investor preference for skewness and show that in equilibrium investors are underdiversified and care about the level of idiosyncratic skewness in their portfolio returns.

Some empirical tests use past return skewness measures and firm characteristics to predict skewness. Harvey and Siddique (2000) predict coskewness using only past coskewness. Chen *et al.* (2001) and Boyer *et al.* (2010)

[5]Scott and Horvath (1980) also consider a Taylor expansion around the expected wealth. They find that for an investor who is consistent in direction of preference of moments, the preference direction is positive (negative) for positive (negative) values of every odd central moment and negative for every even central moment.

use cross-sectional regressions to predict total and idiosyncratic skewness, respectively, based on past risk measures and firm characteristics. Langlois (2020) improves the performance of these regressions by predicting the ordering of individual (systematic or idiosyncratic) skewness and not their values. He confirms the results of Chen *et al.* (2001) and Boyer *et al.* (2010) that skewness is negatively related to firm size. Langlois (2020) shows that predictors of systematic skewness are different from those of idiosyncratic skewness, except for higher momentum, higher price impact, and lower beta, which predict both lower coskewness and idiosyncratic skewness. He finds that his coskewness measure captures future coskewness risk better than other measures, is distinct from leading equity risk factors, and carries a significant risk premium. In contrast, he finds weaker evidence that predicted idiosyncratic skewness is priced in U.S. stocks.

Bali and Murray (2013) and Conrad *et al.* (2013) use options market data to extract estimates of the higher moments of the risk-neutral distribution of stock returns. Bali and Murray (2013) construct "skewness assets," which are combinations of stock and option positions that collectively form a long skewness position. They find a strong negative relation between risk-neutral skewness and the skewness assets' returns, consistent with a positive skewness preference. Conrad *et al.* (2013) find a positive (negative) relation between ex ante kurtosis (volatility) and subsequent returns in the cross-section. Also, more ex ante negatively (positively) skewed returns are associated with higher (lower) subsequent returns. Using also options market data, Schneider *et al.* (2020) find that option-implied ex ante skewness is strongly related to ex post residual coskewness, which allows them to construct coskewness factor-mimicking portfolios. In their sample (January 1996 to August 2014), they show that the returns of portfolios sorted on beta and volatility are driven largely by a single principal component that is largely explained by skewness.

The paper is organized as follows: Section 2 derives a three-moment CAPM following Kraus and Litzenberger (1976) and Harvey and Siddique (2000). Section 3 describes the data and provides the definition of the *HS* factor as well as summary statistics. Section 4 replicates the tests in Harvey and Siddique (2000) and extends them over various sample periods. Section 5 provides asset pricing tests during the extended sample period for the *HS* factor and two persistent coskewness factors (*PSS* and *mPSS*). Section 6 concludes.

2 A Three-Moment CAPM

To obtain the classical CAPM, one usually assumes that either (i) investors' utility has a quadratic or logarithmic expression, (ii) the pricing kernel (marginal rate of intertemporal substitution) is linear in the market return, or (iii) returns are normally distributed, or more generally in the elliptic class of distributions.

To obtain a three-moment CAPM, one similarly assumes that either (i) investors' utility has a cubic expression, as in Kraus and Litzenberger (1976); (ii) the pricing kernel is quadratic in the market return, as in Harvey and Siddique (2000); or (iii) returns have a skewed, normal-exponential distribution, as in Dahlquist *et al.* (2017). We pursue mainly the first approach, as it provide micro-foundations for the risk premia of beta and gamma exposures.

2.1 *Cubic Utility*

We generalize the Kraus and Litzenberger (1976) framework by considering an overlapping-generation (OLG) economy in which investors $i = 1, \ldots, I$ are born each time period t with wealth $W_{i,t}$ and live for two periods. Investors trade risky assets $n = 1, \ldots, N$ whose gross returns at $t+1$ are $R_{n,t+1}$. The risk-free rate has gross return R_f.

In time period t, investor i chooses a portfolio with weights $\omega_{i,t}$ in the risky assets and the rest invested in the risk-free asset to maximize expected utility over wealth at $t+1$, conditional on the information set at t:

$$\max_{\omega_{i,t}} \mathrm{E}_t\Big(U\big(W_{i,t+1}\big)\Big), \quad \text{with} \quad W_{i,t+1} = W_{i,t}R_f + W_{i,t}\omega_{i,t}^{\top}\big(R_{t+1} - R_f\big), \tag{1}$$

where $\omega_{i,t} = \big(\omega_{1,t,1}, \ldots, \omega_{i,t,N}\big)^{\top}$ is the vector of weights in the risky assets, and $R_{t+1} = \big(R_{1,t+1}, \ldots, R_{N,t+1}\big)^{\top}$ is the vector of asset gross returns.

As in Kraus and Litzenberger (1976), we consider a Taylor series expansion of the expected utility function around the expected wealth. To simplify notation, define:

$$\begin{aligned} W &= W_{i,t+1}, \qquad \bar{W} = \mathrm{E}_t(W_{i,t+1}), \qquad W_0 = W_{i,t}, \\ \sigma^2 &= \mathrm{E}_t\left(\Big(\frac{W - \bar{W}}{W_0}\Big)^2\right), \qquad m^3 = \mathrm{E}_t\left(\Big(\frac{W - \bar{W}}{W_0}\Big)^3\right). \end{aligned} \tag{2}$$

Up to higher-order terms, $U(W) = U(\bar{W}) + U'(\bar{W})(W-\bar{W}) + 1/2 \times U''(\bar{W})(W-\bar{W})^2 + 1/6 \times U'''(\bar{W})(W-\bar{W})^3$. Thus, the expected utility $E_t\big(U(W)\big)$ can be written as a function of the moments of W:

$$\begin{aligned}\phi\big(\bar{W},\sigma,m\big) &= E_t\big(U(W)\big)\\ &= U(\bar{W}) + \frac{1}{2}U''(\bar{W})W_0^2\sigma^2 + \frac{1}{6}U'''(\bar{W})W_0^3 m^3 + \text{ higher order terms.}\end{aligned} \tag{3}$$

In the remainder of this section, we ignore the higher-order terms and assume that the formula (3) for expected utility holds with equality. This is equivalent to assuming that the investor's utility is specified directly as a function of the mean, standard deviation, and skewness of portfolio wealth.

If we denote by $R_{P,t+1} = \omega_{i,t}^{\top} R_{t+1}$, Equation (1) implies that:

$$R_{P,t+1} - \mathrm{E}_t(R_{P,t+1}) = \frac{W-\bar{W}}{W_0} = \omega_{i,t}^{\top}(R_{t+1} - \bar{R}_{t+1}). \tag{4}$$

For each asset $n = 1, \ldots, N$, define the following moments:[6]

$$\mu_{n,t+1} = \bar{R}_{n,t+1} - R_f, \quad \beta_{n,P} = \frac{\mathrm{Cov}_t(R_{n,t+1}, R_{P,t+1})}{\sigma_P^2}, \quad \gamma_{n,P} = \frac{\mathrm{Cov}_t(R_{n,t+1}, R_{P,t+1}^2)}{m_P^3}. \tag{5}$$

where $\sigma_P = \sigma$ and $m_P = m$. As $\bar{W} = W_0 R_f + W_0 \omega_{i,t}^{\top} \mu_{t+1}$ and $(W-\bar{W})/W_0 = \omega_{i,t}^{\top}(R_{t+1} - R_f)$, we compute $\partial \bar{W}/\partial \omega_{i,t,n} = W_0 \mu_{n,t+1}$, $\partial \sigma^2/\partial \omega_{i,t,n} = 2\mathrm{E}_t\big((R_{n,t+1} - \bar{R}_{n,t+1})(\omega_{i,t}^{\top}(R_{t+1} - R_f))\big) = 2\beta_{n,P}\sigma^2$, and $\partial m^3/\partial \omega_{i,t,n} = 3\mathrm{E}_t\big((R_{n,t+1} - \bar{R}_{n,t+1})(\omega_{i,t}^{\top}(R_{t+1} - R_f))^2\big) = 3\gamma_{n,P} m^3$.

Thus, the first order condition for maximizing investor i's expected utility in Equation (3) with respect to $\omega_{i,t,n}$ is $\phi_{\bar{W}}\mu_{n,t+1} + \phi_{\sigma^2} 2\beta_{n,P}\sigma^2 + \phi_{m^3} 3\gamma_{n,P} m^3 = 0$, where subscripts indicate partial derivatives. As $\phi_{\sigma^2} = \phi_\sigma/(2\sigma)$ and $\phi_{m^3} = \phi_m/(3m^2)$, the first order condition becomes $\phi_{\bar{W}}\mu_{n,t+1} + \phi_\sigma \beta_{n,P}\sigma + \phi_{m^3}\gamma_{n,P} m = 0$, or:

$$\mu_{n,t+1} = \lambda_1 \beta_{n,P} + \lambda_2 \gamma_{n,P}, \quad \text{with} \quad \lambda_1 = -\frac{\phi_\sigma}{\phi_{\bar{W}}}\sigma_P, \quad \lambda_2 = -\frac{\phi_m}{\phi_{\bar{W}}} m_P. \tag{6}$$

[6] If $m_P = 0$, i.e., the portfolio P has a symmetric return distribution, we omit the denominator in the definition of $\gamma_{n,P}$, and the rest of the derivation is similar. Equations (6) and (7) change: the skewness premium is $\lambda_2 = -3\phi_{m^3}/\phi_{\bar{W}} = -1/2 \times U'''(\bar{W})\, W_0^3/\phi_{\bar{W}}$.

If $\bar{W} = \bar{W}(\sigma, m)$ is the expected wealth at the optimum, the envelope theorem and Equation (3) imply that $\phi_{\bar{W}} \bar{W}_\sigma + \phi_\sigma = 0$, hence $\partial \bar{W}/\partial \sigma = -\phi_\sigma/\phi_{\bar{W}}$. Similarly, we have $\partial \bar{W}/\partial m = -\phi_m/\phi_{\bar{W}}$. Thus, we interpret the coefficients $-\phi_\sigma/\phi_{\bar{W}}$ and $-\phi_m/\phi_{\bar{W}}$ in Equation (6) as the investor's marginal rates of substitution between expected wealth and standard deviation and between expected wealth and skewness, respectively, holding expected utility constant. To determine the signs of these coefficients, we use Equation (3) to compute $\phi_\sigma = U''(\bar{W}) W_0^2 \sigma$ and $\phi_m = 1/2 \times U'''(\bar{W}) W_0^3 m^2$. Thus, the coefficients λ_1 and λ_2 satisfy:

$$\lambda_1 = -\frac{U''(\bar{W})\, W_0^2\, \sigma_P^2}{\phi_{\bar{W}}}, \qquad \lambda_2 = -\frac{U'''(\bar{W})\, W_0^3\, m_P^3}{2\phi_{\bar{W}}}. \tag{7}$$

Thus, if investor i is risk averse ($U'' < 0$) and prudent ($U''' > 0$), λ_1 is positive and λ_2 has the opposite sign to m_P, the skewness of his optimal risky portfolio.

If i is a representative investor, the portfolio P is the market portfolio M.[7] The excess expected return of asset n can be written as:

$$\mathrm{E}_t(R_{n,t+1}) - R_f = \lambda_1\, \beta_{n,M} + \lambda_2\, \gamma_{n,M}, \tag{8}$$

where $\beta_{n,M}$ and $\gamma_{n,M}$ are the market beta and coskewness, respectively, of the n'th asset:

$$\beta_{n,M} = \frac{\mathrm{Cov}_t(R_{n,t+1}, R_{M,t+1})}{\sigma_M^2}, \quad \gamma_{n,M} = \frac{\mathrm{Cov}_t(R_{n,t+1}, R_{M,t+1}^2)}{m_M^3}. \tag{9}$$

This is the three-moment CAPM. The risk premia λ_1 and λ_2 are positive if the market has negative skewness ($m_M < 0$), and the representative investor is risk averse ($U'' < 0$) and prudent ($U''' > 0$). Note that decreasing absolute risk aversion is a sufficient condition for prudence, and therefore it implies a preference for positive skewness.[8]

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[7] Kraus and Litzenberger (1976) explain that a necessary and sufficient condition that each investor's optimal risky portfolio be the same is that each investor's risk tolerance is linear in wealth, i.e., $-U_i'/U_i'' = a_i + bW_i$, with the same cautiousness, b, for all investors.

[8] Indeed, as $U'(W) > 0$, decreasing absolute risk aversion translates into $d(-U''/U')/dW = \left(-U'U''' + (U'')^2\right)/(U')^2 < 0$, which implies $U''' > 0$. A Taylor expansion as in Equation (3) shows that a prudent investor has a preference for positive skewness in returns, holding everything else constant. Kimball (1990) shows that precautionary saving occurs if and only if investors are prudent.

The intuition follows that of the traditional CAPM: Assets with high systematic risk (beta) increase the market portfolio variance, hence are disliked by investors and should command high expected returns. Similarly, if investors prefer positively-skewed portfolios, then assets with low systematic skewness (gamma) decrease the market portfolio skewness, which is less desirable to investors and should command higher expected returns.

2.2 Quadratic Pricing Kernel

To obtain a three-factor CAPM, Harvey and Siddique (2000) specify directly a pricing kernel that is quadratic in the market portfolio return:

$$m_{t+1} = a_t + b_t r_{M,t+1} + c_t r^2_{M,t+1}, \tag{10}$$

where, to simplify notation, $r_{M,t+1}$ denotes the excess return $R_{M,t+1} - R_f$. The pricing kernel (or stochastic discount factor) allows us to price any asset with payoff x_{t+1}:

$$P_t = \mathrm{E}_t\big(m_{t+1} x_{t+1}\big), \tag{11}$$

The pricing kernel is the intertemporal marginal rate of substitution, i.e., $m_{t+1} = \beta U'(c_{t+1})/U'(c_t)$, where β is the discount factor (see Cochrane, 2005).

By definition, the gross return of any asset i is $R_{i,t+1} = x_{t+1}/P_t$, where x_{t+1} is the price next period, including dividends. Thus, the equation $\mathrm{E}_t\big(m_{t+1} R_{i,t+1}\big) = 1$ is true for any asset. In particular, the risk-free asset has gross return $R_f = 1/E_t(m_{t+1})$. Thus, the excess return of any asset i satisfies:

$$\mathrm{E}_t\big(m_{t+1} r_{i,t+1}\big) = 0. \tag{12}$$

We substitute $m_{t+1} = a_t + b_t r_{M,t+1} + c_t r^2_{M,t+1}$ in Equation (12). As $\mathrm{E}(XY) = \mathrm{cov}(X,Y) + \mathrm{E}(X)\mathrm{E}(Y)$, we obtain:

$$\mathrm{E}_t(r_{i,t+1}) = -b_t R_f \,\mathrm{Cov}_t\big(r_{i,t+1}, r_{M,t+1}\big) - c_t R_f \,\mathrm{Cov}_t\big(r_{i,t+1}, r^2_{M,t+1}\big), \tag{13}$$

Thus, we obtain a three-factor CAPM:

$$\mathrm{E}_t(r_{i,t+1}) = \lambda_1 \beta_{i,M} + \lambda_2 \gamma_{i,M}, \quad \text{with} \quad \lambda_1 = -b_t R_f \sigma^2_M, \quad \lambda_2 = -c_t R_f m^3_M. \tag{14}$$

To compute b_t and c_t we need to know the expected return of one more portfolio beside the market portfolio M.[9] Note that the formulas for λ_1 and λ_2 do not provide economic intuition for the sign of the premium. Harvey and Siddique (2000) provide further discussion by considering a Taylor series expansion of the formula $m_{t+1} = \beta U'(W_{t+1})/U'(W_t)$.

3 Portfolio Formation and Summary Statistics

In this section and the next, we replicate the empirical results in Harvey and Siddique (2000) for the "replication sample" period of July 1963 to December 1993, and we report them for other sample periods: the "post-sample" period, January 1994 to December 2019; the "pre-sample" period, July 1926 to June 1963; and the "extended sample" period, July 1963 to December 2019. In principle, we could also report the results for the "maximum sample" period, July 1926 to December 2019, but that would reduce the variables that are available, as the five Fama and French (2015) factors and the *PSS* factor of Langlois (2020) are available only after 1963. Moreover, the quality of the pre-sample return data is often considered to be below the quality of the return data after 1963.[10]

We use monthly and daily U.S. equity returns from CRSP. We retain stocks listed on the NYSE, AMEX, and NASDAQ. Our sample includes only common stocks (Share Code 10 and 11) and stocks not trading on a "when issued" basis. This leads to a total of 14,988 stocks over the replication sample period, while the figure in Harvey and Siddique (2000) is only 9,268 stocks (see the beginning of their Section III.C). Data changes to CRSP in the past two decades are likely responsible for this discrepancy. As communicated privately by Campbell Harvey, the 1997 version of CRSP

[9] To solve the 2-by-2 system of equations, Harvey and Siddique (2000) use the squared market return $r^2_{M,t+1}$ beside the market return $r_{M,t+1}$. This does not appear to be correct, however, as $r^2_{M,t+1}$ is not the return of a tradable asset. Moreover, Equation (12) cannot be satisfied, as the squared return is positive.

[10] Linnainmaa and Roberts (2018) note: "The reason for this difference appears to lie with the corrections to the number of shares data CRSP made in a project started in 2013. As Ken French notes, 'The file [CRSP] released in January 2015 [...] incorporates over 4000 changes that affect 400 Permnos. As a result, many of the returns we report for 1925-1946 change in our January 2015 update and some of the changes are large.' Ken French discusses the repercussions of these changes at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html."

used in Harvey and Siddique (2000) contained significantly fewer NASDAQ stocks than today.[11]

The Fama-French and momentum factors are obtained from Ken French's online data library, and the *PSS* factor is obtained from Hugues Langlois' website, and is available between July 1963 and December 2017. Table IA.1 in the Internet Appendix reports the definitions and the construction details for all variables.

To perform asset pricing tests, we employ several test portfolio groups throughout the paper. All test portfolios used in this paper are value-weighted except the momentum portfolios defined in Table 5, which are equally weighted. The groups that are constructed from multivariate sorts use as cutoffs only the values for the NYSE stocks, which insures a more equitable distribution of large stocks among the portfolios. Independent sorts are used in all portfolio groups except the 27 Carhart portfolios below, which are constructed using dependent sorts. The portfolio groups are: 10 Fama-French portfolios sorted on size (book value of equity); 10 Fama-French portfolios sorted on momentum (past return performance between months $t-12$ and $t-2$); 25 Fama-French portfolios sorted on size and book-to-market (B/M); 25 Fama-French portfolios sorted on size and momentum; 25 portfolios sorted on size and the coskewness measure in Equation (15); and 27 Carhart portfolios with dependent sorts on B/M, size, and momentum.[12] Except for the last two portfolio groups, which we construct ourselves, the other sorted portfolio groups are obtained from Ken French's online data library. We also construct 32 industry portfolios by using the industry names from Panel A in Table I in Harvey and Siddique (2000) and identifying the industry by its SIC code classification.[13]

[11]In the Internet Appendix, we run several tests on portfolios formed after removing all NASDAQ stocks, microcap stocks (i.e., stocks with a market capitalization of less than 250 million U.S. dollars), or penny stocks (i.e., stocks with a price of less than 5 U.S. dollars); see Table IA.10 and Tables IA.29 to IA.34. These restrictions do not change the results qualitatively, but they do lead to significant changes in the estimates.

[12]To construct the 27 Carhart portfolios, we use dependent sorts: we sort all stocks into terciles, first by B/M, then by size, then by momentum. We construct our own book-to-market variable from Compustat data, using the methodology in Davis *et al.* (2000). The portfolios are constructed at the end of every month, using the last known book equity value and the current market equity value.

[13]The only industry that we could not identify is "Distributors," which we replaced by "Other." In the Internet Appendix, we also report summary statistics for the 30 Fama-French industry portfolios (see Table IA.4).

Table 1 shows summary statistics that compare different measures of coskewness for the 25 Fama-French portfolios sorted on size and B/M. We use four ways to measure coskewness. The first measure is the (standardized) coskewness of an asset i:

$$Cosk_{i,t} = \frac{\mathrm{E}_{t-1}[\varepsilon_{i,t}\varepsilon_{M,t}^2]}{\sqrt{\mathrm{E}_{t-1}[\varepsilon_{i,t}^2]\mathrm{E}_{t-1}[\varepsilon_{M,t}^2]}}, \tag{15}$$

where $\varepsilon_{i,t}$ is the residual from regressing i's excess return on the excess market return, and $\varepsilon_{M,t}$ is the market return minus its unconditional average.[14] $Cosk_{i,t}$ represents the contribution of asset i to the coskewness of the market portfolio. A negative value of $Cosk_{i,t}$ indicates that asset i adds negative skewness to the broader portfolio. As investors do not like negative skewness in their portfolio, if asset i has negative coskewness, then it should have a low price and a high average return. The second measure is a non-standardized version of coskewness, β_{i,MKT^2}, where MKT is the excess market portfolio return. This measure is obtained as the coefficient in a univariate regression of i's excess return on MKT^2, which is a non-standardized version of the coskewness measure in Equation (15).[15]

The other two coskewness measures are betas on two value-weighted long-short portfolios that capture differences in coskewness. To construct the portfolios, in each month t we select all stocks listed on NYSE, AMEX, or NASDAQ that have at least 36 monthly returns during the past 60 months ($t-60$ to $t-1$).[16] For each of these stocks, we estimate the coskewness measure from Equation (15) using the past 60 months of returns. We rank the stocks based on estimated coskewness and form three portfolios: S^- with the bottom 30% (lowest coskewness), S^0 with the middle 40%, and S^+ with the top 30% (highest coskewness). The two long-short port-

[14] In Harvey and Siddique (2000), $Cosk_i$ is denoted by $\hat{\beta}_{\mathrm{SKD}_i}$.

[15] Harvey and Siddique (2000) are explicit about using the univariate beta on MKT^2. Nonetheless, as measure of coskewness one may also consider the coefficient on MKT^2 in a bivariate regression of the stock's excess return on MKT and MKT^2. However, this measure is not consistent with the theoretical coskewness measure $\gamma_{n,M}$ in Equation (9), which is proportional to the univariate beta on MKT^2.

[16] The 36-month requirement was communicated privately to the authors by Cambell Harvey, as it is not explicit in Harvey and Siddique (2000). Table IA.35 in the Internet Appendix shows results of Fama-MacBeth regressions similar to those in Table 4 for three different versions of *HS* that have a 36-, 48-, and 60-month requirement, respectively. The most significant results correspond to the 36-month version used in the paper.

folios are $HS = S^{-} - S^{+}$ (denoted *SKS* in Harvey and Siddique (2000)) and $HS^{-} = S^{-} - r_f$.

The significance levels for skewness and coskewness are computed by generating the statistics 10,000 times simulating it under the null using a standard normal distribution for skewness and a bivariate standard normal distribution for coskewness.[17] The cutoff levels used to determine the significance levels for skewness and coskewness are provided in Table IA.2 in the Internet Appendix.

The first thing to notice, which is also true for the other tables in Harvey and Siddique (2000), is that our estimates are fairly different from theirs, even though many of the qualitative results are the same. A possible reason is the large discrepancy between the number of stocks in their sample (9,268) and ours (14,988) over the replication sample period (1963 to 1993).

We compare the summary statistics in Table 1 against the intuition in Section 2: If an asset has negative coskewness with the investor's portfolio, then investing in the asset makes the overall portfolio more negatively skewed, hence makes the asset less desirable and increases its expected return. Therefore, in the data we should observe a negative relation between an asset's average return and its (standardized) coskewness. Also, we should observe a positive relation between an asset's average return and its beta on the coskewness factors (HS and HS^{-}), as these are portfolios of low coskewness stocks minus high/medium coskewness stocks. The last row in each panel in Table 1 shows the correlation of the average return of a test portfolio ("Ave.Ret.") with the other statistics. We check the following results: There is a negative correlation with the portfolio's (i) standardized coskewness, (ii) beta on HS, and (iii) beta on HS^{-}.

The results are mixed: For the replication sample period (1963 to 1993), the replicated results in Panel B are qualitatively similar to the original results in Panel A: The correlations in Panel B that correspond to the columns "Std. Cosk." and "β to HS^{-}" are negative, while the correlation that corresponds to the column "β to HS" is positive, indicating that Results (i) and (iii) are true but Result (ii) is not. Results (i) and (iii) also hold for the extended sample period (1963 to 2019), while only Result (iii)

[17] Harvey and Siddique (2000) mention in the caption of their Table I that the significance level for coskewness is computed using "an ARMA(2,0) process using a bivariate *Normal*." As this specification is unclear, we use instead a bivariate standard normal distribution.

Table 1: Properties of 25 Portfolios Sorted on Size and Book/Market

Panel A. Sample Period 1963 to 1993, Original Results

Size Quin-Tile	B/M Quin-Tile	Std. Skew.	Std. Cosk.	β to HS	β to HS^{-}	β to MKT^2	Time-Var. Cosk.	Ave. Exc.Ret. (%/mo)	β to MKT	Std. Dev. (%/mo)
1	1	−0.303**	−0.276**	0.138**	1.339**	−0.027**	Yes	0.310	1.403**	7.665
	2	−0.274**	−0.330**	0.166**	1.216**	−0.026**	Yes	0.698	1.263**	6.744
	3	−0.359**	−0.349**	0.160	1.098**	−0.024**	Yes	0.818	1.142**	6.135
	4	−0.135	−0.350**	0.196	1.023**	−0.024**	Yes	0.949	1.054**	5.842
	5	0.001	−0.334**	0.234*	1.048**	−0.025**	No	1.082	1.071**	6.142
2	1	−0.416**	−0.196**	0.002	1.337**	−0.020**	No	0.481	1.422**	7.128
	2	−0.445**	−0.332**	0.085	1.185**	−0.022**	Yes	0.720	1.246**	6.250
	3	−0.384**	−0.372**	0.116	1.078**	−0.022**	Yes	0.905	1.124**	5.708
	4	−0.268**	−0.257**	0.131	0.995**	−0.017**	Yes	0.921	1.030**	5.231
	5	−0.306**	−0.328**	0.128	1.085**	−0.022**	Yes	1.095	1.127**	5.943
3	1	−0.353**	−0.181	0.098	1.277**	−0.018**	No	0.439	1.344**	6.512
	2	−0.566**	−0.324**	0.074	1.091**	−0.018**	Yes	0.676	1.146**	5.527
	3	−0.552**	−0.341**	0.059	0.986**	−0.018**	Yes	0.746	1.036**	5.111
	4	−0.277**	−0.173**	0.091	0.930**	−0.013**	Yes	0.857	0.965**	4.794
	5	−0.377**	−0.261**	0.122	1.020**	−0.018**	Yes	1.055	1.060**	5.484
4	1	−0.244*	0.053	0.020	1.174**	−0.010	Yes	0.511	1.241**	5.857
	2	−0.491**	−0.232**	0.066	1.053**	−0.015**	Yes	0.388	1.131**	5.273
	3	−0.314**	−0.158**	0.039	0.991**	−0.013*	Yes	0.638	1.043**	4.975
	4	0.177	0.098	0.123	0.929**	−0.006	Yes	0.799	0.965**	4.811
	5	−0.066	−0.070*	0.154	1.074**	−0.012	Yes	1.039	1.112**	5.664
5	1	−0.069	0.214	0.049	0.974**	−0.005	Yes	0.366	1.025**	4.842
	2	−0.286**	0.013	0.149	0.910**	−0.009	Yes	0.384	0.994**	4.604
	3	−0.104	0.039	−0.206**	0.801**	−0.007	No	0.370	0.883**	4.277
	4	0.189	0.186**	−0.032	0.777**	−0.003	Yes	0.551	0.832**	4.181
	5	0.131	0.020	−0.034	0.839**	−0.007	Yes	0.715	0.889**	4.901
Corr. with Ave.Ret.		0.067	−0.498	0.648	−0.021	−0.319			−0.092	0.122

Panel B. Sample Period 1963 to 1993, Replication Results

Size Quin-Tile	B/M Quin-Tile	Std. Skew.	Std. Cosk.	β to HS	β to HS^{-}	β to MKT^2	Time-Var. Cosk.	Ave. Exc.Ret. (%/mo)	β to MKT	Std. Dev. (%/mo)
1	1	−0.327**	−0.275***	0.686***	1.307***	−0.029***	No	0.267	1.429***	7.667
	2	−0.323**	−0.333***	0.546***	1.142***	−0.028***	Yes	0.712	1.251***	6.712
	3	−0.245*	−0.285***	0.550***	1.060***	−0.023***	Yes	0.757	1.154***	6.148
	4	−0.149	−0.314***	0.473***	0.979***	−0.023***	No	0.912	1.071***	5.819
	5	0.027	−0.273***	0.614***	1.025***	−0.024***	No	1.104	1.105***	6.221
2	1	−0.400***	−0.219**	0.509***	1.296***	−0.024***	No	0.374	1.434***	7.179
	2	−0.477***	−0.333***	0.491***	1.122***	−0.024***	Yes	0.679	1.233***	6.159
	3	−0.499***	−0.391***	0.401***	1.010***	−0.023***	Yes	0.893	1.111***	5.585
	4	−0.199	−0.219**	0.487***	0.953***	−0.017***	No	0.957	1.033***	5.240
	5	−0.155	−0.264***	0.469***	1.032***	−0.021***	No	1.064	1.124***	5.900
3	1	−0.338***	−0.123	0.420***	1.225***	−0.018**	No	0.426	1.363***	6.594
	2	−0.574***	−0.333***	0.405***	1.055***	−0.020***	Yes	0.753	1.161***	5.599
	3	−0.528***	−0.331***	0.359***	0.932***	−0.019***	Yes	0.694	1.028***	5.046
	4	−0.235*	−0.116	0.328***	0.884***	−0.013**	Yes	0.897	0.975***	4.830
	5	−0.189	−0.178**	0.330**	0.967***	−0.017**	No	1.009	1.069***	5.573
4	1	−0.269**	0.054	0.348**	1.104***	−0.012	No	0.458	1.229***	5.807
	2	−0.487***	−0.221**	0.338***	1.021***	−0.016**	Yes	0.423	1.131***	5.308
	3	−0.282**	−0.113	0.329***	0.941***	−0.013**	Yes	0.652	1.037***	4.963
	4	0.105	0.141	0.279**	0.875***	−0.007	No	0.813	0.968***	4.793
	5	−0.109	−0.028	0.313**	0.981***	−0.012*	No	0.909	1.084***	5.543
5	1	−0.037	0.254***	0.225**	0.890***	−0.005	Yes	0.326	0.999***	4.802
	2	−0.230*	0.043	0.245**	0.881***	−0.010*	No	0.374	0.979***	4.597
	3	−0.012	0.057	0.268***	0.777***	−0.008	No	0.361	0.859***	4.265
	4	0.221*	0.298***	0.314***	0.774***	−0.002	No	0.556	0.847***	4.247
	5	−0.033	−0.136	0.082	0.764***	−0.013**	No	0.630	0.859***	4.759
Corr. with Ave.Ret.		0.122	−0.394	0.185	−0.173	−0.249			−0.212	−0.041

(*Continued*)

Table 1: (*Continued*)

Panel C. Sample Period 1994 to 2019, Post-Sample Results

Size Quin-Tile	B/M Quin-Tile	Std. Skew.	Std. Cosk.	β to HS	β to HS^{-}	β to MKT^2	Time-Var. Cosk.	Ave. Exc.Ret. (%/mo)	β to MKT	Std. Dev. (%/mo)
1	1	0.230*	−0.047	−1.060***	1.202***	−0.055***	No	0.270	1.425***	8.055
	2	0.471***	−0.007	−0.787***	1.072***	−0.045***	Yes	0.875	1.245***	7.152
	3	−0.236*	−0.048	−0.477***	0.988***	−0.041***	No	0.780	1.065***	5.721
	4	−0.059	0.017	−0.436***	0.904***	−0.034***	Yes	1.042	0.978***	5.516
	5	−0.584***	−0.131	−0.343***	0.976***	−0.044***	No	0.996	1.017***	5.592
2	1	−0.322**	−0.008	−0.699***	1.223***	−0.049***	No	0.670	1.358***	7.031
	2	−0.433***	0.002	−0.413***	1.058***	−0.040***	No	0.890	1.128***	5.758
	3	−0.481***	0.045	−0.159	1.010***	−0.034***	No	0.856	1.003***	5.121
	4	−0.620***	−0.031	−0.196	1.003***	−0.037***	No	0.823	1.003***	5.244
	5	−0.703***	−0.048	−0.210	1.132***	−0.043***	Yes	0.834	1.137***	6.049
3	1	−0.473***	−0.069	−0.773***	1.128***	−0.050***	No	0.663	1.284***	6.446
	2	−0.388***	0.066	−0.323***	1.042***	−0.036***	No	0.838	1.086***	5.256
	3	−0.415***	0.130	−0.119	1.010***	−0.030***	No	0.818	0.989***	4.855
	4	−0.435***	0.100	−0.042	1.019***	−0.030***	No	0.883	0.982***	5.022
	5	−0.551***	0.037	−0.011	1.119***	−0.035***	No	0.911	1.058***	5.665
4	1	−0.222	0.047	−0.786***	1.080***	−0.041***	No	0.850	1.225***	5.876
	2	−0.688***	−0.005	−0.168	1.025***	−0.036***	No	0.871	1.017***	4.798
	3	−0.820***	−0.096	−0.041	1.023***	−0.039***	Yes	0.761	0.984***	4.864
	4	−0.597***	0.061	−0.081	0.984***	−0.032***	No	0.860	0.959***	4.776
	5	−0.606***	0.078	−0.101	1.123***	−0.034***	No	0.769	1.065***	5.625
5	1	−0.547***	0.110	−0.516***	0.878***	−0.031***	Yes	0.782	0.942***	4.297
	2	−0.657***	−0.012	−0.080	0.902***	−0.031***	Yes	0.702	0.867***	4.077
	3	−0.571***	0.043	0.040	0.920***	−0.029***	Yes	0.817	0.858***	4.256
	4	−1.176***	−0.145	0.221*	1.039***	−0.041***	No	0.477	0.927***	5.041
	5	−0.403***	0.258***	−0.058	1.146***	−0.020*	No	0.668	1.068***	6.127
Corr. with Ave.Ret.		−0.006	0.167	0.227	−0.465	0.366			−0.398	−0.398

Panel D. Sample Period 1963 to 2019, Extended-Sample Results

Size Quin-Tile	B/M Quin-Tile	Std. Skew.	Std. Cosk.	β to HS	β to HS^{-}	β to MKT^2	Time-Var. Cosk.	Ave. Exc.Ret. (%/mo)	β to MKT	Std. Dev. (%/mo)
1	1	−0.052	−0.159**	−0.196	1.264***	−0.038***	No	0.268	1.425***	7.842
	2	0.079	−0.166**	−0.128	1.114***	−0.034***	No	0.787	1.247***	6.913
	3	−0.236**	−0.170**	0.032	1.030***	−0.029***	Yes	0.767	1.113***	5.951
	4	−0.108	−0.152**	0.014	0.949***	−0.027***	No	0.972	1.029***	5.678
	5	−0.197**	−0.204***	0.132	1.005***	−0.031***	No	1.054	1.064***	5.935
2	1	−0.365***	−0.108	−0.102	1.266***	−0.032***	No	0.510	1.399***	7.108
	2	−0.457***	−0.163**	0.034	1.096***	−0.029***	Yes	0.776	1.185***	5.974
	3	−0.484***	−0.176***	0.120	1.009***	−0.027***	No	0.876	1.061***	5.373
	4	−0.393***	−0.121*	0.144	0.972***	−0.024***	No	0.896	1.017***	5.239
	5	−0.421***	−0.159**	0.128	1.071***	−0.029***	No	0.958	1.127***	5.965
3	1	−0.398***	−0.086	−0.183*	1.186***	−0.029***	No	0.535	1.327***	6.523
	2	−0.492***	−0.131**	0.038	1.049***	−0.026***	Yes	0.792	1.126***	5.440
	3	−0.476***	−0.107	0.118	0.963***	−0.023***	Yes	0.751	1.010***	4.956
	4	−0.338***	−0.005	0.143*	0.938***	−0.019***	Yes	0.891	0.976***	4.916
	5	−0.362***	−0.071	0.160*	1.027***	−0.024***	No	0.964	1.062***	5.612
4	1	−0.247***	0.050	−0.226**	1.094***	−0.022***	No	0.638	1.226***	5.838
	2	−0.570***	−0.096	0.081	1.023***	−0.023***	No	0.629	1.080***	5.081
	3	−0.521***	−0.094	0.143*	0.973***	−0.022***	No	0.702	1.012***	4.914
	4	−0.219**	0.104	0.099	0.918***	−0.015***	No	0.834	0.963***	4.782
	5	−0.346***	0.027	0.106	1.037***	−0.020***	No	0.845	1.073***	5.577
5	1	−0.237**	0.205***	−0.151**	0.885***	−0.014***	No	0.535	0.973***	4.579
	2	−0.392***	0.037	0.081	0.890***	−0.017***	Yes	0.525	0.929***	4.365
	3	−0.270***	0.050	0.152**	0.835***	−0.015***	Yes	0.571	0.858***	4.264
	4	−0.635***	0.040	0.269***	0.880***	−0.016***	No	0.520	0.880***	4.626
	5	−0.300***	0.057	0.013	0.917***	−0.016***	No	0.647	0.950***	5.427
Corr. with Ave.Ret.		0.017	−0.347	0.485	−0.226	−0.047			−0.277	−0.154

(*Continued*)

Table 1: (*Continued*)

Panel E. Sample Period 1926 to 1963, Pre-Sample Results

Size Quin-Tile	B/M Quin-Tile	Std. Skew.	Std. Cosk.	β to HS	β to HS^-	β to MKT^2	Time-Var. Cosk.	Ave. Exc.Ret. (%/mo)	β to MKT	Std. Dev. (%/mo)
1	1	3.029***	0.226***	−2.109***	1.900***	0.026***	Yes	1.043	1.751***	16.595
	2	3.160***	0.491***	−2.295***	1.579***	0.026***	Yes	0.554	1.520***	13.033
	3	2.205***	0.901***	−2.292***	1.637***	0.031***	No	1.304	1.545***	12.133
	4	2.885***	0.667***	−2.129***	1.496***	0.025***	Yes	1.439	1.429***	11.140
	5	3.014***	0.757***	−2.426***	1.659***	0.030***	Yes	1.746	1.582***	12.654
2	1	1.527***	0.001	−1.540***	1.227***	0.013***	Yes	0.839	1.169***	9.065
	2	2.140***	0.529***	−1.827***	1.310***	0.019***	Yes	1.181	1.254***	9.306
	3	2.251***	0.829***	−1.825***	1.361***	0.022***	Yes	1.143	1.289***	9.330
	4	2.335***	0.955***	−1.954***	1.421***	0.025***	Yes	1.278	1.341***	9.786
	5	1.955***	0.815***	−2.215***	1.648***	0.028***	Yes	1.623	1.542***	11.539
3	1	1.622***	0.448***	−1.506***	1.269***	0.017***	Yes	1.014	1.189***	8.493
	2	0.391***	−0.186**	−1.392***	1.188***	0.011***	No	1.082	1.124***	7.731
	3	1.378***	0.837***	−1.591***	1.288***	0.018***	Yes	1.177	1.196***	8.249
	4	1.999***	1.081***	−1.824***	1.388***	0.023***	Yes	1.225	1.288***	9.059
	5	1.949***	0.915***	−2.234***	1.676***	0.028***	Yes	1.307	1.583***	11.477
4	1	−0.149	−0.725***	−1.013***	1.038***	0.008***	Yes	0.888	0.997***	6.771
	2	1.043***	0.686***	−1.370***	1.124***	0.015***	No	0.982	1.078***	7.272
	3	1.770***	1.223***	−1.629***	1.263***	0.020***	No	1.096	1.180***	8.094
	4	1.813***	1.072***	−1.699***	1.401***	0.022***	Yes	1.132	1.287***	8.992
	5	2.010***	1.196***	−2.393***	1.773***	0.032***	Yes	1.324	1.637***	11.826
5	1	−0.094	−0.883***	−0.911***	0.988***	0.007***	Yes	0.821	0.937***	6.275
	2	0.658***	0.314***	−1.014***	1.012***	0.012***	Yes	0.784	0.956***	6.393
	3	1.157***	0.764***	−1.332***	1.130***	0.015***	Yes	0.931	1.045***	7.176
	4	1.315***	0.937***	−1.769***	1.363***	0.020***	Yes	0.884	1.257***	8.744
	5	1.681***	0.671***	−1.957***	1.672***	0.026***	Yes	1.378	1.542***	11.751
Corr. with Ave.Ret.		0.426	0.464	−0.612	0.567	0.646			0.568	0.415

Description: The table summarizes properties of the 25 Fama-French portfolios with independent sorts on size and book-to-market, using as cutoffs the stocks listed on NYSE. Standardized (unconditional) skewness is the third central moment about the mean. Standardized (unconditional) coskewness of the i^{th} asset is defined as $E[\varepsilon_{i,t}\varepsilon_{M,t}^2]/(\sqrt{E[\varepsilon_{i,t}^2]}E[\varepsilon_{M,t}^2])$, where $\varepsilon_{i,t}$ is the residual from regressing i's excess return on the excess market return, and $\varepsilon_{M,t}$ is the market return minus its unconditional average. Time-variation in conditional coskewness is captured through the autoregression $E_t[\varepsilon_{i,t+1}\varepsilon_{M,t+1}^2] = \rho_0 + \rho_1\varepsilon_{i,t}\varepsilon_{M,t}^2 + \rho_2\varepsilon_{i,t-1}\varepsilon_{M,t-1}^2$ and whether it is significant at 10% level (i.e., the p-value of the joint test of zero coefficients is below 10%). The coskewness factors are the long-short portfolios $HS = S^- - S^+$ and $HS^- = S^- - r_f$, where S^- (S^+) is the portfolio of the 30% of stocks with the lowest (highest) past standardized coskewness, and r_f is the one-month Treasury bill rate. We also report cross-sectional correlations between the average excess return and other variables. The stars indicate significance at 1% (***), 5% (**), and 10% (*) levels. For the original results, 3-star significance is not available.

Interpretation: Theory predicts that a portfolio's average (excess) return should be negatively correlated to the portfolio's (i) standardized coskewness, (ii) beta on $HS = S^- - S^+$, and (iii) beta on $HS^- = S^- - r_f$. For the replication sample period (1963 to 1993), the replicated results in Panel B are qualitatively similar to Harvey and Siddique (2000)'s original results in Panel A: (i) and (iii) hold, while (ii) does not. Results (i) and (iii) also hold for the extended sample period (1963 to 2019), while only Result (iii) holds for the post-sample period (1994 to 2019), and only Result (ii) holds for the pre-sample period (1926 to 1963).

holds for the post-sample period (1994 to 2019), and only Result (ii) holds for the pre-sample period (1926 to 1963).

Table IA.3 in the Internet Appendix shows weaker results for the 32 industry portfolios: For the replication sample period (1963 to 1993), the replicated results in Panel B are qualitatively similar to Harvey and Siddique (2000)'s original results in Panel A: Results (i)-(iii) do not hold, as the corresponding estimates are either positive or close to zero (and thus likely to be insignificant). Results (i)-(iii) also do not hold in the extended sample period (1963 to 2019) but hold for the post-sample period (1994 to 2019), while in the pre-sample period (1926 to 1963) only Result (ii) holds. The results are also weaker in all sample periods for the 30 Fama-French industry portfolios (Table IA.4); the 10 Fama-French portfolios sorted on size (Table IA.5); the 27 Carhart portfolios sorted on B/M, size, and momentum (Table IA.6); and the 25 Fama-French portfolios sorted on size and coskewness (Table IA.9). The results are however stronger for the 10 Fama-French portfolios sorted on momentum (Table IA.7) and the 25 Fama-French portfolios sorted on size and momentum (Table IA.8): In almost all sample periods considered, Results (i) and (iii) hold, while Result (ii) does not.

If coskewness is priced in the data, then the portfolio *HS* should have a positive and significant premium. Harvey and Siddique (2000) report an average return of *HS* (i.e., the return spread between S^- and S^+) of 3.60% per year over the replication period (1963 to 1993) that is significant at a 95% level. In Table IA.12 in the Internet Appendix, we find this figure to be lower (2.58% per year) and not statistically significant at a 90% level. In Table IA.12 in the Internet Appendix, the average annual return of *HS* is −3.15% over the pre-sample period (1926 to 1963) and 1.64% over the post-sample period (1994 to 2019), but these figures are not statistically significant at a 90% level.

Overall, the summary statistics suggest that coskewness plays a role in explaining the cross-section of asset returns, although the results are mixed. Next, we consider more formal tests of coskewness relative to alternative asset pricing models.

4 Asset Pricing Tests for Coskewness

Traditional asset pricing models such as the CAPM or the Fama-French three-factor model (FF3) often are tested on specific stock portfolios such as those formed on size, book-to-market or momentum. To understand how coskewness contributes to asset pricing, we check whether the addition of a coskewness factor to these models improves the test results.

4.1 GRS and Correlation Tests

First, we carry out GRS tests as in Gibbons *et al.* (1989) by running time-series regressions of excess returns on the FF3 factor:

$$r_{i,t} \;=\; \alpha_i + \hat{\beta}_i MKT_t + \hat{s}_i SMB_t + \hat{h}_i HML_t + e_{i,t}, \tag{16}$$

and compute an F-statistic to test whether the intercepts α_i are jointly zero in the cross-section. Next, we check whether the F-statistic decreases when the coskewness factor HS^- is included in the regression (16). Lower values of the GRS F-test can be interpreted as an indication of less mispricing, although they can be caused also by the noise introduced by the new factor.

We also employ a correlation test, as in Harvey and Siddique (2000), by considering cross-sectional regressions:

$$r_i \;=\; \lambda_0 + \lambda_{MKT}\hat{\beta}_i + \lambda_{SMB}\hat{s}_i + \lambda_{HML}\hat{h}_i + \varepsilon_i, \tag{17}$$

where the coefficients λ are computed every month in a two-step estimation using time-series betas from a Fama and MacBeth (1973) procedure. We compute the correlation between the λ_0 estimates (the pricing errors) and the ex post realizations of the S^- portfolio. The correlation test of Harvey and Siddique (2000) is more difficult to interpret, as it does not test directly whether the cross-sectional alphas are jointly zero.

In Table 2 we show the GRS and correlation test results for several portfolios groups, including 32 industry portfolios;[18] the 25 Fama-French portfolios sorted by size and B/M; the 10 Fama-French portfolios sorted by size; the 27 Carhart portfolios sorted on B/M, size, and momentum; the 10 Fama-French portfolios sorted by momentum; and 10 portfolios sorted by coskewness. The returns are computed over a holding period of 1 month ("1-mo holding") or 6 months ("6-mo holding").

[18] Harvey and Siddique (2000) exclude 5 of the 32 industry portfolios from the regression because these portfolios contain fewer than 10 firms.

Table 2: Tests of Intercepts from the Fama-French 3-Factor Model

Panel A. Sample Period 1963 to 1993, Original Results

Test Portfolios	No. of Portfolios	F-test for FF3	F-test for FF3 + HS^-	Correlation with S^-
Industrial, 1-mo holding	27	8.56 (0.000)	1.40 (0.093)	0.330
Size and B/M, 1-mo holding	25	1.92 (0.006)	1.43 (0.086)	0.340
Size, 1-mo holding	10	12.32 (0.000)	7.56 (0.003)	0.410
B/M, Size, and Momentum, 1-mo holding	27	4.63 (0.000)	1.82 (0.011)	0.312
Momentum, 1-mo holding	10	11.36 (0.000)	1.56 (0.118)	0.610
Momentum, 6-mo holding	10	42.82 (0.000)	2.57 (0.010)	0.120
Coskewness, 1-mo holding	25	74.59 (0.000)	0.698 (0.859)	0.306
Coskewness, 6-mo holding	25	78.85 (0.000)	1.20 (0.235)	0.423

Panel B. Sample Period 1963 to 1993, Replication Results

Test Portfolios	No. of Portfolios	F-test for FF3	F-test for FF3 + HS^-	Correlation with S^-
Industrial, 1-mo holding	32	2.12 (0.001)	2.14 (0.000)	0.336
Size and B/M, 1-mo holding	25	1.75 (0.015)	1.87 (0.008)	0.372
Size, 1-mo holding,	10	2.54 (0.006)	2.65 (0.004)	0.051
B/M, Size, and Momentum, 1-mo holding	27	6.45 (0.000)	6.16 (0.000)	0.415
Momentum, 1-mo holding,	10	6.82 (0.000)	6.45 (0.000)	0.204
Momentum, 6-mo holding	10	5.71 (0.000)	5.34 (0.000)	0.220
Coskewness, 1-mo holding	25	1.58 (0.041)	1.49 (0.064)	0.404
Coskewness, 6-mo holding	25	1.02 (0.441)	1.36 (0.120)	0.447

(*Continued*)

Table 2: Tests of Intercepts from the Fama-French 3-Factor Model

Panel C. Sample Period 1994 to 2019, Post-Sample Results

Test Portfolios	No. of Portfolios	F-test for FF3	F-test for FF3 + HS^{-}	Correlation with S^{-}
Industrial, 1-mo holding	32	1.96 (0.002)	1.91 (0.003)	0.211
Size and B/M, 1-mo holding	25	4.21 (0.000)	4.34 (0.000)	0.161
Size, 1-mo holding,	10	2.30 (0.013)	2.31 (0.012)	0.154
B/M, Size, and Momentum, 1-mo holding	27	3.70 (0.000)	3.76 (0.000)	−0.047
Momentum, 1-mo holding,	10	1.96 (0.038)	1.83 (0.055)	−0.112
Momentum, 6-mo holding	10	1.74 (0.071)	1.62 (0.099)	0.095
Coskewness, 1-mo holding	25	1.46 (0.078)	1.40 (0.101)	0.616
Coskewness, 6-mo holding	25	0.84 (0.695)	0.81 (0.729)	0.609

Panel D. Sample Period 1963 to 2019, Extended-Sample Results

Test Portfolios	No. of Portfolios	F-test for FF3	F-test for FF3 + HS^{-}	Correlation with S^{-}
Industrial, 1-mo holding	32	2.78 (0.000)	2.73 (0.000)	0.285
Size and B/M, 1-mo holding	25	3.86 (0.000)	3.99 (0.000)	0.286
Size, 1-mo holding,	10	2.96 (0.001)	3.06 (0.001)	0.098
B/M, Size, and Momentum, 1-mo holding	27	7.93 (0.000)	7.93 (0.000)	0.234
Momentum, 1-mo holding,	10	4.88 (0.000)	4.58 (0.000)	0.061
Momentum, 6-mo holding	10	3.14 (0.001)	2.99 (0.001)	0.159
Coskewness, 1-mo holding	25	0.96 (0.527)	0.88 (0.639)	0.473
Coskewness, 6-mo holding	25	1.04 (0.413)	1.24 (0.197)	0.483

(*Continued*)

Table 2: (*Continued*)

Panel E. Sample Period 1926 to 1963, Pre-Sample Results

Test Portfolios	No. of Portfolios	F-test for FF3	F-test for FF3 + HS^-	Correlation with S^-
Size and B/M, 1-mo holding	25	1.23	1.23	0.251
		(0.207)	(0.205)	
Size, 1-mo holding,	10	1.30	1.30	0.341
		(0.231)	(0.230)	
Momentum, 1-mo holding,	10	2.75	2.75	0.205
		(0.003)	(0.003)	
Momentum, 6-mo holding	10	2.41	2.39	−0.032
		(0.009)	(0.009)	
Coskewness, 1-mo holding	25	0.71	0.73	0.609
		(0.843)	(0.825)	
Coskewness, 6-mo holding	25	0.74	0.75	0.534
		(0.815)	(0.803)	

Description: The table presents the Gibbons-Ross-Shanken (GRS) F-statistic of the Fama and French (1993) three-factor model (FF3) and another model (FF3 + HS^-) that includes $HS^- = S^- - r_f$, the excess return on the portfolio of the 30% of stocks with the lowest past standardized coskewness. The table also reports the correlation between the return on the S^- portfolio and intercepts obtained from month-by-month cross-sectional regressions on the FF3 factors. In parentheses are the p-values. The 1-month or 6-month holding periods refer to the period over which test portfolio returns are computed. The test portfolios are the 32 industry portfolios constructed as in Harvey and Siddique (2000), the 25 Fama-French portfolios sorted on size and book-to-market, the 10 Fama-French portfolios sorted on size, the 10 Fama-French portfolios sorted on momentum (i.e., on the past performance from month $t-12$ to month $t-2$), and the 25 portfolios sorted on size and coskewness.

Interpretation: If coskewness is priced in the cross-section of stocks, the intercepts (alphas) from a three-factor model (FF3) should become closer to zero when we introduce a coskewness factor (HS^-), and that would imply (i) a significant decrease of the corresponding F-statistic and (ii) a positive correlation between FF3 alphas and the coskewness factor. Both (i) and (ii) hold in Panel A, while in our replication only (ii) holds, both in the replication sample period (1963-1993, Panel B) and in the other sample periods.

The results of the GRS test are inconclusive: Harvey and Siddique (2000) report significant decreases in the GRS F-statistics for all portfolio groups once the coskewness factor is introduced (see Panel A of our Table 2), while our replication shows little change in the GRS F-statistics (see Panel B of Table 2) for all portfolio groups, both in the replication sample period and in the other sample periods.

For the correlation test, as in Harvey and Siddique (2000), we generally find a positive replicated correlation between the coskewness portfolio S^- and intercepts from regressions on the FF3 factors, for all portfolio groups and all sample periods. By contrast, the more formal GRS test does not show an improvement in pricing once HS^{-} is introduced, possibly because of the noise in this coskewness proxy, a feature that is discussed in Section 5.

4.2 R^2 Tests

Another way to test the importance of coskewness to asset pricing is to check whether the R^2 increases in asset pricing regressions when a coskewness factor is introduced in addition to the CAPM or FF3 factors. We use two estimation methods: the full information maximum likelihood (FIML), and the cross-sectional regressions (CSR) introduced in Fama and MacBeth (1973). The main difference between the FIML and CSR methods is that the former method assumes that betas are constant over time. The FIML method is a multivariate version of Equation (17), which allows the intercepts and beta estimates to vary across the cross-section, although they remain constant in time. We maximize the likelihood function and use the beta estimates to run the cross-sectional regressions:

$$\hat{\mu}_i \; = \; \lambda_0 + \lambda_{MKT}\hat{\beta}_i + \lambda_{SMB}\hat{s}_i + \lambda_{HML}\hat{h}_i + \varepsilon_i, \tag{18}$$

where $\hat{\mu}_i$ are the unconditional average excess returns of asset i. This is a two-stage estimation procedure where the average excess returns and betas are estimated using all the returns, and the risk premia are estimated from the average excess returns and betas. In contrast, the CSR method uses 60 time-series observations to estimate the betas. These betas are then employed in cross-sectional regressions using the 61st period returns to estimate the risk premia, i.e., the λs.

In Table 3, we show the adjusted R^2 of FIML and CSR regressions.[19] The tests are performed for several portfolio groups, including 32 industry portfolios; the 25 Fama-French portfolios sorted by size and B/M; the 10 Fama-French portfolios sorted by momentum, with returns computed for a holding period of 6 months; the 10 Fama-French portfolios sorted by size; and the 27 Carhart portfolios sorted on B/M, size, and momentum.

The results of the R^2 test are mixed: By comparing Panels A and B in Table 3, we confirm qualitatively the results of Harvey and Siddique (2000) that the adjusted R^2 increases when a coskewness factor (HS^{-} or

[19] In the first row and first column of Panel A, we replace the original value of 1.53 with 15.3, since table values have only one digit after the period, and the new value (15.3) is closer to the value in the first row and second column (13.2).

Table 3: R^2 of Regressions for Portfolio Groups

Panel A. Sample Period 1963 to 1993, Original Results

	FIML (constant betas)			2-step CSR (rolling betas)		
Test Portfolios	CAPM	CAPM + HS^-	CAPM + HS	CAPM	CAPM + HS^-	CAPM + HS
Size and B/M (25)	11.4	68.1	62.7	21.5	25.2	25.2
Industrial (32)	15.3	13.2	9.2	9.6	17.9	18.2
Size (10)	44.7	84.9	81.3	25.6	54.1	55.6
B/M, Size, and Mom (27)	−3.9	1.4	1.4	11.3	19.3	18.2
Mom (10), 6-mo holding	3.5	61.1	59.6	30.0	46.9	45.6

	FIML (constant betas)			2-step CSR (rolling betas)		
Test Portfolios	FF3	FF3 + HS^-	FF3 + HS	FF3	FF3 + HS^-	FF3 + HS
Size and B/M (25)	71.8	82.5	81.0	46.0	48.9	49.5
Industrial (32)	25.3	30.1	28.5	18.3	28.1	29.5
Size (10)	84.7	83.0	81.7	62.9	65.1	65.7
B/M, Size, and Mom (27)	6.8	3.2	8.5	32.3	38.6	37.4
Mom (10), 6-mo holding	89.1	95.8	86.9	57.3	67.1	61.8

Panel B. Sample Period 1963 to 1993, Replication Results

	FIML (constant betas)			2-step CSR (rolling betas)		
Test Portfolios	CAPM	CAPM + HS^-	CAPM + HS	CAPM	CAPM + HS^-	CAPM + HS
Size and B/M (25)	0.4	32.9	21.2	22.8	31.3	31.4
Industrial (32)	2.9	3.0	2.7	11.6	17.1	17.5
Size (10)	81.9	79.5	79.4	28.4	43.7	45.9
B/M, Size, and Mom (27)	−3.9	47.9	30.8	13.9	21.6	21.2
Mom (10), 6-mo holding	−1.7	86.3	74.1	20.4	36.5	35.1

	FIML (constant betas)			2-step CSR (rolling betas)		
Test Portfolios	FF3	FF3 + HS^-	FF3 + HS	FF3	FF3 + HS^-	FF3 + HS
Size and B/M (25)	73.1	72.0	72.5	46.4	49.2	49.4
Industrial (32)	22.2	19.4	20.9	22.6	26.5	27.0
Size (10)	77.0	80.2	80.5	54.8	55.9	56.4
B/M, Size, and Mom (27)	17.2	59.4	49.7	32.8	36.9	36.9
Mom (10), 6-mo holding	79.1	82.9	76.4	40.1	45.4	44.4

Panel C. Sample Period 1994 to 2019, Post-Sample Results

	FIML (constant betas)			2-step CSR (rolling betas)		
Test Portfolios	CAPM	CAPM + HS^-	CAPM + HS	CAPM	CAPM + HS^-	CAPM + HS
Size and B/M (25)	12.2	14.6	11.8	13.6	22.5	20.3
Industrial (32)	−1.3	5.1	6.5	11.2	14.7	15.1
Size (10)	26.8	16.4	16.7	23.7	41.9	38.2
B/M, Size, and Mom (27)	0.2	−2.8	−3.9	16.4	25.5	25.6
Mom (10), 6-mo holding	33.1	26.3	24.6	19.9	36.7	38.9

	FIML (constant betas)			2-step CSR (rolling betas)		
Test Portfolios	FF3	FF3 + HS^-	FF3 + HS	FF3	FF3 + HS^-	FF3 + HS
Size and B/M (25)	33.0	30.4	30.3	41.7	43.4	43.6
Industrial (32)	9.6	11.8	8.1	17.7	20.8	21.4
Size (10)	18.1	37.6	22.7	53.7	56.6	53.6
B/M, Size, and Mom (27)	30.1	60.8	50.0	38.4	43.1	42.4
Mom (10), 6-mo holding	21.4	8.8	20.1	47.7	49.6	49.8

(*Continued*)

Table 3: (*Continued*)

Panel D. Sample Period 1963 to 2019, Extended-Sample Results

	FIML (constant betas)			2-step CSR (rolling betas)		
Test Portfolios	CAPM	CAPM + HS^-	CAPM + HS	CAPM	CAPM + HS^-	CAPM + HS
Size and B/M (25)	3.7	8.5	17.8	18.5	27.1	26.8
Industrial (32)	−3.2	−6.7	−6.6	10.6	15.3	15.6
Size (10)	86.1	84.2	84.5	25.3	43.7	43.5
B/M, Size, and Mom (27)	−2.6	−0.8	19.9	14.3	22.5	22.4
Mom (10), 6-mo holding	28.0	65.1	30.8	19.8	35.0	34.9

	FIML (constant betas)			2-step CSR (rolling betas)		
Test Portfolios	FF3	FF3 + HS^-	FF3 + HS	FF3	FF3 + HS^-	FF3 + HS
Size and B/M (25)	60.9	59.2	59.7	44.7	47.3	47.4
Industrial (32)	14.1	36.0	28.4	19.6	23.1	23.7
Size (10)	89.0	87.2	87.2	54.8	56.9	55.9
B/M, Size, and Mom (27)	30.8	37.2	28.5	35.3	39.5	39.3
Mom (10), 6-mo holding	81.0	78.1	79.0	43.0	46.5	46.0

Panel E. Sample Period 1926 to 1963, Pre-Sample Results

	FIML (constant betas)			2-step CSR (rolling betas)		
Test Portfolios	CAPM	CAPM + HS^-	CAPM + HS	CAPM	CAPM + HS^-	CAPM + HS
Size and B/M (25)	29.3	32.6	31.7	16.9	22.0	21.1
Industrial (32)	−0.7	15.1	11.0	8.8	12.0	11.7
Size (10)	93.6	93.6	94.6	33.8	40.4	39.8
B/M, Size, and Mom (27)	−3.3	−7.1	−5.4	2.0	0.7	2.4
Mom (10), 6-mo holding	76.0	85.7	78.4	17.5	27.1	25.6

	FIML (constant betas)			2-step CSR (rolling betas)		
Test Portfolios	FF3	FF3 + HS^-	FF3 + HS	FF3	FF3 + HS^-	FF3 + HS
Size and B/M (25)	58.7	60.1	55.8	32.0	35.3	35.1
Industrial (32)	3.4	19.9	16.1	16.7	19.0	18.6
Size (10)	97.0	96.3	96.7	47.1	47.5	47.1
B/M, Size, and Mom (27)	2.8	0.8	0.2	7.5	10.2	8.3
Mom (10), 6-mo holding	83.8	80.6	81.2	33.7	36.8	36.1

Description: This table presents the adjusted R^2 from the estimation of risk premia in different asset pricing models. We use two methods of estimation: full-information maximum likelihood estimation ("FIML"), which assumes constant betas; and month-by-month Fama-MacBeth cross-sectional regressions ("CSR"), where betas are estimated in rolling regressions using 60 months at a time. In FIML for n factors $F_1, \ldots, F_n$, we run regressions of the form: $\hat{\mu}_i = \lambda_0 + \lambda_1 \hat{\beta}_{F_1,i} + \ldots + \lambda_n \hat{\beta}_{F_n,i} + e_i$ of test portfolio average excess returns on the factor betas. In CSR, we run regressions of test portfolio next period's excess return on the factor betas. We report the adjusted R^2 for FIML and the time-series average adjusted R^2 for CSR. The holding period refers to the period over which test portfolio returns are computed: 1 month (the default) or 6 months. The test portfolios are: 32 industry portfolios constructed as in Harvey and Siddique (2000), the 25 Fama-French portfolios sorted on size and book-to-market; the 10 Fama-French portfolios sorted on size; the 10 Fama-French portfolios sorted on momentum (i.e., on the past performance from month $t-12$ to month $t-2$), and the 27 Carhart portfolios with dependent sorts on book-to-market, size, and momentum. As risk factors, we use either the factors from the CAPM model or from the Fama-French three-factor model (FF3), with the optional addition of $HS^- = S^- - r_f$ or $HS = S^- - S^+$, where S^- (S^+) is the portfolio of the 30% of stocks with the lowest (highest) past standardized coskewness.

Interpretation: The adjusted R^2 in Panel B generally increases when a coskewness factor (HS^- or HS) is added to the CAPM or FF3 regressions, but not as significantly as reported originally by Harvey and Siddique (2000) in Panel A for the replication sample period (1963 to 1993).

HS) is added to the CAPM or FF3 regressions (FIML or CSR), although the replicated increases in R^2 are generally smaller than the original increases. The results are similar in the other sample periods (1994 to 2019 and 1926 to 1963), with even smaller increases in R^2. Similar to the interpretation of the correlation test in Table 2, the results of the R^2 test in Table 3 suggest that the coskewness factors (HS^{-} or HS) contain pricing information in addition to the CAPM or FF3 factors.

4.3 Fama-MacBeth Tests for Individual Stocks

Another way to test whether coskewness is priced in the cross-section of stocks is to perform Fama and MacBeth (1973) tests at the individual stock level. The advantage of using portfolio groups (as we have done thus far) is that (i) the return data are usually available for the whole sample period, and (ii) the errors made on estimating betas in the first step are smaller than when using individual stocks, which creates smaller biases on the risk premium estimates in the second stage (caused by the error-in-variable problem). The disadvantage, however, is that the reduction in the number of cross-sectional observations may lead to information loss. By contrast, using individual stocks leads to better cross-sectional information but, as the stocks have different return histories, also leads to large idiosyncratic variation.

Using the terminology in Section 4.2, we describe a FIML method to estimate risk premia for individual securities. In the first step, we compute the individual betas by running a regression of stocks' excess return on the model factors. In the second step, we average the excess returns for each stock over the available months and we run one regression on the betas from the first step and obtain the risk premia.[20] To mitigate the large idiosyncratic variation, we follow Harvey and Siddique (2000) and weight each security i by the number $1/\sigma(\hat{e}_j)$, where $\sigma(\hat{e}_i)$ is the standard deviation of residuals from the beta estimation.

[20] In unreported tests, we use a second method. Following the first step in which we compute individual betas, we run cross-sectional regressions of the available stocks' excess returns on the betas, to obtain risk premia. Then, we average the monthly risk premia. Both methods should generate the same coefficients provided all stocks returns were available over all months. However, the errors are more correlated if they are computed with the first method, which leads to smaller standard errors of coefficients and larger T-statistics. When the second estimation method is used, the standard errors are several times larger.

Table 4 shows the estimated risk premia for three models: the Fama-French three-factor model (FF3); the FF3 model in which a stock's coskewness $Cosk_i$ is added in the second step to the betas; and a model in which the *HS* factor is added to the FF3 factors. As Harvey and Siddique (2000) find that cross-sectional correlations between the estimated betas are different for stocks with different lengths of return histories, we permit the risk premia to vary by the length of return history available. E.g., for the replication sample period (July 1963 to December 1993), we use all 15,129 individual equities in the CRSP NYSE, AMEX, and NASDAQ files, and estimate the models with four indicator variables that allow the slopes to differ for the following return histories: fewer than 24 months, 24 to 59 months, 60 to 89 months, and greater than or equal to 90 months.[21]

As higher coskewness should be associated with smaller expected returns, in Table 4 we expect a negative premium on coskewness (in Panel B) and a positive premium on the *HS* factor (in Panel C).

The results in Table 4 Panel B are mixed: For the replication sample period (1963 to 1993), the original risk premia on *Cosk* are negative and significant only for the first stock group ("All stocks"), while the replicated results are true almost for all five stock groups. The results are similar for the extended sample (1963 to 2019), but weaker for the post-sample (1994 to 2019) and the pre-sample (1926 to 1963).

The results in Table 4 Panel C are mixed as well: For the replication sample period (1963 to 1993), the original and replicated risk premia on *HS* are positive and significant only for the first two stock groups ("All stocks" and "$T < 24$"). The results are similar for the post-sample (1994 to 2019) and the extended sample (1963 to 2019), while no risk premium is positive for the pre-sample (1926 to 1963).

4.4 Momentum Strategies and Skewness

As momentum strategies are known to generate negative skewness (e.g., in the first half of 2009, there was a crash in momentum strategies as the market recovered strongly at the end of the financial crisis), it is natural to study the expected returns and skewness of various momentum strategies.

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[21] In Panel C of their Table IV, Harvey and Siddique (2000) label the variable λ_{HS} (or λ_{SKS} in their notation) as being multipled by 100. However, this is probably a typo, because their estimates are closer to ours without any multiplication.

Table 4: Estimation of Risk Premia from Individual Stocks

Panel A. Risk Premia for FF3

Stock Selection	No. of Stocks	λ_{MKT}	λ_{SMB}	λ_{HML}	Time Period/ Source
All stocks	9,268	0.290**	0.013	−0.060**	1963.07-1993.12/ Original
		(0.023)	(0.012)	(0.012)	
T < 24	1,707	0.308**	0.005	−0.060**	
		(0.025)	(0.013)	(0.013)	
24 ≤ T < 60	2,283	0.250**	0.113**	−0.054	
		(0.064)	(0.045)	(0.051)	
60 ≤ T < 90	1,240	0.321**	−0.066	−0.132	
		(0.116)	(0.087)	(0.097)	
T ≥ 90	4,038	0.088	0.072	−0.269**	
		(0.077)	(0.062)	(0.088)	
All stocks	15,129	0.248***	−0.322***	0.089***	1963.07-1993.12/ Replicated
		(0.018)	(0.009)	(0.013)	
T < 24	2,680	0.152***	−0.355***	0.140***	
		(0.040)	(0.020)	(0.028)	
24 ≤ T < 60	3,609	0.312***	0.091***	−0.208***	
		(0.058)	(0.035)	(0.035)	
60 ≤ T < 90	2,443	0.311***	0.094***	−0.212***	
		(0.059)	(0.034)	(0.033)	
T ≥ 90	6,397	0.436***	0.071***	−0.281***	
		(0.028)	(0.014)	(0.017)	
All stocks	15,103	0.102***	0.083***	−0.076***	1994.01-2019.12/ Post-Sample
		(0.018)	(0.011)	(0.013)	
T < 24	2,125	0.072	0.077***	−0.105***	
		(0.046)	(0.027)	(0.033)	
24 ≤ T < 60	4,390	0.035	−0.020	0.111***	
		(0.052)	(0.042)	(0.035)	
60 ≤ T < 90	2,540	0.017	−0.096**	−0.184***	
		(0.058)	(0.046)	(0.037)	
T ≥ 90	6,048	0.178***	0.073***	−0.222***	
		(0.025)	(0.021)	(0.018)	
All stocks	24,272	−0.119***	−0.205***	0.201***	1963.07-2019.12/ Extended Sample
		(0.013)	(0.007)	(0.011)	
T < 24	3,064	−0.304***	−0.273***	0.276***	
		(0.034)	(0.018)	(0.028)	
24 ≤ T < 60	6,095	0.005	0.036	−0.092***	
		(0.046)	(0.032)	(0.030)	
60 ≤ T < 90	3,586	0.119**	−0.007	−0.161***	
		(0.049)	(0.033)	(0.031)	
T ≥ 90	11,527	0.287***	0.030**	−0.191***	
		(0.019)	(0.012)	(0.013)	
All stocks	2,492	0.564***	−0.217***	−0.028	1926.01-1963.06/ Pre-Sample
		(0.061)	(0.032)	(0.020)	
T < 24	969	0.615***	−0.223***	−0.025	
		(0.102)	(0.054)	(0.032)	
24 ≤ T < 60	206	−0.854***	−0.188	−0.426***	
		(0.282)	(0.136)	(0.127)	
60 ≤ T < 90	119	0.458	−0.099	−0.585***	
		(0.323)	(0.142)	(0.156)	
T ≥ 90	1,198	0.325***	0.053**	0.007	
		(0.041)	(0.022)	(0.026)	

(*Continued*)

Table 4: (*Continued*)

Panel B. Risk Premia for FF3 + *Cosk*

Stock Selection	No. of Stocks	λ_{MKT}	λ_{SMB}	λ_{HML}	λ_{Cosk}	Time Period/ Source
All stocks	9,268	0.293** (0.023)	0.012 (0.012)	−0.060** (0.012)	−0.019** (0.009)	1963.07-1993.12/ Original
T < 24	1,707	0.308** (0.025)	0.006 (0.013)	−0.059** (0.013)	−0.022 (0.019)	
24 ≤ T < 60	2,283	0.255** (0.064)	0.114** (0.045)	−0.055 (0.051)	−0.011 (0.019)	
60 ≤ T < 90	1,240	0.334** (0.117)	−0.067 (0.087)	−0.137 (0.097)	−0.018 (0.023)	
T ≥ 90	4,038	0.098 (0.078)	0.071 (0.062)	−0.268** (0.089)	−0.011 (0.015)	
All stocks	15,129	0.245*** (0.018)	−0.326*** (0.009)	0.088*** (0.013)	−0.466*** (0.097)	1963.07-1993.12/ Replicated
T < 24	2,680	0.148*** (0.041)	−0.356*** (0.020)	0.141*** (0.029)	−0.247 (0.286)	
24 ≤ T < 60	3,609	0.325*** (0.058)	0.078** (0.036)	−0.219*** (0.036)	−0.343** (0.171)	
60 ≤ T < 90	2,443	0.352*** (0.062)	0.089*** (0.034)	−0.225*** (0.034)	−0.273** (0.127)	
T ≥ 90	6,397	0.508*** (0.029)	0.047*** (0.014)	−0.288*** (0.017)	−0.435*** (0.058)	
All stocks	15,103	0.102*** (0.018)	0.083*** (0.011)	−0.076*** (0.013)	0.047 (0.091)	1994.01-2019.12/ Post-Sample
T < 24	2,125	0.072 (0.046)	0.076*** (0.027)	−0.103*** (0.033)	−0.199 (0.251)	
24 ≤ T < 60	4,390	0.014 (0.052)	−0.004 (0.042)	0.123*** (0.034)	1.235*** (0.199)	
60 ≤ T < 90	2,540	0.008 (0.058)	−0.087* (0.046)	−0.173*** (0.037)	0.416** (0.170)	
T ≥ 90	6,048	0.175*** (0.026)	0.073*** (0.021)	−0.222*** (0.018)	0.039 (0.088)	
All stocks	24,272	−0.119*** (0.013)	−0.205*** (0.007)	0.201*** (0.011)	−0.169** (0.076)	1963.07-2019.12/ Extended Sample
T < 24	3,064	−0.303*** (0.034)	−0.273*** (0.018)	0.276*** (0.028)	0.064 (0.230)	
24 ≤ T < 60	6,095	−0.002 (0.047)	0.043 (0.032)	−0.087*** (0.030)	0.280* (0.158)	
60 ≤ T < 90	3,586	0.139*** (0.050)	−0.012 (0.033)	−0.169*** (0.031)	−0.267* (0.139)	
T ≥ 90	11,527	0.307*** (0.020)	0.023* (0.012)	−0.194*** (0.013)	−0.198*** (0.054)	
All stocks	2,492	0.577*** (0.061)	−0.212*** (0.033)	−0.026 (0.020)	−0.146 (0.118)	1926.01-1963.06/ Pre-Sample
T < 24	969	0.628*** (0.103)	−0.215*** (0.055)	−0.022 (0.032)	−0.235 (0.221)	
24 ≤ T < 60	206	−0.864*** (0.284)	−0.193 (0.138)	−0.439*** (0.133)	0.176 (0.547)	
60 ≤ T < 90	119	0.455 (0.346)	−0.100 (0.144)	−0.586*** (0.165)	0.017 (0.647)	
T ≥ 90	1,198	0.290*** (0.045)	0.057*** (0.022)	−0.016 (0.028)	0.094* (0.050)	

(*Continued*)

Table 4: (*Continued*)

Panel C. Risk Premia for FF3 + *HS*						
Stock Selection	No. of Stocks	λ_{MKT}	λ_{SMB}	λ_{HML}	λ_{HS}	Time Period/ Source
All stocks	9,268	0.278** (0.023)	0.022* (0.012)	−0.051** (0.012)	0.058** (0.011)	1963.07-1993.12/ Original
T < 24	1,707	0.294** (0.026)	0.109* (0.013)	−0.049** (0.013)	0.074** (0.011)	
24 ≤ T < 60	2,283	0.343** (0.066)	0.144** (0.046)	−0.012 (0.052)	−0.232** (0.048)	
60 ≤ T < 90	1,240	0.349** (0.116)	−0.051 (0.087)	−0.136 (0.097)	−0.054 (0.099)	
T ≥ 90	4,038	0.122* (0.077)	0.075 (0.062)	−0.271** (0.088)	−0.027 (0.173)	
All stocks	14,988	0.289*** (0.017)	0.135*** (0.011)	−0.083*** (0.010)	0.047*** (0.011)	1963.07-1993.12/ Replicated
T < 24	2,539	0.255*** (0.038)	0.127*** (0.024)	−0.056** (0.024)	0.073*** (0.025)	
24 ≤ T < 60	3,609	0.285*** (0.057)	0.116*** (0.035)	−0.197*** (0.033)	−0.022 (0.029)	
60 ≤ T < 90	2,443	0.296*** (0.057)	0.103*** (0.033)	−0.224*** (0.033)	−0.183*** (0.033)	
T ≥ 90	6,397	0.419*** (0.027)	0.086*** (0.014)	−0.277*** (0.017)	−0.067*** (0.019)	
All stocks	15,030	0.291*** (0.021)	0.093*** (0.010)	−0.110*** (0.011)	0.048*** (0.009)	1994.01-2019.12/ Post-Sample
T < 24	2,052	0.352*** (0.052)	0.094*** (0.023)	−0.141*** (0.028)	0.028 (0.022)	
24 ≤ T < 60	4,390	0.022 (0.051)	−0.026 (0.040)	0.112*** (0.034)	0.287*** (0.032)	
60 ≤ T < 90	2,540	0.025 (0.058)	−0.111** (0.046)	−0.181*** (0.037)	0.178*** (0.039)	
T ≥ 90	6,048	0.184*** (0.025)	0.085*** (0.021)	−0.236*** (0.018)	0.027 (0.022)	
All stocks	24,160	0.276*** (0.015)	0.055*** (0.008)	−0.151*** (0.010)	0.032*** (0.008)	1963.07-2019.12/ Extended Sample
T < 24	2,952	0.246*** (0.040)	0.028 (0.020)	−0.157*** (0.027)	0.014 (0.020)	
24 ≤ T < 60	6,095	0.004 (0.046)	0.067** (0.030)	−0.087*** (0.028)	0.090*** (0.025)	
60 ≤ T < 90	3,586	0.119** (0.049)	0.006 (0.033)	−0.178*** (0.031)	0.019 (0.030)	
T ≥ 90	11,527	0.282*** (0.019)	0.043*** (0.012)	−0.201*** (0.013)	−0.002 (0.015)	
All stocks	2,389	−0.029 (0.058)	−0.133*** (0.034)	−0.007 (0.019)	−0.037 (0.027)	1926.01-1963.06/ Pre-Sample
T < 24	921	−0.091 (0.099)	−0.127** (0.060)	−0.010 (0.030)	−0.040 (0.043)	
24 ≤ T < 60	155	−0.589** (0.298)	−0.122 (0.137)	−0.418*** (0.122)	−0.189 (0.143)	
60 ≤ T < 90	115	−0.107 (0.256)	−0.062 (0.109)	−0.433*** (0.133)	0.090 (0.124)	
T ≥ 90	1,198	0.302*** (0.041)	0.048** (0.021)	0.020 (0.025)	−0.055* (0.032)	

(*Continued*)

Table 4: (*Continued*)

Description: This table presents risk premia for the individual stocks in CRSP over the extended sample period (1963 to 2019). As risk factors, in Panel A we use the factors from the Fama-French three-factor model (FF3), to which we add in Panel B the (standardized) coskewness *Cosk*, and in Panel C the long-short portfolio $HS = S^{-} - S^{+}$, where S^{-} (S^{+}) is the portfolio of the 30% of stocks with the lowest (highest) past standardized coskewness. For each stock, we run a time-series regression of its excess returns on a constant and the factor returns. Then we run a cross-sectional regression of the average stock excess returns on a constant and the betas obtained from the time-series regression, and we obtain the factor premia. We weight each stock i by $1/\sigma(\hat{e}_i)$, where $\sigma(\hat{e}_i)$ is the standard deviation of residuals from the beta estimation. The first line reports the estimated premia and the second line the weighted least squares (WLS) standard errors in parentheses. The stars indicate significance at 1% (***), 5% (**), and 10% (*) levels. For the original results, 3-star significance is not available.

Interpretation: As theory predicts a negative risk premium associated to coskewness, we should observe (i) a negative premium on *Cosk* in Panel B and (ii) a positive premium on $HS = S^{-} - S^{+}$ in Panel C. For the replication sample period (1963 to 1993), Result (i) holds significantly in our replication but less significantly in the original, while Result (ii) holds only for the first two groups ("All stocks" and "T < 24"). The evidence for Results (i) and (ii) in the other sample periods is relatively weak, except for the post-sample period (1994 to 2019), where Result (ii) holds significantly for three of the five stock groups.

In Table 5 we show summary statistics for different momentum strategies. These momentum strategies differ in two aspects. First, the return history over which we measure past returns is either $t-24$ to $t-2$, $t-12$ to $t-2$, or $t-2$ to $t-1$. Second, the holding period is either 1 month, 3 months, 6 months, or 12 months. Decile 1 represents "losers" and decile 10 represents "winners."

A typical momentum strategy that buys past winners (portfolio 10) and sells short past losers (portfolio 1) is profitable but induces significant negative skewness for investors. Indeed, for each pair of losers and winners in Table 5, the winners have both a higher average return and a lower skewness than the losers, which is true both in the original Harvey and Siddique (2000) results (Panel A) and in the replicated results (Panel B). One notable exception is that the average return difference between the winner and loser portfolios is not always positive when the portfolio return history is estimated over the past 24 months. This is not surprising, however, as momentum portfolios are usually constructed by sorting on performance over a period of 12 months or less. Interestingly, Table 5 also shows that the fourth moment (kurtosis) is larger for the winners than for the losers. Table IA.11 in the Internet Appendix shows that the same qualitative results hold for all the other sample periods considered: 1926

Table 5: Summary Statistics on Momentum Strategies

Panel A. Sample Period 1927 to 1997, Original Results

Holding Period	Momentum	Decile	Average (%/yr)	Volatility (%/yr)	Skewness	Kurtosis
1	6	1	2.47	34.43	2.60	19.62
1	6	10	20.26	29.26	1.05	8.49
3	6	1	−5.09	20.91	1.50	12.67
3	6	10	6.25	18.57	0.50	3.84
1	12	1	2.47	35.81	2.45	18.54
1	12	10	23.64	27.83	0.67	6.74
3	12	1	−6.38	21.45	1.30	10.55
3	12	10	10.40	18.00	0.71	6.25
6	12	1	−6.06	14.28	0.07	1.93
6	12	10	6.48	12.94	0.13	2.37
1	24	1	12.55	39.85	3.25	25.36
1	24	10	19.81	24.84	−0.22	3.24
3	24	1	−0.05	23.01	1.72	12.34
3	24	10	10.04	15.91	−0.24	2.47
6	24	1	−0.88	14.91	0.36	2.49
6	24	10	7.06	11.58	−0.33	1.47
12	24	1	0.86	0.21	0.14	2.17
12	24	10	4.11	8.67	−0.63	0.72

Panel B. Sample Period 1927 to 1997, Replication Results

Holding Period	Momentum	Decile	Average (%/yr)	Volatility (%/yr)	Skewness	Kurtosis
1	6	1	13.48	37.25	3.00	25.64
1	6	10	17.64	25.09	−0.16	7.57
3	6	1	12.60	36.69	2.90	24.60
3	6	10	18.03	25.59	0.07	9.24
1	12	1	10.63	38.07	3.08	27.93
1	12	10	22.41	25.42	0.22	9.93
3	12	1	10.56	37.65	3.35	30.65
3	12	10	21.45	25.86	0.25	9.13
6	12	1	11.30	36.87	3.06	27.83
6	12	10	20.36	26.46	1.02	15.38
1	24	1	16.36	41.23	3.73	33.34
1	24	10	17.58	24.48	−0.40	6.07
3	24	1	16.47	40.44	3.63	31.77
3	24	10	16.40	24.53	−0.43	6.14
6	24	1	17.48	39.77	3.46	30.00
6	24	10	15.47	24.71	−0.10	7.23
12	24	1	18.46	39.01	3.72	34.01
12	24	10	13.88	25.00	−0.02	7.28

Description: This table presents summary statistics for selected momentum portfolios. Among the U.S. stocks listed on NYSE, AMEX, and NASDAQ, we form equally weighted portfolios by sorting on performance over the past (monthly) return history of 24 months ($t-24$ to $t-2$), 12 months ($t-12$ to $t-2$), or 6 months ($t-6$ to $t-2$). The holding period refers to the period over which test portfolio returns are computed: 1 month, 3 months, 6 months, or 12 months. The summary statistics are the average return, volatility, skewness, and kurtosis, and are reported for decile 1 (the "loser" portfolio) and decile 10 (the "winner" portfolio).

Interpretation: Buying the winner portfolio 10 and selling the loser portfolio 1 requires acceptance of significant negative skewness, as shown both in the original Harvey and Siddique (2000) results in Panel A and in the replicated results in Panel B. The average return of the winner portfolio 10 is larger than the average return of the loser portfolio 1 in both Panel A (original) and Panel B (replicated), except for the 24-month momentum portfolios in Panel B.

to 1963 (pre-sample), 1963 to 1993 (replication sample), 1994 to 2019 (post-sample), 1963 to 2019 (extended sample), and 1926 to 2019 (maximum sample).

The results in Table 5 offer additional evidence that investors prefer stocks with high skewness and thus require a lower expected return to hold them. Overall, however, the evidence of the coskewness factor *HS* being priced is inconclusive.

5 Persistent Coskewness

In this section, we recast the results of Harvey and Siddique (2000) in light of the recent literature and compare *HS* with two alternative coskewness factors: the predicted systematic skewness (*PSS*) in Langlois (2020), and a "modified *PSS*" (*mPSS*) that we build to better compare the performance of coskewness in asset pricing tests.

In order for all the coskewness proxies to be defined, we study the asset pricing performance of *HS*, *mPSS*, and *PSS* over the extended sample (1963 to 2019). In the Internet Appendix, we also study the performance of *HS* and *mPSS* over the pre-sample period (1926 to 1963).

5.1 Persistent Factors

Langlois (2020) calls a tradable coskewness factor F "persistent" if its coskewness, as defined in Equation (15), is negative. To understand why this is a desirable feature, suppose $F = S^- - S^+$, where S^- (S^+) is the value-weighted portfolio of the 30% of stocks with the smallest (largest) coskewness estimate, and suppose that the coskewness estimate is a noisy version of a "true" coskewness that changes only very slowly.

If we observed the true coskewness, then the composition of the true portfolios S^- and S^+ would change only very slowly. In the extreme case when the composition of these portfolios does not change over time, and if some additional assumptions are satisfied, then the coskewness of F must be negative, as it is equal to the difference between the weighted coskewness of the stocks in S^- and S^+, respectively.[22] Note that, by the

[22] The additional assumptions arise from the fact that the coskewness measure in Equation (15) is standardized. Thus, if we want the coskewness of a portfolio to be equal to the weighted average of the coskewness of the portfolio's component stocks, we need to impose a requirement, e.g., that the market residuals have the same volatility for all stocks.

law of large numbers, the weighted coskewness estimate of the stocks in S^- (S^+) is likely closer to the true coskewness than the coskewness estimate of an individual stock in S^- (S^+), and therefore it is less important which proxy we use to estimate the coskewness of F.

Thus, in order to make sure that the factor F captures the true coskewness, it is important to ensure that the composition of the S^- and S^+ portfolios is not too volatile, and this fact can be measured by estimating the coskewness of F: If this value is negative rather than positive, then F is more likely to be close to the true coskewness factor.

Table 6 shows that the Harvey and Siddique (2000) factor *HS* is not persistent, as its coskewness is positive (0.138) and significant for the extended sample period (1963 to 2019). This fact is likely related to the low persistence of skewness in the time series, as documented, e.g., in Bali *et al.* (2016).

To build a less noisy coskewness proxy, we follow the construction of the predicted systematic skewness (*PSS*) factor by Langlois (2020). This factor is built on two ideas: First, instead of using realized coskewness to build long-short portfolios, we use predicted coskewness from a regression on many stock moments. Second, instead of values we use normalized cross-sectional ranks, which should reduce the noise.[23]

We call our factor "modified *PSS*" and we denote it by *mPSS*. Differently from Langlois (2020), our factor involves conditioning only on moments of the realized stock returns and not on any other firm characteristics. As data on firm characteristics are usually available only after 1962, *mPSS* is defined for the pre-sample period (1926 to 1963), while *PSS* is not.

Specifically, we build *mPSS* in two steps. First, we estimate the coefficients in a regression of coskewness rank on past ranks of certain return moments: market beta, idiosyncratic volatility, coskewness, and idiosyncratic skewness. Coskewness is estimated as beta on the squared market return using daily data going from month $t-12$ to month $t-1$, while the return moments are calculated from daily data going from month $t-24$ to month $t-13$. Second, we predict the coskewness rank of a stock in month t using the $t-12$ to $t-1$ values of the stock moments and the

[23]The second idea is not enough to produce a persistent coskewness factor. Indeed, in unreported results, we find that a modified version of *HS* that uses normalized cross-sectional ranks instead of values still does not have negative coskewness.

coefficients calculated in the first step. The details of this construction are given in Appendix A.

The *mPSS* factor is a long-short portfolio calculated as the return difference between the value-weighted bottom 30% of stocks with the lowest predicted coskewness rank and the top 30% of stocks with the highest predicted coskewness rank. As the compositions of the long and short portfolios in the *mPSS* factor are determined by how stocks are ordered, one only needs to forecast the cross-sectional ranks of stock coskewness, and not the actual coskewness values.

5.2 Summary Statistics of Coskewness Factors

If a coskewness factor is priced, it should first be persistent, i.e., it should have negative realized coskewness. If a factor is not persistent, Section 5.1 shows that the factor is less likely to capture actual coskewness. Second, the factor should have a significant and positive risk premium. Note that a positive premium is not a substitute for performing more formal asset pricing tests.[24]

Table 6 shows statistics for the coskewness factors and standard risk factors, as well as alphas with respect to the CAPM model, the four-factor model (FF4), and the five-factor model (FF5), over the extended sample period (1963 to 2019).

For the coskewness factors, we should observe a positive and significant premium (if the factor is priced) and a negative realized coskewness (if the factor is persistent). The coskewness factors *HS*, *mPSS*, and *PSS* have positive risk premia (2.145%, 1.594%, and 5.366%, respectively), although these are not statistically significant. The coskewness of *HS* is positive and significant (0.138), while the coskewness of *mPSS* and *PSS* is negative and significant (−0.193 and −0.274, respectively), which shows that the latter two factors are persistent.

Table IA.12 in the Internet Appendix shows that during the pre-sample period (1926 to 1963), *HS* has a negative risk premium (−3.152%) while

[24]Indeed, the average return of a (traded) risk factor is usually different from the risk premium estimated from a Fama-MacBeth cross-sectional regression of average returns on factor betas. An illustration of this difference is provided by stocks that have a large premium (the equity market premium) but, as shown for example in Fama and French (1992), the premium for equity market exposure is much smaller and even negative if one controls for the size and value factors, indicating that equity risk is not priced within the cross section of stocks.

Table 6: Coskewness and Risk Factors, July 1963 to December 2019

Factor	Av.Ret. (%/yr)	Volat. (%/yr)	Sharpe Ratio	β_{MKT}	Std. Skew.	Std. Cosk.	CAPM α (%/yr)	FF4 α (%/yr)	FF5 α (%/yr)
HS	2.145	8.018	0.27	−0.007	−2.501***	0.138**	2.192**	0.991	0.393
mPSS	1.594	16.401	0.10	0.457***	0.691***	−0.193***	−1.372	−3.631**	2.422*
PSS	5.366	14.191	0.38	0.212***	1.279***	−0.274***	4.016**	−0.340	5.500***
MKT	6.496	15.194	0.43	1.000	0.429***				
SMB	2.339	10.536	0.22	0.204***	2.200***	−0.219***	1.013		
HML	3.650	9.729	0.38	−0.161***	4.049***	−0.032	4.693***		
RMW	3.090	7.516	0.41	−0.110***	−0.343***	0.056	3.804***	3.678***	
CMA	3.293	6.899	0.48	−0.175***	0.292***	0.041	4.430***	1.934***	
MOM	7.750	14.495	0.53	−0.135***	−4.223***	−0.255***	8.627***		8.515***

Description: This table presents statistics for the coskewness factors and for some common factors used in asset pricing tests. We include alphas from several models: the CAPM with the market factor (*MKT*); the Carhart-Fama-French four-factor model (FF4) with *MKT*, size (*SMB*), value (*HML*), and momentum (*MOM*) factors; and the Fama-French five-factor model (FF5) with *MKT*, *SMB*, *HML*, profitability (*RMW*), and investment (*CMA*) factors. $HS = S^- - S^+$ is the Harvey and Siddique (2000) factor, *PSS* is the Langlois (2020) factor, and *mPSS* is the modified *PSS* factor. The stars indicate significance at 1% (***), 5% (**), and 10% (*) levels.

Interpretation: All three coskewness factors have positive risk premia but not statistically significant. *mPSS* and *PSS* have negative coskewness, while *HS* does not, which shows that the Harvey and Siddique (2000) coskewness proxy is not persistent. With respect to FF5, *PSS* has the largest and most significant annualized alpha (5.5%), followed by *mPSS* (2.42%) and *HS* (0.39%).

mPSS has a positive risk premium (0.363%), and both *HS* and *mPSS* have negative and significant coskewness (−1.036 and −0.548, respectively). Table IA.12 also shows that the significance of the coskewness results in Table 6 comes from the replication sample period (1963 to 1993), while in the post-sample period (1994 to 2019) the coskewness of the factors *HS*, *mPSS*, and *PSS* is not statistically significant.

To avoid multicollinearity issues in more formal asset pricing tests, in Table 7 we show the correlations between the coskewness factors (*HS*, *PSS*, and *mPSS*) and standard factors used in asset pricing tests.

The correlations of *HS* with all other factors are relatively low, including with the other coskewness factors *mPSS* and *PSS*. The persistent coskewness factors *mPSS* and *PSS*, however, are highly correlated with each other (85%) and with size (69% and 73%, respectively). Table IA.13

Table 7: Factor Correlations, July 1963 to December 2019

	HS	*mPSS*	*PSS*	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*MOM*
HS	1.00								
mPSS	−0.13	1.00							
PSS	−0.11	0.85	1.00						
MKT	−0.02	0.43	0.24	1.00					
SMB	0.02	0.65	0.71	0.28	1.00				
HML	0.19	−0.31	−0.18	−0.25	−0.07	1.00			
RMW	0.27	−0.51	−0.45	−0.23	−0.35	0.08	1.00		
CMA	0.04	−0.34	−0.19	−0.38	−0.10	0.70	−0.02	1.00	
MOM	0.02	0.18	0.32	−0.14	−0.04	−0.18	0.11	−0.03	1.00

Description: This table presents correlations between the market (*MKT*), size (*SMB*), value (*HML*), momentum (*MOM*), profitability (*RMW*), and investment (*CMA*) factors, as well as the Harvey and Siddique (2000) factor ($HS = S^- - S^+$), the Langlois (2020) factor (*PSS*), and the modified persistent coskewness (*mPSS*).

Interpretation: The persistent coskewness factors *mPSS* and *PSS* are highly correlated with each other (85%) but not with *HS*. Size is very correlated with *mPSS* and *PSS* (69% and 73%, respectively) but not with *HS* (-4%).

in the Internet Appendix shows that the correlation results are robust to the sample period used. Overall, these results suggest that the asset pricing tests for *mPSS* and *PSS* should be performed with the size factor removed.

5.3 Asset Pricing Tests with Coskewness Factors

Table 8 shows the risk premia corresponding to the Fama-French five-factor model FF5, with an additional coskewness factor included (*HS*, *mPSS*, or *PSS*).[25] In Panel A, the test assets are the 25 Fama-French portfolios sorted on size and book-to-market. In Panel B, the test assets are the 25 Fama-French portfolios sorted on size and momentum. In Panel C, the test assets are the 25 portfolios sorted on size and coskewness. As the size factor (*SMB*) is highly correlated with the persistent coskewness factors (*mPSS* and *PSS*), we also report test results when the size factor is omitted.

If coskewness is priced in the cross-section of stocks, the risk premium

[25]Tables IA.14 to IA.25 in the Internet Appendix show results similar to those in Table 8 for other sample periods and for other standard asset pricing models: the CAPM and the Carhart-Fama-French four-factor model (FF4).

Table 8: Risk Premia from the 5-Factor Model and Coskewness, 1963 to 2019

Model	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*HS*	*mPSS*	*PSS*
Panel A. the 25 portfolios Sorted by Size and B/M								
FF5	−0.474	0.260***	0.264***	0.402**	0.035			
	(0.357)	(0.053)	(0.056)	(0.169)	(0.205)			
FF5	−0.307	0.238***	0.254***	0.257*	0.451**	−1.116***		
+ *HS*	(0.272)	(0.040)	(0.042)	(0.133)	(0.187)	(0.336)		
FF5 − *SMB*	0.360		0.270***	−0.399**	0.625*	−0.696		
+ *HS*	(0.470)		(0.073)	(0.202)	(0.335)	(0.582)		
FF5	−0.610*	0.290***	0.263***	0.625***	0.140		0.847	
+ *mPSS*	(0.350)	(0.054)	(0.053)	(0.208)	(0.205)		(0.617)	
FF5 − *SMB*	−0.557		0.249***	0.590***	0.078		0.578***	
+ *mPSS*	(0.351)		(0.050)	(0.187)	(0.174)		(0.125)	
FF5	−0.504*	0.265***	0.283***	0.399***	0.030			1.367***
+ *PSS*	(0.288)	(0.043)	(0.045)	(0.139)	(0.171)			(0.405)
FF5 − *SMB*	−0.498		0.256***	0.446***	−0.009			0.616***
+ *PSS*	(0.317)		(0.046)	(0.152)	(0.174)			(0.111)
Panel B. 25 Portfolios Sorted on Size and Momentum								
Model	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*HS*	*mPSS*	*PSS*
FF5	0.162	0.448***	−0.661***	0.300	0.760			
	(0.432)	(0.081)	(0.191)	(0.314)	(0.476)			
FF5	−0.343	0.410***	−0.673***	0.079	1.053**	−1.677*		
+ *HS*	(0.523)	(0.081)	(0.184)	(0.333)	(0.494)	(0.996)		
FF5 − *SMB*	0.465		−0.274	−0.474	1.439**	0.865		
+ *HS*	(0.712)		(0.228)	(0.481)	(0.712)	(1.263)		
FF5	0.196	0.432***	−0.519	0.269	0.819		0.046	
+ *mPSS*	(0.459)	(0.096)	(0.596)	(0.338)	(0.526)		(0.441)	
FF5 − *SMB*	0.324		0.080	0.067	1.071**		0.487***	
+ *mPSS*	(0.439)		(0.162)	(0.282)	(0.421)		(0.161)	
FF5	0.250	0.340***	0.068	−0.049	0.987**			0.533
+ *PSS*	(0.427)	(0.105)	(0.550)	(0.381)	(0.479)			(0.348)
FF5 − *SMB*	0.239		−0.001	−0.023	0.902**			0.504***
+ *PSS*	(0.404)		(0.143)	(0.238)	(0.400)			(0.126)
Panel C. 25 Portfolios Sorted on Size and Coskewness								
Model	*MKT*	*SMB*	*HML*	*RMW*	*CMA*	*HS*	*mPSS*	*PSS*
FF5	0.408**	0.215***	0.342**	0.329***	−0.008			
	(0.168)	(0.030)	(0.137)	(0.068)	(0.131)			
FF5	0.409**	0.218***	0.326**	0.325***	−0.005	0.207***		
+ *HS*	(0.180)	(0.033)	(0.164)	(0.077)	(0.134)	(0.037)		
FF5 − *SMB*	0.869***		0.439**	0.073	−0.155	0.216***		
+ *HS*	(0.203)		(0.218)	(0.063)	(0.173)	(0.044)		
FF5	0.397**	0.229***	0.397***	0.322***	0.022		0.273	
+ *mPSS*	(0.166)	(0.032)	(0.143)	(0.067)	(0.132)		(0.288)	
FF5 − *SMB*	0.392**		0.395***	0.324***	0.022		0.386***	
+ *mPSS*	(0.164)		(0.123)	(0.061)	(0.125)		(0.102)	
FF5	0.361**	0.220***	0.402***	0.297***	−0.022			0.178
+ *PSS*	(0.183)	(0.031)	(0.152)	(0.073)	(0.130)			(0.206)
FF5 − *SMB*	0.325*		0.450***	0.271***	−0.039			0.316***
+ *PSS*	(0.186)		(0.123)	(0.058)	(0.125)			(0.085)

Description: This table presents risk premia for the Fama-French five-factor model and an additional coskewness factor: the Harvey and Siddique (2000) factor ($HS = S^- - S^+$), the Langlois (2020) factor (*PSS*), or the modified persistent coskewness factor (*mPSS*). The test portfolios are the 25 Fama-French portfolios sorted on size and book-to-market (Panel A), the 25 Fama-French portfolios sorted on size and momentum (Panel B), and the 25 portfolios sorted on size and coskewness (Panel C). For each test portfolio, we run a time-series regression of the portfolio excess return on a constant and the market (*MKT*), size (*SMB*), value (*HML*), profitability (*RMW*), and investment (*CMA*) factors. The factor premia are obtained from a cross-sectional regression of the average portfolio excess return on a constant and the betas obtained from the time-series regression. Standard errors are in parentheses. The stars indicate significance at 1% (***), 5% (**), and 10% (*) levels.

Interpretation: When adding a coskewness factor to the five-factor model, its risk premium should be positive and significant. This is true for *HS* only in Panel C, but it is generally true for *mPSS* and *PSS* as long as we remove the size factor *SMB*, which is highly correlated with the persistent coskewness factors.

of the coskewness factor should be positive and significant. Table IA.16 shows that, when added to the CAPM, *PSS* has a positive and significant risk premium in all three panels, *mPSS* only in Panels B and C, and *HS* only in Panel C.[26] Table IA.24 shows that, when added to FF4, the risk premium of *HS* is not positive and significant in any of the panels, while *PSS* and *mPSS* have a positive and significant risk premium in all three panels but only if the size factor is omitted. Similarly, Table 8 shows that, when added to FF5, the risk premium of *HS* is not positive and significant in any of the panels, while *PSS* and *mPSS* have a positive and significant risk premium in all three panels but only if the size factor is omitted.

Table IA.16 in the Internet Appendix shows the risk premia during the pre-sample period (1926 to 1963) when either *HS* or *mPSS* is added to the CAPM model (*PSS* is not defined during this period): *mPSS* has a positive and significant risk premium in Panels B and C of Table IA.16 and a positive but insignificant risk premium in Panel A, while *HS* has a negative and significant risk premium in all three panels. Table IA.21 in the Internet Appendix shows the risk premia during the pre-sample period when either *HS* or *mPSS* is added to the FF4 model. The risk premia for *mPSS* are no longer significant in any of the panels, and the same is true for *HS*, probably because the less reliable pre-sample data leads to more noisy test results.

In the Internet Appendix, we perform robustness checks for the results on *HS* in Table 8. We modify *HS* in several ways: First, we reduce the estimation window of *HS* from 5 years to 3 years. Second, we exclude microcap stocks (i.e., stocks with a market capitalization of less than 250 million U.S. dollars) from the portfolio formation of *HS*. Third, we exclude penny stocks (i.e., stocks with a price of less than 5 U.S. dollars) from the portfolio formation of *HS*. Fourth, we require the stocks included in the definition of *HS* to have a number of observations over the past 60 months of at least N, where N is equal to 36, 48, or 60. The results remain qualitatively the same.

Overall, *HS* has a mixed asset pricing performance and is not persistent, while *mPSS* and *PSS* perform significantly better in asset pricing tests and are persistent. Thus, coskewness appears to be priced in the cross sec-

[26] In Panel A, the *mPSS* premium is positive but not significant, which can be explained by the relatively high correlation between *mPSS* and the market (42%), compared the lower correlations for *HS* (−2%) and *PSS* (23%).

tion, as long as we use a more persistent empirical proxy for coskewness.

6 Conclusion

We reexamine the role of systematic skewness in asset pricing, by replicating and extending the seminal paper of Harvey and Siddique (2000). In addition to their *HS* factor, we consider two more coskewness proxies suggested by recent literature: (i) *PSS*, the predicted systematic skewness factor of Langlois (2020), which uses many stock characteristics as conditioning information in predicting coskewness, and (ii) *mPSS*, a modified version of *PSS* that we construct by conditioning only on return-based characteristics (and is thus available for a longer time period).

We evaluate a coskewness factor by its performance in various asset pricing tests but also by its persistence, i.e., whether a factor has negative realized coskewness—otherwise, the prediction is less reliable and the factor is less likely to capture true coskewness. This is especially relevant since, compared to many other factors in the "zoo," coskewness is theoretically motivated and has a compelling economic intuition.

Our results confirm the intuition in Harvey and Siddique (2000) that coskewness is priced in the cross section of stocks. However, to that end one needs to use a more persistent coskewness proxy, like *PSS* or *mPSS*. A direct comparison shows that *PSS* generally performs better than *mPSS*, suggesting that conditioning only on market data is not sufficient to capture all the information on coskewness that matters to investors.

Appendix A Construction of the mPSS Factor

We detail the construction of the *mPSS* factor. First, define the coskewness of a stock as:

$$Csk_{i,t} = \mathrm{Cov}_{t-1}\left(r_{i,t}, r_{M,t}^2\right). \tag{A1}$$

If $i = 1, 2, \ldots, N_t$, define the normalized rank for a variable $x_{i,t}$ by:

$$F(x_{i,t}) = \frac{Rank(x_{i,t})}{N_t + 1}, \tag{A2}$$

where $Rank(x_{i,t})$ is the order $(1, 2, \ldots, N_t)$ of variable $x_{i,t}$ in all x_t values sorted in ascending order. The normalized rank is a number between 0 and 1.

Each month t (say, January 2019), we run the following panel regression using all available stocks and historical data:[27]

$$F\big(Csk_{i,k-12\to k-1}\big) = \kappa + F\big(Y_{i,k-24\to k-13}\big)\theta + \varepsilon_{i,k-12\to k-1}, \quad k = 25, 26, \ldots, t, \quad i = 1, \ldots, N_k, \tag{A3}$$

where:

- N_k is the number of stocks available at time k.
- $Csk_{i,k-12\to k-1}$ is the coskewness of stock i estimated from Equation (A1) using daily returns from month $k-12$ to month $k-1$ (e.g., January to December 2018).
- The K_Y-by-one vector of variables $Y_{i,k-24\to k-13}$ is composed of stock moments (idiosyncratic volatility, beta, coskewness, idiosyncratic skewness) computed using stock i's daily returns from month $k-24$ to month $k-13$ (e.g., January to December 2017).
- $\varepsilon_{i,k-12\to k-1}$ are random shocks.
- κ is a constant.

To make sure that our factor is defined using more liquid stocks, we also require at least 50 daily return observation over a 1 year period. The period of 12 months to estimate risk measures is chosen to provide a reasonable trade-off between having enough returns while allowing for variations over time.

We run the panel regression (A3) using all monthly observations from month 25 to month t. Each month, we use all stocks in the cross-section for which the values of Csk and Y are available. By estimating the regression (A3), we model how past cross-sectional ranks of stock moments predict future coskewness ranks.

To form our coskewness factor, *mPSS*, we first compute the model-predicted coskewness ranks for month t using our regression estimates, $\hat{\kappa}$, $\hat{\theta}$, and $\hat{\phi}$, as:

$$F\Big(\widehat{Cosk_{i,t\to t+11}}\Big) = \hat{\kappa} + F\big(Y_{i,t-12\to t-1}\big)\hat{\theta} \tag{A4}$$

[27] To define the *PSS* factor, Langlois (2020) adds another term to the regression, $F\big(X_{i,k-13}\big)\phi$, where $X_{i,k-13}$ is a K_X-by-one vector composed of characteristics (size, book-to-price ratio, etc.) of stock i observed at the end of month $k-13$ (e.g., at the end of December 2017).

Thus, we form our coskewness factor as the return spread between the value-weighted portfolio containing the bottom 30% of stocks with the lowest predicted coskewness cross-sectional ranks and the value-weighted portfolio containing the top 30% of stocks with the highest predicted coskewness cross-sectional ranks. We denote the resulting factor as the modified predicted systematic skewness factor or *mPSS* for short.[28]

In our empirical implementation, we use market beta, idiosyncratic volatility, coskewness, and idiosyncratic skewness as past risk measures Y. These measures capture the systematic and idiosyncratic second and third order moments and therefore describe the shape of the distribution of past returns.

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[28]Note that any inference about the panel regression coefficients is complicated by two aspects: there is overlap in the periods used for computing the risk measures, and cross-sectional ranks are estimated with error. These complications are avoided because we rely only on the predicted values to form a long-short portfolio and we do not make any inference about the regression coefficients.

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